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The Chern-Simons Coefficient in the Higgs Phase

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ABSTRACT

We study one-loop corrections to the Chern-Simons coefficient κ in abelian self-dual Chern-Simons Higgs systems and their $N = 2$ and $N = 3$ supersymmetric generalizations in both symmetric and asymmetric phases. One-loop corrections to the Chern-Simons coefficient of these systems turn out to be integer multiples of $1/4\pi$ in both phases. Especially in the maximally supersymmetric $N = 3$ case, the correction in symmetric phase vanishes and that in asymmetric phase is $\kappa/(2\pi|\kappa|)$. Our results suggest that nonabelian self-dual systems might enjoy similar features. We also discuss various issues arising from our results.

We consider one-loop correctons to the Chern-Simons coefficient in abelian Chern-Simons Higgs systems. In broken phase of these systems there are topological vortices of fractional spin and statistics. The quantum correction to the Chern-Simons coefficient could lead to a change in vortex dynamics given in the tree approximation. If there are charged particles in broken phase, there would be a nontrivial Aharonov-Bohm-type interaction between vortices and charged particles, which could also get a quantum correction. Thus, to find the quantum correction to the Chern-Simons coefficient would be interesting as a step to understand quantum physics in broken phase.

There have been recently some controversies related to the nature of the correction to the Chern-Simons coefficient. In symmetric phase of abelian systems, there is a theorem that the correction originates from the fermion one-loop and no more [1]. When there are massless charged particles or when the gauge symmetry is spontaneously broken, the theorem breaks down and the loop corrections turn out to be in general complicated functions of couplings and particle masses [2,3]. When the gauge symmetry is nonabelian, the coefficient should be quantized to have a consistent quantum theory[4]. While this is indeed the case in the symmetric phase [5], the quantization is not respected by one-loop correction in asymmetric phase [6]. However, there has been no clear understanding of this puzzle.

In this paper, we calculate one-loop correction to the Chern-Simons coefficient in abelian self-dual Chern-Simons Higgs systems and their $N = 2, N = 3$ supersymmetric generalizations [7,8,9] in both symmetric and asymmetric phases. The corrections in these systems turn out to be 'quantized' in the sense that they are integer multiples of the Dirac fermion correction, $1/4\pi$. This suggests immediately an interesting possibility that the one-loop corrections in nonabelian self-dual systems [10] and their supersymmetric systems [8,9] would be also quantized. While the quantum consistency condition of nonabelian systems does not imply the quantization of the Chern-Simons coefficient in asymmtric phase, we will argue that our result is no coincidence and that there may be other reasons why the correction should be quantized. These points and other aspects arising from our calculation would be discussed at the end.

We first start with a simple model of a gauge field A_μ, a complex Higgs field

ϕ, and a complex fermion field ψ. The most general symmetric lagrangian which is renormalizable is given as

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4e^2}F_{\mu\nu}^2 + \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + |D_\mu\phi|^2 + i\bar{\psi}\gamma^\mu D_\mu\psi \\ & - U(|\phi|) - (M + 2g_1|\phi|^2)\bar{\psi}\psi - g_2[\phi^2\bar{\psi}\psi^* + \phi^{*2}\bar{\psi}^*\psi]\end{aligned} \tag{1}$$

where $D_\mu = \partial_\mu + iA_\mu$ and all coupling constants are real. (An additional possible term $g'i[\phi^2\bar{\psi}\psi^* - \phi^{*2}\bar{\psi}^*\psi]$ can be transformed to the last term in Eq.(1) by a suitable global phase rotation on ψ.) For this theory to be renormalizable, a simple dimensional counting shows that $U(|\phi|)$ is at most sixth order in ϕ. The metric is given by $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1,)$ and $\epsilon^{012} = \epsilon_{012} = 1$. The gamma matrices are pure imaginary, $\gamma^\mu = (\tau^2, i\tau^3, i\tau^1)$, and satisfy $\gamma^\mu\gamma^\nu = \eta^{\mu\nu} - i\epsilon^{\mu\nu\rho}\gamma_\rho$.

We are interested in calculating one-loop correction to the photon propagator in a constant background scalar field. We rewrite the scalar and spinor fields in real and Majorana fields, $\phi = (\phi_R + i\phi_I)/\sqrt{2}$ and $\psi = (\psi_R + i\psi_I)/\sqrt{2}$. We separate the scalar fields ϕ_a into the constant background field φ_a and the quantum fluctuations ϕ_a. The background lagrangian is then given by

$$\mathcal{L}_B = \mathcal{L}(A_\mu, \varphi_a + \phi_a, \psi) - \mathcal{L}(0, \varphi_a, 0) - \phi_a\frac{\partial\mathcal{L}}{\partial\phi_a}(0, \varphi_a, 0) \tag{2}$$

In addition, we use the R_ξ gauge fixing,

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}(\partial^\mu A_\mu + \xi\epsilon_{ab}\phi_a\varphi_b))^2 \tag{3}$$

which leads to the Fadeev-Popov ghost lagrangian

$$\mathcal{L}_{FP} = \bar{\eta}(-\partial_\mu^2 - \xi(\varphi_a^2 + \varphi_a\phi_a))\eta \tag{4}$$

The ghost loop contributes to the photon propagator in one-loop via the ϕ_a field tadpole diagram, but no correction to the Chern-Simons coefficient.

Putting the lagrangian (2) and the gauge fixing terms (3) and (4) together, we get the quadratic term

$$\begin{aligned}\mathcal{L}_0 \ &= \frac{1}{2}A^\mu \left((\frac{1}{e^2}\partial_\rho^2 + \varphi_a^2)\eta_{\mu\nu} - (\frac{1}{e^2} - \frac{1}{\xi})\partial_\mu\partial_\nu - \kappa\epsilon_{\mu\nu\rho}\partial^\rho \right) A^\nu \\ &+ \frac{1}{2}\phi_a \left(-\partial_\mu^2\delta_{ab} - m_1^2\hat{\varphi}_a\hat{\varphi}_b - (m_2^2 + \xi\varphi_c^2)(\delta_{ab} - \hat{\varphi}_a\hat{\varphi}_b) \right) \phi_b \\ &+ \frac{1}{2}\bar{\psi}_a \left(i\gamma^\mu\partial_\mu\delta_{ab} - M_1\hat{\varphi}_a\hat{\varphi}_b - M_2(\delta_{ab} - \hat{\varphi}_a\hat{\varphi}_b) \right) \psi_b \\ &+ \bar{\eta}(-\partial_\mu^2 - \xi\hat{\varphi}_a^2)\eta \end{aligned} \tag{5}$$

where the terms quadratic in ϕ_a of $U(\sqrt{(\varphi_a + \phi_a)^2})$ is $m_1^2\hat{\varphi}_a\hat{\varphi}_b\phi_a\phi_b/2 + m_2^2(\delta_{ab} - \hat{\varphi}_a\hat{\varphi}_b)\phi_a\phi_b/2$, and we have denoted $M_1 \equiv M + g_1\hat{\varphi}_a^2$, and $M_2 \equiv g_2\hat{\varphi}_a^2$. The interaction parts involving the gauge field become

$$\begin{aligned}\mathcal{L}_I = \varphi_a\phi_a A_\mu^2 - \frac{1}{2}\phi_a^2 A_\mu^2 - \frac{i}{2}A_\mu\epsilon_{ab}\bar{\psi}_a\gamma^\mu\psi_b \\ - \epsilon_{ab}\phi_b\partial^\mu\phi_a A_\mu - \xi\varphi_a\phi_a\bar{\eta}\eta\end{aligned} \tag{6}$$

From the quadratic terms (6) it is straightforward to get the propagators for the gauge, scalar and fermion fields, $\Delta_{\mu\nu}(p), D_{ab}(p), S_{ab}(p)$. Here we just display the propagator for the gauge field,

$$\begin{aligned}\Delta_{\mu\nu}(p) \ &= \frac{-i(p^2/e^2 - \varphi_a^2)}{(p^2/e^2 - \varphi_a^2)^2 - \kappa^2p^2}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \\ &+ \frac{\kappa\epsilon_{\mu\nu\rho}p^\rho}{(p^2/e^2 - \varphi_a^2)^2 - \kappa^2p^2} + \frac{-i\xi p_\mu p_\nu}{p^2(p^2/e^2 - \xi\varphi_a^2)}\end{aligned} \tag{7}$$

By the Lorentz invariance and gauge invariance, the full inverse photon propagator may be expressed as

$$\begin{aligned}i\Delta_{\mu\nu}^{-1}(p) = (p^2g_{\mu\nu} - p_\mu p_\nu)\Pi_1(p^2) + i\epsilon_{\mu\nu\rho}p^\rho\Pi_2(p^2) \\ + p_\mu p_\nu\Pi_3(p)\end{aligned} \tag{8}$$

The one-loop graphs would be linearly divergent, but become finite after the Pauli-Villar regularization. With the convention $D_\mu = \partial_\mu + iA_\mu$, there is no wave function renormalization of the gauge field. Among many Feynman graphs contributing to the photon self-energy, there are two graphs for possible corrections to the Chern-Simons term Π_2: a mixed loop of the ϕ and gauge fields and a fermion loop. The Chern-Simons coefficient appears in the full photon propagators as

$$\Pi_2(p^2) = \kappa + \Pi^B(p^2) + \Pi^F(p^2) \tag{9}$$

Here we have chosen our regularization so that $\Pi_2(p^2 \to -\infty) = \kappa$; at short distance, the correction would vanish. The one-loop corrected Chern-Simons coefficient would be then $\Pi_2(p = 0)$.

If we try to understand the Chern-Simons coefficient in an effective action, there would be many gauge invariant terms in the effective action which could appear as a Chern-Simons term at large distance in *asymmetric* phase. One of such terms is $\epsilon^{\mu\nu\rho} i(D_\mu\phi^*\phi - \phi^* D_\mu\phi)F_{\nu\rho}$ with an appropriate coupling constant. Similar term for fermions describes the interaction between an anomalous magnetic moment and the magnetic field. In asymmetric phase, this term would contribute to the Chern-Simons coefficient. The bosonic correction to the Chern-Simons coefficient can be seen as the result of many such gauge-invariant terms in the effective action. Since these terms are invariant also under large gauge transformations, there seems no need for such bosonic corrections to the Chern-Simons term should be quantized in general. (Similar point of view appeared in the first paper of Ref.[6].) However, we will later argue that there may be other reasons why the sum of these bosonic corrections should be quantized.

From the interaction terms (6), we get the mixed bosonic 1-loop contribution to the photon self-energy

$$i\pi^B_{\mu\nu}(p) = 4i^2\varphi_a\varphi_b \int \frac{d^3q}{(2\pi)^3}\Delta_{\mu\nu}(q)D_{ba}(q-p) \tag{10}$$

In pure Chern-Simons limit ($e^2 \to \infty$) the above bosonic correction leads to

$$\Pi^B(p^2) = \frac{\mu}{4\pi}\int_0^1 d\alpha \frac{1}{\sqrt{(1-\alpha)\mu^2 + \alpha m_1^2 - \alpha(1-\alpha)p^2}} \tag{11}$$

where $\mu \equiv \varphi_a^2/\kappa$. Taking the zero momentum limit, we get the bosonic correction to the Chern-Simons coefficient,

$$\Pi^B(0) = \frac{1}{4\pi}\left\{\frac{4\mu(2|\mu| + |m_1|)}{3(|\mu| + |m_1|)^2}\right\} \tag{12}$$

In the limit where $\kappa \to 0$ or $\mu \to \infty$, it becomes $2\kappa/(3|\kappa|) \times 1/4\pi$, which is an old result [3]. When two masses μ, m_1 become identical, $\Pi^B(p^2 = 0) = \kappa/(4\pi|\kappa|)$. Such coincidence of the two masses is realized in abelian self-dual Chern-Simons Higgs systems[7] and so we have just shown that these self-dual systems in asymmetric phase would acquire a quantized one-loop correction to the Chern-Simons coefficient.

From the interaction terms (6), it is also straightforward to get the fermionic one-loop contribution

$$i\pi^F_{\mu\nu}(p) = -\epsilon_{ab}\epsilon_{cd}\int \frac{d^3q}{(2\pi)^3}\,\mathrm{tr}\gamma_\mu S_{bc}(q)\gamma_\nu S_{da}(q-p) \tag{13}$$

which leads to the following correction to Π_2:

$$\Pi^F(p^2) = -\frac{1}{4\pi}\int_0^1 d\alpha \frac{M_1\alpha + M_2(1-\alpha)}{\sqrt{M_1^2\alpha + M_2^2(1-\alpha) - p^2\alpha(1-\alpha)}} \tag{14}$$

Taking the zero momentum limit, we see that the fermionic one-loop correction to the coefficient is given as

$$\Pi^F(0) = -\frac{1}{6\pi}\left\{\frac{|M_1|(M_1 + 2M_2) + |M_2|(2M_1 + M_2)}{(|M_1| + |M_2|)^2}\right\} \tag{15}$$

We will later be interested in two special cases. When $M_1 = M_2$, we have a Dirac fermion of mass M_1 and the correction becomes $-M_1/(4\pi|M_1|)$. When $M_1 = -M_2$ on the other hand, our fermion system consists of two Majorana fermions of spin $1/2$ and $-1/2$ and then the correction (15) vanishes.

Let us now apply the above results to an $N = 3$ self-dual Chern-Simons Higgs system[9]. Once we know the correction in the $N = 3$ theory, we will see that it is trivial to read off the correction for systems with lesser supersymmetry. The lagrangian for the $N=3$ theory is given by

$$
\begin{aligned}
\mathcal{L} &= \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + |D_\mu\phi_a|^2 + i\bar{\psi}\gamma\cdot D\psi + i\bar{\chi}\gamma\cdot D\chi \\
&- \frac{1}{\kappa^2}|\phi_a|^2[(|\phi_a|^2)^2 + 2v^2(|\phi_1|^2 - |\phi_2|^2) + v^4] \\
&- \frac{v^2}{\kappa}(\bar{\psi}\psi - \bar{\chi}\chi) + \frac{3|\phi_a|^2}{\kappa}(\bar{\psi}\psi + \bar{\chi}\chi) \\
&- \frac{4}{\kappa}(\phi_1\bar{\psi} - \phi_2\bar{\chi})(\phi_1^*\psi - \phi_2^*\chi) \\
&- \frac{1}{\kappa}(\phi_1\bar{\psi} - \phi_2\bar{\chi})(\phi_1\psi^* - \phi_2\chi^*) \\
&- \frac{1}{\kappa}(\phi_1^*\bar{\psi}^* - \phi_2^*\bar{\chi}^*)(\phi_1^*\psi - \phi_2^*\chi)
\end{aligned} \tag{16}
$$

where $D_\mu = \partial_\mu + iA_\mu$. This lagrangian is invariant under an abelian gauge symmetry and an additional global symmetry corresponding to a uniform phase rotation of $\phi_1, \phi_2^*, \psi, \chi^*$. The central charge of the supersymmetric algebra is proportional to the magnetic flux. This theory is obtained by maximally supersymmetrizing the abelian self-dual Chern-Simons Higgs systems. There are degenerate symmetric and asymmetric vacua, and the self-dual configurations are topological vortices in asymmetric phase and nontopological q-balls in symmetric phase [7].

In symmetric phase, there are no particle degrees of freedom for the gauge field. Two fermions are of the opposite mass $\pm v^2/\kappa$ and also of the opposite spin. Scalar bosons carry mass v^2/κ. The matter fields form a reduced supermultiplet $(2,4,2)$ of spin $(1/2, 0, -1/2)$, saturating the energy bound given by the central charge. Explicit one-loop calculation is rather straightforward. In addition, our theory is finite at one-loop and needs no regularization. Since the two Dirac fermions ψ, χ come with the opposite mass, there is no correction to the Chern-Simons term. There is a nontrivial generation of the Maxwell term,

$$
i\pi_{\mu\nu} = \frac{i}{\pi p^2}\left\{\frac{|m|}{2} + \frac{m^2}{4p}\ln\left(\frac{|m|-p}{|m|+p}\right)\right\}(p^2\eta_{\mu\nu} - p_\mu p_\nu) \tag{17}
$$

where $m \equiv 2v^2/\kappa$. Naively, this term leads to a Maxwell term when $p \to 0$, leading to a dynamical generation of photons. However, the photon mass is of order $|m|\kappa$ which lies far above the branch point $p = |m|$ for the small coupling constant $\kappa >> 1$. Hence, there is no dynamical generation of photons at least in the perturbative regime. Calculating one-loop correction to mass of various fields is straightforward and follows the standard procedure. In symmetric phase, all one-loop corrections to masses of elementary particles turn out to vanish.

In asymmetric phase, the calculation is more delicate. Introducing the new variable $\phi_2 = v + (f + ig)/\sqrt{2}$, the free lagrangian in the broken phase becomes

$$
\begin{aligned}
\mathcal{L}_1 = & \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + |\partial_\mu\phi_1|^2 + \frac{1}{2}(\partial_\mu f)^2 + \frac{1}{2}(\partial_\mu g)^2 \\
& + i\bar{\psi}\gamma\cdot\partial_\mu\psi + \frac{i}{2}[\bar{\chi}_R\gamma\cdot\chi_R + \bar{\chi}_I\gamma\cdot\partial\chi_I] \\
& + \frac{\kappa m}{2}A_\mu^2 - m^2|\phi_1|^2 - \frac{1}{2}m^2 f^2 + m\bar{\psi}\psi \\
& - \frac{m}{2}(\bar{\chi}_R\chi_R - \bar{\chi}_I\chi_I) + \sqrt{2}v\partial^\mu g A_\mu
\end{aligned}
\tag{18}
$$

We need a gauge fixing term (3) and the corresponding ghost lagrangian (4) with $\varphi_a = \sqrt{2}v\delta_{a0}$. The gauge field absorbs the Goldstone boson to become a massive vector boson. The particles form a supermultiplet $(1, 3, 3, 1)$ of spin $(1, 1/2, 0, -1/2)$.

Now it is straightforward to calculate the one-loop correction to the Chern-Simons coefficient. We may in fact use our earlier results (12) and (15). First we note that there is no correction from $\chi_{R,I}$ because their masses are opposite to each other in the sign. The gauge boson and the Higgs field have the same mass, i.e., $\mu = m, m_1 = m$, and so they yield the correction, $\kappa/(4\pi|\kappa|)$. For the ψ fermions, the parameters $M_{1,2}$ becomes $-m$ and its contribution to the Chern-Simons coefficient is $m/(4\pi|m|)$. Thus, the bosonic and fermionic contributions come with the *same* sign. One loop corrected Chern-Simons coefficient (9) is then

$$
\Pi_2(p = 0) = \kappa + \frac{1}{2\pi}\frac{\kappa}{|\kappa|}
\tag{19}
$$

This is the main result of this letter. Note that the correction is quantized and has the same sign as the classical term. Any higher order correction, if exists, would be of order $1/\kappa$ and spoil the quantization feature in the small coupling limit $\kappa >> 1$.

If there are many coupling constants (as in non-supersymmetric or non-self-dual theories), the loop expansion would be more complicated.

The $N=2$ supersymmetric systems [8] can be obtained from Eq.(16) by dropping the terms depending on ϕ_1, χ. Thus, there would be a nonzero correction $-\kappa/(4\pi|\kappa|)$ in the symmetric phase and the correction (19) in the asymmetric phase.

Since our result depends crucially on the sign of the terms in the lagrangian (16) and the sign of the results (12) and (15), we have performed an independent check to confirm the result (19). As the Chern-Simons coefficient gets a nonzero one-loop correction (19) in asymmetric phase, there would be also a correction to the mass of massive vector bosons. If the supersymmetry is preserved to one-loop, the mass corrections to other fields would be identical to that of massive gauge bosons. This turns out to be indeed the case.

Since the calculation is rather straightforward and the method is in standard textbooks, we simply summerize our findings. First, we calculated the correction to the vacuum expectation value both by the Feynman diagram method and by the effective potential method and obtained the identical result, $\langle f \rangle = -\sqrt{2}(m - \sqrt{2\xi v^2})/(16\pi v)$. This correction is again finite because of the $N=3$ supersymmetry. (For the case with $N=2$, it is infinite and needs a renormalization [11].) The vacuum energy degeneracy between symmetric and asymmetric phases is preserved in one-loop. Second, we calculated the correction to the propagators of various fields in asymmetric phase. Mass of an elementary particle would appear as the pole of each propagator and is gauge invariant and independent of the gauge fixing parameter ξ. The corrections to the propagators assume particularly simple forms for the choice $2\xi v^2 = m^2$. For example, the photon propagator correction in asymmetric phase would be

$$\pi_{\mu\nu} = \left\{ -\eta_{\mu\nu} p^2 + p_\mu p_\nu + 2im\epsilon_{\mu\nu\rho} p^\rho \right\} \frac{1}{4\pi p} \ln \frac{2|m| + p}{2|m| - p} \tag{20}$$

From Eq.(20) it is easy to confirm the result (19). We can also calculate the mass shift from the above expression (20). The mass of the vector boson gets shifted from m to $m + \Delta m$ with $\Delta m = -3m(\ln 3)/(4\pi\kappa)$. We have found the identical mass shift for other particles in asymmetric phase.

In this letter we have calculated one-loop correction to the Chern-Simons coefficient in self-dual Chern-Simons Higgs systems and their $N=2, N=3$ supersymmetric generalizations in symmetric and asymmetric phases. These corrections turn out to be "quantized" in the sense that they are integer times the Dirac fermion contribution $1/4\pi$. While it is not clear whether this feature will be preserved in higher loops, our result suggests various interesting possibilities. Some self-dual models might have higher order correction of order $1/\kappa$ which might spoil this 'quantization' feature. Some supersymmetric models, especially the $N=3$ case, might have no higher order corrections. To support or refute these speculations need a further effort. The Maxwell kinetic term for the gauge field complicates our result and we hope to explore this theory in near future.

Our results suggest that one-loop correction to the Chern-Simons coefficient in nonabelian self-dual Chern-Simons Higgs systems or their supersymmetric generalizations might be quantized. Again, there may be no higher loop correction in some of these nonabelian models. If we break supersymmetry or self-duality, the one-loop correction would not in general be quantized. There seem to be two possibilities in such cases. The first possibility is that the corrected Chern-Simons coefficient might again be quantized if we include all higher loop corrections and possible nonperturbative corrections. The second possibility is that the fully corrected Chern-Simons coefficient is not quantized, with a possible theoretical difficulty discussed below.

As argued before, the bososnic contribution to the Chern-Simons coefficient originates from the parity violating gauge-invariant terms in the effective action. Hence the bosonic correction does not need to be quantized to be consistent with the global gauge transformations. On the other hand, there may be another reason why one demands the correction to the Chern-Simons coefficient to be quantized. First, we can consider an abelian system. If the Chern-Simons coefficient is a rational number, we can put vortices on a large sphere, being consistent with quantized magnetic flux, charge and angular momentum. However, a nonquantized correction to the Chern-Simons coefficient could make the system quantum mechanically inconsistent one. Second, in nonabelian systems the flux and charge quantization is necessary to get a finite representation of a braid group. Such feature would be spoiled if the quantum correction is not quantized appropriately.

Finally there are some interesting implications we like to point out. First, the one-loop correction in asymmetric phase is nonzero and quantized. This would lead to the corrections to the spin-statistics of vortices and to the interaction between vortices and charged particles in broken phase. Second, there is a diffecence between corrections in symmetric and asymmetric phases. If we break the degeneracy between symmetric and asymmetric vacua or if we think about the theory on a large sphere, there could be tunneling between two vacua. The possible tunneling does not seem consistent naively with the fact that the corrected Chern-Simons coefficient is different between two phases.

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