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Two-Dimensional Frequency Estimation for Fringe Analysis by Phase Gradient Detection

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A new frequency estimation technique is proposed, which enables us to estimate two-dimensional frequencies of interferometric fringes with high accuracy and low computational cost. It is accomplished by phase gradient detection, where phases are calculated by a local model fitting algorithm for carrier pattern analysis. The algorithms used and experimental results are presented.

Key words: frequency estimation, phase gradient, interferometry, fringe analysis, surface profiler, single-shot

1. Introduction

In recent years, in various industrial fields such as semiconductors and LCD panels, there is an increasing demand for accurate measurement of surface profiles on the order of nanometers. The surface profile measurement method using optical interference is the most promising measurement method from the viewpoint of speed, measurement accuracy, and maintainability.

In the phase shifting method [1)], which is a typical optical interferometry method, multiple interference images are taken while changing the relative distance between the measurement surface and the reference surface of the interferometer, and the surface profile is estimated from that information. With this method, it is necessary to capture multiple images, so there is a problem that the accuracy is greatly reduced in an environment with disturbance such as vibration. As a solution to this problem, a single-shot measurement method for obtaining the surface profile from a single image has been proposed. A typical method is to generate carrier fringes by tilting the reference plane, which is called spatial carrier fringe method [2)-8)] (**Fig. 1**). From one interference fringe image (**Fig. 2**) obtained by this method, the surface profile can be obtained by Fourier transform method [3)], spatial phase synchronization method [4)-6)], or local model fitting method (LMF method) [7)8)].

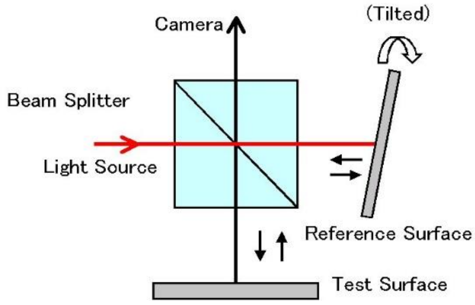

Fig. 1 Optics of spatial carrier method.

Fig. 2 Interferogram with carrier fringes.

However, since the fringe order cannot be determined from one fringe image, there is a problem that correct phase unwrapping cannot be performed when there is a step difference of 1/4 or more of the light source wavelength between adjacent pixels. In order to solve this problem, the author realized a dual-wavelength simultaneous imaging system using a color camera and two blue and red color LEDs, and succeeded in measuring a step of 350 nm by two-wavelength single-shot interferometry.[9)10)] In this method, the captured color image is separated into B and R components, the phase at each pixel is obtained by the local model fitting method, and the height is obtained by the two-wavelength unwrapping using the equivalent wavelength method or the extended order determination method.[†1] The equivalent wavelength of the two wavelengths used is 1877 nm, and the maximum measurable step is about 470 nm.

In order to extend this method and further expand the measurement range, we added a green (center wavelength 530 nm) LED illumination and studied a three-wavelength one-shot measurement method that also uses the G signal of the camera. [11)] To realize this method, it was necessary to solve new problems such as crosstalk correction between RGB signals, highly accurate frequency estimation, and three-wavelength unwrapping. The final schematic flow obtained is shown in **Fig. 3.** This paper describes frequency estimation in this flow. The frequency referred to here is the carrier fringe frequency in the x and y directions that is determined by the relative tilt angle between the sample surface and the reference surface and the wavelength.

Here, the purpose of frequency estimation is described. The local model fitting method developed by the authors (the outline is given in Appendix I) is used to calculate the phase of the interference fringe image. In this method, of the parameters (amplitude, frequency, phase, DC component) included in the sinusoidal model function, the frequency is a known parameter and the remaining three parameters are found by the least squares method. Therefore, it is necessary to estimate the frequency before calculating the phase.

Furthermore, we consider the required accuracy of frequency estimation. If there is an error in the frequency, the phase changes linearly as described in Section 2.1. Considering the effect of this error on the measurement results, in the case of the one-wavelength method and the

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†1 Unwrapping as used here refers to the use of multiple wavelengths and is different from the usual use of neighboring pixel information.

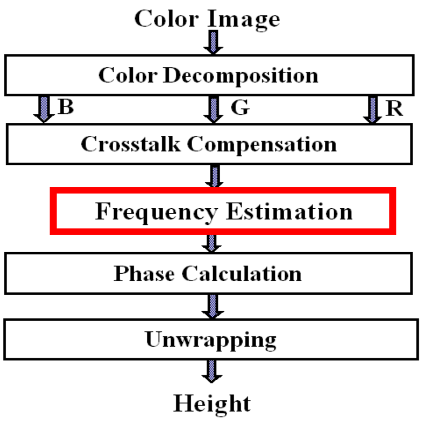


Fig. 3 Flowchart of three-wavelength single-shot interferometry.

two-wavelength method that uses the equivalent wavelength method for unwrapping, the surface shape is measured only with an inclination, which is no problem. However, in the case of the three-wavelength method, since the fringe order is determined by using the coincidence method for unwrapping, a slight phase error may expand as the order changes. According to the consideration described in Appendix II, the required accuracy of frequency estimation is about 0.4%, a fairly high accuracy.

As the frequency estimation method for sinusoidal signals, many estimation methods such as autocorrelation method, Fourier transform method and Prony method[12] have been proposed. However, there is no practical method with high accuracy and low computational cost. Furthermore, as shown in Fig. 2, when the fringes are parallel to the coordinate axis, the fringe frequency in the axial direction is close to zero, and it is difficult to estimate with high accuracy by the one-dimensional estimation method.

The authors devised a frequency estimation method that uses a phase gradient and solved the above problems.[11] [13] This paper describes the estimation principle, computer experiment results, implementation problems and their solutions, and actual experimental results.

2. Frequency estimation principle

2.1 One-dimensional estimation

The proposed method (called the phase gradient method) uses the relationship between the frequency error and the phase gradient. Although this method can be applied to two-dimensional frequency estimation, the principle is described with one-dimensional frequency estimation at first.

Assume that the observed signal is represented by the following equation, with the horizontal direction in Fig. 2 as the x-axis.

$$g(x) = A\cos(2\pi f x + \phi(x)) \tag{1}$$

where A is the amplitude, f is the desired frequency, and $\phi(x)$ is the phase at point x, which changes with height.

Model function is represented by the following equation with the initial frequency estimate f_0.

$$g'(x) = A'\cos(2\pi f_0 x + \phi'(x)) \tag{2}$$

By fitting this model to the observed data, we can obtain the phase $\phi'(x)$. For this phase calculation, the local model fitting method (see Appendix I) previously developed by the authors is used.

The following equation holds from equations (1) and (2):

$$2\pi f x + \phi(x) = 2\pi f_0 x + \phi'(x) \tag{3}$$

Therefore, the obtained phase $\phi'(x)$ is expressed by the following equation.

$$\phi'(x) = 2\pi(f - f_0)x + \phi(x) \tag{4}$$

Here, considering the region that can be regarded as $\phi(x)$ = constant (called the reference plane region), the phase becomes a linear expression of x, and its slope is proportional to the frequency error (f - f_0). Therefore, the frequency can be estimated by the following equation obtained by differentiating both sides of Eq. (4).

$$f = f_0 + (d\phi'(x)/dx)/2\pi \tag{5}$$

Here, since the phase gradient is obtained from the data of at least two points, the simplest is to calculate the phase ϕ'_1, ϕ'_2 at two different points x_1, x_2 in the reference plane, and obtain the frequency f by the following equation:

$$f = f_0 + (1/2\pi)(\phi'_2 - \phi'_1)/(x_2 - x_1) \tag{6}$$

The estimation accuracy can be improved by setting the obtained frequency as the initial value and repeating the estimation. Note that phase unwrapping is usually required to obtain the correct phase gradient. This problem is described in Section 4.2.

2.2 2-D estimation

Next, when this method is extended to two dimensions, equations (1), (2), and (4) become the following equations, respectively.

$$g(x, y) = A\cos(2\pi f_x x + 2\pi f_y y + \phi(x, y)) \tag{7}$$

$$g'(x, y) = A'\cos(2\pi f_{x0} x + 2\pi f_{y0} y + \phi'(x, y)) \tag{8}$$

$$\phi'(x, y) = 2\pi(f_x - f_{x0})x + 2\pi(f_y - f_{y0})y + \phi(x, y) \tag{9}$$

Also, the frequency estimation equation corresponding to equation (5) is as follows, and the frequencies in the x and y directions can be estimated simultaneously.

$$f_x = f_{x0} + (d\phi'(x, y)/dx)/2\pi \tag{10}$$

$$f_y = f_{y0} + (d\phi'(x, y)/dy)/2\pi \tag{11}$$

3. Computer experiment

A computer experiment was conducted to verify the proposed method.

3.1 One-dimensional estimation experiment

3.1.1 Experimental conditions

The observed data was generated from a function $g(x) = \cos(2\pi f x)$, with the true frequency $f = 0.020$. Uniform random noise (-0.1, +0.1) was added to $g(x)$. The initial frequency f_0 was 0.019 (relative error 5%), and the phase calculation data size was 25 pixels.

3.1.2 Experimental results

Figure 4 shows the observed data and the model function (where amplitude $A' = 1$ and phase $\phi'(0) = 0$). **Figure 5** shows the phase $\phi'(x)$ and the regression equation obtained by fitting. From the phase gradient of 0.00628, the frequency estimate was 0.019 + 0.00628/2π = 0.02000. This is in good agreement with the true value, and the error is 0.00001 or less, Similar results were obtained over a wide range of initial frequencies.

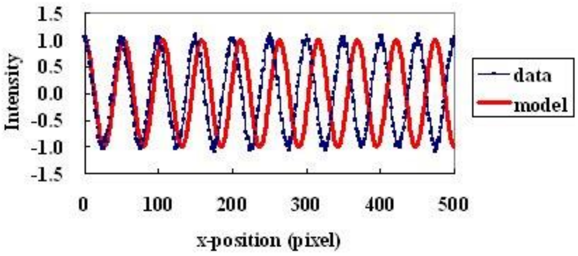


Fig. 4 Observed data and model function.

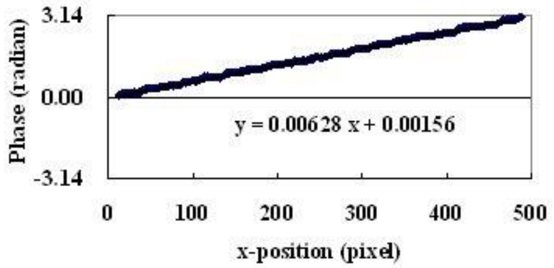


Fig. 5 Phase gradient.

3.2 Two-dimensional estimation experiment

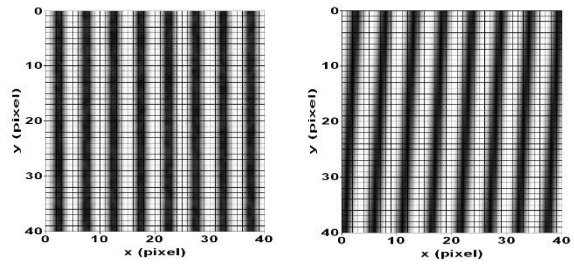
2D frequency estimation is performed when the frequency in the y direction is zero. This is a case where frequency estimation is extremely difficult with conventional one-dimensional signal processing.

3.2.1 Experimental conditions

The observed data was generated from a function $g(x,y) = \cos(2\pi f x + 2\pi f y) + 1$, with the true frequency $f_x = 0.2$, $f_y = 0.0$. Uniform random noise (-0.1, +0.1) was added to $g(x,y)$. The observed image is shown in **Fig. 6(a)**. The initial frequency f_{x0} was 0.019 (relative error 5%), and f_{y0} was =0.01. **Figure 6(b)** shows a model image (where amplitude $A' = 1$ and phase $\phi'(x,y) = 0$). The phase calculation data size was 3×3 pixels.

3.2.2 Experimental results

The phase $\phi'(x,y)$ obtained by the fitting is shown in **Fig. 7**, and its profile is shown in **Fig. 8(a)(b)**. The phase gradients in the x and y directions were calculated from the coefficients a and b by fitting the plane $Z = aX + bY + c$ to the phase data, and they were 0.06275 and -0.06298 (rad/pixel), respectively. From this gradients, the x-direction frequency estimate $f_x = 0.19999$ and the y-direction frequency estimate $f_y = -0.00002$ were obtained. When the estimation was repeated 10 times, the average value ± standard deviation was $f_x = 0.20000 \pm 0.00003$, $f_y = -0.00002 \pm 0.00002$. It was also confirmed that similar results were obtained over a wide range of initial frequencies. It was shown that the two-dimensional frequency estimation can be performed with high accuracy regardless of the frequency and direction of the interference fringes.



(a) Observed image (b) Model image

Fig. 6 Two-dimensional frequency estimation.

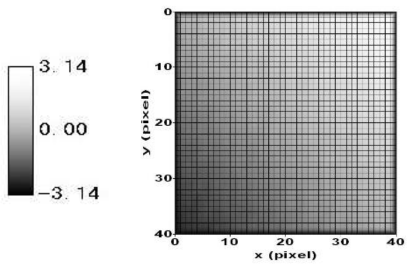
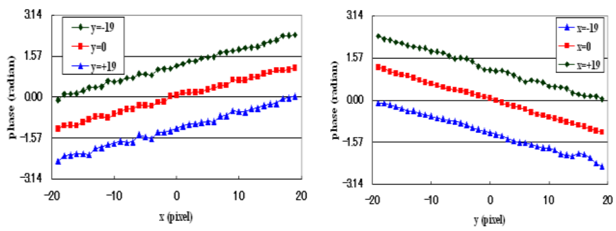


Fig. 7 Two-dimensional phase map.



(a) x-direction (b) y-direction

Fig. 8 Phase profiles along three selected lines.

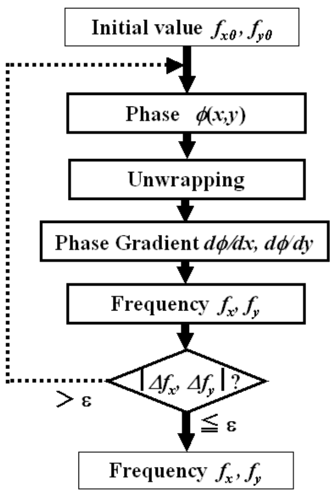


Fig. 9 Flowchart of frequency estimation.

4. Implementation problems and solutions

4.1 Estimation flow

The estimation flow is shown in **Fig. 9**. The dotted line shows the loop that repeats the estimation with the obtained estimated value as the initial value. The convergence condition is that the absolute values of the frequency correction values Δf_x, Δf_y are below the convergence threshold ε. The threshold ε is set considering that the required accuracy of frequency estimation is about 0.4% (see Appendix II).

4.2 Problems in implementation

As mentioned in Section 2.1, phase unwrapping is usually required to obtain the correct phase gradient. For example, the phase in **Fig. 10(a)** must be unwrapped. Although the phase gradient is small, the discontinuity occurs because the phase value is near ±π. There are many proposals for phase unwrapping, but most of the robust unwrapping methods have high computational cost. Therefore, we investigated a method that eliminates the need for phase unwrapping.

4.3 Solution (1): Origin shift method

To eliminate the need for phase unwrapping, it is effective to bring the phase value obtained by Eq. (9) close to zero, and this can be achieved by "origin shift" described below. That is, as shown in Fig. **10(b)**, when the phase of the model signal matches the phase of the observed signal, the obtained phase is near zero. Therefore, the solution is to obtain the maximum point of the observation data, and to calculate the phase with that point as the origin of coordinates. This method eliminates the need for phase unwrapping, as shown in Fig. 10(b).

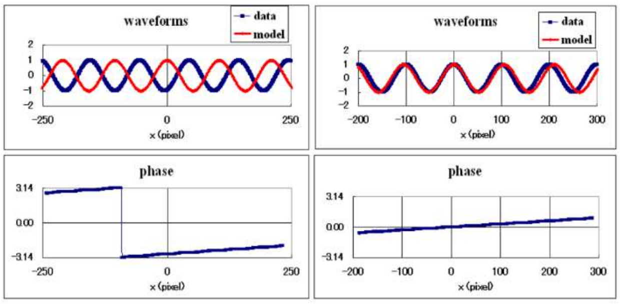
4.4 Solution (2): Two-step method

If the error in the initial value is large, the phase gradient will be large, and if the estimation region is large, the phase may exceed $\pm\pi$. Therefore, as the first step, rough estimation is performed in a plurality of small areas as shown by the blue dotted line in **Fig. 11(a)**. At this time, phase unwrapping is made unnecessary by using the "origin shift method" described in the previous section together. The solid yellow line in Fig. 11(a) shows the origin and coordinate axes. Next, in the second step, the average value of the frequencies obtained by rough estimation is used as the initial value, and frequency estimation in a wide area is performed as shown by the dotted line in **Fig. 11(b)**. Again, the "origin shift method" can be adopted. This method will eliminates the need for phase unwrapping.

5. Actual experiment

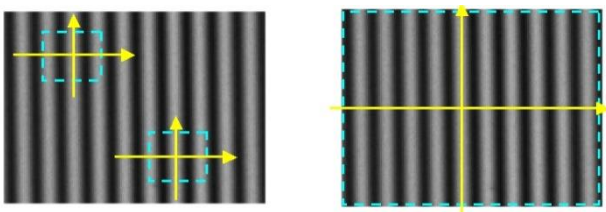
5.1 Experimental method

The experimental setup is shown in **Fig. 12**. It was made for three-wavelength single-shot measurement, the light source is a three-color (RGB) LED lighting device, and the interference image is captured by a color camera. **Figure 13** shows the R component of the color interference image of a 1µm standard step. The rectangular part in the lower center is the step.



(a) With non-optimized origin (b) With optimized origin

Fig. 10 Effects of origin optimization.



(a) 1st step: Rough estimation (b) 2nd step: Fine estimation

Fig. 11 Two-step estimation.

5.2 Experimental results

The frequency estimation was performed while changing the initial value by setting the rectangular area (400 × 200 pixels) at the top of Fig. 13, shown by the white line, as the reference plane area†2. The data size for phase calculation was 25 × 5 pixels. As a result, f_x =0.030064 and f_y =0.000361 were obtained with stable convergence for a wide range of initial values. When converted to the number of stripes in the area with horizontally 512 pixels, they are 15.393 in x-direction and 0.185 in y-direction. **Figure 14** shows the relationship between the initial value and the estimation result in one estimation (that is, when the estimation is not repeated). Since the regression coefficient is 0.0029, it can be seen that the frequency error is reduced to about

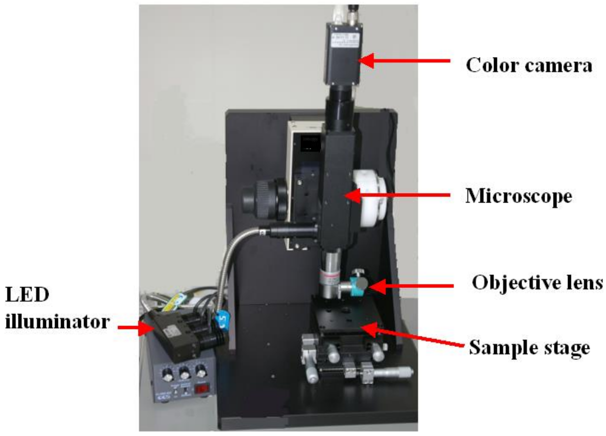


Fig. 12 Experimental setup.

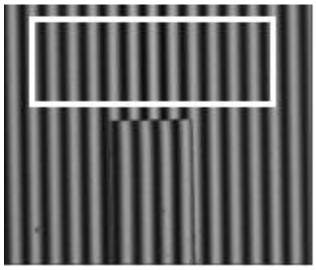


Fig. 13 Observed red image of 1µm step.

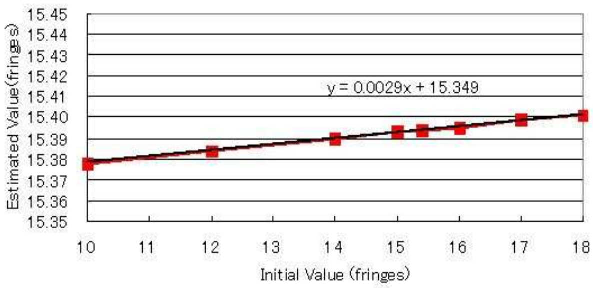


Fig. 14 Estimated frequencies versus initial values.

†2 As mentioned in Section 2.1, the reference plane area is set to the area where the phase can be regarded as constant (that is, the height is constant). There is no need for a single area, and it may be scattered at multiple locations on the image..

3/1,000 by one estimation. When the convergence threshold ε is set to 0.4%, convergence occurs in the first estimation, and the repetition of estimation does not occur. The time required for estimation was 23 ms on a PC (Pentium 1.6 GHz). In addition, the phase calculation was sufficiently accurate even if it was thinned out to 100×100 pixels instead of all pixels, and the calculation time in this case was 8 ms.

6. Summary

In this paper, we proposed the phase gradient method as a new frequency estimation method for fringe images. The frequency is estimated by utilizing the fact that the local phase value of the image depends on the frequency error. It is characterized by high accuracy, light calculation cost, and simultaneous estimation of two-dimensional frequencies. It is also applicable when the frequency is near zero. The validity and effectiveness of the proposed method were confirmed by computer experiments and actual experiments. Furthermore, as an implementation problem, we took up phase unwrapping and devised a method to eliminate it.

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Appendix I: Phase estimation method

The outline of the local model fitting method [7)8)] used for the phase estimation of each point of the fringe image is described below.

Suppose that the intensity $g(x,y)$ of each point is represented by the following formula.

$$g(x,y) = a(x,y) + b(x,y)\cos(\phi(x,y) + 2\pi f_x x + 2\pi f_y y) \quad \text{(A1)}$$

where $a(x,y)$ is the DC component, $b(x,y)$ is the amplitude, and $\phi(x,y)$ is the phase. f_x, f_y are the carrier fringe frequencies in the x and y directions, respectively, and are assumed to be known.

In the local model fitting method, the sinusoidal model function is defined by the following equation, assuming that $a(x,y)$, $b(x,y)$ and $\phi(x,y)$ are locally constant near each point.

$$g(x,y) = a + b\cos(\phi + 2\pi f_x x + 2\pi f_y y) \quad \text{(A2)}$$

This model function is least squares fitted to the intensity data of n points ($n \geqq 3$) in the vicinity of each point. However, since the above model is non-linear, its fitting is computationally expensive. Therefore, by the variable transform of $\xi_c = b\cos\phi$ and $\xi_s = b\sin\phi$, the linearization is performed as follows.

$$g(x,y) = a + \xi_c\phi_c(x,y) + \xi_s\phi_s(x,y) \quad \text{(A3)}$$

where $\phi_c(x,y) = \cos\left(2\pi f_x x + 2\pi f_y y\right)$ and $\phi_s(x,y) = -\sin\left(2\pi f_x x + 2\pi f_y y\right)$.

As a result, the problem to find a, b, ϕ is converted into the linear least squares problem to find a, ξ_c, ξ_s, and by solving the simultaneous linear equation with three unknowns, a, ξ_c, ξ_s can be calculated. The phase ϕ is calculated by the following equation:

$$\phi = \arctan(\xi_s/\xi_c) \quad \text{(A4)}$$

Appendix II: Required accuracy of frequency estimation

Consider the frequency estimation accuracy required when determining the fringe order using the coincidence method for three-wavelength unwrapping. For simplicity, consider one-dimensional measurement in the x-axis direction. Also, pay attention to the absolute value of the error and ignore the sign. Then, the phase error $\Delta\phi$ due to the frequency error Δf is expressed by the following equation by modifying Eq. (4).

$$\Delta\phi = 2\pi(\Delta f)x \quad \text{(A5)}$$

The height error Δh due to the phase error is represented by

$$\Delta h = \lambda(\Delta\phi)/4\pi \quad \text{(A6)}$$

where λ is the wavelength. Then, from the formulas (A5) and (A6)

$$\Delta h = (\lambda x/2)(\Delta f) \quad \text{(A7)}$$

is obtained.

Under the experimental conditions of this paper, the maximum wavelength λ is about 600 nm, and if the zero point of the coordinate system is placed in the center of the image, the maximum value of x is 256 pixels, so the approximate maximum value of Δh is expressed by the following equation:

$$\Delta h \cong (300*256)(\Delta f) \quad \text{(A8)}$$

In the coincidence method, the height candidate values are found by changing the fringe order from the phase of each wavelength, and the combination in which the height candidate values of the three wavelengths best match is found. From experience, the height error required to avoid selecting the wrong order is about 10 nm. Therefore, if the allowable height error is 10 nm, the allowable frequency error is about 0.00013(1/pix) from Eq. (A8). This is equivalent to 0.067 when converted to the number of stripes in the image of 512 pixels. In the experiment of this paper, the allowable relative error is about 0.4% because the number of stripes is about 15.

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Graduated from the Department of Engineering, University of Tokyo in 1964. In the same year, joined Toray Industries, Inc. Since 1989, engaged in R&D of semiconductor inspection equipment based on the image processing technology. Since 2000, worked at Toray Industries, Inc. Received the 2001 Technology Award of SICE (the Society of Instrument and Control Engineers), the ViEW2003 Odawara Award, the Tejima Memorial Foundation Invention Award. Advanced Measurement and Control Engineer (authorized by SICE).

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