

# Revised technical guidelines for reliable dc measurements of the quantized Hall resistance

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## Abstract

This paper describes the main tests and precautions necessary for both reproducible and accurate results in the use of the quantum Hall effect as a means to establish a reference standard of dc resistance having a relative uncertainty of a few parts in  $10^9$ .

## 1. Introduction

This paper is a revised version of the *Technical Guidelines for Reliable Measurements of the Quantized Hall Resistance* established in 1988 [1]. The 1988 text was based on the suggestions of a Working Group on the Quantum Hall Effect<sup>3</sup> established by the Comité Consultatif d'Électricité (CCE). At its 22nd meeting (September 2000), the Comité Consultatif d'Électricité et Magnétisme (CEEM, new denomination of the CCE) asked the authors of this paper to prepare a revised version of the Guidelines taking into account comments and suggestions received from the national metrology institutes (NMIs).

Indeed, since 1988 considerable progress was made in the NMIs on the subject of accurate comparisons of quantized Hall resistances (QHRs) as realized using different types of quantum Hall effect (QHE) devices [2–5]. Also, it was possible to confirm, in particular through on-site comparisons of resistance standards based on the QHE [6], that the reproducibility of the QHR, as realized by the different NMIs, is as good as a few parts in  $10^9$ . A generally admitted conclusion is that the 1988 Guidelines were found adequate to ensure accurate QHR measurements, in the sense that every QHE device that gave a discrepant result was also found to fail at least one of the tests suggested in the Guidelines. In

particular, it was confirmed that an important criterion is the absence of a longitudinal voltage drop along both sides of the QHE device.

The aim of this paper is not to recommend strict rules but rather to propose guidelines to serve as a reminder of the main tests and precautions necessary to assure reliable measurements of the QHR at a relative uncertainty of a few parts in  $10^9$ . Also, this paper is not intended to be a review paper on the subject of the metrological application of the QHE. The interested reader is referred to recently published reviews [7–9].

## 2. Device choice

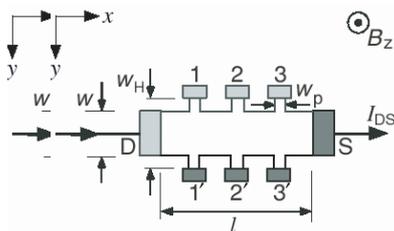
Metal-oxide semiconductor field-effect transistors (MOSFETs) or GaAs/AlGaAs devices (and possible alternative heterostructures) can be used for accurate measurements of the QHR. An important feature is the value of the measuring current that can be used without producing significant longitudinal dissipation in the device. It has been shown that, from this point of view, specially designed MOSFETs can compete with GaAs-based heterostructures and accept measuring currents as high as  $50\ \mu\text{A}$  [3]. It was demonstrated that QHRs measured on both types of devices are in agreement to better than 1 part in  $10^9$  [3, 4]. However, GaAs devices are usually preferred for routine QHR measurements, and this for a number of reasons: GaAs devices can be used at a relatively high temperature (of the order of 1.5 K instead of 0.5 K for MOSFETs) and at a relatively low magnetic flux density  $B$  (as low as 6 T); they are simpler to operate as no gate electrode is needed; moreover, it is reasonably easy to obtain suitable GaAs devices as there are several fabrication sources.

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In the case of GaAs/AlGaAs devices, a mobility  $\mu$  higher than  $10\text{ T}^{-1}$  and a carrier concentration  $n$  in the range  $3 \times 10^{15}\text{ m}^{-2}$  to  $5.5 \times 10^{15}\text{ m}^{-2}$  are suitable in order to obtain wide and well-quantized  $i = 2$  plateaux for the values of temperature mentioned above and with  $B$  in the range 6 T to 11 T. If  $n$  is increased to values above  $6 \times 10^{15}\text{ m}^{-2}$ , the second electrical sub-band in the potential well at the interface between GaAs and AlGaAs is populated as well. As a consequence, a second current path develops in the device, producing interference with the usual quantum Hall picture. If good quantization conditions for the  $i = 4$  plateau are important, a mobility of  $10\text{ T}^{-1}$  is not sufficient. As shown in [10], the minimum longitudinal resistivity for  $i = 4$  rapidly increases when the mobility decreases below  $30\text{ T}^{-1}$ , occurring for current levels required for high-accuracy QHR measurements. Other parameters to be considered are the critical current and the plateau width. At the critical current, the QHE breaks down and the longitudinal resistivity abruptly increases by several orders of magnitude. It was shown [10] that the critical current is independent of the mobility when  $\mu$  is between  $15\text{ T}^{-1}$  and  $130\text{ T}^{-1}$  for  $i = 2$ , and between  $30\text{ T}^{-1}$  and  $130\text{ T}^{-1}$  for the case of  $i = 4$ . On the other hand, the plateau width decreases with increasing mobility although not as dramatically as predicted previously. Considering the different aspects, a mobility of  $40\text{ T}^{-1}$  to  $80\text{ T}^{-1}$  seems to be an optimal choice for GaAs devices, especially for high-accuracy measurements on plateaux other than the  $i = 2$  plateau.

In the case of silicon MOSFETs, a mobility of about  $0.8\text{ T}^{-1}$  and a carrier concentration of  $13 \times 10^{15}\text{ m}^{-2}$  were found adequate to obtain a well-quantized  $i = 4$  plateau at a temperature of 0.4 K and for  $B$  of the order of 13 T [3].

The devices should be fitted with source (S) and drain (D) contacts (gate and substrate for MOSFETs) and with at least two, preferably three, pairs of Hall-voltage contacts (figure 1). As the critical current scales linearly with the sample width, for width  $w$  at least up to 1.5 mm [10], the width should be chosen as large as possible. The current contacts (S and D), where the electrons are injected into the two-dimensional electron gas (2DEG), should extend over the whole width of the device to reach the desired critical current. Deviations from the nominal QHR can be caused if the populations of the electronic edge states are not equilibrated (see section 4). In order to prevent the formation and detection of non-equilibrium distributions, narrow side arms ( $w_p < 100\text{ }\mu\text{m}$ ) along the edge of the device should be avoided and the distances between the contacts should be as large as possible.



**Figure 1.** Device with three pairs of Hall-voltage contacts. For the magnetic field pointing out of the sample in the  $z$  direction, the drain contact D and the Hall potential contacts 1–3 are at the same potential.

### 3. Device cooling and handling

Devices should be cooled slowly in the dark ( $>15$  min), at a constant rate and in an environment that is shielded from rf radiation.

MOSFETs should be cooled with a gate voltage applied from the very beginning of cooling or, alternatively, with the gate short-circuited to the source or drain contact.

Output wires attached to the device should be handled cautiously, as connecting them to accidental environmental noise sources may induce longitudinal dissipation (the longitudinal resistivity,  $\rho_{xx}$ , assumes a finite value) in a device previously in a dissipationless state ( $\rho_{xx} \approx 0\text{ }\Omega$ ). This is particularly true for MOSFETs but has also been observed on some occasions for GaAs devices. Restoration to a dissipationless state is normally possible, however, by cycling the device to room temperature for a short time.

### 4. Contact resistance

Non-ideal contacts to the 2DEG are often the major device-related limitation encountered by metrologists. Poor contacts are characterized by a high contact resistance,  $R_C$ , and in the worst case by a non-linear behaviour. High  $R_C$  values may be caused by structural defects in the metallization of the contact. In the case of the voltage contacts, another possibility is the partial depletion of the 2DEG in the narrow channel connecting the metallic pad to the main channel of the Hall bar (potential probe). Such a local reduction of the carrier concentration may be caused by cooling a device too fast, by passing a current above the critical current through the potential probe or, sometimes, even by leaving the device in the cold for several days. In most of these cases, the original contact properties are restored by cycling the device through room temperature or by illuminating the device at low temperature with a short pulse of infrared light [11].

The perturbing effects of poor contacts may include the following four characteristics:

- Poor source–drain contacts induce noise in the measuring current  $I_{SD}$  despite the use of a current source with a relatively high (with respect to the QHR) internal impedance. This noise makes precise measurements impossible. Also, the source–drain contact resistance may be different for the two polarities of  $I_{DS}$ , resulting in a measuring current that is different for the two polarities if the current source internal impedance is not high enough.
- Potential contacts may themselves generate excessive voltage noise when connected to a nanovoltmeter.
- Even in the case of an acceptable level of voltage noise, imperfect potential contacts can generate dc offset voltages (possibly by a process of rectification of noise) that depend on the polarity of  $I_{DS}$  and can introduce systematic errors in measurements of the QHR.
- The combined effect of an imperfect source or drain contact and an imperfect potential contact can produce a deviation of the measured QHR from its nominal value, through a mechanism involving unequal population of the Landau levels in the 2DEG [12, 13].

The influence of non-ideal voltage contacts on the QHR was extensively studied by Jeckelmann *et al* [4, 15]. These authors studied evaporated AuGeNi contacts on GaAs/AlGaAs devices. It was shown that deviations of the QHR from its nominal value of up to 1 part in  $10^6$  can occur as a consequence of contact resistance values in the kilohm range. At the same time, a corresponding positive or negative longitudinal voltage is measured along the side of the device to which the bad contacts are connected. In a device with good current contacts ( $R_C$  in the milliohm range), the QHR deviations caused by non-ideal voltage contacts decrease with increasing temperature and drain source current in the Hall bar. At a temperature of 0.3 K, the deviations were always below 1 part in  $10^9$  if the resistance of the voltage contacts was below  $100 \Omega$  for the  $i = 2$  plateau and below  $10 \Omega$  in the case of the  $i = 4$  plateau [4].

The following tests can be used to detect imperfect contacts. It is assumed that  $B$  (or the gate voltage in the case of a MOSFET device) is first adjusted to a value corresponding to the centre of a Hall plateau of resistance  $R_K/i$  where  $R_K$  is the von Klitzing constant. The measuring current used for these tests should be adapted to the type of contact under test: for the source or drain contacts the current should be of same order as that used for the QHR measurements ( $I_{DS}$ ); for the Hall-voltage contacts the current should be significantly lower since these contacts are used with virtually no current. In all cases, the value of the current should be low enough to avoid degrading the device. Also, the problem of possible noise contamination, mentioned above, should be kept in mind while making these tests. Special care should be taken if a mains-operated digital ohmmeter or voltmeter is used (see also section 6.1):

(1) The resistance between any two contacts of the device is determined by two-terminal measurements, made for both polarities of the measuring current. This is the simplest method but it has the disadvantage that the measured resistance is the sum of three terms: the Hall resistance  $R_K/i$ , the resistance of the leads,  $R_L$ , and the two contact resistances. The first term is the largest one, which makes the contact resistance measurement somewhat imprecise. The measured values depend on the material of the sample and on the nature and thickness of the contacts and the way they are made [14]. For example, for diffused tin contacts or for AuGeNi contacts on GaAs/AlGaAs devices, the values should be ideally within  $1 \times 10^{-4}$  of  $R_K/i + R_L$  and independent of current polarity.

(2) The contact resistance is measured using a three-terminal measurement technique. For instance, to evaluate the resistance of the drain contact (D, figure 1), the measuring current ( $I_{DS}$ ) is passed through contacts D and S and the potential difference between contacts D and, for instance, 1 is measured. The second contact used (here contact 1) is a contact at the same nominal potential as the contact under test (here D), taking into account the direction of the magnetic flux density vector  $B$ . This method provides a measurement of  $R_L + R_C$  where  $R_L$  is the resistance of the lead attached to the contact under test and  $R_C$  the contact resistance.

(3) The contact resistance is measured using a four-terminal measurement technique [16]. This gives the most precise results but is not very practical since the method requires that two wires be attached to the contact under test.

(4) The voltage noise across contact pairs (with  $I_{DS} = 0$ ) is evaluated using a nanovoltmeter with a sufficiently high input resistance ( $>10 \text{ k}\Omega$ ), sufficiently low offset current ( $<1 \text{ pA}$ ) and sufficiently low voltage noise for source impedance of the order of  $10 \text{ k}\Omega$  in the frequency band from 0 Hz to 1 Hz. The noise measured across pairs should be less than or equal to that observed with the meter's leads connected to the terminals of a high-quality, wire-wound resistor of nominal resistance  $R_K/i$  at room temperature. A higher level of noise may be due to poor contacts, and also possibly to microphonic noise in the leads connected to the device.

## 5. Conditions of quantization

The quantity to be measured,  $R_K/i$ , is believed to be the value of the Hall resistivity  $\rho_{xy}$  on a plateau of a 2DEG in a dissipationless state, i.e. with  $\rho_{xx} = 0$ . The following tests are useful for detecting a possible imperfect quantization.

### 5.1. Evaluation of the residual longitudinal resistivity

The condition for absence of dissipation can be tested by measuring the longitudinal voltage,  $V_x$ , between two contacts on the same side of the device while sweeping the magnetic flux density (in the case of heterostructures) or the gate voltage (in the case of MOSFETs) through the range corresponding to the plateau of Hall resistance. This measurement must be done with the current  $I_{DS}$  equal to that which will be used for QHR measurements. It is very important to carry out this measurement on both sides of the device (with two contacts on one side, for instance 1 and 2, and with two corresponding contacts on the other side, 1' and 2').

Ideally,  $V_x$  should be 'non-measurable', within the limit of resolution of the measuring instrument (possibly as low as a few nanovolts), for a central region of the sweeping range, a requirement for both sides of the device. Under practical conditions of temperature and magnetic field this is not always the case;  $V_x$  may present only a finite minimum value,  $V_x^{\min}$ , when the range is swept. The value of the minimal longitudinal resistivity,  $\rho_{xx}^{\min}$ , corresponding to  $V_x^{\min}$  is given by

$$\rho_{xx}^{\min} = \frac{V_x^{\min} w}{I_{DS} l},$$

where  $w$  is the width of the device and  $l$  the distance between the  $V_x$  contacts. (Note that this equation will always yield an approximate value for  $\rho_{xx}^{\min}$  because of the possible device inhomogeneities and because  $w$  and  $l$  are never precisely defined.) In the more favourable case where  $V_x$  becomes 'non-measurable', the above formula can also be used to calculate an upper limit to the possible residual longitudinal resistivity with  $V_x^{\min}$  taken as the limit of resolution of the measurements (for instance, 2 nV). This upper limit may be as low as  $10 \mu\Omega$  in the case where  $w/l = 1/4$  and for a measuring current  $I_{DS}$  of  $50 \mu\text{A}$ .

### 5.2. Possible temperature dependence

Varying the temperature with  $I_{DS}$  held constant is an important test for the characterization of a device. It is recommended that it be carried out at least once for a given device.

Ideally, the plateau value of  $\rho_{xy}$  should be invariant, within the limit of resolution of the measurements, over an appreciable range of temperature starting from the lowest temperature attainable with the cryogenic equipment used,  $T_1$ . This is not always the case and, indeed, a sufficiently large increase in temperature produces measurable and increasing values of  $\rho_{xx}$  and measurable variations of  $\rho_{xy}$ . The variation of  $\rho_{xy}$  as a function of  $\rho_{xx}$  can be quite different in magnitude, sign and character depending on the set of Hall contacts used, the magnetic field direction, position on the plateau and the value of  $I_{DS}$ . It has often been observed [17–20] that  $\rho_{xy}$  varies linearly with  $\rho_{xx}$ , over typically three decades in  $\rho_{xx}$ . In a limited range of temperature, where the longitudinal conductivities are thermally activated [20], the behaviour of  $\rho_{xy}(T)$  can be described by

$$\Delta\rho_{xy}(T) = \rho_{xy}(\rho_{xx}(T)) - \rho_{xy}(0) = s\rho_{xx}(T),$$

where  $s$  is a constant and  $\rho_{xy}(0)$  is the extrapolated value of  $\rho_{xy}$  at  $\rho_{xx} = 0$ , which is believed to be equal to  $R_K/i$ . The parameter  $s$  usually assumes values between  $-0.1$  and  $-1$ . Occasionally, however, positive values have been observed [7].

The temperature dependence of the transverse resistivity  $\rho_{xy}$  can only partly be ascribed to the effects of thermal activation. Another cause is the geometrical mixing [21] of the longitudinal voltage  $V_x$  into the Hall voltage  $V_H$ . As the Hall voltage is effectively sensed between diagonally opposite edges of the probe arms, the longitudinal voltage over the finite probe arm width  $w_p$  is compounded into  $V_H$ , yielding a measured value of  $\rho_{xy}$  given by

$$\rho_{xy}^{\text{meas}} = \rho_{xy}(T) - \frac{w_p}{w_H}\rho_{xx}(T),$$

where  $w_H$  is the distance between the Hall contacts (see figure 1). For a typical device, the ratio  $w_p/w_H$  is of the order of 10%.

For our purposes,  $s$  is best determined around the plateau centre where  $\rho_{xx}$  has its minimum value  $\rho_{xx}^{\text{min}}$ . For the measurement of  $s$ , the current has to be held constant. Ideally, current should be low enough so that the current-induced elevation of electron temperature is lower than the lowest bath temperature,  $T_1$ . It should be remembered that  $s$  can be a function of the chosen set of Hall contacts and usually depends on the direction of  $\mathbf{B}$ . Furthermore, the determination of  $s$  is very time-consuming and it is not necessarily reproducible with thermal cycling.

As a consequence, a device showing a measurable temperature dependence of  $\rho_{xy}$  near  $T_1$  can be used for accurate QHR measurements only if it has been verified beforehand that  $s$  is reasonably reproducible. Furthermore, the relative value of the correction applied to  $\rho_{xy}$ , i.e.  $-s\rho_{xx}/\rho_{xy}$ , should not exceed a few parts in  $10^8$ .

It is, of course, much better to use a device for which  $\rho_{xy}$  is invariant with respect to a significant increase in the temperature above  $T_1$ . This is usually associated with a ‘non-measurable’ value of  $\rho_{xx}$  on both sides of the device at  $T_1$ . A knowledge of  $s$  is not necessary for such a device when the  $\rho_{xy}$  measurements are made at  $T_1$ .

### 5.3. Possible current dependence

The check of the invariance of  $\rho_{xy}$  with respect to significant changes in  $I_{DS}$  is important as it may reveal imperfect quantization. It also offers a means of detecting leakage currents that are not proportional to  $I_{DS}$ .

By varying the current at a fixed temperature and correcting for the current dependence of the reference resistor, the current-driven relation between the measured  $\rho_{xy}(I)$  and  $\rho_{xx}(I)$  is obtained. As in the temperature-driven relation, a linear behaviour of  $\rho_{xy}$  as a function of  $\rho_{xx}$  is found [20], typically over three decades of  $\rho_{xx}$ . It should be noted that, when observed, the current-induced variations of  $\rho_{xx}$  result in variations  $\Delta\rho_{xy}$  of the QHR that may differ from those associated with temperature-induced variations of  $\rho_{xx}$ .

### 5.4. Possible magnetic-field (or gate-voltage) dependence

The flatness of the Hall plateau should be verified, at least once for a given device, by making measurements of  $\rho_{xy}$  not only at the centre of the plateau but at a few points on either side of the centre. Flatness is necessary for useful measurements but does not imply that the correction due to finite  $\rho_{xx}^{\text{min}}$  is negligible. Also,  $\rho_{xx}^{\text{min}}$  should occur at the same value of  $B$  when it is measured on both sides of the device. If  $\rho_{xx}^{\text{min}}$  does not occur at the centre of the plateau,  $\rho_{xy}$  should be determined at the value of  $B$  corresponding to  $\rho_{xx}^{\text{min}}$ , but extra precautions should be taken to demonstrate the flatness of the plateau.

Another test for imperfect quantization, which should be made at least once, is that of the invariance of  $\rho_{xy}$  with respect to the direction of  $\mathbf{B}$ .

### 5.5. Possible geometric dependence

To ensure that the finite aspect ratio of the QHE device does not cause a significant error in the measurement of  $\rho_{xy}$ , measurements of this quantity at all three Hall-contact pairs along the length of the device (or at least the pair at the centre and one pair on the end) should yield the same value. It has been demonstrated [22] that there are no significant differences, to within 1 part in  $10^9$ , in the QHRs measured with devices with channel width in the range from 1 mm down to 10  $\mu\text{m}$ .

## 6. Measurement of the QHR

All the tests mentioned above are very time-consuming and cannot be repeated each time a QHR measurement is made. This section suggests the minimum number of measurements that might be made, per day, during a particular QHR measurement. It is assumed that the device has already been thoroughly characterized and has been found to be free from significant corrections arising from a finite value of  $\rho_{xx}^{\text{min}}$  at the operating value of temperature (usually  $T_1$ ) and  $I_{DS}$ .

### 6.1. Fast check of the contacts

Three-terminal measurements between contact pairs, as described in section 4, which are not very time-consuming, can easily be made before each QHR measurement. The resolution of the ohmmeter should be 10  $\Omega$  or better. Special care should

be taken if a digital ohmmeter is used. These instruments typically have a very high input impedance ( $>10\text{ G}\Omega$ ) and considerable voltages may develop across the sense input terminals when left open. Thus, a connection of an open instrument to the QHE device may damage the contacts, especially at  $B = 0$ . As a consequence, a digital ohmmeter should be properly configured (meter on, four-wire ohms function and range selected) and its input short-circuited before connecting it to the device and finally removing the short circuit. The measuring current delivered by the ohmmeter for the selected measuring range should be tested beforehand and be low enough to avoid deteriorating the device and its contacts. From this point of view the use of an auto-ranging ohmmeter is not recommended. The source and drain contacts should be evaluated with a current equal to the effective measuring current  $I_{DS}$ . For the voltage contacts, the current should preferably be below  $10\text{ }\mu\text{A}$ . In the best circumstances all three-terminal resistances should be equal to the lead resistance to within the resolution of the ohmmeter.

### 6.2. Measurements of $\rho_{xx}^{\min}$ and $\rho_{xy}$

The resistivity  $\rho_{xx}^{\min}$  should be evaluated on both sides, along the full length of the device, using two opposite pairs of  $V_x$  contacts, for instance, 1–3 and 1'–3' (figure 1). The measurement should be made with a current equal to  $I_{DS}$  and for both polarities of the current. This checks that the characteristics of  $\rho_{xx}^{\min}$  in the area of the channel delimited by these pairs are reasonably homogeneous.

We propose two possible methods for the evaluation of  $\rho_{xx}^{\min}$ :

The first one is classical: a current is passed through the device and the residual potential differences between contacts 1–3 and contacts 1'–3' are evaluated using a nanovoltmeter.

The second method based on Kirchhoff's laws consists in carrying out four  $\rho_{xy}$  measurements using, for instance, the following four pairs of Hall voltage contacts: 1–1', 3–3', 1–3' and 3–1'. The residual longitudinal resistance between contacts 1 and 3 is then estimated as the difference between the  $\rho_{xy}$  values measured with pairs 1–1' and 3–1' (or with pairs 1–3' and 3–3') and that between contacts 1' and 3' as the difference corresponding to pairs 1–3' and 1–1' (or to pairs 3–3' and 3–1'). The closure of the four  $\rho_{xy}$  measurements along the loop formed by contacts 1, 3, 3' and 1' can be checked by verifying that the average of the measurements corresponding to pairs 1–1' and 3–3' is equal to that corresponding to pairs 1–3' and 3–1' to within the resolution of the measurements. The advantage of this second method is that all the time spent in the measurements contributes to the reduction of the final type A uncertainty associated with the QHR measurement: the average of the four  $\rho_{xy}$  measurements is taken as the final result. The method described here is for the loop formed by contacts 1, 3, 3' and 1', but it can be applied as well to the loop formed by contacts 1, 2, 2' and 1' or by contacts 2, 3, 3' and 2'.

When the first method is used, the  $\rho_{xx}$  and  $\rho_{xy}$  measurements are made separately. Here also the  $\rho_{xy}$  measurements should be made at least on two different pairs of facing Hall contacts. These two pairs should be either those delimiting the area formed by the contacts used for the  $\rho_{xx}$  measurements (here, for instance, 1–1' and 3–3'), or, where

applicable, a delimiting pair (1–1' or 3–3') and a central one (2–2'). However, it may be argued that to ensure unequivocally contact reliability, the two pairs of  $V_x$  contacts should involve the same pads as the two  $V_H$  pairs. Additionally, the voltages of the two  $V_x$  pairs and those of the two  $V_H$  pairs should sum to zero around the loop, to within the resolution of the measurements. Close agreement between the values of  $\rho_{xy}$  obtained with two different pairs is a confirmation that there is no significant problem arising from the contacts.

## 7. Consistency of QHR measurements for different devices and different quantum numbers

A last but essential criterion for judging a particular QHR measurement is its agreement with measurements made on other devices, preferably from different wafers, and with different quantum numbers.

Such comparisons test for shunting resistances (parallel conduction) across the device, especially those between source and drain that are not revealed by  $\rho_{xx}$  measurements. In the case of different quantum numbers, this is also an excellent test for leakage resistances or other imperfections in the measuring equipment. For example, a very good test is to compare QHR measurements as obtained from the  $i = 2$  and  $i = 4$  plateaux on the same device and during the same cooling cycle.

When plateaux with different quantum numbers are not attainable, there is still the possibility to put in series or parallel QHRs corresponding to a given quantum number in order to obtain multiple or sub-multiple quantized resistance values, useful for testing the measuring bridge for different nominal resistance ratios. This is possible when several devices are available from the same wafer, developing plateaux for the same nominal values of  $B$ . The technique to use is that of multiple series or parallel connections of QHE devices [23] that provide accurate multiple or sub-multiple QHR values. The connections to be realized can be made inside or outside the cryostat.

Recently, arrays consisting of a large number of QHE devices connected in series or parallel and integrated on a single chip have been developed [24, 25]. This technique can provide a broad range of quantized resistances, in particular resistances as low as  $100\text{ }\Omega$ , which should be very useful for testing the accuracy of resistance bridges.

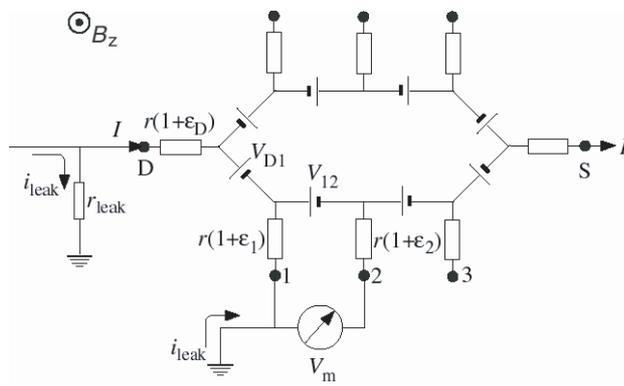
## 8. Comments on the measuring equipment

(1) All electronic apparatus used in the experiment should introduce a minimum amount of extraneous electrical noise to prevent possible noise rectification and damage to the device.

(2) All the components of the equipment used to measure the QHR, including the sample holder<sup>4</sup>, should have leakage resistances as high as possible.

For the accuracies that concern us here, minimum leakage resistances of  $5 \times 10^{12}\text{ }\Omega$  would normally be required. However, gated devices in which the gate voltage is much greater than the Hall voltage require a minimum leakage resistance of  $10^{14}\text{ }\Omega$  in the leads connected to the gate.

<sup>4</sup> Many laboratories mount devices in non-magnetic ceramic headers of the TO-8 type (diameter  $\approx 15\text{ mm}$ ) and use the mating sockets for easy connection to the device holder.



**Figure 2.** Equivalent circuit representation of a quantum Hall device and measurement configuration for the detection of leakage currents.

Other voltage sources (usually batteries) incorporated in the equipment and which may drive excessive leakage current should also have leakage resistances of the order of  $10^{14} \Omega$ . In all cases, it is a necessary precaution to check the leakage resistances or currents before each precise QHR measurement, as they may deteriorate with time.

A powerful method for checking leakage to ground in a measurement set-up is illustrated in figure 2. The quantum Hall device is represented by its equivalent circuit as described in [23]. Let us assume that a leakage resistance to ground is present in the circuit at the ‘high’ side of the device (terminal D). A leakage current  $i_{\text{leak}} \approx I_{\text{DS}} R_{\text{H}} / R_{\text{leak}}$  flows to ground at this point. If the longitudinal voltage along the device is measured in the quantum Hall regime as illustrated in figure 2, the leakage current flows back into the circuit through the grounded ‘low’ terminal of the detector (terminal 1). The measured voltage  $V_{\text{m}} = V_2 - V_1$  is given by

$$V_{\text{m}} = -r(1 + \varepsilon_1)(i_{\text{leak}} + i_{\text{D}}) - V_{12} - r(1 + \varepsilon_2)i_{\text{D}},$$

where  $r = R_{\text{H}}/2$  and  $\varepsilon_i$  is equal to the ratio of the resistance of contact  $i$  and the Hall resistance. The correction terms  $\varepsilon_1$  and  $\varepsilon_2$  are very small and are neglected in the following. The term  $i_{\text{D}}$  denotes the bias current of the detector and the voltage generator  $V_{12}$  has the value  $V_{12} = r i_{\text{leak}}$ . We thus have

$$V_{\text{m}} = -R_{\text{H}}(i_{\text{leak}} + i_{\text{D}}).$$

As the offset voltage due to the bias current of the detector can be eliminated by measuring in both polarities of  $I_{\text{DS}}$ , the measurement of the longitudinal voltage in this configuration yields the leakage current directly. If the detector is connected in reversed polarity (grounded ‘low’ terminal at contact 2), the leakage term is not seen by the detector, and we have  $V_{\text{m}} = R_{\text{H}}i_{\text{D}}$ .

(3) Guarding techniques limiting the effect of the leakage currents are recommended, but the guard circuits themselves should be carefully analysed and designed so that they introduce no extra perturbation.

(4) A number of laboratories use, as the first step of the scaling process from the QHR to  $1 \Omega$  standards, a potentiometric method whereby the QHR is placed in series with a standard resistance  $R$  nominally equal to the QHR or to  $10 \text{ k}\Omega$ . It is important to test the residual ‘interchange error’ of

such potentiometric systems: the ratio between the QHR and resistance  $R$  is measured for the two possible relative positions of the resistors in the series circuit.

If the two resistors (QHR and  $R$ ) in the series circuit do not have the same nominal values (for instance, if  $R = 10 \text{ k}\Omega$ ), the voltage drops across them are significantly different. Consequently, the linearity errors of the potentiometer used to measure them must be carefully checked. In particular, commercially available potentiometers that use a fixed internal resistor to generate the output voltage may introduce significant errors due to the power coefficient of this resistor.

The use of a seven-digit numerical voltmeter to evaluate the two voltage drops is a simple method, adequate for accuracies of the order of a few parts in  $10^7$ . A much more sophisticated method is the use of a Josephson potentiometer (see [9] for a review).

(5) When the lowest possible uncertainty (say a few parts in  $10^9$ ) is desirable, the use of a bridge based on a cryogenic current comparator (CCC, see [9] for a review) is recommended. Indeed, the problem is to measure a resistance ratio and bridge techniques, designed to measure ratios, are inherently superior to potentiometric methods for such an application. In particular, a bridge method is totally insensitive to drift of the measuring current. Also, the use of a bridge based on a CCC is very flexible in the sense that it allows accurate and direct measurements of various resistance ratios, including ratios corresponding to resistances of very different nominal values, such as, for instance, the ratio between the QHR and a conventional  $100 \Omega$  resistance standard.

(6) A particularity of bridges based on a CCC is the use of a superconducting quantum interference device (SQUID) as a magnetic flux detector. Such detectors are rather sensitive to electromagnetic interference, and there is a risk of noise rectification by the SQUID. This is usually minimized by careful shielding of the equipment and by appropriate filtering. When significant noise rectification is present, the result is an offset voltage at the bridge output. The offset is usually different for the two polarities of the measuring current and produces an error in the QHR measurements. This can be detected by recording the bridge output for the two polarities as well as in the absence of current. The output value in the absence of current should be equal to the average of the values for the two polarities. If this is not the case, there is probably some significant noise rectification.

(7) A final general comment concerns the concept of ‘dc resistance measurement’, which is, in fact, rather ill-defined. Low-value resistances (below  $100 \Omega$ ) may have some significant frequency dependence at very low frequency (say between  $0.01 \text{ Hz}$  and a few hertz) due to thermal effects such as the Peltier effect. High-value resistances (above  $10 \text{ k}\Omega$ ) may also have some residual very low frequency dependences arising from dielectric losses. It is important to mention an estimation of the effective ‘equivalent frequency’ of the so-called dc measurement. This can be estimated as the reciprocal of the period corresponding to the reversal of the measuring current.

Moreover, ‘dc measurements’ have a number of other disadvantages: influence of thermal voltages, possible noise rectification,  $1/f$  noise. It should be noted that all these effects can be reduced or suppressed if true low-frequency

ac measurements (say at frequencies from 0.1 Hz to a few hertz) are carried out instead of ‘dc measurements’. In the case of QHR measurements, it was demonstrated that the Hall resistance is still perfectly quantized (to better than 1 part in  $10^9$ ) at frequencies up to at least 1 Hz and that CCC bridges can be successfully operated with ac currents of frequency up to at least 1 Hz [26, 27]. Also, bridges using room temperature ac current comparators can be successfully used [28]. Thus, we encourage the laboratories to consider the possibility of carrying out low-frequency ac measurements instead of ‘dc measurements’.

## 9. Reporting results

In order to compare QHR values obtained in different laboratories, the assigned uncertainties must be estimated in a uniform manner. This should be done following the ‘*Guide to the Expression of Uncertainty in Measurement*’. In particular, a detailed and complete listing of the type A and type B uncertainties should be given, along with the measurement dates, the number of measurements made, and a clear statement as to the units or conventional value for the von Klitzing constant that are used to express the results. As noted above, special attention should be given to describing the characteristics of the devices used and the tests and procedures employed to address the possible sources of error discussed in these guidelines.

## References

- [1] Delahaye F 1989 *Metrologia* **26** 63–8
- [2] Delahaye F and Bournaud D 1991 *IEEE Trans. Instrum. Meas.* **40** 237–40
- [3] Hartland A, Jones K, Williams J M, Gallagher B L and Galloway T 1991 *Phys. Rev. Lett.* **66** 969–73
- [4] Jeckelmann B, Jeanneret B and Inglis D 1997 *Phys. Rev. B* **55** 13124–34
- [5] Pierz K and Schumacher B 1999 *IEEE Trans. Instrum. Meas.* **48** 293–5
- [6] Comparison BIPM.EM-K12, <http://www.bipm.org>, appendix B of the MRA
- [7] Hartland A 1992 *Metrologia* **29** 175–90
- [8] Witt T J 1998 *Rev. Sci. Instrum.* **69** 2823–43
- [9] Jeckelmann B and Jeanneret B 2001 *Rep. Prog. Phys.* **64** 1603–55
- [10] Jeckelmann B, Rüfenacht A, Jeanneret B, Overney F, Pierz K, von Campenhausen A and Hein G 2001 *IEEE Trans. Instrum. Meas.* **50** 218–22
- [11] Jeanneret B, Jeckelmann B, Bühlmann H-J and Ilegems M 1997 *IEEE Trans. Instrum. Meas.* **46** 285–8
- [12] Büttiker M 1988 *Phys. Rev. B* **38** 9375–89
- [13] Komiyama S and Hirai H 1989 *Phys. Rev. B* **40** 7767–75
- [14] Rikken G L J A, van Haaren A M N, van der Wel W, van Gelder A P, van Kempen H, Wyder P, André J P, Ploog K and Weimann G 1988 *Phys. Rev. B* **37** 6181–6
- [15] Jeckelmann B and Jeanneret B 1997 *IEEE Trans. Instrum. Meas.* **46** 276–80
- [16] Piquemal F, Genevès G, Delahaye F, André J P, Patillon J N and Frijlink P 1993 *IEEE Trans. Instrum. Meas.* **42** 264–8
- [17] Yoshihiro K, Kinoshita J, Inagaki K, Yamanouchi C, Endo T, Murayama Y, Koyanagi M, Yagi A, Wakabayashi J and Kawaji S 1986 *Phys. Rev. B* **33** 6874–96
- [18] Cage M E, Field B F, Dziuba R F, Girvin S M, Gossard A C and Tsui D C 1984 *Phys. Rev. B* **30** 2286–8
- [19] Delahaye F, Dominguez D, Alexandre F, André J P, Hirtz J P and Razeghi M 1986 *Metrologia* **22** 103–10
- [20] Furlan M 1998 *Phys. Rev. B* **57** 14818–28
- [21] van der Wel W, Harmans C J P M and Mooij J E 1988 *J. Phys. C: Solid State Phys.* **21** L171–5
- [22] Jeanneret B, Jeckelmann B, Bühlmann H J, Houdré R and Ilegems M 1995 *IEEE Trans. Instrum. Meas.* **44** 254–7
- [23] Delahaye F 1993 *J. Appl. Phys.* **73** 7914–20
- [24] Poirier W, Hayashi K, Fhima H, Piquemal F and André J P 2000 *CPEM2000 Conf. Digest* pp 517–18
- [25] Poirier W, Bounouh A, Piquemal F and André J P 2002 *CPEM2002 Conf. Digest*
- [26] Delahaye F 1991 *IEEE Trans. Instrum. Meas.* **40** 883–8
- [27] Seppä H, Satrapinski A, Varpula T and Saari J M 1995 *IEEE Trans. Instrum. Meas.* **44** 276–80
- [28] Delahaye F 1993 *IEEE Trans. Instrum. Meas.* **42** 287–91