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Confinement-Deconfinement Transition in 3-Dimensional QED

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Abstract

We argue that, at finite temperature, parity invariant non-compact electrodynamics with massive electrons in 2+1 dimensions can exist in both confined and deconfined phases. We show that an order parameter for the confinement-deconfinement phase transition is the Polyakov loop operator whose average measures the free energy of a test charge that is not an integral multiple of the electron charge. The effective field theory for the Polyakov loop operator is a 2-dimensional Euclidean scalar field theory with a global discrete symmetry Z, the additive group of the integers. We argue that the realization of this symmetry governs confinement and that the confinement-deconfinement phase transition is of Berezinskii-Kosterlitz-Thouless type. We compute the effective action to one-loop order and argue that when the electron mass m is much greater than the temperature T and dimensional coupling e^2, the effective field theory is the Sine-Gordon model. In this limit, we estimate the critical temperature, $T_{\mathrm{crit.}} = e^2/8\pi(1 - e^2/12\pi m + \ldots)$.

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Gauge field theories in 2+1-dimensions have many interesting field theoretical features such as gauge invariant local topological mass [1, 2], fractional spin [3] and statistics [4]. They have been of interest as model systems with somewhat more severe infrared divergences than their 3+1-dimensional relatives [1, 5] and where one can study the dynamics of chiral symmetry breaking [6, 7, 8]. Gauge field theories arise naturally in the description of lower dimensional statistical models such as spin systems and the Hubbard model [9] and an understanding of both their ground state and thermodynamic properties are essential to the applications of mean field theory there. In this Letter, we shall show that parity invariant 2+1-dimensional quantum electrodynamics exhibits one more interesting property - when the electron has a large enough mass - it has a finite temperature confinement-deconfinement phase transition.

The question of confinement has been extensively investigated in the more familiar context of quantized Yang-Mills theory at finite temperature and it is intimately related to the realization of a global symmetry involving the center of the gauge group [10, 11]. This symmetry transforms the Polyakov loop operator

$$P(\vec{x}) \equiv \mathrm{tr}\mathcal{P}\exp\left(\int_0^{1/T} d\tau A_0(\tau,\vec{x})\right) \tag{1}$$

which measures gauge group holonomy in the periodic Matsubara time in the Euclidean path integral description of finite temperature gauge theory.[2] Consider a gauge transformation

$$A_\mu{}'(\tau,\vec{x}) = g^{-1}(\tau,\vec{x})A_\mu(\tau,\vec{x})g(\tau,\vec{x}) + ig^{-1}(\tau,\vec{x})\nabla_\mu g(\tau,\vec{x}) \tag{2}$$

under the group $SU(N)$ which has center Z_N, the additive group of the integers modulo N. In Eq.(2) $g(\tau,\vec{x})$ can be periodic up to an element of the center of the group, $g(1/T,\vec{x}) = g(0,\vec{x})e^{2\pi in/N}$. Under a gauge transformation of the form (2),

$$P'(\vec{x}) = P(\vec{x})e^{2\pi in/N} \tag{3}$$

Therefore, if the Z_N symmetry is not spontaneously broken, the correlators of Polyakov loop operators $< P(\vec{x}_1)\ldots P(x_m)P^\dagger(\vec{y}_1)\ldots P^\dagger(\vec{y}_n) >$, are zero

[2] We use units where Planck's constant, the speed of light and Boltzmann's constant are one. For a discussion of the path integral formulation of finite-temperature gauge theory, see [12].

unless $m = n$ modulo N. The quantity

$$F(\vec{x}_1, \ldots, \vec{x}_m, \vec{y}_1, \ldots, \vec{y}_n) = -T \ln \left(< P(\vec{x}_1) \ldots P(x_m) P^\dagger(\vec{y}_1) \ldots P^\dagger(\vec{y}_n) > \right) \tag{4}$$

is the free energy of the gluodynamic system in the presence of an external fundamental representation quark sources at positions $\vec{x}_1, \ldots \vec{x}_m$ and anti-quark sources at $\vec{y}_1, \ldots, \vec{y}_n$. If the Z_N symmetry is unbroken, the expectation value of a single loop, $< P(\vec{x}) >$, vanishes and consequently the free energy $F(\vec{x})$ of a single quark source is infinite. This is a signal of confinement - introducing a colored source into the confining system requires infinite energy. On the other hand, if the Z_N symmetry is spontaneously broken, $F(\vec{x})$ is finite and characterizes the deconfined phase [13].

When dynamical quarks in the fundamental representation of the gauge group are present, the Polyakov loop operator does not characterize the confining phase, since, even if quarks are confined, screening can still take place. The coupling of gluons to quarks which are in the fundamental representation, is invariant only under strictly periodic gauge transformations and therefore breaks the Z_N symmetry explicitly. This is interpreted as the possibility of fundamental quarks screening the color of an external source, so that the free energy of the source is always finite.

In this Letter, we shall argue that the Polyakov loop operator can be used to study confinement in non-compact quantum electrodynamics even when dynamical electrons are present. In this case, the free energy of a distribution of external charges is

$$e^{-F(\vec{x}_i)/T} = \frac{\int dA_\mu d\psi d\bar{\psi} e^{-\int_0^{1/T} \left(\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}(\gamma\cdot(\nabla - ieA)+m)\psi\right)} e^{i\sum e_i \int_0^{1/T} d\tau A_0(\tau,\vec{x}_i)}}{\int dA_\mu d\psi d\bar{\psi} e^{-\int_0^{1/T} \left(\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}(\gamma\cdot(\nabla - ieA)+m)\psi\right)}} \tag{5}$$

with (anti-)periodic boundary conditions $A_\mu(1/T, \vec{x}) = A_\mu(0, \vec{x})$, $\psi(1/T, \vec{x}) = -\psi(0, \vec{x})$, $\bar{\psi}(1/T, \vec{x}) = -\bar{\psi}(0, \vec{x})$. The gauge transformation

$$A_\mu{}'(\tau, \vec{x}) = A_\mu(\tau, \vec{x}) + \nabla_\mu \chi(\tau, \vec{x}) \ , \ \psi'(\tau, \vec{x}) = e^{ie\chi(\tau,\vec{x})}\psi(\tau, \vec{x}) \ , \ \bar{\psi}'(\tau, \vec{x}) = \bar{\psi}(\tau, \vec{x}) e^{-ie\chi(\tau,\vec{x})} \tag{6}$$

is a symmetry of the action and measure if it preserves the (anti-)periodic boundary conditions, $\vec{\nabla}_\mu \chi(1/T, \vec{x}) = \nabla_\mu \chi(0, \vec{x})$ and $\chi(1/T, \vec{x}) = \chi(0, \vec{x}) + 2\pi n/e$. The coset of the group of all time-dependent gauge transformations modulo those which are periodic is the group Z, the additive group of the integers. The Abelian Polyakov loop operator transforms under this global symmetry as

$$\exp\left(i\sum_i e_i\int_0^{1/T} d\tau A_0{}'(\tau,\vec{x}_i)\right)=\exp\left(i\sum_i e_i\int_0^{1/T} d\tau A_0(\tau,\vec{x}_i)\right)\exp\left(\frac{2\pi i n}{e}\sum_i e_i\right) \tag{7}$$

Thus, if Z is not spontaneously broken, $F(\vec{x}_i)$ defined by (5) is infinite when the total charge of the external distribution is not an integral multiple of the electron charge, $\sum_i e_i \neq \text{integer}\cdot e$. When the symmetry is broken, $F(\vec{x}_i)$ can be finite. Thus, the nature of the realization of Z tests the ability of the electrodynamic system to screen charges which are not integral multiples of the electron charge and therefore probes the confining nature of the electromagnetic interaction.

Z is the analog of the global Z_N symmetry of gluo-dynamics. However, in contrast to the case of a compact non-Abelian gauge theory, where Z_N is explicitly broken by dynamical quarks, and, due to compactness of the gauge group, charges which are not integer multiples of the quark charge are not available, the non-compactness of the gauge group of QED allows the Z symmetry to exist even in the presence of dynamical electrons. The electrons could be viewed as the analog of adjoint particles, either gluons or adjoint quarks, in QCD.

At $T = 0$, and for the physical value of the electromagnetic coupling constant, 3+1-dimensional electrodynamics does not exhibit a confining phase. It is in the deconfined Coulomb phase at zero temperature and forms a Debye plasma at finite temperature and density. There is a conjecture that, if the electron charge is increased so that $e^2/4\pi \sim 1$, there is a phase transition to a chiral symmetry breaking and perhaps confining phase [14]. This phase, being in the strong coupling region, is difficult to analyze. In 1+1-dimensions, the massive Schwinger model is confining and the Z symmetry is not broken, at least when the temperature is much greater than the electron mass and the confinement scale is set by the dimensional electron charge e. On the other hand, it is known that when the electron mass is zero, the Z symmetry is spontaneously broken [15]. The symmetry breaking can be attributed to nonlocal effects of massless fermions. It can be argued that the phase transition between the broken Z and unbroken Z phases occurs for all temperatures at some value of the electron mass.

One might also expect a confinement-deconfinement transition in the intermediate case of $2+1$-dimensional electrodynamics. In that case, even at the classical level, the Coulomb potential is a marginally confining logarithm. Its entire spectrum is bound states, but the bound states can have arbitrarily large size. The free energy of a gas of charged particles is

$$F_{\rm cl.} = -\frac{1}{2}\sum_{i,j} e_i e_j \frac{1}{2\pi} \ln |\vec{x}_i - \vec{x}_j| \tag{8}$$

It is straightforward to compute the correlators of Polyakov loop operators (which are simply exponentials of the appropriate free energies, $e^{-F_{\rm cl.}/T}$). They have the scaling form

$$< \prod_i e^{ie_i \int_0^{1/T} d\tau A_0(\tau,\vec{x}_i)} >= \text{const.} \prod_{i<j} |\vec{x}_i - \vec{x}_j|^{e_i e_j/2\pi T} \tag{9}$$

with temperature dependent exponent, reminiscent of the spin-wave correlators in Gaussian spin wave theory in 2 dimensions [16].

It is interesting to ask how this result would be changed by radiative corrections and by thermal fluctuations. This can be done by computing the effective action for the Polyakov loop operator. In electrodynamics at finite temperature, it is possible to use a gauge transformation to set the temporal component of the gauge field, A_0, independent of the Euclidean time. Then, the effective 2-dimensional field theory for A_0 is obtained by integrating the other degrees of freedom from the path integral. What remains is an effective action for a static field $A_0(\vec{x})$. In general, it was shown in ref. [17] that the effective action for the Polyakov loop operator in D+1-dimensional gauge theory is a D-dimensional sigma model with group-valued fields. In the present case of electrodynamics, the group-valued variables are $e^{ieA_0/T}$ and the effective field theory for A_0 describes the appropriate sigma model. The Z symmetry is a periodicity of the effective action under the field translation $A_0(\vec{x}) \to A_0(\vec{x}) + 2\pi T/e$.

The effective action obtained from integrating propagating fields from the path integral is non-local and non-polynomial in the remaining fields. It can only be regarded as a local field theory when the momenta of interest are much smaller than the masses of the fields which have been eliminated. In that case the effective action has a local expansion in powers of derivatives divided by masses. The effective action for A_0 possesses such a local expansion. If the electron mass is sufficiently large that this expansion is accurate, the effective field theory for A_0 can be approximated by a local field theory.

In 2+1-dimensions, the fermion mass operator constructed from the minimal 2-component Dirac fermions is a pseudoscalar and therefore violates parity [1, 18]. If included in the action, they can generate a parity violating topological mass for the photon by radiative corrections [19, 20]. In this paper, we wish to study the case where the photon is massless. For this purpose, we study the model with Euclidean action

$$S = \int d^3x \left[\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}_1(\gamma \cdot (\nabla + ieA) + m)\psi_1 + \bar{\psi}_2(\gamma \cdot (\nabla + ieA) - m)\psi_2 \right] \tag{10}$$

Where the parity transformation is the spacetime parity as well as $\psi'_{1,2}(x') = \gamma_1 \psi_{2,1}(x)$, $\bar{\psi}'_{1,2}(x') = -\bar{\psi}_{2,1}(x)\gamma_1$ with $x' = (-x_1, x_2)$. The mass term $\bar{\psi}_1\psi_1 - \bar{\psi}_2\psi_2$ is a scalar. If $m = 0$ in (10) there is a 'chiral' symmetry under the transformation ${\psi_i}'(x) = u_{ij}\psi_j(x)$ where $u \in SU(2)$. The latter symmetry is broken at $T = 0$ [6, 7], however, since it is a continuous symmetry, it must be unbroken at any finite temperature in 2+1-dimensions. It has been argued that at finite temperature, the chiral transition is replaced by a Berezinskii-Kosterlitz-Thouless (BKT) transition [21]. In this paper, we shall consider the opposite limit of large mass, and show that there is indeed a BKT-transition corresponding to a confinement-deconfinement transition at some value of the temperature T. It is likely that this transition is in some way related to the chiral transition.

At finite temperature, 2+1-dimensional QED contains three parameters with the dimension of mass, the electron mass m, the gauge coupling e^2, and temperature T. The loop expansion is super-renormalizable [1, 5] and is an expansion in the dimensionless ratios e^2/m and e^2/T. We can compute the effective action for $A_0(\vec{x}) \equiv a(\vec{x})\sqrt{T}$ in a double expansion in the number of loops and in powers of derivatives of $a(\vec{x})$. To order 1-loop and up to quadratic order in derivatives the effective action is

$$S_{\text{eff}}[A_0] = \int d\vec{x} \left(Z(m, ea/\sqrt{T}) \frac{1}{2} \vec{\nabla} a \cdot \vec{\nabla} a - V(m, ea/\sqrt{T}) \right) \quad . \tag{11}$$

Here V is the effective potential for A_0 arising from the fermion determinant and Z is obtained from expansion of the temporal components of the vacuum polarization function to linear order in $-\vec{\nabla}^2$. To order one-loop, the effective potential is obtained from the fermion determinant in constant background

A_0,

$$V(m, eA_0/T) = \frac{1}{(\text{Vol.})} \log\det((-i\partial_0 - eA_0)^2 - \nabla^2 + m^2) \tag{12}$$

where the fermions have anti-periodic boundary conditions in the 0 direction. The determinant can be computed by considering the ratio [23]

$$\Delta(m, eA_0/T) = \det((-i\partial_0 - eA_0)^2 - \nabla^2 + m^2)/\det(-\partial_0^2 - \nabla^2 + m^2) \tag{13}$$

One finds

$$\Delta(m, eA_0/T) = \prod_{\vec{k}} \left[1 - \frac{\sin^2(eA_0/2T)}{\cosh^2(\lambda_k/2T)}\right] \equiv \prod_{\vec{k}} \Delta_{\vec{k}}(m, eA_0/T) \quad , \tag{14}$$

where $\lambda_k^2 = \vec{k}^2 + m^2$ are the positive eigenvalues of the operator $-\nabla^2 + m^2$. Eq.(14) holds in any dimensions. In 2+1-dimensions however one can perform the integral on $\vec{k}$ arising in $\log\Delta(m, eA_0/T)$, after taking the infinite volume limit.

$$\begin{aligned} V(m, eA_0/T) &= \frac{1}{(\text{Vol.})} \int_{-\infty}^{+\infty} \frac{d^2\vec{k}}{(2\pi)^2} \log\Delta_{\vec{k}}(m, eA_0/T) = \\ &= -\frac{T^2}{\pi}\left[\frac{m}{T} Li_2(e^{-m/T}, eA_0/T + \pi) + Li_3(e^{-m/T}, eA_0/T + \pi)\right] \end{aligned} \tag{15}$$

where $Li_2(r,\theta) = -\int_0^r dx \ln(1 - 2x\cos\theta + x^2)/2x$ and $Li_3(r,\theta) = \int_0^r dx Li_2(x,\theta)/x$ are the real parts of the dilogarithm and trilogarithm according to the convention of Ref. [24]. Eq.(15) shows the periodicity of the effective potential for $eA_0/T \to eA_0/T + 2\pi$. This is the residual gauge invariance. In 1 and 3 dimensions the integral in $\vec{k}$ can only be performed for $m = 0$ in which case it gives simple polynomial expressions. In the limit $m = 0$, the effective potential for A_0 has been discussed in [22].

It is also straightforward to compute the term which contributes the leading order in derivatives to the effective action,

$$Z(m, ea/\sqrt{T}) = \frac{e^2}{12\pi m}\left(\frac{\sinh m/T}{\cosh m/T + \cos ea/\sqrt{T}} - \frac{m}{T}\frac{e^{-m/T}}{\left(\cosh m/T + \cos ea/\sqrt{T}\right)^2}\right) \tag{16}$$

The critical behavior of the 2-dimensional model defined by Eqs.(11), (15) and (16), can be understood by comparing it with the sine-Gordon model in two-dimensions. That this comparison can be reliably performed can be seen by the study of the harmonic content of (15).

$$V(m, ea/\sqrt{T}) = -\frac{T^2}{\pi}\sum_{n=1}^{\infty} e^{-nm/T}\left(1+\frac{nm}{T}\right)\cos(n(ea/\sqrt{T}+\pi)) \quad . \tag{17}$$

Consider then the large m limit, T/m and e^2/m small with finite e^2/T. In this limit, the higher harmonics are small perturbations to the potential

$$V(m, ea/\sqrt{T}) = \frac{Tm}{\pi}e^{-m/T}\cos(ea/\sqrt{T}) \quad , \tag{18}$$

which is the sine-Gordon potential. Amit et al. [25] showed that in the sine-Gordon model any perturbation of the type $\cos(n\beta\phi)$ to a sine-Gordon potential $\alpha\cos(\beta\phi)/\beta^2$ are irrelevant for the critical behavior of the model. By analogy with the spin wave plus Coulomb gas model, it was also proven in Refs. [25] that a critical line for a BKT [26, 27] phase transition in the sine-Gordon model with a logarithmic potential starts at the point $(\alpha, \beta^2) = (0, 8\pi)$. We can then conclude that also in 2+1-QED at finite temperature there is a BKT phase transition, with a critical line in the $(m/T, e^2/T)$ plane starting at $(m/T, e^2/T) = (\infty, 8\pi)$. The critical temperature for this transition (up to 1-loop order) can be computed from Eq.(16) and Eq.(18) as

$$T_{\text{crit.}} = \frac{e^2}{8\pi}\left(1 - \frac{e^2}{12\pi m} + \ldots\right) \quad . \tag{19}$$

This is the critical value of the coupling constant originally found by Coleman in his discussion of bosonization of the massive Thirring model [28].

Note that the vacuum expectation value of A_0 in the deconfined phase, where the Z symmetry is spontaneously broken, is

$$< A_0 >= \frac{2\pi nT}{e} \quad . \tag{20}$$

In a semiclassical analysis, this expectation value contributes an imaginary chemical potential for the electron action. However, this chemical potential can be absorbed by shifting the Matsubara frequency. Thus, the semiclassical thermodynamics do not suffer from the difficulties of the meta-stable Z_N phases of QCD [30, 31].

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