# Thermal expansion of granular superconductors based on elastic response of Josephson junction arrays

**G Rotoli [1], S Sergeenkov [2], G Filatrella [3], and F. M. Araujo-Moreira [4]**

[1] CNISM and DIMEG, Università di L'Aquila, Località Monteluco, I-67040 L'Aquila, Italy E-mail: rotoli@ing.univaq.it
[2] Departamento de Física, CCEN, Universidade Federal da Paraíba, Cidade Universitária, 58051-970 João Pessoa, PB, Brazil
[3] Laboratorio Regionale SuperMat CNR-INFM Salerno and Dipartimento di Scienze Biologiche ed Ambientali, Università del Sannio, via Port'Arsa, 11, I-82100 Benevento, Italy
[4] Grupo de Materiais e Dispositivos, Centro Multidisciplinar para o Desenvolvimento de Materiais Ceramicos, Departamento de Física, Universidade Federal de São Carlos, 13565-905 São Carlos, SP, Brazil

**Abstract.** We introduce the concept of thermal expansion (TE) in Josephson systems as an elastic response to an effective stress field. The temperature and magnetic field dependences of TE coefficient can be studied both analytically and numerically in a single junction and in a square array of Josephson junctions (JJA) [1]. We found that in addition to field oscillations due to Fraunhofer-type dependence of the critical current, both single junction and JJAs may exhibit flux driven temperature oscillations of the TE coefficient near $T_c$ provided the applied magnetic field is strong enough to compensate for the screening-induced effects. We briefly discuss possible consequences of TE coefficient temperature oscillations for the local penetration of magnetic field in granular systems modelled as JJAs.

## 1. Introduction

The effects of penetration of magnetic flux in high-$T_c$ superconductors have been a subject of many investigations in recent years, especially in view of their unconventional pairing properties [2]. On the other hand, the important applications for this class of materials strongly depend on their magnetization properties as a critical state is desired to appear at the highest possible fields, which means that the material can support a large supercurrent [3]. Usually, theoretical models for magnetic penetration emphasize some particular properties, such for example as the role of inhomogeneities or defects [4], multi-connected paths [5], presence of unconventional junctions [6], surface effects [7], etc. More recently, special attention has been given to the numerous thermal effects arising from the concurrence between the local temperature oscillations and the presence of defects [8], and resulting in generation of giant flux avalanches [9]. At the same time, a rather unusual temperature behaviour of the field-dependent thermal expansion coefficient (TEC) in Josephson junctions and their arrays (JJA) has been predicted which is based on elastic response of Josephson system to an effective stress field [9]. Here we will analyze the last phenomenon for its potential applications in real materials, using a well-established analogy between JJA and granular superconductors.

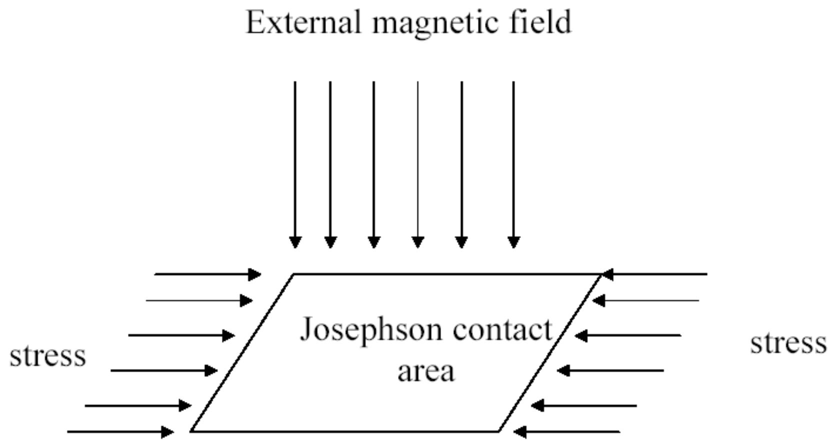


**Figure 1.** Schematic view of a Josephson contact area in the presence of applied stress and external magnetic field.

## 2. The model

A JJ contact is characterized by the critical tunnel current $I_C$ (the maximum current that can flow without resistance through the contact). This current depends upon external parameters, such as the temperature and the magnetic field. More recently, it was realized that an applied mechanical stress σ could also influence the critical current [1]. The physical origin of this phenomenon stems from the effect the applied stress has upon the separation between the superconducting electrodes, see Fig. 1. The change of the distance has two main consequences. On one side, the tunnel current increases exponentially with σ

$$I_c \propto e^{\beta t(\sigma)} \tag{1}$$

while on the other side, the weak superconducting region shrinks, see Fig. 1. In turn, this results in a strong effect for the modulation of the critical current upon external magnetic field since the period of the modulation depends upon the flux trapped into the contact. For a given field the effect is proportional to the effective area *A* (if effects such as magnetic field focusing are neglected). Hence:

$$\Phi_{ext} = H_{ext} A\left(1 - \chi \frac{\sigma}{\sigma_0}\right) \tag{2}$$

So far, we have analyzed an isolated contact. Some additional effects are expected to arise when several junctions are connected. Such a system is physically relevant as a prototype of a granular superconductor which can be modelled as an array of superconducting islands (grains) which are separated by the normal areas and connected via weak links, see Fig. 2.
The behaviour of such a granular system is electrically equivalent to a regular square array of Josephson junctions (JJA) of the type shown in Fig. 3. Assuming the applied stress perpendicular to the magnetic field, as it is depicted in Fig. 3, we also have a third effect, namely a stress-induced variation of the geometrical loop inductance, since the loop area deforms, approximately as follows:

$$L = L_0\left(1 - \chi_L \frac{\sigma}{\sigma_0}\right) \tag{3}$$

Journal of Physics: Conference Series **97** (2008) 012235 doi:10.1088/1742-6596/97/1/012235

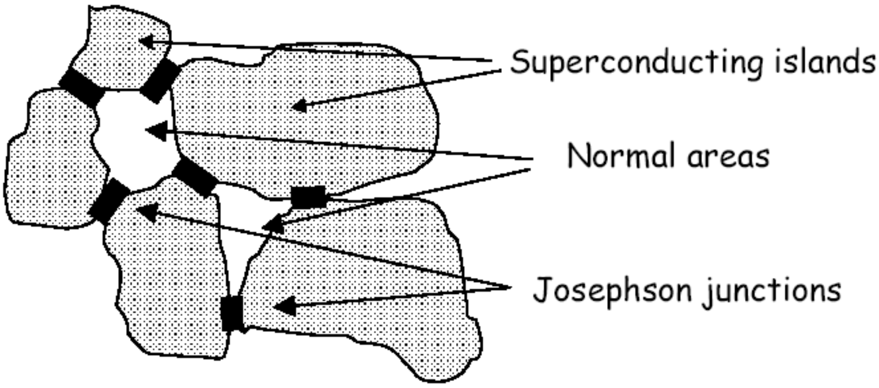


**Figure 2.** A schematic view of a granular superconductor. The grey areas are the superconducting islands separated by white areas of normal material. Josephson contacts are dark rectangles in the connections between the islands.

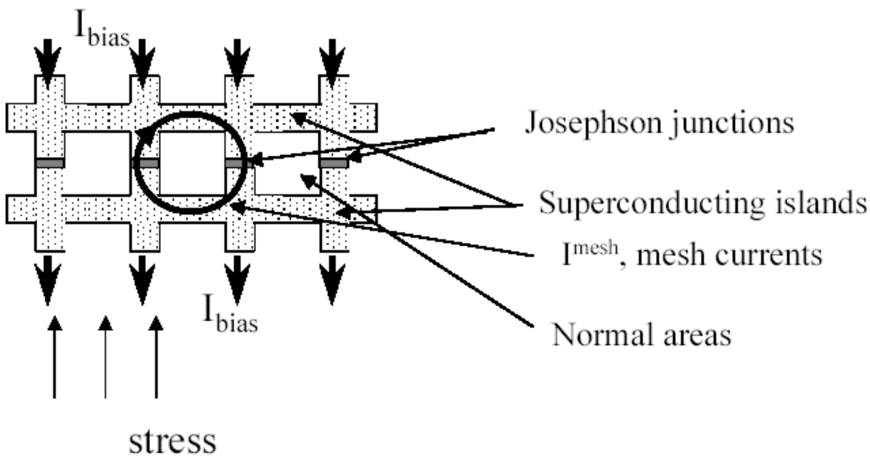


**Figure 3.** The simplified topology of a regular square array.

Eqs. (1)-(3) form the basis for studying the electrodynamics of JJA under applied stress [1]. Here, we will primarily discuss the temperature, field and bias current behavior of the so-called thermal expansion coefficient (TEC) which is defined via the induced strain field ε (i.e. the change of the energy per unit volume of the system under stress)

$$\varepsilon = -\frac{1}{V}\left[\frac{dE}{d\sigma}\right]_{\sigma=0} \tag{4}$$

as follows

$$\alpha = \frac{d\varepsilon}{dT} \tag{5}$$

To accomplish this plan, we must derive the stress dependent energy $E$ as a function of the temperature. Let us recall that the Josephson energy of a small junction [10] depends on the current $I$ flowing in the junction as [11]:

$$E_J = I_c\left[1-\sqrt{1-\left(\frac{I}{I_c}\right)^2}\right] \tag{6}$$

Besides, for any array (artificial or natural), another contribution must be included into the basic equation for ε. Namely, in addition to the external bias, in JJA's the mesh currents also contribute to the current through the junction. For the array depicted in Fig. 4 the current $I_{b,i}$ flowing in the junction with phase $\varphi_i$ is the sum of three terms:

$$I_{b,i} = I_B - I_i^{mesh} + I_{i-1}^{mesh} \tag{8}$$

To determine mesh currents one has to impose fluxoid quantization for each array mesh:

$$\sum_{loop} \varphi = 2\pi \frac{\Phi_{ext}}{\Phi_0} - 2\pi \frac{LI^{mesh}}{\Phi_0} \tag{9}$$

Even though the effect of the change in the mesh inductance is indirect and is through the fluxoid quantization (9), the mere presence of a circulating current requires the energy:

$$E_{ind} = \frac{1}{2} L_0 \left( 1 - \chi_L \frac{\sigma}{\sigma_0} \right) \sum_{array} \left( I^{mesh} \right)^2 \tag{10}$$

where the summation is extended to all elementary loops of the array.

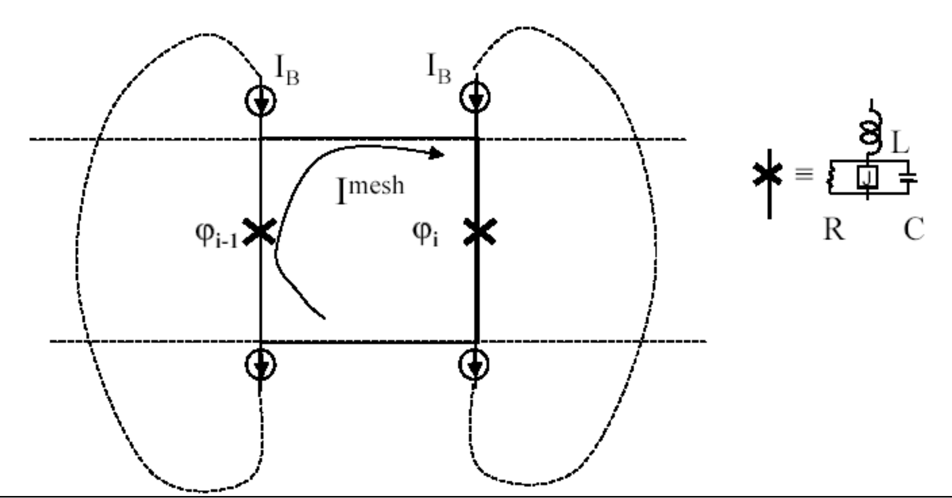


**Figure 4.** The electrical scheme of the square array depicted in Fig. 3. The bias current is fed via virtual loops external to the array.

## 3. Numerical results

Clearly, the inclusion of all these effects requires a numerical approach. Let us show the relative contribution of the various terms as a function of the parameters to determine the regions where one or the other effect is more relevant in the calculation of the total energy. In particular, in Fig.5 we compare the Josephson energy, given by Eq. (6), and the screening energy contribution, given by Eq. (10) for different values of the inductance parameter $\beta_L = 2\pi LI_c/\Phi_0$, and for different values of external flux given in units of flux quantum, i.e., $f = \Phi_{ext}/\Phi_0$.

By introducing the dependence on temperature via a standard BCS approach [1], the resulting overall behaviour of the TEC, given by Eq. (5), is reported in Fig. 6. It is clear that the oscillations induced by the applied stress are more pronounced near the critical temperature. So, the measurements should be performed in the region very close to the transition to observe the predicted here phenomena.

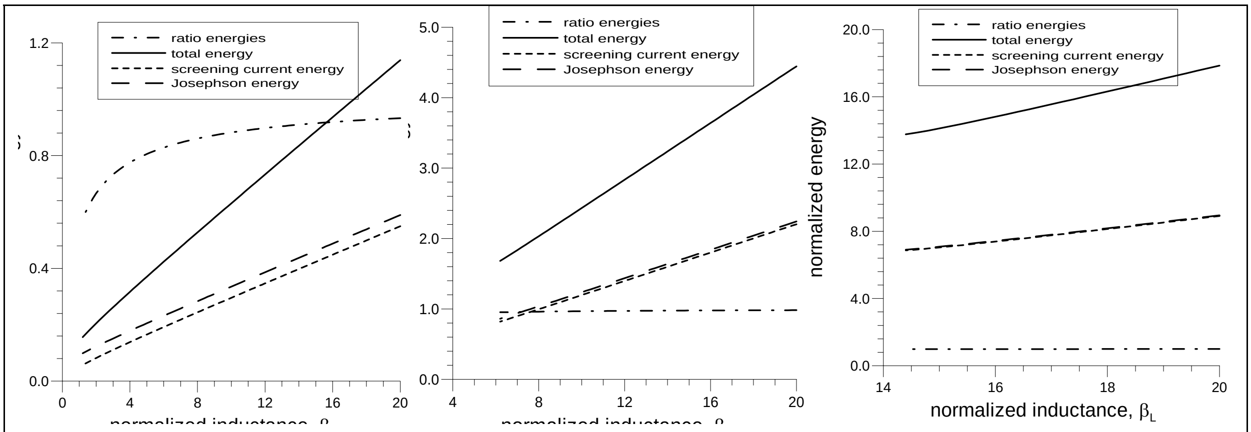


**Figure 5.** The behaviour of the Josephson energy (dashed line), the screening current contributions (dotted line) and the total energy (solid line) for different values of the external applied field in the loops [$f = \Phi_{ext}/\Phi_0 = 0.25$ (a), 0.5 (b), 1 (c)] as a function of the inductance parameter $\beta_L$ of a 5x5 array.

## 4. Conclusions

The elastic response of an array of Josephson junctions to a stress field is introduced in order to study the temperature, magnetic field and bias current behaviour of the thermal expansion (TE) of Josephson systems and granular superconductors. The competition between two main contributions (due to stress-induced modification of the inductance parameter) is found to determine the overall TE properties. Since the geometrical inductance is related to the size of the grains (see Fig. 2), the obtained here results are expected to be relevant for behaviour of granular superconductors under stress as well. This is especially true when $\beta_L$ goes to zero, making the contribution due to screening currents relatively strong. In this regime, the difference between arrays and single junctions becomes important. The consequence is that the TEC coefficient begins to exhibit flux driven temperature oscillations close to the transition temperature (see Fig. 6) just for a relatively large $\beta_L$. This occurs for a 5x5 JJA. For larger arrays the same oscillations should be exist at smaller $\beta_L$ values. We observe that during the cooling a negative value of $\alpha$ can induce flux penetration due to positive strain of JJA.

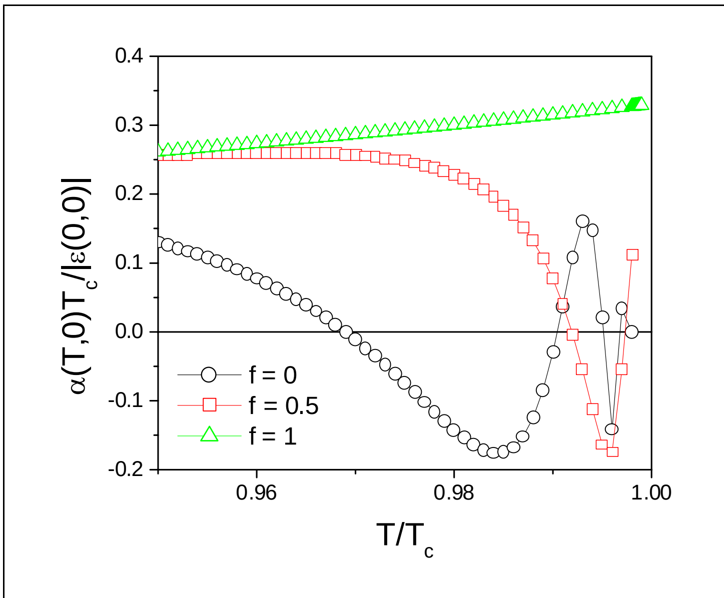


**Figure 6.** Oscillations of the normalized TEC coefficient [Eq. (5)] for an array 5x5 for three different values of the magnetic field in the junctions loops ($f$=0, 0.5, 1) which causes the oscillation of the critical current $I_c$ [see Eq. (6)] with a Fraunhofer-like pattern The bias and inductance parameter are as follows: $I_{bias}/I_C$=0.5, $\beta_L$=20.

### References

[1] Sergeenkov S, Rotoli G, Filatrella G and Araujo-Moreira F M 2007 Phys. Rev. B **75** 014506.
[2] Prozorov R and Giannetta R W 2006 Supercond. Sci. Technol. **19** R41–R67.
[3] Palau A, Puig T,Gutierrez J, Obradors X and de la Cruz F 2006 Phys. Rev B **73** 132508.
[4] Fehrenbacher R, Geshkenbein V B, Blatter G 1992 Phys. Rev. B **45** 5450.
[5] Araùjo-Moreira F M, Barbara P, Cawthorne A B and Lobb C J 1997 Phys.Rev.Lett.**78** 4625; De Leo C, Rotoli G, Nielsen A, Barbara P and Lobb C J 2001 Phys.Rev.B **64**, 144518; De Leo C, Rotoli G 2002 Supercond. Sci. & Technol. **15** 1716.
[6] Rotoli G 2003 Phys. Rev. B **68** 052505.
[7] Gurevich A, Cooley L D 1994 Phys. Rev. B **50** 13563; Koshelev A E and Larkin A I 1995 Phys.Rev.B **52** 13559; Deo P S, Schweigert V A, Peeters F M and Geim A K 1997 Phys.Rev.Lett.**79** 4953.
[8] Rakhmanov A L, Shantsev D V, Galperin Y M, and Johansen T H 2004 Phys. Rev. B **70**, 224502.
[9] Denisov D V, Shantsev D V, Galperin Y M, Eun-Mi Choi, Hyun-Sook Lee, Sung-Ik Lee, Bobyl A V, Goa P E, Olsen A A F, and Johansen T H 2006 Phys. Rev. Lett. **97** 077002; Denisov D V, Rakhmanov A L , Shantsev D V, Galperin Y M, and Johansen T H 2006 Phys. Rev. B **73** 014512.
[10] We assume that array is formed by small junctions, i.e., which Josephson length is larger than physical dimension of the junction. This applies as long as the magnetic field is sufficiently low that self-field effects do not came into play at junction level. This happens if $HA \ll \Phi_0$, thus being junction area much smaller than loop area $A_{loop}$ this occurs for relatively large fields with respect to field relevant for array loop effects $HA_{loop} = \Phi_0$.
[11] Tinkham M 1997 *Introduction to Superconductivity* 2nd edition (Singapore: McGraw-Hill).