

Nomenclature

AR	aspect ratio	T_f	final time
C_c	Cunningham correction factor	T	temperature
c_k	discrete lattice velocity in direction (k)	u	horizontal components of velocity (m/s)
c_s	speed of sound in lattice scale	v	vertical components of velocity (m/s)
DPF	deposited particle fraction	<i>Greek symbols</i>	
d^p	particle diameter (μm)	β	thermal expansion coefficient (1/k)
F_k	external force in direction of lattice velocity	ρ	density (kg/m^3)
f_k^{eq}	equilibrium distribution function	τ	lattice relaxation time
g	gravity (m/s^2)	$\tau_p = \frac{\rho^p c_s (d^p)^2}{18\mu}$	particle relaxation time (s)
G	Gaussian random number	Δt	lattice time step
Gr	Grashof number	ν	kinematic viscosity (m^2/s)
$H = Ro - Ri$	length scale	λ	gas mean free path (μm)
k	thermal conductivity (w/m k)	<i>Subscripts</i>	
$kn_p = 2\lambda/d^p$	particle Knudsen number	c	cold
K_{Th}	thermophoretic coefficient	h	hot
L	Saffman lift force (N/kg)	Th	thermophoretic
MSV	maximum stream function value	p	particle
N_p	number of particles	g	gas
N	Brownian force (N/kg)	<i>Superscript</i>	
$Pr = \nu/\alpha$	Prandtl number	i	vector axis indicators
$Ra = g\beta\Delta TH^3/\alpha\nu$	Rayleigh number		
r	Radial coordinate		
Ri	inner radius		
Ro	outer radius		
$S = \rho^p/\rho^g$	particle Specific density		
SPF	suspended particle fraction		
Th	thermophoretic force (N/kg)		

The objective of the present study is to determine the effect of aspect ratio of annulus and particles diameter on the particles removal and dispersion. The Lattice Boltzmann Method is used to simulate laminar flow in a concentric annulus. Then particle equation of motion is solved to investigate dispersion and removal of particles. Drag, Saffman lift, gravity, buoyancy, Brownian motion and thermophoretic are forces that included in Lagrangian particle tracing procedure.

2. Lattice Boltzmann Method

The general form of Lattice Boltzmann equation with external force can be written as:

$$f_k(\vec{x} + \vec{c}_k \Delta t, t + \Delta t) - f_k(\vec{x}, t) = \Delta t \frac{f_k^{eq}(\vec{x}, t) - f_k(\vec{x}, t)}{\tau} + \Delta t \cdot \vec{F} \quad (1)$$

where Δt is lattice time step, \vec{c}_k is the lattice velocity in direction k , F_k is the particle distribution function which corresponds to the external force in direction of \vec{c}_k and τ denotes the lattice relaxation time and it is defined as:

$$\tau = \left(\frac{1}{\Delta t c_s^2} \right) \nu + 1/2 \quad (2)$$

where ν is the kinematic viscosity, f_k^{eq} is the equilibrium distribution function and depends on the type of problem. The equilibrium distribution functions for fluid field are calculated with following equation:

$$f_k^{eq} = \omega_k \cdot \rho \left[1 + \frac{\vec{c}_k \cdot \vec{u}}{c_s^2} + \frac{1}{2} \frac{(\vec{c}_k \cdot \vec{u})^2}{c_s^4} - \frac{1}{2} \frac{\vec{u} \cdot \vec{u}}{c_s^2} \right] \quad (3)$$

where ρ and \vec{u} are the macroscopic fluid density and velocity and are calculated as below:

$$\text{Flow density: } \rho = \sum_k f_k \quad (4)$$

$$\text{Momentum: } \rho \vec{u} = \sum_k f_k \vec{c}_k \quad (5)$$

ω_k are weighting factors. The values of $w_0 = 4/9$ for $|c_0| = 0$, $w_{1-4} = 1/9$ for $|c_{1-4}| = 1$ and $w_{5-8} = 1/36$ for $|c_{5-8}| = \sqrt{2}$ are assigned in this model. The thermal Lattice Boltzmann equation could be written as below:

$$g_k(\vec{x}, t + \Delta t) - g_k(\vec{x}, t) = \Delta t \frac{g_k^{eq}(\vec{x}, t) - g_k(\vec{x}, t)}{\tau_c} \quad (6)$$

where the thermal equilibrium distribution functions are given as:

$$g_k^{eq} = \omega_k T \left[1 + \frac{\vec{c}_k \cdot \vec{u}}{c_s^2} \right] \quad (7)$$

T is the fluid temperature and evaluated from:

$$T = \sum_k g_k^{eq} \quad (8)$$

The temperature relaxation time is calculated as a function of diffusivity coefficient:

$$\tau_c = \left(\frac{1}{\Delta t c_s^2} \right) \alpha + 1/2 \quad (9)$$

In order to incorporate buoyancy force in the model, the Boussinesq approximation was applied, therefore the force term in the Eq. (1) need to be calculated as below in vertical direction (y):

$$\vec{F}_y = 3\omega_k \vec{g}_y \beta \Delta T \quad (10)$$

where \vec{g}_y is the acceleration of gravity acting in the y -direction of the lattice links; β is the thermal expansion coefficient and ΔT is temperature difference.

2.1. Curved boundary treatment

In the present study, a second-order accurate method to define curve boundary condition is used [20]. Fig. 1 shows the lattice node

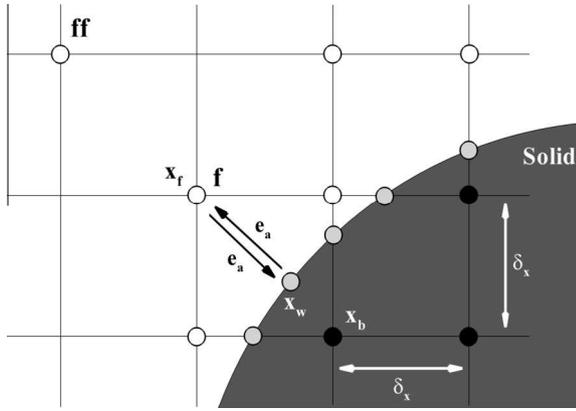


Fig. 1. Layout of the regularly spaced lattices and curved wall boundary.

treatment on curved boundary condition, the solid block nodes indicates the boundary nodes x_b , the white nodes shows the first fluid nodes x_f and the gray nodes on the boundary x_w indicates the intersections of the wall with various lattice links.

The fraction of an intersected link in the fluid region Δ , is determined by:

$$\Delta = \frac{|x_f - x_w|}{|x_f - x_b|} \tag{11}$$

At the collision step, the fluid side distribution function on the fluid side \tilde{f}_k , is determined but the solid side distribution function at the opposite direction \tilde{f}_k , is unknown. On the other hand to finish the streaming step, we need to know \tilde{f}_k at the boundary node x_b . More detail about the velocity and temperature boundary conditions over the curved body can be found at the study of Fattahi et al. [15].

3. Dispersed phase

A particle suspended in buoyancy driven flow is affected by some forces. Drag, Saffman lift, gravity, buoyancy, Brownian motion and thermophoretic are included in particle equation of motion in this study. The corresponding particle equation of motion in i direction is given as:

$$\frac{du_p^i}{dx^i} = \frac{1}{\tau_p} (u_g^i - u_p^i) + L^i + N^i + \left(1 - \frac{1}{S}\right) g^i + Th^i \tag{12}$$

The first term in right hand side of Eq. (12) is drag force that is due to relative velocity between particles and carrier gas. The relaxation time τ_p is the characteristic time scale (response time) of particle. The particle needs to this time to response the variation of flow field. u_p^i and u_g^i are the corresponding particle and velocity indirection of i . S is specific gravity, m^p is the particle mass and g^i is gravity acceleration. By increasing the relaxation time, particle reaction decreases to variation of flow parameters. For submicron particles with particle Knudsen number $kn_p = \frac{2\lambda}{d_p}$ greater than 0.1, when the particle diameter is in the range of gas mean free path (λ), flow slips over the particle surface. Therefore, the Stokes drag must be modified by Cunningham correction factor (C_c) as [21]:

$$C_c = 1 + \frac{2\lambda}{d_p} \left[1.257 + 0.4e^{\left(\frac{-1.1d_p}{2\lambda g}\right)} \right] \tag{13}$$

Small particles in a shear field experience a force perpendicular to the direction of flow. The shear lift originates from the inertia effects in the viscous flow around the particle and is different from

aerodynamic lift force that originates from shape asymmetry of particles. The first equation for this force was obtained by Saffman [22]:

$$L^i = \frac{2k_L v^{0.5} d_{ij}}{S d_p (d_{ik} d_{kl})} (u_g^i - u_p^i) \tag{14}$$

ρ^p and ν are density and kinematic viscosity of gas phase, respectively. $k = 2.549$ is constant coefficient of Saffman lift force and d_{ij} is deformation tensor and define as [21]:

$$d_{ij} = \frac{1}{2} (u_{ij} + u_{ji}) \tag{15}$$

Finally, the Brownian motion per unit mass in direction i is denoted by N^i . The instantaneous random momentum that imparts to the ultrafine particles due to the impaction of the gas molecules, which causes the particle to move on an erotic path known as Brownian motion [21,23]:

$$N^i = G \sqrt{\frac{\pi S_0}{\Delta t}} \tag{16}$$

$$S_0 = \frac{216 \nu K_1 T_g}{\pi^2 \rho_p (d_p)^5 S^2 C_c} \tag{17}$$

T_g is gas temperature and $K_1 = 1.38 \times 10^{-23} \frac{j}{k}$. G is unit variance zero mean Gaussian random numbers as:

$$G = \sqrt{-2 \ln U_i} \cos(2\pi U_j) \tag{18}$$

where U^i and U^j are random numbers (between 0 and 1). Finally, Th^i denotes thermophoretic force which is defined as:

$$Th = K_{Th} \frac{\partial T}{\partial x^i} \tag{19}$$

where K is the thermophoretic coefficient suggested by Talbot et al. [24]:

$$K_{Th} = \frac{2C_s C_c \left[\left(\frac{K_g}{K_p}\right) + C_t K n_p \right]}{(1 + 3C_m K n_p) (1 + 2\frac{K_f}{K_p} + 2C_t K n_p)} \tag{20}$$

The values of C_t , C_m and C_s are 2.18, 1.14 and 1.17, respectively [25]. The K_f and K_p are the thermal conductivity of fluid and particle, respectively. By solving particle equation of motion for particles, particles path is obtained by:

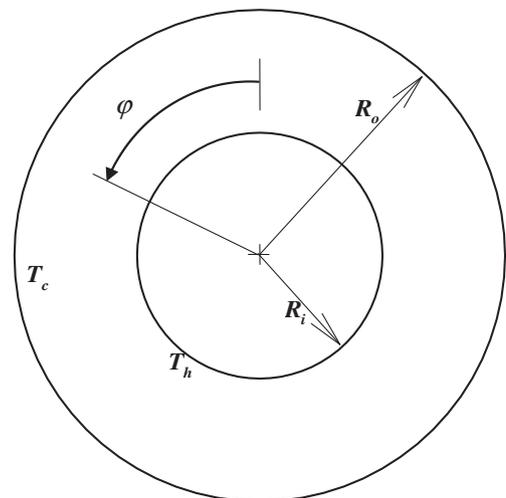


Fig. 2. Computational domain of concentric annulus.

Table 1
Annulus characteristic at $Ra = 10^4$, ΔT and $R_o = 0.5$ (cm).

Aspect ratio (AR)	Inner cylinder diameter (cm)
2	0.25
3	0.167
4	0.125

$$\frac{dx^i}{dt} = u_p^i \quad (21)$$

4. Numerical procedure

In the present study an in-house code is developed to use hybrid Lattice Boltzmann Method and discrete phase model for simulating flow characteristics and particle trajectories in the buoyancy driven natural convection laminar flow. Computational domain is considered a two dimensional concentric cylindrical annulus with inner radius R_i and outer radius R_o (Fig. 2). The 200×200 grid resolution is used to perform the simulation.

The aspect ratio is defined as the ratio of outer radius to the inner radius. The cylinder surfaces are maintained at two different uniform temperatures. The Rayleigh number Eq. (22) is fixed at 10^4 :

$$Ra = \frac{g\beta\Delta TH^3}{\nu_g\alpha_g} \quad (22)$$

When the flow reached to the steady state, with the flow velocity distribution known, tracking of random distributed particles are then performed in a Lagrangian frame with assumption of one-way

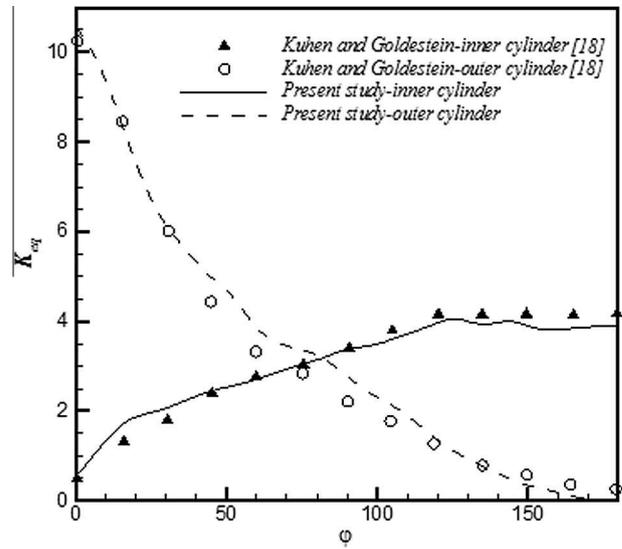


Fig. 4. Comparison of equivalent thermal conductivity on inner and outer cylinder with experimental results of Kuehn and Goldstein [18] for annulus at $Ra = 5 \times 10^4$.

coupling. In the one way coupling assumption, Particles do not impart momentum to the carrying fluid and also there is no collision of particles. These effects are considered for high particles concentration [26]. It is necessary to convert the all lattice parameters to real ones to use them in the particle equation of motion. It is due to presence of dimensional parameters in the equation of motion. For example with equality of Fourier number as dimensionless time of LBM results and real condition, the relation of real time and Lattice time step obtained as:

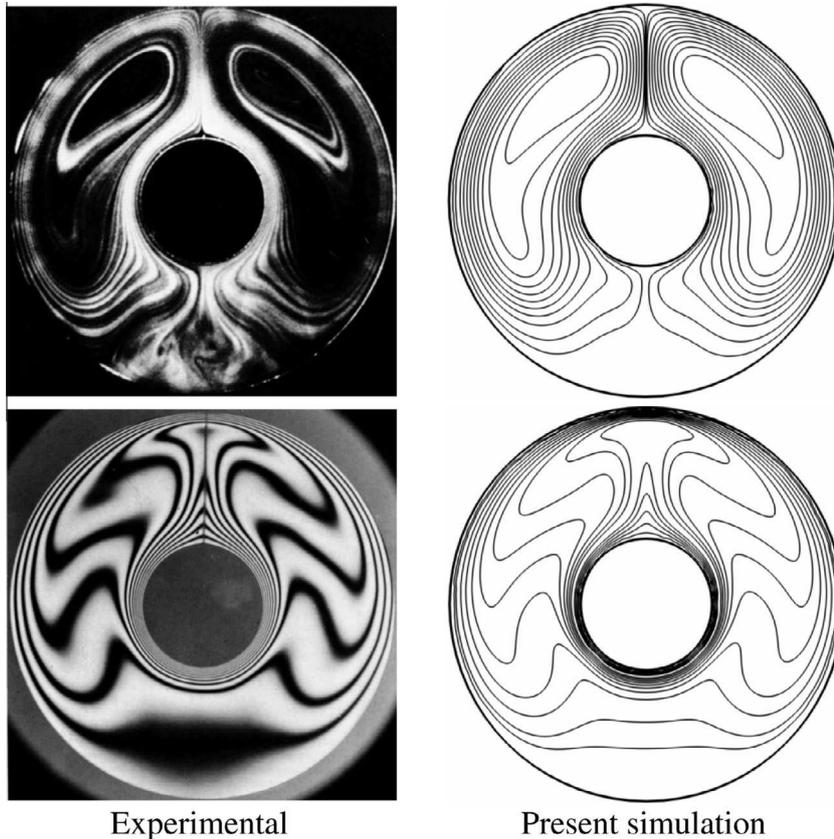


Fig. 3. Comparison with experimental result of Grigull and Hauf [27]. The streamlines for $Gr = 120,000$ (top) and Isotherms for $Gr = 122,000$ (bottom).

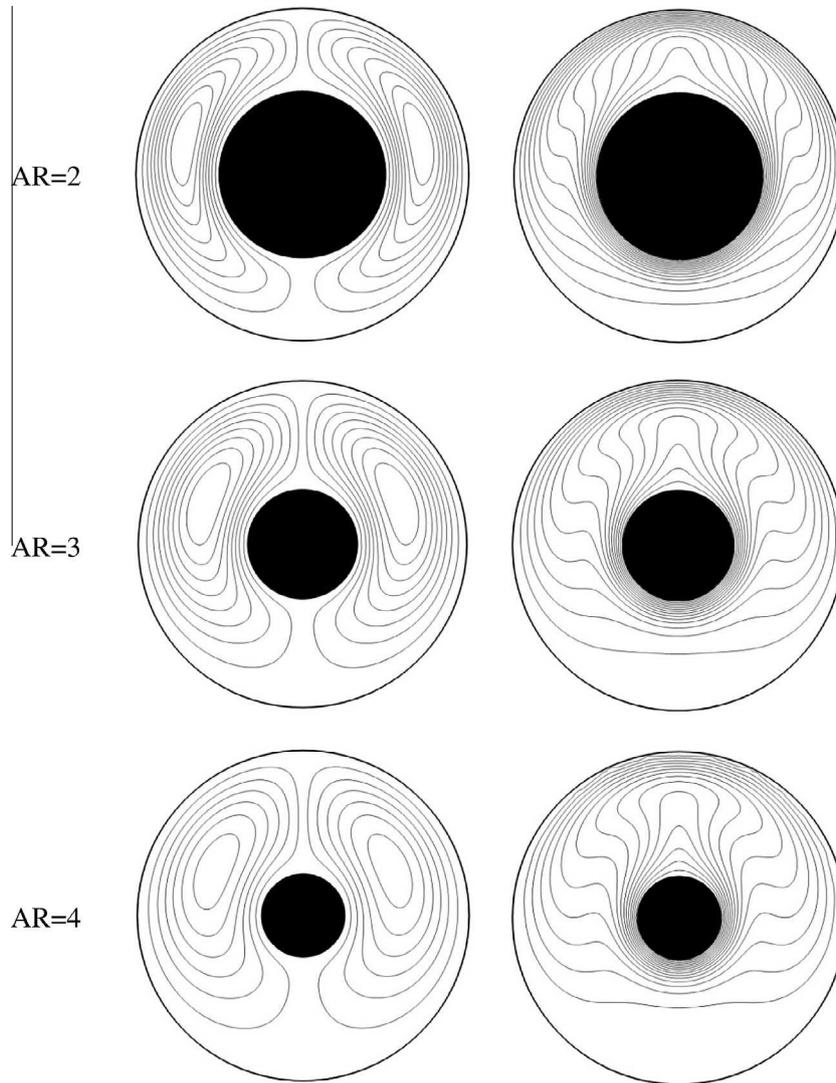


Fig. 5. The streamlines (right) and isotherms (left) at $Ra = 10^4$.

Table 2
Annulus Maximum stream function value versus aspect ratio at $Ra = 10^4$.

Aspect ratio (AR)	MSV
2	3.97
3	3.31
4	2.96

$$t_{(Red)} = t_{(Lattice)} \times \frac{\alpha_{(Lattice)}}{\alpha_{(Red)}} \times \frac{H^2}{N^2} \quad (23)$$

Particles trajectories are studied at different aspect ratio (AR). The particle initial slip velocity is set to zero and the temperature of particles is assumed by a linear distribution between the cold and the hot walls temperature. This assumption is employed to obtain particles properties. The characteristics of annulus are presented in Table 1. Particles are selected with different diameter at $S = 1000$ to obtain a wide range of microsize aerosol behavior.

Standard 4th-order explicit Runge–Kutta (ERK4) method is implemented in order to time integrates the particles trajectory. Consider the first order ODE:

$$\frac{dy}{dx} = f(x, y) \quad (24)$$

The formula for 4th order Runge–Kutta approximation to the above ODE is given as:

$$y_{n+1} = y_n + \frac{h}{6}(K_1 + 2(K_2 + K_3) + K_4) + O(h^5) \quad (25)$$

where:

$$\begin{aligned} K_1 &= f(x_n, y_n) \\ K_2 &= f\left(x_n + \frac{h}{2}, y_n + h\frac{K_1}{2}\right) \\ K_3 &= f\left(x_n + \frac{h}{2}, y_n + h\frac{K_2}{2}\right) \\ K_4 &= f\left(x_n + h, y_n + hK_3\right) \end{aligned} \quad (26)$$

Perfect absorbing wall condition is assumed at the wall boundaries. This implies that when particles distance with cylinders surfaces becomes smaller than radius of particles they stick to cylinders and do not reenter in the flow domain because of rebound or lubrication effects. In the particle calculations the Δt was 10^{-5} at all cases. Since the effect of randomness may become important in the simulation of Brownian force, simulation of some cases with $d_p = 1 \mu m$ are performed with different time steps to ensure that calculations are independent of time step.

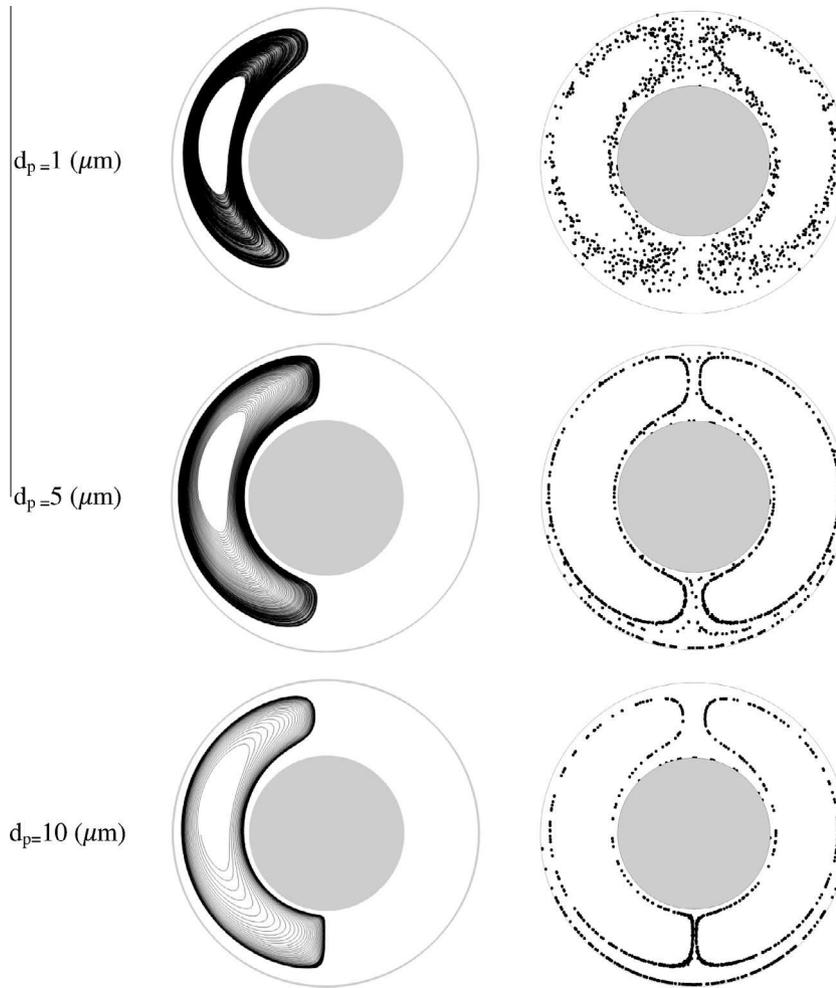


Fig. 6. A particle trajectories (left) and all particles end positions (right) at AR = 2.

5. Result and discussion

The present study considers two horizontal circular cylinder of inner and outer radius, R_i and R_o , respectively. A temperature difference is applied with subjecting the walls of the inner cylinder to a higher temperature (T_h) than its outer cylinder counterpart (T_c). The effect of aspect ratio and particles diameter are investigated on aerosol particles behavior. An experimental study is selected to validate flow field and isotherms. Gr number of 120000 and 122000 are selected to compare streamlines and isotherms of present simulation with experimental results of Grigull and Hauf [27]. Results show agreement between the present computations and their experiments as shown in Fig. 3. For furthermore validation, the local equivalent thermal conductivity is obtained. Results have been compared with the study of Kuehn and Goldstein [18]. This parameter is defined as the actual heat flux divided by the heat flux that would be occurred by pure conduction in the absence of fluid motion. Results of local equivalent thermal conductivity are shown in Fig. 4 and represent good agreement between the present computations and their experiments results. In particular, the present local equivalent thermal conductivity results are well within $\pm 3\%$ of the benchmark data by Kuehn and Goldstein [18]. The streamlines and isotherms are presented for aspect ratio of 2, 3 and 4 at $Ra = 10^4$ in Fig. 5. Two large recirculation areas form between cylinders at the left and right sides. The flow moves clockwise in the right section of annulus. It is due to the inner wall push flow to the top of

the annulus and cold outer wall push it downward. Finally flow moves counterclockwise in the left section of annulus. On the basis of same reason.

Variation of aspect ratio effects on flow field and heat transfer characteristics. At smaller aspect ratio (AR = 2), buoyancy driven flow is limited to circulate in a smaller region. So the buoyancy power is great. By increasing the aspect ratio to 3 and 4, the speed (strength) of recirculation zones decreases. It is clear in the behavior of maximum Stream function value at different aspect ratios (Table 2).

5.1. Particles dispersion and removal

The particle with sizes in the range of 1–10 μm at $S = 1000$ are selected for simulating. Drag, Saffman lift, gravity, buoyancy, Brownian motion and thermophoretic are forces that included in particle equation of motion. The effect of aspect ratio and particles diameter are investigated on the particles behavior such as removal and dispersion. Hydrodynamic force, gravity and inertia of particles are primary contributors in the microparticles dynamic. Since the flow velocity vectors are changing sharply, relative velocity between carrier (gas) and discrete phases (particle), causes slip between particles and gas molecules to develop. The flow passes through curve lines in annulus and particles cannot follow it exactly that is due to inertial force. Particles which are near the solid boundaries, contact with walls and are captured there. This behavior is called inertial impaction.

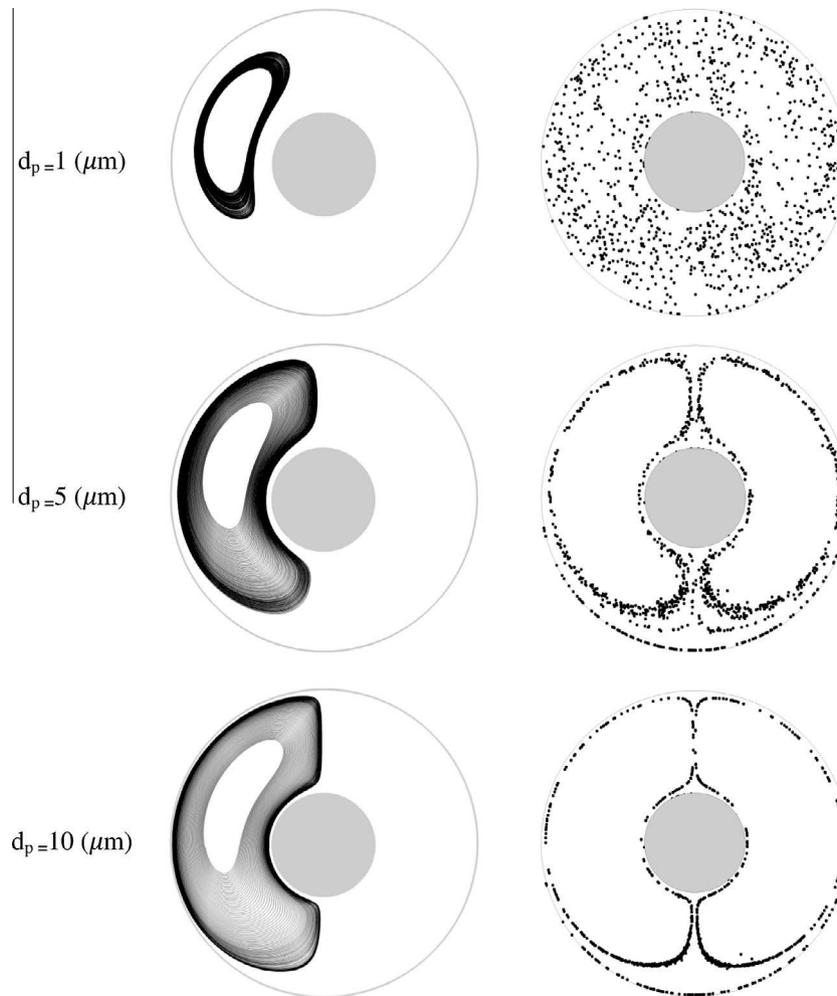


Fig. 7. A particle trajectories (left) and all particles end positions (right) at $AR = 3$.

This phenomenon is more sensible for heavy particles. Brownian motion is dominant removal mechanism for ultrafine particles with diameter less than $1 \mu\text{m}$. However, Brownian motion comes into account for particles with diameter in the range of present simulation.

A specific particle trajectory and all particles end positions with diameter of 1, 5 and 10 micrometer are presented for different aspect ratios at $Ra = 10^4$ in Figs. 6–8. Hydrodynamic forces of the two recirculation regions tend to hold the particles in themselves and Inertia force pushes the particles outward. Finer particles are caught up in the recirculating flow and remain suspended in the gas in two quasi-equilibrium zones at both right and left sides of annulus. It is due to the outward and inward forces are nearly balanced. As particles diameter increases, the inertial force causes particles to deviate from inner streamlines to outers. So quasi-equilibrium zones form in a thinner bounded curve near the cylinder surfaces. By increasing the aspect ratio to 3 and 4, recirculation power decreases (Table 2). So particles acceleration and their inertia decreases and quasi-equilibrium zones become thicker and cover the large part of flow. It should be mentioned that hydrodynamic forces decrease too.

The progress of particles dispersion is shown over the time in Fig. 9. Figures provide the instantaneous snapshots of particles locations for the two different sets ($d_p = 3 \mu\text{m}$, $d_p = 6 \mu\text{m}$) and the two aspect ratio $AR = 2, 3$ at time $t = 1, 10, 50$. Figure indicates that particles reach the quasi equilibrium region at aspect ratio (AR) of

2 in a shorter time with respect to $AR = 3$ (red¹ and green dots). It means that particle disperses in a quasi-steady state in first few seconds.

Time histories of particles removal fraction (PRF) for particles with three different diameters are presented in Fig. 10 at different aspect ratios. For particles with medium ($d_p = 5 \mu\text{m}$) and large ($d_p = 10 \mu\text{m}$) sizes PRF shows three different time behaviors clearly: during the first few seconds, high and almost linear trends (constant rate) are observed in PRF. It is due to the initial velocity of particles are set zero. So the effect of gravitation plays the important role in the deposition process. As time passes, increasing of particles velocity reduces this effect in later seconds. The instant when the removal rate changes sharply and become almost horizontal, are proportional to the time needed by most number of particles that will be deposited, to remove from the equilibrium regions. For medium and large Particles also, removal process takes less time by increasing of particle diameters. It is due to the effect of gravity that helps inertia to enforce particles deviate from their streamlines to move from inner streamlines to outer ones in the recirculation region and hit the wall in a shorter time. As aspect ratio increases, particles removal fraction (PRF) reaches its maximum value in a longer time. In fact particle must cover larger distance to hit the wall as the gap of cylinders become greater.

¹ For interpretation of color in Fig. 9, the reader is referred to the web version of this article.

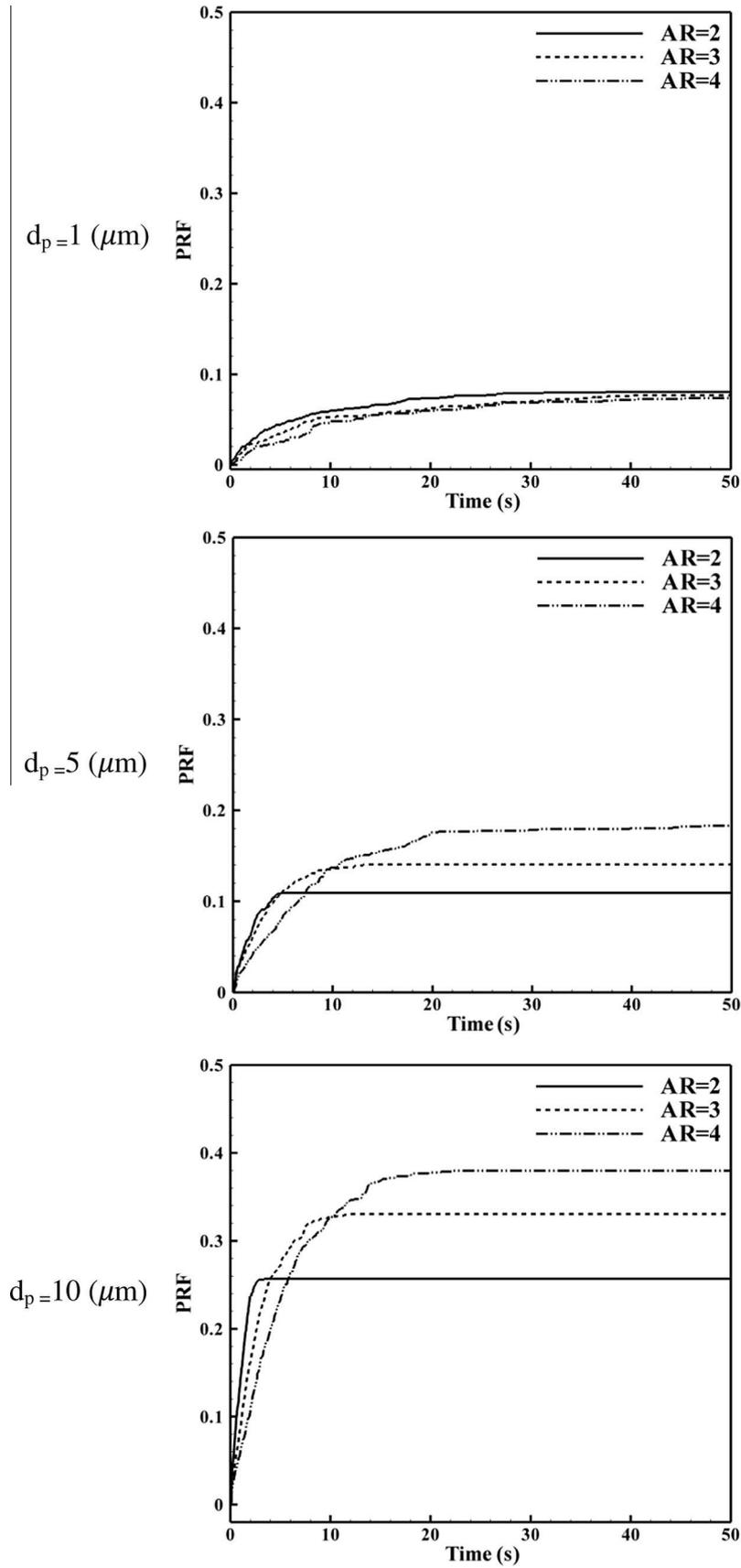


Fig. 10. Time histories of deposited particle fraction.

