Investigations on Extending the Frequency Range of PJVS based AC Voltage Calibrations by Coherent Subsampling

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***Abstract* — This paper describes a method to extend the frequency range of Programmable Josephson Voltage Standard based ac voltage calibrations. It requires coherent (synchronous) signal generation and sampling to a common reference time-base of a digitizer and accurate phase alignment of ac signals. Practical and physical limitations of the underlying principle are covered in the text. Experimental investigations corroborate its feasibility in the range of some hertz towards audio- and well up to the MHz-range.**

***Index Terms* — Adaptive signal processing, digital signal processing, electronic circuits, Josephson effect, measurement techniques, precision measurements.**

I. INTRODUCTION

Programmable Josephson Voltage Standards (PJVS) or Synthesizers are used in many National Metrology Institutes (NMIs) to ensure the traceability of low frequency ac signals (up to a few hundred hertz) [1] to the International System of Units. Present research in leading NMIs aims at the development of pulse-driven Josephson standards towards higher frequencies [2]-[3]. There is however, in the opinion of the authors, room for research in exploiting special methods of digital signal sampling and processing [4], which aided by hardware and software design, indeed allow extending the frequency range of PJVS based ac voltage calibrations.

The method here proposed uses coherent subsampling, with the advantage of keeping a PJVS working at low frequencies in its best operation range, as treated next.

II. THE UNDERLYING PRINCIPLE OF THE METHOD

The classical use of a PJVS considers the calibration of an ac signal v_{AC} of fundamental frequency f_{o} (dashed curve in Fig. 1), which is frequency- and phase-synchronized to the PJVS signal v_{PJVS} (generated with N plateaus) of fundamental frequency f_{J}, where $f_{\mathrm{J}} = f_{\mathrm{o}}$. The ac signal crosses the dc plateaus of the v_{PJVS} (e.g., at their middle point) on the points 0 to (N/2-1) for a half period, as shown in Fig. 1. The digital sampling at equally time-spaced intervals T_{S} of the differential signal v_{PJVS}-v_{AC} allows one to infer on v_{AC} to a very high accuracy and precision, since each PJVS plateau is known with fundamental (or quantum) accuracy. Now, let us use the same principle for a steady state sine waveform v_{AC} of frequency f_{n} (for $f_{\mathrm{n}} \geq f_{\mathrm{J}}$) that is finely adjusted so that within a sampling interval T_{S} a number $(n + 1/N)$ times its period T_{n} (equal to $1/f_{\mathrm{n}}$) elapses. The required fundamental frequency f_{n} that satisfies this condition can be expressed as

$$f_{\mathrm{n}} = f_{\mathrm{J}}\,(nN+1)\,, \tag{1}$$

where n is any positive integer equal to or greater than zero. Fig. 1 shows the case for n equal to 2, N is 16 (steps), v_{AC} is accurately phase aligned with v_{PJVS} by digital phase regulation [4], as is $f_{\mathrm{S}} = 1/T_{\mathrm{S}}$ predefined. Therefore, the first sample crossing the 0 V plateau (i.e., point no. 0) coincides with the zero point 0′of v_{AC}. The second sample (no. 1), although taken exactly at $T_{\mathrm{S}} = (2{+}1/16)T_{\mathrm{n}}$ seconds from point 0 apart, coincides numerically and thus exactly with the hypothetical sample 1′ of v_{AC} of frequency f_{n}. The same occurs for the remaining crossing points of v_{AC} (points 2 to 15 per period), although only a half period of v_{PJVS} is shown in Fig. 1. It is thus evident, that for a PJVS signal sampled at a sampling rate $f_{\mathrm{S}} = Nf_{\mathrm{J}}$ over its period T_{J} (for $T_{\mathrm{J}} = 1/f_{\mathrm{J}} = NT_{\mathrm{S}}$), a total number of N samples per period of the higher frequency v_{AC} will be harvested, as if it were sampled at a N-times higher sampling rate $f_{\mathrm{S}}' = Nf_{\mathrm{n}} = Nf_{\mathrm{J}}\,(n\mathrm{N}{+}1)$. In fact, the sampled v_{AC} (dashed waveform of $f_{\mathrm{o}} = f_{\mathrm{J}}$ in Fig. 1) may be viewed as being the "alias" of the higher frequency signal v_{AC}, of which fundamental is folded back at the frequency f_{J} of the PJVS. Hence, the sampled points may be determined by direct sampling of v_{AC} or by differential sampling of $v_{\mathrm{DIFF}} = v_{\mathrm{PJVS}} - v_{\mathrm{AC}}$, leading to the determination of v_{AC} as $v_{\mathrm{AC}} = v_{\mathrm{PJVS}} - v_{\mathrm{DIFF}}$ as usual [4].

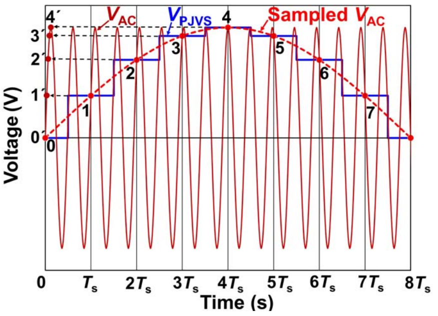

Fig. 1. Half of a cycle of a PJVS step-approximated sine waveform v_{PJVS} (blue) with 16 plateaus used to calibrate an ac signal v_{AC} (dark red) of a 33 times higher fundamental frequency.

III. METROLOGICAL CHALLENGES OF THE METHOD

Although there are no theoretical limits imposed on the outlined principle, practical limitations in its realization

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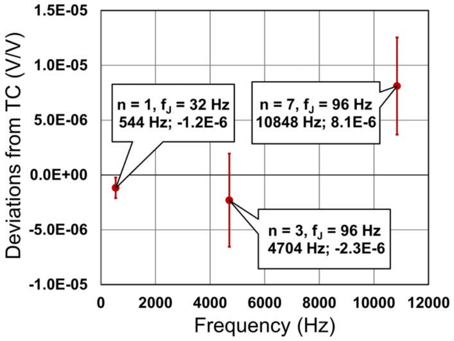

Fig. 2. Deviations of the method applied to three frequencies for 20 measurements of 2.5 V in magnitude with respect to ac-dc transfer measurements, and $N = 16$. The inserts show the values of n, f_J, the signal frequency and deviations (V/V). The error bars indicate type A measurement uncertainties for 95% confidence level ($k = 2$).

always do exist as ever and they can be enumerated here with a short comment on solutions: 1) The differential voltage attains levels equal to the sum of magnitudes of v_{PJVS} and v_{AC}, although the sampling occurs close to zero crossings. The digitizer must thus be operated at a higher voltage range, what worsens the resolution. To allow digitizer's lower ranges to be used, a special precision clipping amplifier with delays as low as 1 ns and bandwidth of nearly 600 MHz was developed for this purpose. 2) Higher signal slew rates, i.e., $d(v_{PJVS}-v_{AC})/dt$ degrade the performance of the input circuits of the digitizer. The clipping amplifier above helps to alleviate this problem. 3) Jitter effects of v_{AC} due to a higher f_n requires a large number of averages. 4) The digitizer's bandwidth must be characterized in amplitude and phase with care, and its effect must be numerically compensated, before calculating v_{AC} from differential sampled data. 5) Phase alignment of v_{AC} with v_{PJVS} must be attained very accurately. This was solved by modifying the adaptive synchronization algorithms of our system [4] to deal with different frequencies. 6) In some cases, either the f_n signals or the sampling rates f_S (or both) cannot be generated without numerical truncation. The use of (1) should thus be done with integer numbers only. In other cases, the cascading of direct digital synthesizers to attain higher frequency resolution may be required, what asks for special hardware construction. 7) Cabling and connections must be carefully chosen. Special shielding and guarding of relays are necessary as well as the proper definition of connection planes and impedance loading. 8) Interference and undesired coupling occur, e.g., charge injection through the input impedance/capacitance of the digitizer into the PJVS. To protect the PJVS, a sine waveform of frequency $f_0 = f_J$ is first calibrated against v_{PJVS}. After this calibration, v_{PJVS} is disconnected and substituted by this sine of frequency f_J, which is then compared with the signal of higher frequency v_{AC}. This "ac-ac transfer technique" [5] is used for f_n values above 600 Hz.

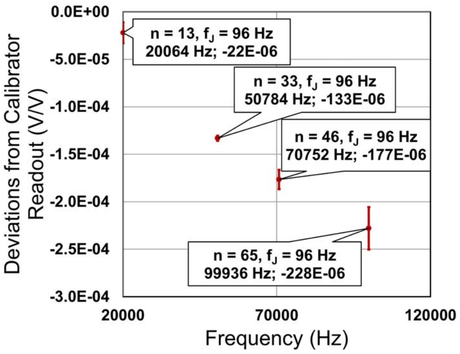

Fig. 3. Deviations of measurements as in Fig. 3 for higher frequencies with respect to the calibrator readout (see text).

IV. EXPERIMENTAL VERIFICATIONS

Fig. 2 shows preliminary data on the deviations of amplitude from magnitude determinations using the proposed method with respect to ac-dc transfer measurements with a thermal converter (TC). The digitizer employed was a Keysight digital voltmeter 3458A operating in direct sampling dc mode (DSDC) with an aperture of 2 ns and a 12 MHz bandwidth. Deviations above 10 kHz are attributed to our ac-dc transfer buffer amplifier, which needs further improvements. Above 20 kHz the calibrator readout, of a Fluke 5730A, was used as the reference as shown in Fig. 3. The values in Fig. 3 are thus shown for illustration purposes only, irrespective of their traceability.

VI. CONCLUSION

Although many investigations are in progress, especially those concerning type B uncertainties, the feasibility of the method was corroborated. The system of [4] is being improved to cover higher frequencies (up to 1 MHz). Additional details will be available at the conference.

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