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SU(9) grand unified theory

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Frampton's SU(9) model is considered in detail as a grand unified theory with SU(4) horizontal symmetry. We find a correlation among neutrino, horizontal-gauge-boson, and new-fermion masses. With neutrino mass around 10 eV, the horizontal gauge boson is estimated to be as heavy as 6×10^{10} GeV. The theory also contains new charged-current processes, which are $B+L$ conserving and $\Delta L = 2$.

I. INTRODUCTION

The grand unified theories (GUT's) have been proposed to unify strong, weak, and electromagnetic interactions. Some of the models[1] not only retain the phenomenologically successful features of $SU(3)\times SU(2)\times U(1)$ gauge theory but also predict very interesting phenomena such as proton decay and baryon-number asymmetry in the universe. The present form of GUT's would, however, be incomplete in the sense that gravitational interaction is not unified and that the fact of light family replication cannot be explained. The aim of this paper is to introduce a local horizontal symmetry among families and unify it with a conventional GUT and study its consequences. Although there is as yet no clear indication of the need of a local horizontal symmetry, we propose it as one way of understanding the family structure. Then the first question to be asked is what is the right horizontal symmetry.[2] It was once assumed to be $SU^H(3)$ (Ref. 3) (H denotes "horizontal") the motivation for which was to incorporate only three light families into the fundamental representation. On the other hand, if we believe in the sequence of successful physical gauge symmetry

$$U(1)\subset SU^W(2)\subset SU^c(3)\subset \cdots,$$

$SU^H(4)$ may be the next symmetry to be exploited. By adding $SU^H(4)$ on top of SU(5) we arrive at SU(9) GUT. In this paper we study an SU(9) GUT proposed by Frampton.[4] The main purpose of our analysis has been to present the general features of GUT's with a horizontal symmetry by taking explicit examples of GUT's. By comparing SU(8) Ref. (3) and SU(9) models one will see that most of the features are shared by both models but they are very different as far as the fermion-mass spectrum is concerned. We pay particular attention to the fermion mass since it could serve the purpose of eliminating some candidates for the GUT. In fact, it is pointed out[3] that the SU(8) vectorlike model is unlikely to survive unless a novel way of producing mass is found. The fermion mass is, in this paper, assumed to arise in the standard manner, i.e., through Yukawa couplings of fermions and scalars with spontaneous breaking. Yukawa coupling constants are assumed to be smaller than the weak-gauge coupling constant in the lower-energy region to secure the asymptotic freedom in the region. (As we shall see later the asymptotic freedom is lost in this model in the high-energy limit.) The masses of the weak gauge boson, 85 GeV, and of the $SU^H(4)$ gauge boson, larger that 10^4 GeV, serve as very effective constraints on the fermion masses. We point out that small neutrino mass appears naturally and also that the dynamical creation of mass is possible in this model (see Sec. III).

Other features of the model, such as renormalization effects and charged currents, will be studied in Secs. IV and V, respectively.

II. MODEL

A. Charge operator

To avoid exotic fermions Q (= charge operator) is chosen to be

$$Q=\mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},1,0,0,0,0) \tag{1}$$

for the fundamental representation. This assignment guarantees $\sin^2\theta_W=\frac{3}{8}$ at the symmetric limit.

B. Fermions

With left- and right-handed parts counted separately there are 165 fields,

$$165=84_L+9\times 9_R\,. \tag{2}$$

A reasonable assignment is as follows:

$9\times 9_R$:

$$(d_\alpha,e^c,\nu_e^c,F_{1i})\,, \tag{3a}$$

$$(s_\alpha, \mu^c, \nu_\mu^c, F_{2i}) \,, \tag{3b}$$

$$(b_\alpha, \tau^c, \nu_\tau^c, F_{3i}) \,, \tag{3c}$$

$$(q_{1\alpha}, l_1^c, \nu_1^c, F_{4i}) \,, \tag{3d}$$

$$(q_{2\alpha}, l_2^c, \nu_2^c, F_{5i}) \,, \tag{3e}$$

$$(q_{3\alpha}, l_3^c, \nu_3^c, F_{6i}) \,, \tag{3f}$$

$$(q_{4\alpha}, l_4^c, \nu_4^c, F_{7i}) \,, \tag{3g}$$

$$(q_{5\alpha}, l_5^c, \nu_5^c, F_{8i}) \,, \tag{3h}$$

$$(q_{6\alpha}, l_6^c, \nu_6^c, F_{9i}) \,; \tag{3i}$$

$84_L = (4,10) + (1,\overline{10}) + (6,5)$
$+ (\overline{4},1)$ [with respect to $SU^H(4)\times SU(5)$]:

$$(4,10): \left[u^{\alpha c}, \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}, e^c \right] \left[t^{\alpha c}, \begin{pmatrix} t_\alpha \\ b_\alpha \end{pmatrix}, \tau^c \right], \tag{4a}$$

$$\left[c^{\alpha c}, \begin{pmatrix} c_\alpha \\ s_\alpha \end{pmatrix}, \mu^c \right] \left[t'^{\alpha c}, \begin{pmatrix} t'_\alpha \\ b'_\alpha \end{pmatrix}, \tau'^c \right].$$

$$(1,\overline{10}): \left[t'_\alpha, \begin{pmatrix} t'^{\alpha c} \\ b'^{\alpha c} \end{pmatrix}, \tau' \right]; \tag{4b}$$

$$\begin{aligned} (6,5):\ & (q_{1\alpha}, l_1, \nu_1)\ (q_{4\alpha}, l_4, \nu_4)\,, \\ & (q_{2\alpha}, l_2, \nu_2)\ (q_{5\alpha}, l_5, \nu_5)\,, \\ & (q_{3\alpha}, l_3, \nu_3)\ (q_{6\alpha}, l_6, \nu_6)\,; \end{aligned} \tag{4c}$$

$$(\overline{4},1):\ F_{10}^i \,. \tag{4d}$$

In (3) and (4), α $(=1,2,3)$ and i $(=1,2,3,4)$ are color and $SU^H(4)$ indices, respectively, and C denotes charge conjugation. As for F_{pi} $(p=1-10)$ they could be all independent or some of them could be antiparticles. In the former case they are all one handed and they are massless in the ordinary sense, i.e., Dirac mass is zero. But as will be seen in Sec. III they can acquire a large Majorana mass and thus not contradict the experimental result and cosmological consideration which predicts the number of massless fields to be not more than four. (Majorana mass was first mentioned in the context of GUT's by Gell-Mann *et al.*[5] and has recently been discussed by many people.[6,7])

TABLE I. Decomposition of $9\times 9 = 36 + 45$ with respect to $SU^H(4)\times SU(5)$.

	(4,1)	(1,5)
(4,1)	$(6,1)=\psi_1$ $(10,1)=\psi_2$	$(4,5)=\psi_3$
(1,5)		(1,10) (1,15)

C. Gauge fields

New gauge bosons appear besides those of SU(5):

$$\begin{array}{lcc} \text{New gauge bosons} & SU^c(3)\times SU(3)\times SU^H(4) & \text{Charge} \\ \hline (A_\mu)^i_j & (1,1,15) & 0 \\ (Z_\mu)_{i\alpha} & (3,1,4) & -\frac{1}{3} \\ \begin{pmatrix} V_\mu^\dagger \\ V_\mu^0 \end{pmatrix}_i & \begin{matrix}(1,2,4)\\(1,1,1)\end{matrix} & \begin{pmatrix}1\\0\end{pmatrix} \\ B'_\mu & & 0 \end{array} \tag{5}$$

A_μ are $SU^H(4)$ gauge bosons and cause the change of flavor. Z_μ and $V_\mu^\pm$ are charged bosons and mediate $B+L$ conserving and $\Delta L = 2$ processes. Being a singlet with respect to $SU^c(3)\times SU(2)\times SU^H(4)$ B'_μ behaves in a similar manner as B_μ of the SU(5) model. But we note there is no new mixing angle with respect to weak and electromagnetic interaction because the charge operator is chosen as in Eq. (1).

D. Breaking pattern and gauge-boson mass

We assume the following breaking pattern:

$$\begin{aligned} SU(9) & \xrightarrow[A]{} SU(5)\times SU^H(4)\times U(1) \\ & \xrightarrow[B]{} SU^c(3)\times SU(2)\times SU^H(4)\times U(1)\times U(1) \\ & \xrightarrow[C]{} SU^c(3)\times SU(2)\times U(1) \\ & \xrightarrow[D]{} SU^c(3)\times U(1)\,. \end{aligned} \tag{6}$$

Steps A and B could be realized by two adjoint scalar multiplets, ϕ_1 and ϕ_2,

$$\phi_1 = v_1\,\mathrm{diag}(1,1,1,1,1,-\tfrac{5}{4},-\tfrac{5}{4},-\tfrac{5}{4},-\tfrac{5}{4})\,, \tag{7a}$$

$$\phi_2 = v_2\,\mathrm{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2},0,0,0,0)\,, \tag{7b}$$

where

$$10^{15}\ \mathrm{GeV} \simeq v_1 < v_2 < 10^{19}\ \mathrm{GeV}\,. \tag{7c}$$

Steps C and D are assumed to be effected by the scalar multiplets which also couple with fermion bilinears (adjoint scalars do not couple with fermion bilinears).

In this setting we have

$$10^{14}\ \mathrm{GeV} \simeq m_{X_\mu(Y_\mu)} < m_{V_\mu(Z_\mu)} < 10^{19}\ \mathrm{GeV} \tag{8}$$

and

$$m_{A_\mu(B_\mu)} < m_{Z_\mu(V_\mu)}\,.$$

Also the suppression of flavor-changing neutral currents implies a lower bound for A_μ:

$$m_{A_\mu} > 10^4\ \mathrm{GeV}\,. \tag{9}$$

III. FERMION MASS

We start the discussion on the fermion-mass spectrum with the constraints imposed by the mass of weak and $SU^H(4)$ gauge bosons. It should be noted that the scalar components which contribute to the fermion mass also contribute to the gauge-boson mass. Namely, any vacuum expectation value (VEV) of SU(2) [$SU^H(4)$] nonsinglet scalar components contribute to the weak [$SU^H(4)$] gauge-boson mass and thus its value is restricted. This observation greatly simplifies the analysis and we can easily tell which fermions can be heavy or light. In a realistic model we would expect that the mass matrix is so polarized that many or all of the new fermions are much heavier than familiar ones. We restate the criterion[3] for extracting heavy fermions in the present context:

Fermions can be heavy if they are SU(2) singlet and $SU^H(4)$ nonsinglet or if both left- and right-handed parts are SU(2) nonsinglet and their fermion bilinears are SU(2) singlet and $SU^H(4)$ nonsinglets.

In practice our criterion presumably leads to the same conclusion as Georgi's[8] but ours applies to any type of models besides the $SU(N)$ chiral model. Its content becomes clearer once we write out fermion bilinears and decompose them with respect to $SU^H(4)\times SU(5)$ as is presented in Tables I–III. By making use of Table I we obtain the following results. [ψ_i ($i=1$–12) are defined in Tables I–III.]

(1) F_{pi} ($p=1$–10, $i=1$–4). F_{pi} becomes heavy by acquiring a large Dirac and/or a Majorana mass through scalar components defined as ψ_1, ψ_2, ψ_4, ψ_5, ψ_9, ψ_{10} in Tables I–III.

(2) ν_s, L_s ($s=4$–9). Through ψ_{12} ν_s and L_s obtain a large Dirac mass.

(3) t', b', τ'. ψ_6 gives a large Dirac mass for t', b', and τ'. At this stage all familiar fermions are massless. To make them massive SU(5)-nonsinglet scalar components should develop a small VEV.

(4) u, c, t. u, c, t become massive through ψ_7 and ψ_8.

(5) d, s, b, e, μ, t. ψ_{11} makes these massive. If flavor mixings are neglected we recover the familiar SU(5) mass relation

$$\frac{m_e}{m_d}=\frac{m_\mu}{m_s}=\frac{m_\tau}{m_b}\,. \tag{10}$$

(6) ν_e, ν_μ, ν_τ. These neutrinos are one handed and do not have Dirac mass. We point out, however, that they may acquire very small Majorana masses through ψ_3. [ψ_3 is SU(5) nonsinglet and its VEV is small.] The reason is the following. F_i may acquire large Majorana mass through $\psi_1, \psi_2, \psi_4, \psi_5, \psi_9, \psi_{10}$: ψ_3 causes mixings among ν_e (ν_μ, ν_τ) and F_i. Then forgetting about other possible mixings we have for example, a mass matrix[6,7]

$$\begin{array}{cc} & \begin{array}{cc}\nu & F_1\end{array} \\ \begin{array}{c}\nu \\ F_2\end{array} & \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \end{array}\,. \tag{11}$$

m (M) comes from ψ_3 ($\psi_1, \psi_2, \psi_4, \psi_5, \psi_9, \psi_{10}$) and thus is small (can be very large). If $M \gg m$, ν_e acquires a Majorana mass $\sim m^2/M$. If neutrino mass is less than 10 eV, then we obtain a lower bound for the $SU^H(4)$ gauge-boson mass, $\sim 6\times 10^{10}$ GeV. In such a case all new fermions become superheavy with the mass around 10^9 GeV or larger.

Next we comment on the scalars to be used. To break SU(2) only the 5-dimensional representation [with respect to the SU(5) subgroup] is used to maintain the normal mixing relation between photon and neutral weak gauge boson. One result associated with this choice is that the neutrinos are necessarily massive in this model because, as Tables I–III show, the scalar component which is

TABLE II. Decomposition of $84\times 84=\overline{84}+1050+2520+3402$ with respect to $SU^H(4)\times SU(5)$.

	(4,10)		(6,5)		$(1,\overline{10})$	(4,1)
(4,10)	$(6,5)=\psi_7$ (5,45) (6,50)	$(10,5)=\psi_8$ (10,45) (10,50)	(4,10) (4,40)	(20,10) (20,40)	$(4,1)=\psi_6$ (4,24) (4,75)	(1,10) (15,10)
(6,5)			(1,10) (1,15) (15,10) (15,15)	(20,10) (20,15)	(6,10) (6,40)	(4,10) (20,5)
(1,10)					(1,1) (1,24) (1,75)	(4,10)
(4,1)						$(6,1)=\psi_4$ $(10,1)=\psi_5$

TABLE III. Decomposition of $\bar{9}\times 84=36+\overline{720}$ with respect to $SU^H(4)\times SU(5)$.

	(4,10)	(6,5)	$(1,\overline{10})$	$(\bar{4},1)$
$(\bar{4},1)$	(1,10)	(4,5)	(4,10)	$(6,1)=\psi_9$
	(15,10)	(10,5)		$(10,1)=\psi_{10}$
$(1,\bar{5})$	$(4,5)=\psi_{11}$	$(6,1)=\psi_{12}$	(1,10)	(4,5)
	(4,45)	(6,24)	(1,40)	

5 dimensional with respect to SU(5) is not singlet with respect to SU(4) and thus neutrinos always mix with F's. Also, it should be mentioned that 84-dimensional scalar multiplet does not couple with 84×84 fermion bilinear and 36-dimensional scalar multiplet does not couple with 9×9 in case two 9's are identical. This is due to the anticommuting nature of fermion fields and is explained in Ref. 9.

We may, incidentally, turn around the way of thinking on creating fermion mass and look for the possibility of creating it dynamically.[10] This possibility is not realized in an arbitrary model but interestingly in this particular model it is. One could forbid some of 9_R's to couple with 84_L, so that there is no direct coupling between left- and right-handed parts. Then such fermions can become massive only through radiative corrections and it is known that the radiative corrections can give sizable contributions.[10] This way of creating mass is very attractive because it will naturally explain the smallness of ordinary fermion masses. The actual implementation of the idea is rather involved and will be discussed elsewhere.

IV. ASYMPTOTIC FREEDOM AND RENORMALIZATION EFFECTS

We summarize in this section the behavior of the effective coupling constants. Their behavior depends on β which is given at the one-loop level[11] by

$$\beta=-\tfrac{11}{3}G+\tfrac{4}{3}F+\tfrac{1}{6}S\,, \tag{12}$$

where G, F, and S denote gauge, fermion, and scalar contributions, respectively. In our model we have, neglecting scalars,

$$\begin{aligned}&\beta(SU(2))=\tfrac{8}{3}\,,\\ &\beta(SU^c(3))=-1\,,\\ &\beta(SU^H(4))=-\tfrac{14}{3}\,.\end{aligned} \tag{13}$$

Three comments are due.

(1) Without scalars quantum chromodynamics is asymptotically free, and so is the entire theory in the high-energy limit. However, if scalars are included asymptotic freedom is lost because 1050, 2520, or 3402 must be introduced. As far as we have studied there is no asymptotically free GUT based on the single group with its rank larger than seven.

(2) Crossing of coupling constants can take place. Suppose the breaking pattern is stepwise:

$$\begin{aligned}SU(9)&\to SU(5)\times SU^H(4)\times U(1)\\ &\to SU(3)\times SU(2)\times SU^H(4)\times U(1)\times U(1)\end{aligned} \tag{14}$$

and mass of $SU^H(4)$ gauge bosons is less than 10^{10} GeV then the $SU^H(4)$ coupling constant crosses with both $SU^c(3)$ and SU(2) coupling constants. Crossing has been noted in the SU(8) model.[3] There, the $SU^H(3)$ coupling constant crosses with that of SU(2) but not $SU^c(3)$. If the breaking pattern is

$$\begin{aligned}SU(9)&\to SU^c(3)\times SU(6)\\ &\to SU^c(3)\times SU(2)\times SU^H(4)\times U(1)\times U(1)\end{aligned} \tag{15}$$

then $SU^c(3)$ and SU(2) cross each other.

(3) The prediction of the mixing angle and g_s (gluon coupling constant) at the present energy remains almost the same as that of the SU(5) model even though each effective coupling constant behaves significantly different from those of SU(5).

V. CHARGED CURRENTS

The feature of GUT's which is of great physical interest is the existence of the processes which break both B (baryon number) and L (lepton number). The SU(5) model contains $B-L$ conserving processes mediated by $X_\mu^{\pm 4/3}, Y_\mu^{\pm 1/3}$ charged superheavy vector bosons and they lead to proton decay. Also it is hoped that they may explain the baryon-number asymmetry in the universe.

In our model new charged bosons $V_\mu^{\pm}$ and $Z_\mu^{\pm 1/3}$ appear besides $X_\mu^{\pm 4/3}$ and $Y_\mu^{\pm 1/3}$. Their couplings with fermions turn out to be as follows:

	9_R	84_L
V_μ^+	$\bar{l}^c F_i$ and/or $\bar{l}^c T_i^c$	$\bar{l}^c F_{10,i}, \bar{u}d, \bar{t}'d$
$Z_\mu^{-1/3}$	$\bar{d}F_i$ and/or $\bar{d}F_i^c$	$\bar{d}F_{10,i}, \bar{u}l^c, \bar{d}\nu^c$
		$\bar{u}^c d$

where F, l, ν, d, and u denote (F_1-F_9), $(l_1-l_6$ and $e,\mu,\tau)$, $(\nu_1-\nu_6$ and $\nu_e,\nu_\mu\,\nu_\tau)$, (d,s,b), and (u,c,t), respectively. And/or is due to the arbitrariness of particle and antiparticle assignment. The above Table shows the existence of new type of processes. V_μ mediate $\Delta L=2$ process and Z_μ mediate $B+L$ conserving and $\Delta L=2$ processes at the lowest order. These processes have been found in the SU(8) model[3] and also in SO(18) and E(8) models. We expect it to be a general feature of GUT's with a horizontal symmetry. These new processes may affect the estimate of B asymmetry[12] in the universe although we have not performed numerical calculation.

VI. SUMMARY AND COMMENTS

We discussed consequences of SU(4) horizontal symmetry by taking Frampton's model as an example. The model is similar to the SU(8) model as far as the charged-current structure and the renormalization effects are concerned. We have noted $\Delta L = 2$ and $B+L$ conserving processes. The difference shows up in the fermion-mass spectrum. The SU(9) model has three light families whereas the SU(8) model contains five. The model also provides the possibility of creating small Majorana mass for neutrinos and of creating light fermion masses dynamically. Majorana mass of neutrino may resolve the problem of missing mass in the universe, and dynamical creation of small mass would solve the problem of families in a very appealing manner, even though it is achieved at the cost of introducing a local horizontal symmetry and superheavy fermions. The picture emerging from the analysis is the following: There are superheavy bosons and fermions and their existence is reflected on the light fermions and perhaps weak gauge bosons in the form of small masses, flavor mixings, Weinberg angle, etc.

Our analysis is admittedly incomplete. In discussing symmetry breaking and the fermion mass spectrum we assumed, without proof, that certain components of scalars develop suitable vacuum expectation values. This problem of gauge hierarchy becomes more difficult as we go to larger groups since the theory could undergo multistage breaking. We would like to come back to the problem in the future.

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