

# Measurement of Transparent Plate Surface Topography using Wavelength Scanning Interferometry

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**Abstract:** Wavelength scanning interferometry is known as a method to measure the surface roughness and the thickness of a transparent object such as a glass plate with high accuracy. However, in the conventional technology, it is necessary to strictly adjust the distance between the reference surface and the measurement target, and it is difficult to measure the absolute value. We have developed a new collective measurement algorithm for the front, back, and thickness distributions using the Fizeau interferometer and wavelength scanning interferometry. It solves the problems of the conventional method and is excellent in practicality. The validity of the proposed method was confirmed by computer experiments.

## 1. Introduction

Wavelength scanning interferometry is known as a method of three-dimensionally measuring the surface irregularities of a transparent object such as a glass plate and its plate thickness with high accuracy. In wavelength scanning interferometry, a tunable laser is used as the light source, and the wavelength is linearly scanned. The distance to the object can be obtained by using the fact that the frequency of the intensity signal obtained at this time is proportional to the optical distance. In addition, even if there are multiple interfaces such as when the measurement target is a transparent plate, it is possible to analyze them separately.

As a measurement algorithm for transparent objects by wavelength scanning interferometry, the ones that have been proposed so far are (1) model fitting method, (2) Fourier transform method, and (3) window function method (Calculate the phase from the brightness value similar to the phase shift method).

Despite the most basic method, the model fitting method [1, 2] is almost ignored because of the heavy computational cost. The Fourier transform method [4,10,11] is a method that takes advantage of the characteristics of the wavelength scanning method and is used in practical products, but the measurement resolution is low only from the amplitude information of the frequency spectrum. Therefore, the use of phase information has been devised. However, the Fourier transform method, like the following window function method, has restrictions on the sample position. That is, the distance between the reference surface and the sample surface must be strictly adjusted so that the frequency of the superimposed sine wave has an integer ratio. The window function method [3, 5-9] is an extension of the method of the phase shift method and is characterized by a light calculation cost.

In other words, in the conventional technology that has been put into practical use, it is necessary to strictly adjust the distance between the reference surface and the measurement target, and it is not possible to measure absolute values. In this report, we propose a new method for simultaneous measurement of front, back, and plate thickness distribution that solves these problems.

## 2. Measurement principle

Configure a Fizeau interferometer as shown in Fig. 1. A

tunable laser (wavelength range 630 to 640 nm) is used as the light source, and the wavelength can be scanned at any speed. When the sample is transparent, the interference image is a superposition of three interference images of (1) reference surface and sample surface, (2) reference surface and sample back surface, and (3) sample surface and back surface (Fig. 2). The purpose of this research is to separate these three signals.

In the Fizeau interferometer, if the reference surface R, the front surface S, and the back surface B are arranged as shown in Fig. 3, if the RS surface distance is  $S$  and the RB surface distance is  $B$ , then, considering the phase inversion of RS interference and SB interference, the observed brightness  $I$  is expressed as the following equation.

$$I = I_S + I_B + I_R - 2\sqrt{I_S I_R} \cos(4\pi S/\lambda) + 2\sqrt{I_B I_R} \cos(4\pi B/\lambda) - 2\sqrt{I_S I_B} \cos[4\pi(B - S)/\lambda] \quad (1)$$

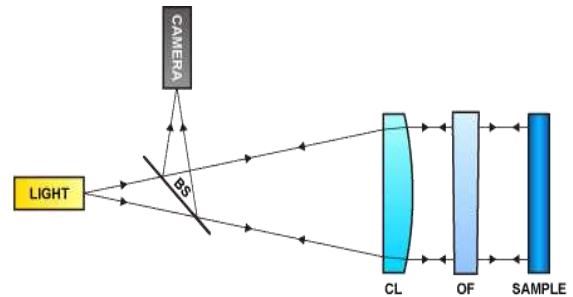


Fig.1 Fizeau interferometer.

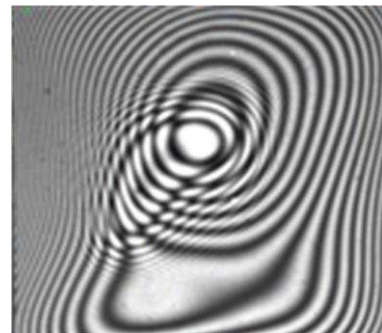


Fig.2 Interference image.

Now consider the amount of reflected light at each interface of the Fizeau interferometer (Fig. 3). If the reference plate refractive index is  $n_R$  and the sample refractive index is  $n$ , the interface reflectivities are:

$$R_R = ((n_R - 1)/(n_R + 1))^2 \quad (2)$$

$$R_S = ((n - 1)/(n + 1))^2 \quad (3)$$

Also, if the amount of incident light on the reference surface is  $I_0$ , the following equations hold for the reflected light amounts of reference surface, surface and back surface, respectively (see Appendix 1 for ignoring internal multiple reflected light).

$$I_R = I_0 R_R \quad (4)$$

$$I_S = I_0 (1 - R_R)^2 R_S \quad (5)$$

$$I_B = I_0 (1 - R_R)^2 R_S (1 - R_S) \quad (6)$$

Next, consider wavelength scanning. When the initial wavelength is  $\lambda_0$  and the wavelength scanning speed is  $c$ , the wavelength at time  $t$  is:

$$\lambda(t) = \lambda_0 + ct \quad (7)$$

At this time, assuming  $\lambda_0 \gg ct$ , the phase value  $\phi(t)$  of the distance  $L$  is:

$$\begin{aligned} \phi(t) &= 4\pi L / \lambda(t) = 4\pi L / (\lambda_0 + ct) \\ &\approx 4\pi L / \lambda_0 - 4\pi ct L / \lambda_0^2 \\ &= \phi_0 - 2\pi f t \end{aligned} \quad (8)$$

where

$$\phi_0 = 4\pi L / \lambda_0 \quad (9)$$

$$f = 2cL / \lambda_0^2 \quad (10)$$

Rewriting Eq. (1) using Eqs. (4)-(6) and (8), and setting the optical plate thickness as  $T = B - S$ ,

$$\begin{aligned} I(t) &= I_0 \{ a - b_S \cos(2\pi f_S t - \phi_{0S}) \\ &\quad + b_B \cos(2\pi f_B t - \phi_{0B}) \\ &\quad - b_T \cos(2\pi f_T t - \phi_{0T}) \} \end{aligned} \quad (11)$$

where

$$a = (I_S + I_B + I_T) / I_0 = R_R + (1 - R_R)^2 R_S + (1 - R_R)^2 R_S (1 - R_S)^2 \quad (12)$$

$$b_S = 2\sqrt{I_R I_S} / I_0 = 2(1 - R_R)\sqrt{R_R R_S} \quad (13)$$

$$b_B = 2\sqrt{I_R I_B} / I_0 = 2(1 - R_R)(1 - R_S)\sqrt{R_R R_B} \quad (14)$$

$$b_T = 2\sqrt{I_S I_B} / I_0 = 2(1 - R_R)^2 (1 - R_S) R_S \quad (15)$$

Substituting Eqs. (9)-(10) into equation Eq. (11),

$$\begin{aligned} I(t) &= I_0 \{ a - b_S \cos(4\pi c S t / \lambda_0^2 - 4\pi S / \lambda_0) \\ &\quad + b_B \cos(4\pi c B t / \lambda_0^2 - 4\pi B / \lambda_0) \\ &\quad - b_T \cos(4\pi c (B - S) t / \lambda_0^2 \\ &\quad - 4\pi (B - S) / \lambda_0) \} \end{aligned} \quad (16)$$

This model formula is the sum of three cosine waves with different frequencies, and the unknowns are  $I_0$ ,  $S$ , and  $B$ . These are found by the least-squares fit with the observed values  $I_i$  ( $i = 1, 2, \dots, n$ ); that is, by minimizing the sum of squared errors (SSE) shown below:

$$SSE(I_0, S, B) = \sum_{i=1}^n (I_i - I(t_i))^2 \quad (17)$$

From the obtained  $S$  and  $B$  and the refractive index  $n$ , the physical thickness  $T'$  and the physical back surface position  $B'$  can be calculated by the following equations:

$$T' = (B - S) / n \quad (18)$$

$$B' = S + T' \quad (19)$$

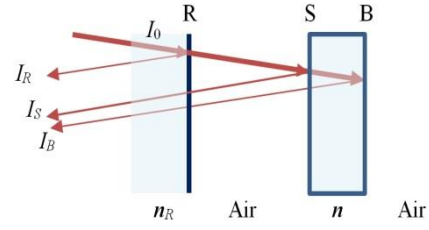


Fig.3 Reflections in Fizeau interferometer.

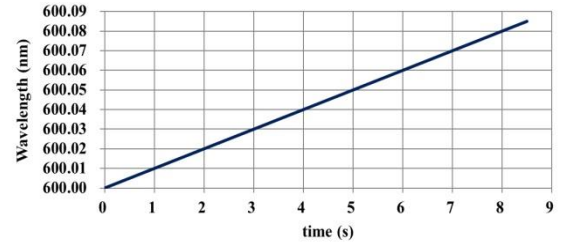


Fig.4 Wavelength scanning.

### 3. Computer experiment (1): Point measurement

#### 3.1 Experimental method

The theoretical interferogram was created under the following conditions.

- (1) Wavelength scanning speed  $c$ : 0.01 nm/s (Fig. 4)
- (2) Initial wavelength  $\lambda_0$ : 600 nm
- (3) Number of observation data: 256
- (4) Camera imaging speed: 30 fps
- (5) Surface distance  $S$ : 10 mm
- (6) Physical plate thickness  $T'$ : 10 mm
- (7) Refractive index of reference surface and sample: 1.46

The waveform obtained is shown in Fig. 5(a). For the least squares fit, we used Solver, the optimization tool of MS Excel. The initial values are  $I_0 = 95$ ,  $S = 100,000,050$  nm,  $B = 24,600,025$  nm.

#### 3.2 Experimental results

The results are shown in Table 1. The distances to the front and back surfaces are correctly estimated with an estimation error of 1 nm or less. Also, the separated signal waveforms of the front surface, back surface and thickness are shown in Fig. 5(b)(c)(d).

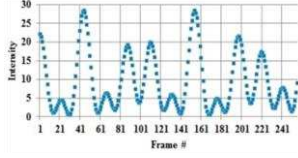
### 4. Computer experiment (2): Line measurement

#### 4.1 Experimental method

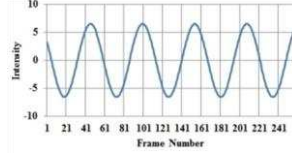
The theoretical interferogram was created by giving the front and back surface profiles as shown by the dotted lines in Fig. 6. The number of pixels in the horizontal direction ( $x$  coordinate) was set to 164. Other conditions are the same as in the previous section. The initial values are  $I_0 = 100$ ,  $S = 100,000,000$  nm, and  $B = 24,600,000$  nm.

Table 1 Computer experiment results

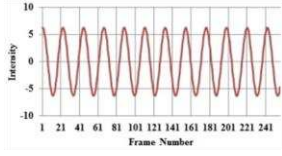
	True	Initial	Estimated	Error
$I_0$	100	95	100.00	0.00
$S$	10,000,000	10,000,050	10,000,000	0
$B$	24,600,000	24,600,025	24,600,000	0



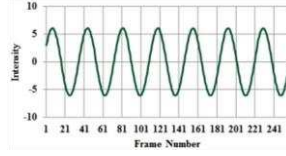
(a) Theoretical interferogram



(b) Surface interferogram



(c) Back-surface interferogram



(d) Thickness interferogram

Fig.5 Computer experiment results.

## 4.2 Experimental results

The results are shown by the solid lines in Fig. 6. As the difference between the initial value and the true value increases, an error that is an integral multiple of about half the wavelength occurs. This is because the model function in Eq. (16) is a periodic function and there are many local solutions at intervals of about  $1/2$  wavelength in the least-squares fit.

## 5. Absolute measurement

### 5.1 Challenge of absolute measurement

As described in the previous section, this algorithm has local solutions at intervals of about  $1/2$  wavelength. In order to measure the absolute value correctly, it is necessary to give an initial value with an accuracy within 300 nm, but preliminary estimation with this accuracy is difficult (see Appendix 2 for the estimation accuracy of Fourier transform method). Therefore, we propose a new method.

### 5.2 Solution to local minima problem

The model formula (16) is the sum of cosine functions, and, assuming  $ct \ll \lambda_0$ , we can approximate as

$$\cos(4\pi cSt/\lambda_0^2 - 4\pi S/\lambda_0) \cong \cos(4\pi S/\lambda_0) \quad (20)$$

Thus, the unknown variable  $S$  has local solutions at intervals of  $\lambda_0/2$ .

Pay attention to the first term in the cosine function on the left side of Eq. (20). When this cannot be ignored, there is a difference in  $SSE$  among the local solutions, and it becomes possible to obtain the correct global optimum solution. Since there is noise in the real data, the difference in  $SSE$  needs to be significantly larger than the noise. If the wavelength scanning width  $\Delta\lambda = c\tau$  ( $\tau$  is the scanning time) is made large and the above conditions are satisfied, the global

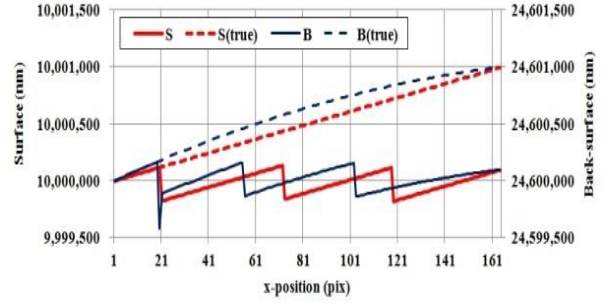


Fig.6 Line measurement results.

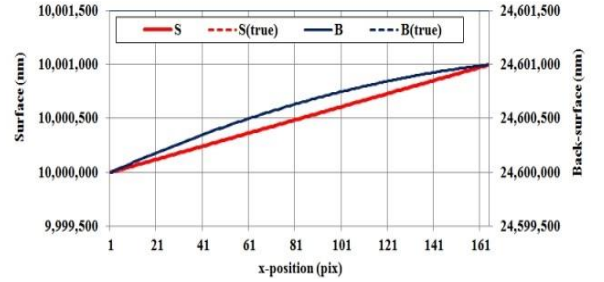


Fig.7 Line measurement results by multi-start method.

optimum solution can be obtained by the multi-start method.

## 5.3 Computer experiment: Multi-start method

Absolute value estimation experiments were carried out with the following conditions:

- (1) wavelength scanning speed  $c$ : 0.1 nm/s
- (2) number of images: 256
- (3) noise amplitude: 0
- (4) multi-start step: 250 nm
- (5)  $S, B$  initial values: [-500, -500] [-250, -250] [0, 0] [250, 250] [500, 500] nm of the true value at  $x = 82$ .

Figure 7 shows the obtained results. Absolute values are correctly measured for both  $S$  and  $B$ .

## 6. Summary

We proposed a simultaneous measurement method for the surface profile, back surface profile, and plate thickness distribution of transparent plates using a Fizeau interferometer and wavelength scanning interferometry. The least squares fit of the interference signal model is applied to the wavelength scanning intensity waveform, and the front surface position, back surface position, and plate thickness can be obtained [12]. The validity of the proposed method was confirmed by computer experiments.

This method does not require the sample position adjustment, which was necessary in the conventional methods, and has excellent operability. It also has the feature that it can measure absolute values.

## Supplement 1

In Fig. 3, the multiple reflected lights of the second and subsequent orders in the Fizeau interferometer are ignored. This validity is shown below.

In the interferometer shown in Figure 1-1, the amount of secondary reflected light is expressed by the following equation.

$$S_2 = SR_R R_S \quad (1-1)$$

$$B_2 = BR_S^2 \quad (1-2)$$

$R_R$  and  $R_S$  defined by Eqs. (2)-(3) are about 4% for glass. Therefore, the amount of secondary reflected light can be ignored.

## Supplement 2

The distance resolution  $\Delta L$  obtained by the discrete Fourier transform (FFT) depends on the initial wavelength  $\lambda_0$  and the wavelength scanning width  $\Delta\lambda$ , and is expressed by the following equation.

$$\Delta L = \lambda_0^2 / 2\Delta\lambda \quad (2-1)$$

If the initial wavelength  $\lambda_0$  is 600 nm,  $\Delta L$  becomes 18  $\mu\text{m}$  when  $\Delta\lambda = 10$  nm, and  $\Delta L = 1.8$   $\mu\text{m}$  when  $\Delta\lambda = 100$  nm. That is, it is difficult to obtain an accuracy of 300 nm, which is half the wavelength.

The process of deriving equation Eq. (2-1) is shown below:

The frequency resolution of FFT is

$$\Delta f = 1/\tau \quad (2-2)$$

where  $\tau$  is the observation time.

On the other hand, from Eq. (10),

$$\Delta f = 2c\Delta L / \lambda_0^2 \quad (2-3)$$

From equation Eqs. (2-2)-(2-3),

$$\Delta L = (\Delta f)\lambda_0^2 / 2c = \lambda_0^2 / 2c\tau = \lambda_0^2 / 2\Delta\lambda \quad (2-4)$$

Note 1: Eq. (10) is an approximate equation for  $\lambda_0 \gg c\tau$ . The exact expression without this approximation is

$$\Delta L = \lambda_{\max} \lambda_{\min} / 2\Delta\lambda \quad (2-5)$$

Note 2: Since FFT generally uses a window function, the actual resolution is lower than the theoretical resolution.

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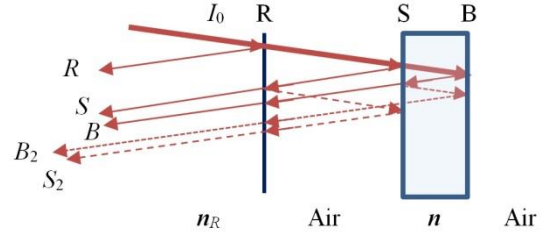


Fig.1-1 Multiple reflections in Fizeau interferometer.

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- [12] Patent pending.

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