Refractive Index Profiling of Multi-layer Optical Fibers using Mach-Zehnder Interferometer

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We propose a method for determining the refractive-index profile of axially symmetric optical fibers from transverse interferograms obtained by a Mach-Zehnder interferometer. The profile is obtained from the interferogram data by a model-fitting algorithm which uses the Abel transform of an approximate polynomial expression of the profile. This algorithm is simple to use, easily applicable to multi-layer fibers, and does not require the Fourier transform. The proposed method is validated through computer simulations and actual experiments.

1. Introduction

Much research has been done on the measurement of the refractive index profile of optical fibers. In particular, the transverse interferometry method in which a ray is irradiated transversely onto the fiber is nondestructive and is highly practical [1-4].

However, the conventional technology put to practical use in the industry has a problem that complicated calculations such as Fourier transform are required. In this report, we propose a new measurement algorithm to obtain the refractive index profile by the model fitting method from the phase difference data obtained by the Mach-Zehnder interferometer. The validity of the proposed method was confirmed by computer experiments.

2. Measurement principle

Configure a Mach-Zehnder interferometer as shown in Fig. 1. A laser (wavelength 546 nm) is used as the light source, and irradiation is performed from the side of the optical fiber. The sample is immersed in index-matching fluid. The interferogram shown in Fig. 2 represents the optical path difference (OPD) due to the internal refractive index profile. Figure 3 shows the OPD obtained by the phase shifting method. The purpose of this research is to obtain the refractive index profile from this data.

2.1 Abel transformation

For a cylindrical object such as an optical fiber, where the refractive index is symmetrical, the OPD observed through the lateral direction is expressed as follows.

$$f(y) = 2\int_y^R \left[n(r)r/\sqrt{r^2-y^2}\right] dr \qquad (1)$$

where the periphery is a liquid for refractive index compensation, and the object refractive index *n(r)* is represented by the difference from the periphery.

Here, the coordinate system is as shown in Fig. 4. This expression is called the Abel transform or Abel integral.

2.2 Abel inverse transformation

Obtaining $n(r)$ from $f(y)$ is the inverse Abel transform and is expressed by the following equation:

$$n(r) = -(1/\pi)\int_r^R \left[(df(y)/dy)/\sqrt{y^2-r^2}\right] dy \qquad (2)$$

The inverse transform of Eq. (2) is a definite integral that includes the derivative of the observed value function, is weak against noise, and is not easy in practice. Therefore, various methods have been proposed, including Fourier transform method [1-4] and Polynomial approximation method [5, 6].

These methods make the OPD data $f(y)$ differentiable by approximating it with a Fourier polynomial, etc. However, the Fourier transform has a problem that the calculation cost is high.

2.3 Proposed method

We have devised a model fitting method that assumes a refractive index profile model for optical fibers. The observed values and the corresponding values predicted by the Abel transform of the refractive index profile model are least-squares fitted to obtain the unknown parameters in the model.

As the refractive index profile model, a polynomial model is adopted because its Abel transform is analytically possible. Here, we describe the case where the difference between the optical fiber refractive index and the immersion liquid refractive index is expressed as a function of the radius *r* by the fourth-order polynomial eqation shown below:

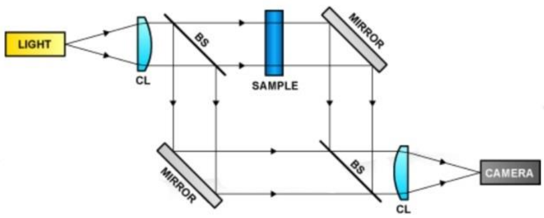

Fig.1. Mach-Zehnder interferometer

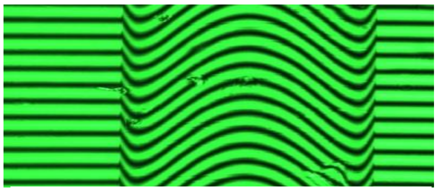

Fig.2. Interferogram

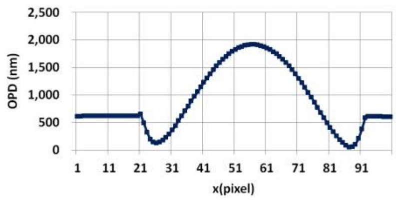

Fig.3. Optical path difference profile

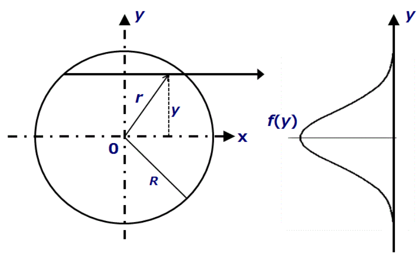

Fig.4. Coordinate system of Abel transform

$$\Delta n(r) = \Delta n_0[1 - a(r/R)^2 + b(r/R)^4\,] \tag{3}$$

For this Abel transform we use the following indefinite integral formula:

$$\int \left[(1 - ax^2 + bx^4)x/\sqrt{x^2-y^2}\right] dx = \sqrt{x^2-y^2}\left\{1 - \frac{a(x^2+2y^2)}{3} + \frac{b(3x^4+4x^2y^2+8y^4)}{15}\right\} \tag{4}$$

Then the Abel transform of Eq.(3) becomes:

$$f(y) = 2\Delta n_0\sqrt{R^2-y^2}\left\{1 - \frac{a(R^2+2y^2)}{3R^2} + \frac{b(3R^4+4R^2y^2+8y^4)}{15R^4}\right\} \tag{5}$$

The parameters $\Delta n_0, a, b$ are obtained by the following least squares fitting of this equation to the observed values f_i (i=1,2,$\cdots$,m).

$$SSE(\Delta n_0, a, b) = \sum_{i=1}^{m}\left(f_i - f(y_i)\right)^2 \tag{6}$$

The refractive index profile can be calculated from the obtained parameters using Eq.(3).

3. Computer experiment

3.1 Experimental method

The OPD data was created under the following conditions.

- Central refractive index n_0: 1.0
- Refractive index model: 4th order polynomial (Eq.(3))
- Parameter a: 1.0
- Parameter b: 0.0
- Radius R: 1.0
- Observation interval: 1/20 of the diameter
- Number of observation data (one side): 21

The obtained OPD data is shown in Fig.5. For the nonlinear least-squares fitting, the Solver, which is an optimization tool of Microsoft Excel, was used. The initial values was set to +10% of the true values. Note that this least-squares fitting problem can also be solved analytically by a variable transformation to a linear equations (see Appendix).

3.2 Experimental results

The results are shown in Table 1. The estimation error of the unknown parameter was less than 0.00001. Figure 5 shows the matching results. Figure 6 shows the estimated refractive index profile obtained from the results. The estimation error (difference from the true value) was less than 0.00001.

Table 1 Fitting results

	True	Initial	Estimated	Error
n_0	1.0	1.1	1.0000	0.0000
a	1.0	1.1	1.0000	0.0000
b	0.0	0.0	0.0000	0.0000

4. Actual experiment

4.1 Experimental method

Using the Mach-Zehnder interferometer shown in Fig. 1, we measured the OPD of a fiber of 340 mm in diameter, whose estimated central refractive index is 1.512, immersed in index-matching fluid (refractive index 1.4875). The refractive index distribution was estimated by using the proposed method. The Solver was used for least squares fitting. The initial values of unknown parameters were n_0 = 0.01 , a = 0 and b = 0.

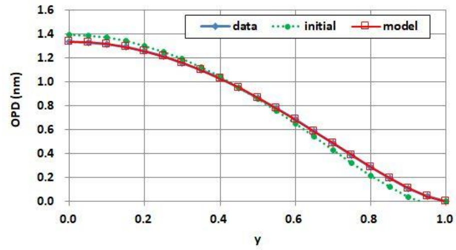

Fig.5. Fitting results of OPD profile

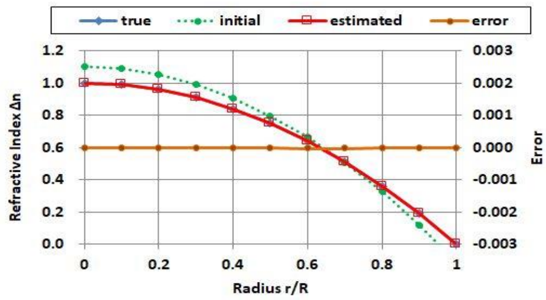

Fig.6. Refractive index profiles

4.2 Experimental results

Fig. 7 shows the obtained interferogram of the sample. The OPD profile of one line indicated in Fig. 7 is shown in Fig. 8 as "data". It also shows the initial values and the fitting results with the data (71 points on each side). The obtained parameters were n_0 = 0.0142 , a = 0.999 and b = 0.0449. Figure 9 shows the estimated refractive index profile obtained from the results. Note that the data close to the edge (y>0.85) was excluded because it was considered unreliable.

5. Extension to multi-layer optical fibers

5.1 Algorithm

The proposed method can be extended to a multi-layer fiber with two or more layers. Here, consider the case of a two-layer structure as shown in Fig. 10. For example, there is a cladding with a refractive index $n_2(r)$ and a thickness R-R_1 outside the center (core) with a refractive index $n_1(r)$ and radius R_1.

The Abel transform in this case is expressed by the following formula:

(1) $0 \le y < r_1$:

$$f(y) = 2R\int_y^{r_1}\left[n_1(r)r/\sqrt{r^2-y^2}\right]dr + 2R\int_{r_1}^{1}\left[n_2(r)r/\sqrt{r^2-y^2}\right]dr \tag{7}$$

(2) $r_1 \le y$:

$$f(y) = 2R\int_y^{1}\left[n_2(r)r/\sqrt{r^2-y^2}\right]dr \tag{8}$$

Here, as in the case of a single layer, if the refractive index of each layer can be approximated by a polynomial equation, the above Abel transform is analytically possible and the refractive index can be estimated by the model fitting method.

Below, we describe the case where the refractive index of the core part is a 4th order polynomial as in Eq.(3) and the cladding part is represented by the following 1st order equation:

$$n_2(r) = n_{20}\{1 - c(1 - r)\} \tag{9}$$

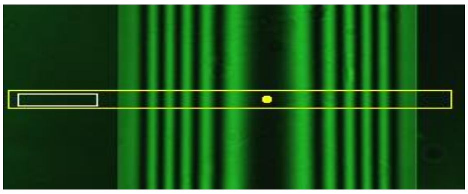

Fig.7. Interferogram of actual sample

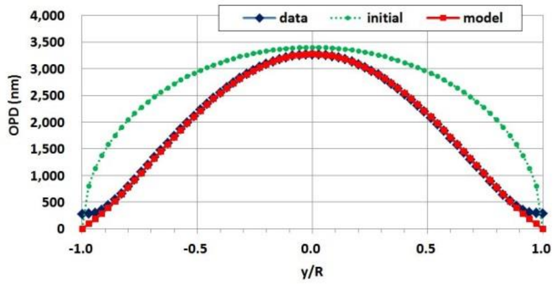

Fig.8. Results of fitting

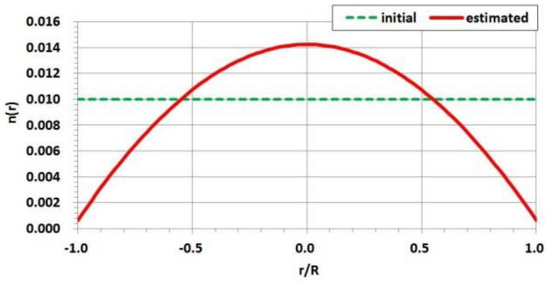

Fig.9. Results of index profile estimation

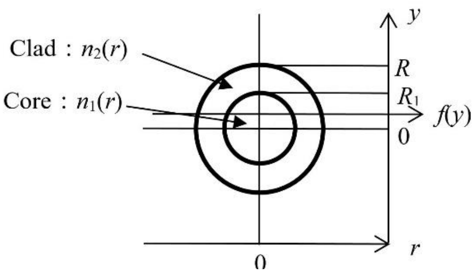

Fig.10. Model of two-layer fiber

where n_{20} is the refractive index at the outer edge, and c is the refractive index gradient.

Considering the Abel transform of Eq.(7), the first term on the right side is expressed as the following equation by a modification of Eq.(5).

$$f_1(y) = 2n_0R\sqrt{r_1^2 - y^2}\left\{1 - \frac{a(r_1^2 + 2y^2)}{3} + \frac{b(3r_1^4 + 4r_1^2y^2 + 8y^4)}{15}\right\} \tag{10}$$

For the second term on the right side of Eq.(7), we use the following indefinite integral formula:

$$\int\left[(a + bx)x/\sqrt{x^2 - y^2}\right]dx = (a + bx/2)\sqrt{x^2 - y^2} + (by^2/2)\log\left\{2\left(\sqrt{x^2 - y^2} + x\right)\right\} \tag{11}$$

Then the second term becomes:

$$f_2(y) = 2n_{20}R\left[\sqrt{1 - y^2}\left(1 - \frac{c}{2}\right) - \sqrt{r_1^2 - y^2}\left(1 - c + \frac{cr_1}{2}\right) + \left(\frac{cy^2}{2}\right)\log\left(\frac{\sqrt{1 - y^2} + 1}{\sqrt{r_1^2 - y^2} + r_1}\right)\right] \tag{12}$$

Similarly, Eq. (8) becomes the following equation using Eq. (11).

$$f(y) = 2n_{20}R\left[\sqrt{1 - y^2}\left(1 - \frac{c}{2}\right) + \left(\frac{cy^2}{2}\right)\log\left(\frac{\sqrt{1 - y^2} + 1}{y}\right)\right] \tag{13}$$

Using Eqs. (10), (11), and (12) as models, the unknown parameters n_0, n_{20}, a, b, c, r_1 are obtained by the least-squares fit with the observed values f_i ($i = 1, 2, \ldots, m$), and the refractive index can be estimated.

5.2 Computer experiment

The OPD data was created under the following conditions: radius R = 1.0, inner layer radius R_1 = 0.8, inner layer central refractive index n_0 = 1.0, second-order coefficient a = 1.0, fourth-order coefficient b = 0.0, outer-layer edge refractive index n_{20} = 0.1, first-order coefficient c = -5, observation interval = R/100, number of observation data (one side) = 101.

The OPD data obtained is shown in Fig. 11 as "data". The Solver was used for least-squares fitting. The initial value was +5% of the true value.

The fitting result is shown in Fig. 11. Figure 12 shows the estimated refractive index profile obtained from the result. The refractive index estimation error (difference from the true value) was 0.000002 or less.

5.3 Actual experiment

Using the Mach-Zehnder interferometer shown in Fig. 1, a commercially available multimode optical fiber with a diameter of 250 µm was immersed in a liquid with a refractive index of 1.458, and the OPD was measured. Figure 13 shows the interferogram. The refractive index profile was estimated by the proposed method from the OPD data of the area surrounded by the rectangle. In the refractive index profile model, the inner layer is a quartic equation and the outer layer is a linear equation. The Solver was used for least-squares fitting. The initial values of unknown parameters were n_0 = 0.01 , a = 5.0 , b = 0.0 , r_1 = 0.5 , c = 0.0 , n_{20} = 0.004.

Figure 14 shows the OPD profile data (total 115 on both left and right sides), the initial value of the fit, and the fit result. The parameters obtained were n_0 = 0.0171 , a = 4.620 , b = -4.309 , r_1 = 0.512 , c =0.0057 , n_{20} = 0.0014. Figure 15 shows the estimated refractive index profile obtained from the results. The core part has a graded index profile, and the clad part is almost constant.

6. Summary

We proposed a new inverse Abel transform algorithm for measuring the refractive index profile of an optical fiber using a Mach-Zehnder interferometer and Abel transform. Assuming an axially symmetric refractive index profile model, the observed OPD values and the corresponding values predicted by the Abel transform of the model are least-squares fitted to estimate the

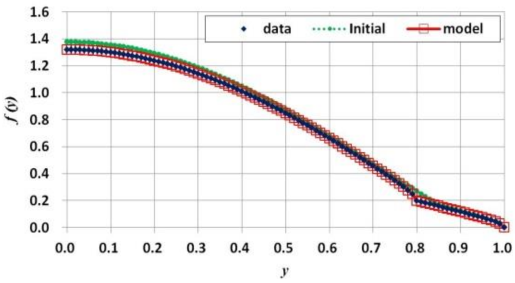

Fig.11. Fitting results of two-layer fiber

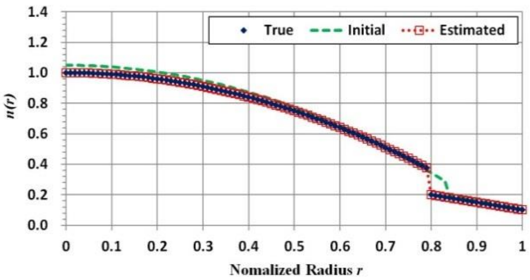

Fig.12. Estimated index of two-layer fiber

unknown parameters in the model. Polynomial approximation is effective as a refractive index profile model. Furthermore, it can be applied to a multi-layer optical fiber. The validity of the proposed method was confirmed by computer experiments and actual experiments. This method does not require complicated calculations such as Fourier transform, and the algorithm is simple.

Acknowledgment

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Appendix: Analytical solution of least-squares fitting problem

Using the variable transformations of $X_1 = \Delta n_0$, $X_2 = a\Delta n_0$, $X_3 = b\Delta n_0$ in Eq.(5), the model becomes linear as follows:

$$f(y) = A_1X_1 + A_2X_2 + A_3X_3 \quad \text{(A1)}$$

where

$$A_1 = 2\sqrt{R^2 - y^2} \quad \text{(A2)}$$

$$A_2 = -2\sqrt{R^2 - y^2}(R^2 + 2y^2)/3R^2 \quad \text{(A3)}$$

$$A_3 = 2\sqrt{R^2 - y^2}(3R^4 + 4R^2y^2 + 8y^4)/15R^4 \quad \text{(A4)}$$

This linear least squares problem can be solved analytically as a simultaneous linear equation with three unknowns.

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