Measurement Techniques, Vol. 55, No. 7, October, 2012

ELECTROMAGNETIC MEASUREMENTS

AN AUTOMATIC PRECISION SYSTEM FOR THE METROLOGICAL BACKUP OF MEASUREMENTS OF IMPEDANCE PARAMETERS. PART. 1. OPERATING PRINCIPLES

M. N. Surdu, A. L. Lameko,
D. M. Surdu, and S. N. Kursin

UDC 621.317:537.311.6

A system for reproducing and transferring the units of impedance parameters, including a set of precision automatic comparators and thermostatted measures-carriers is described. The apparatus enables the dimensions of any parameters of impedance to be transferred for any value of the loss tangent or phase angle over a whole range of values of impedance, to reproduce the unit of inductance in terms of the units of capacitance and frequency, and to compare the impedances of the standards of capacitance and resistance at frequencies of 1.0 kHz and 1.59 kHz.

Keywords: *impedance, standard, reproduction and transfer of units, quadrature and autotransformer bridges.*

Impedance, as a characteristic of each component of an electric circuit, is the most important measured electrical quantity. A feature of impedance is the fact that it is characterized by a set of parameters: capacitance, inductance, resistance, loss factor, Q-factor, time constant, etc. Such a set of parameters requires a corresponding set of standards and means of transferring the dimensions of the units, as well as highly qualified service personnel. For these reasons, and in view of the availability of new measuring instruments, metrological backup for measurements of the parameters of impedance becomes considerably more complex and expensive. Hence, the design of specialized automatic equipment, which could simplify the processes of reproducing the units of the parameters of impedance and of transferring their dimensions over a range of values, is a pressing problem.

Description and Formulation of the Problem. Standards, specially developed for a specific parameter (capacitance, inductance, resistance, etc.), and complex systems for reproducing and transferring the dimensions of the units over a range of values of the measured parameters, have been traditionally employed for reproducing the units of the parameters of impedance. For this purpose, numerous measures-carriers, possessing high short-term stability, are also necessary.

During the second half of the last century, three important discoveries were made in physics, which touch on electrical measurements: Lampard's theorem has been proved [1] and a standard theoretical capacitor has been constructed based on it [2–5], the capacitance of which is directly tied to the meter; the discovery of the Josephson effect [6] and the construction, based on it, of a standard of dc voltage [7], the value of which is determined by the frequency and the ratio of Planck's constant to the charge of the electron; and the discovery of the quantized Hall effect [8], which has enabled a standard of resistance to be produced, connected with the same ratio [9, 10].

All-Ukraine State Scientific and Production Center for Standardization, Metrology, Certification, and Consumer Protection (Ukrmetrteststandart), Kiev, Ukraine; e-mail: michaelsurdu1941@yandex.ru. Translated from Izmeritel'naya Tekhnika, No. 7, pp. 51–57, July, 2012. Original article submitted November 7, 2011.

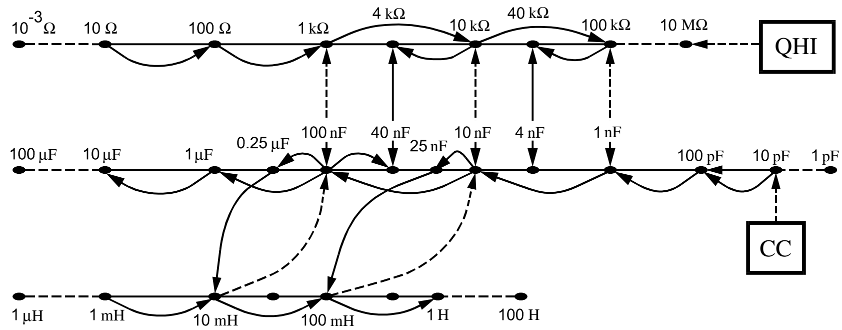

Fig. 1. Structure of the transfer of the dimensions of the units and the mutual relation between the parameters of impedance: QHI – quantized Hall resistance; CC – calculable capacitor.

These discoveries have led to the emergence of new measuring instruments and have further complicated the systems for reproducing and transferring the dimensions of the units of impedance. Corresponding precision apparatus for comparing the new standards are used daily in all the leading metrological laboratories of the world [11–13]. These take the form of specialized ac bridges of high accuracy with manual control, each of which performs separate functions and is designed to measure one of the physical quantities, as a rule, over a narrow range of values. The cost of such apparatus, naturally, is extremely high.

The use of this apparatus is completely justified when carrying out unique experiments, directed towards fundamental investigations, where the measurement errors amount to 10^{-8}–10^{-9} or less. However, in the majority of cases, even in scientific research, automatic miniature apparatus with a measurement error of the order of 10^{-7}–10^{-6} would be sufficient. Such apparatus, without being the most accurate in the world, could considerably simplify the process of reproducing and transferring the dimensions of the units of impedance parameters in the majority of practical cases. For this purpose, it must be optimum with respect to the set of metrological and operational parameters.

The reproduction and transfer of the dimensions of the units of impedance involve solving several general problems (Fig. 1).

The first problem consists of the highly accurate transfer of the dimensions of the unit over a range of values of the measured quantities for many forms of the measured parameters. At the present time, this problem is solved separately for the different impedance parameters. Moreover, as a rule, different apparatus is used for operation in different parts of the measurement range. The most important feature of the transfer of impedance parameters is the fact that its phase-angle tangent $\tan\Phi$ or the loss tangent $\tan\delta$ may vary over a wide range: from zero to units or even greater. This is particularly characteristic for inductance standards.

The second problem reduces to comparing standards having unlike parameters, i.e., it reduces to comparing the impedance of capacitance and resistance standards, of inductance and resistance standards, and capacitance and inductance standards. Various quadrature bridges (for transfers of the form $C \leftrightarrow R$) and resonance or four-arm bridges (for transfers of the form $C \to L$) are usually employed to solve this problem.

The third problem consists of the need to automate a large volume of the measurements, which is not only important to increase their productivity, but it should also help to increase the accuracy, reduce the probability of the occurrence of errors in the measurements, and relax the requirements as regards the qualifications of service personnel. This problem has been partially solved in the area of measurements of small capacitances using Anden–Hagerling automatic bridges, but it is necessary to solve this problem for all impedance parameters over the whole range of measurement.

The fourth problem is the fact that a considerable number of standard measures, which must possess high short-term stability, is used when transferring the dimensions of a parameter over a range of values or when comparing different types of parameters of the standards. Investigations show that, to transfer the dimensions of a unit using comparators over different ranges of the parameters, it is necessary to have measures with decimal values of the resistance from 1 Ω to 10 MΩ, mea-

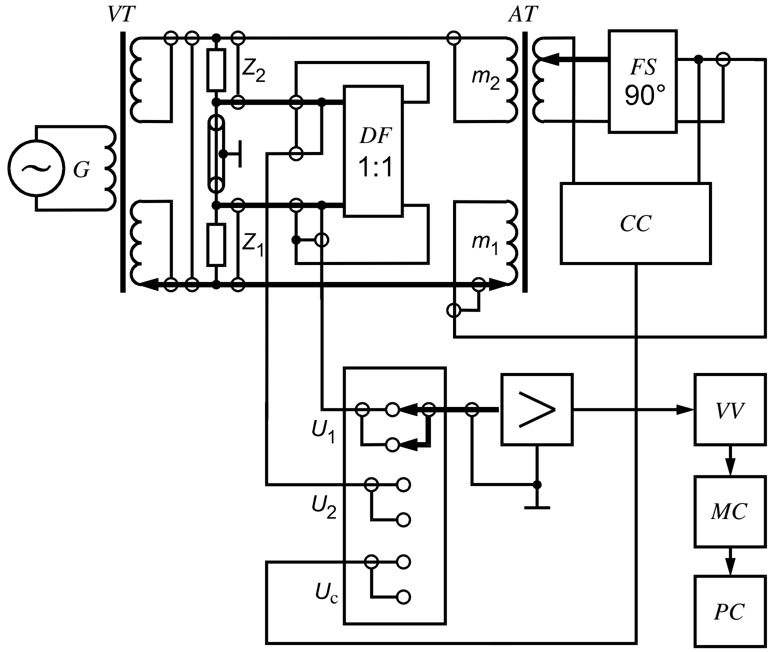

Fig. 2. Block diagram of an autotransformer bridge-comparator: *G* – generator; *VT* – voltage transformer; *AT* – autotransformer voltage divider; *DF* – differential follower; *PS* – phase shifter; *CC* – calibration circuit; *VV* – vector voltmeter; *MC* – microcontroller; *PC* – personal computer.

sures of capacitance from 1 nF to 100 μF (here it is taken into account that high-quality thermostatted measures of capacitance of 1, 10, and 100 pF from different manufacturers in many laboratories are already available), and the inductances of 1, 10, 100 mH, and 1 H.

Measures with decimal values of the parameters can also be employed to compare different kinds of parameters at a frequency of 1.59 kHz. To compare diverse standards at a frequency of 1 kHz in an equinominal comparison mode, it is necessary to use measures having fractional values of the parameters, which gives rise to serious difficulties. In the present paper, for this purpose we propose to carry out a quadrature comparison of the parameters of capacitance and resistance standards for values that are multiples of four: 0.4, 4, and 40 nF for capacitance standards and 4, 40, and 400 kΩ for resistance standards, while to reproduce the unit of inductance we will use capacitance measures with values of 2.5, 25, 250 nF, and 2.5 μF. The transfer of dimensions for measures with decimal values in all the measures mentioned above is carried out for a ratio of the comparator arms of 1.0:0.4.

To ensure the necessary short-term stability, these measures must be maintained at a thermostatically controlled temperature.

To solve the above problems within the framework of an international project, we developed automatic apparatus, including an autotransformer bridge-comparator, which serves to transfer the dimensions of the units of any parameter over the whole range of values for a value of the second parameter (tanδ or tanΦ), which varies from zero to a few units, and also to reproduce the unit of inductance from the units of capacitance and frequency (a transfer of the form $C \to L$); a quadrature bridge-comparator, which executes $C \leftrightarrow R$ for small angle tangents; a set of thermostatted measures of capacitance, resistance and inductance, containing measures with given nominal values. Below we describe the principles for constructing a working system.

An Autotransformer Bridge-Comparator. Among the different configurations of bridges with tight inductive coupling, bridges with transformer or autotransformer connection of an inductive voltage divider are the most common. The theory of these bridges is well developed [14, 15].

In Fig. 2, we show a block diagram of the balancing of an autotransformer bridge-comparator, which enables accurate measurements to be made of the parameters of impedances over a wide range of values using two-circuit balancing with respect to two parameters.

The bridge contains a voltage generator *G*, from which a supply is taken to the voltage transformer *VT*, which serves to provide galvanic decoupling between the generator and the measuring circuit of the bridge. The transformer has two series-connected output windings. The number of turns in one of these is constant, while the other is regulated. The middle point of these windings is used as a "Wagner ground."

The standards Z_1 and Z_2 to be compared are connected in series and also connected to the secondary windings of the transformer *VT* by their high-voltage current terminals, while the autotransformer voltage divider *AT* is connected to their potential terminals. It has two primary windings m_1 and m_2, one of which is regulated. The regulated windings of the *VT* and the *AT* are connected to adjacent terminals of one of the standards being compared and are varied in synchronism in such a way that their number of turns is always the same. By regulating the number of turns of these windings, the bridge is balanced with respect to the main parameter, which, depending on the form of the standards being compared, can be a capacitance, an inductance or a resistance.

To balance the bridge with respect to the second parameter, a secondary regulated winding, wound on the autotransformer, to which a quadrature phase shifter *PS* is connected, is used. Its output is connected in series with the regulated primary winding of the autotransformer. The phase shifter is calibrated using the calibration circuit *CC* using a special procedure [16, 17], for which the output voltage U_c of this circuit is analyzed.

In order to eliminate the effect of the impedance Z_c of the low-voltage current conductors of the impedances being compared and of the corresponding connecting cables, double autotransformer bridges are usually employed [14] (see, for example, the UDMK-1M double autotransformer bridge). These bridges contain two autotransformer voltage dividers with switchable windings. The main divider divides the voltage, applied to the series-connected objects being compared, while the other is connected to the equilibrium indicator and divides, in the same ratio, the voltage drop on the cable connecting these objects. In the high-voltage autotransformer bridges, the voltage dividers are constructed using a two-stage circuit, and hence to balance the bridge one must simultaneously change the number of windings of the four regulated windings of these dividers (ignoring the cost in setting up the bridge protection circuits). It should also be noted that these dividers are multi-decade devices (for accurate measurements the number of decades can reach eight-nine). Hence, there are also four switches, and the transformers themselves are complex and very large structures. In addition to this, the inductive divider, connected to the indicator, considerably reduces the noise immunity of the measurements with respect to interference at the working frequency and from the power supply network. This particularly affects the medium and high-resistance limits (we recall that a resistance of the connecting wire of 0.5 Ω leads to an error of $5 \cdot 10^{-7}$ when measuring an impedance of 1 MΩ).

We propose a new solution here, which serves two purposes: it halves the number of voltage dividers and reduces the number of decades in the divider by at least the same factor. To do this, the voltage drop on the conductor, connecting the impedances being compared, is introduced, by means of a differential repeater *DR* (see Fig. 2), into the circuit between the windings of the main divider, and is connected either in one or the other arm of the bridge, depending on the unbalanced signal being analyzed.

To analyze the state of the bridge and to obtain an accurate measurement result, irrespective of Z_c and the error of the differential follower, two unbalanced voltages U_1 and U_2 between the low-voltage leads of the windings of the autotransformer and the corresponding low-voltage potential leads of the standards being compared are measured. The voltages U_1 and U_2 are amplified by a preliminary selective amplifier, and their orthogonal components are measured by the vector voltmeter *VV*. The microcontroller *MC* transmits the results of a measurement to a personal computer *PC*, which, by analyzing these voltages via an interface and a microcontroller, controls the whole measurement and calibration process.

The autotransformer bridge operates in a not entirely balanced mode. The main process in its balancing and measuring is the change in the unbalanced signal [18] by varying the ratio of the number of turns in the arm windings. In this case, the number of turns in the winding m_1 is changed by an amount Δm_v. One then measures the unbalanced signal U_{2v}. The system of equations, which describes the measured voltages U_1, U_2, and U_{2v} is then solved. Using this procedure, one obtains the difference between the actual current and the balancing current of the bridge, which is used for rapid balancing using the leading decades and improving the result of a measurement in the neighborhood of the bridge balancing point using the lowest decade.

The system of equations describing the set of measurements when the number of turns in arm m_1 is varied and the voltage U_{2v} is simultaneously measured, has the form

$$\dot{U}_1 = U_0\left[1 - \frac{Z_c}{Z_d}(1+\delta)\right]\frac{m_1}{m_1 + m_2} - U_0\frac{Z_1}{Z_d};$$

$$\dot{U}_2 = -U_0\left[1 - \frac{Z_c}{Z_d}(1+\delta)\right]\frac{m_2}{m_1 + m_2} + U_0\frac{Z_2}{Z_d};$$

$$\dot{U}_{2v} = U_0\left[1 - \frac{Z_c}{Z_d}(1+\delta)\right]\frac{m_2}{m_1 + m_2 + \Delta m_v} - U_0\frac{Z_2}{Z_d}, \tag{1}$$

where U_0 is the generator voltage; $Z_d = Z_1 + Z_2 + Z_c$; and δ is the error in the transfer constant of the differential follower *DF*.

Solving this system of equations, we obtain corresponding expressions which determine the deviation from relative equilibrium, expressed in the windings or in the relative deviations of the ratio of the measured impedances:

$$\delta Z = -\frac{m_1 + m_2}{m_2}\frac{C + (m_1 - m_2)D/(m_1 + m_2)}{1 + (C + D)\delta_v}\delta_v, \tag{2}$$

where

$$C = \frac{\dot{U}_2 + \dot{U}_1}{\dot{U}_{2v} - \dot{U}_2}; \quad D = \frac{\dot{U}_2 - \dot{U}_1}{\dot{U}_{2v} - \dot{U}_2}; \quad \delta_v = \frac{\delta m_v}{1 + \delta m_v}; \quad \delta m_v = \frac{\Delta m_v}{m_1 + m_2},$$

or $\delta Z = Z_1/Z_2 - m_1/m_2$.

The error in determining the coordinates of the balance point depends only on the sensitivity and nonlinearity of the vector voltmeter and does not exceed 0.01–0.1%. This enables us to balance the bridge in the minimum time up to the fourth decade and to improve the result in the last decade, so that the final result of the measurement is produced in the form of a nine-digit number.

Calibration of the phase shifter. The quadrature phase shifter used for balancing with respect to the second parameter must possess high accuracy: the permissible error is less than $(1–2)\cdot 10^{-6}$. The direct construction of such a phase shifter is a complex technical problem, particularly if it must operate at a number of frequencies. Attempts to solve this "head on" using existing technical facilities lead to instruments with considerable dimensions and mass and high cost. Hence, the phase shifter is constructed in the form of an automatically balanced unit in which the necessary value of the transfer factor is established at each frequency (Fig. 3).

The phase shifter consists of two series-connected sections with a nominal phase shift of 45° and a modulus of the transfer constant of unity (amplifiers *A1* and *A2*), and a precision transformer inverter *Inv*. The transfer constants of the sections in modulus and phase at each frequency are established by regulating the digital-to-analog converters *DAC11*, *DAC12* of the direct connection and *DAC21* and *DAC22* of the feedback, occurring in these sections, shunted by capacitance measures *C1* and *C2*, respectively. This construction of the phase shifter is necessary to obtain a frequency characteristic which ensures its stability at low frequencies when operating in the autotransformer comparator as well as the necessary short-term stability of its parameters. In order to establish the required transfer constant of the phase shifter at each working frequency, a calibrated *RC*-circuit is used, in which the resistor *R* is in the form of *DAC3*. Its transfer impedance is equal to Z_1 while the impedance of the capacitance measure *C* is Z_2.

To balance the bridge, made up of the calibration circuit and the phase shifter with the inverter, the interchange method for alternating current is used [16, 17] and a variational estimate of the error [18]. The orthogonal components of the bridge unbalance signal U_c are measured by the vector voltmeter *VV* (see Fig. 2). The microcontroller *MC* uses the results of

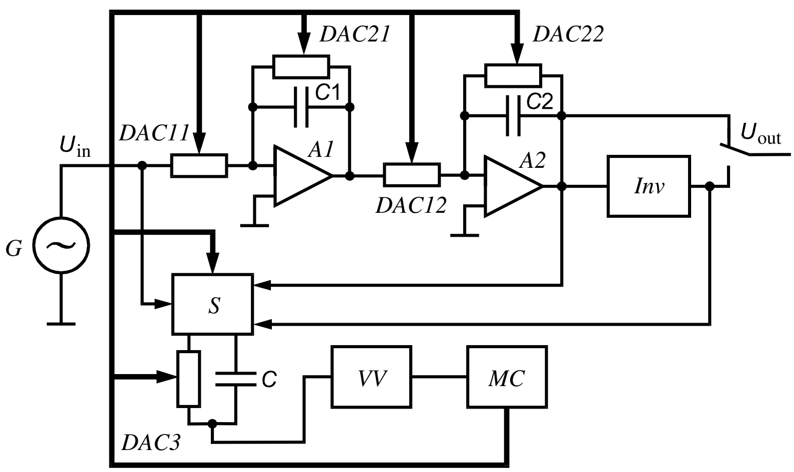

Fig. 3. Block diagram of the quadrature phase shifter: *DAC11*, *DAC12*, *DAC21*, *DAC22*, *DAC3* – digital-to-analog converters; *S* – switch; *A1*, *A2* – amplifiers; *Inv* – inverter.

these measurements to balance the bridge by simultaneous regulation of the transfer constant of the phase shifter and of the calibration circuit.

The algorithm for balancing the bridge and estimating the error of the phase shifter with respect to time is divided into two parts. In the first part, the bridge is balanced by setting the required parameters of the phase shifter and the calibration circuit, while in the second the error in establishing the transfer constant of the phase shifter (the residual error in balancing the calibrated bridge, due to the error in the digital-to-analog circuits employed) is determined. The transfer constant of the phase shifter will then be determined with an error expressed by the product of the error of the balancing DAC, equal to 0.01–0.1%, by twice the error of the vector voltmeter (less than 10^{-4}).

In each of these parts, the same sequence is used. This consists of the following three actions:

1. The calibration circuit of the phase shifter is connected between its output and input so that the capacitor *C* of the calibration circuit is connected to the output. The vector voltmeter measures the unbalanced signal U_{c1} at the output of the calibration circuit.

2. The transfer constant of the phase shifter is varied by a known amount δ_v. The vector voltmeter measured the unbalanced signal U_{c2}.

3. The calibration circuit of the phase shifter is connected between the input of the phase shifter and the output of the inverter so that the resistor *R* (*DAC3*) of the calibration circuit is connected to the output of the inverter. The vector voltmeter measures the unbalanced signal U_{c3}.

These actions are described by the following system of equations:

$$\left.\begin{aligned} &U_0 - \frac{U_0 + jU_0(1+\delta_f)}{Z_1 + Z_2} Z_1 - \dot{U}_{c1} = 0; \\ &U_0 - \frac{U_0 + jU_0(1+\delta_f+\delta_v)}{Z_1 + Z_2} Z_1 - \dot{U}_{c2} = 0; \\ &U_0 - \frac{U_0 - jU_0(1+\delta_f)}{Z_1 + Z_2} Z_2 - \dot{U}_{c3} = 0, \end{aligned}\right\} \tag{3}$$

where U_0 is the input voltage of the phase shifter, δ_f is the error of the transfer constant of the phase shifter, $Z_1/Z_2 = j\omega C_n R_n(1 + \delta_c) = j(1 + \delta_c)$, C_n and R_n are the nominal values of the capacitance and resistance of the calibration circuit ($\omega C_n R_n = 1$), and δ_c is the error in establishing the parameters of the calibration circuit.

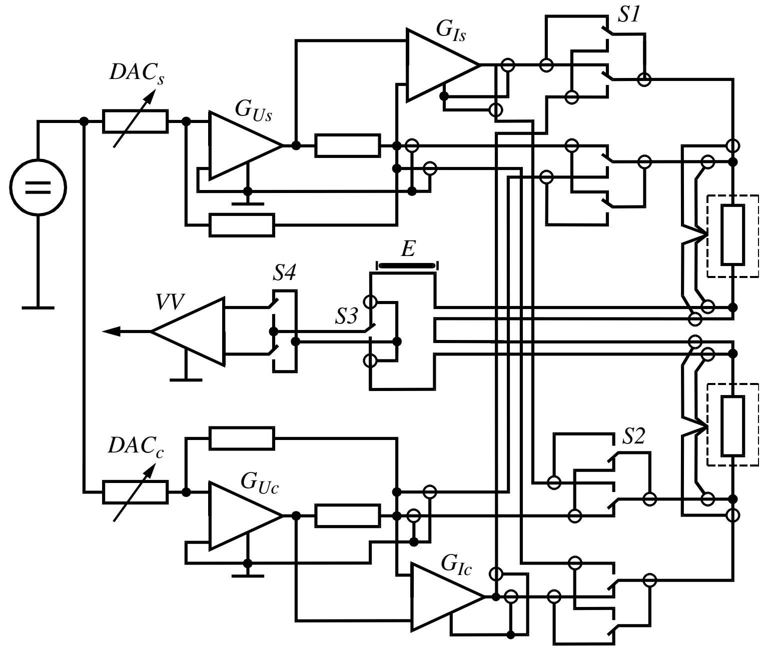

Fig. 4. The quadrature bridge-comparator: G_{Us}, G_{Uc}, G_{Is}, and G_{Ic} – voltage and current generators; DAC_s, DAC_c – digital-to-analog converters; E – equalizer.

Solving this system of equations for δ_c and δ_f, we obtain their exact values. In view of the complexity of the analysis of the expressions obtained, we use the approximate solution of system (3):

$$\delta_f = \frac{\delta_v}{2}\frac{\dot{U}_{c1} + j\dot{U}_{c3}}{\dot{U}_{c2} - \dot{U}_{c1}}; \quad \delta_c = \frac{\delta_v}{2}\frac{\dot{U}_{c1} - j\dot{U}_{c3}}{\dot{U}_{c2} - \dot{U}_{c1}}. \tag{4}$$

As a result of these procedures, the true value of the transfer constant of the phase shifter in this bridge is determined with an error of less than $(1–2)\cdot 10^{-6}$. This is sufficient for an exact measurement of the impedance parameters when tgδ or tgφ change over a wide range.

The divider of the autotransformer bridge is constructed so that its division factors are –0.1, 0, +0.1, +0.2, ..., +1.0. This enables us to transfer the dimensions of the units of standards of any physical quantities of practically any nominal value with the maximum possible accuracy. The division factor of –0.1 is used to reproduce the unit of inductance with respect to the units of capacitance and frequency. The compared impedances of the capacitance and inductance standards are then in the ratio of 10:–1.

The Quadrature Bridge-Comparator. The impedances of the capacitance and resistance standards are compared using a quadrature bridge-comparator [17], a block diagram of which is shown in Fig. 4.

The quadrature bridge consists of two voltage generators G_{Us} and G_{Uc} and two controlled current generators G_{Is} and G_{Ic}. The voltage generators are based on highly stable high-speed DACs and are constructed with an iterational structure. This practically eliminates the effect of the amplifier gains on the accuracy with which the stepwise approximated sinusoidal voltage is generated (the equivalent loop gain at the working frequency exceeds 10^8).

In order to eliminate the effect on the transfer constant of the generators of short-duration instability of DAC_s and DAC_c, the latter were placed in a thermostat with a short-duration temperature instability of less than 10^{-2}°C. Taking into account the temperature coefficient of the DAC, equal to $(1–2)\cdot 10^{-6}$, by keeping them in a thermostat we were able to reduce the instability of the transfer constant to 10^{-8} during the measurements.

To ensure four-terminal connection of the standards in the quadrature bridge, we used two controlled current generators G_{Is} and G_{Ic}. The output signals for these were the voltage drops across resistors connected at the output of the corresponding voltage generators.

The output current of generators G_{Is} and G_{Ic} is used to supply the standards being compared. Here the current consumed from the voltage generator is reduced by a factor equal to the gain of the operational amplifier of the current generator. This means that the equivalent output resistance of the voltage generator (including the resistance of the conductors and the switches) is reduced to values of less than 10^{-6} Ω and thereby enables the corresponding component of the comparison error to be reduced to a negligibly small value (not greater than 10^{-8}) for an impedance of the compared standards of greater than 0.1–1 kΩ.

For the four-terminal connection of the compared standards, we also used an equalizer *E*, which is a transformer, wound on a core of amorphous iron, connected in the current branch of the comparator. The vector voltmeter *VV* is connected in turn to the low-voltage potential terminals of the standards and measures the corresponding unbalance signals. The ratio of the impedances of the compared standards is measured using the following algorithm.

1. The unbalance signal U_1 is measured in the state of the bridge shown in Fig. 4.

2. The voltage of one of the quadrature generators is changed by an amount δ_v and the voltage U_2 is measured.

3. The switch *K3* is switched and a new value of the signal U_3 is measured.

4. The switches *K1* and *K2* are switched to the opposite state, the phase of the voltage at the cosinusoidal output of the quadrature generator is inverted and the voltage U_4 is measured.

Each measurement is carried out with the vector voltmeter connected to the output of the measuring circuit in the forward and reverse positions (paired measurements with reversal of switch *K4*). The unbalance signal U_i is calculated as the half-difference of the results of the paired measurements, which enables the effect of additive interference at the input circuits of the voltmeter to be eliminated without loss of speed. The solution of the system of equations, describing the above set of measurements, gives the ratio of the impedances of the compared standards, so that the error of the ratio of the voltages of the quadrature generators and the effect of the resistance of the connecting wires are eliminated. The simplified solution of the system has the form

$$\delta Z \approx \mathrm{j}\frac{\delta_v}{2}\left[\frac{\dot{U}_4 - \dot{U}_3 + (1+\mathrm{j})\dot{U}_1}{\dot{U}_2 - \dot{U}_1}\right]. \tag{6}$$

Here $\delta Z = Z_1/Z_2 - \mathrm{j}$.

Calibration of the Vector Voltmeter. To measure unbalance signals in the bridges described above, we used a highly selective vector voltmeter with a sensitivity threshold of less than 10^{-5} and an integral nonlinearity of less than $3 \cdot 10^{-5}$, while the degree of suppression of all the harmonic components up to the 128th reaches 10^5. Higher harmonics are additionally suppressed by a selective frequency-tunable filter at the voltmeter input. These parameters of the vector voltmeter enable bridge-comparators to be balanced with high accuracy in those special cases when they are frequency-dependent circuits.

To increase the speed of response of the apparatus, the vector voltmeter was designed with a two-channel structure. The fact that its channels are nonidentical is taken into account when it is periodically calibrated. For this purpose, a generator voltage U_g with a working frequency ω and initial phase φ_0 is applied to the measuring input of the voltmeter, while two sinusoidal or two cosinusoidal voltages U_0 are applied in turn to the reference inputs of its synchronous detectors.

Under these conditions, one measures the orthogonal components U_1, U_2, U_3, and U_4 of the input voltage U_g, which, taking into account the ratio of the moduli of the transfer constants of the channels *K* and the phase delays $\Delta\varphi$ of the detectors, are described by the following system of equations:

$$\left.\begin{aligned} U_1 &= 0.5KU_gU_0\cos\varphi_0; \\ U_2 &= 0.5U_gU_0\cos(\varphi_0 - \Delta\varphi); \\ U_3 &= 0.5KU_gU_0\sin\varphi_0; \\ U_4 &= 0.5U_gU_0\sin(\varphi_0 - \Delta\varphi). \end{aligned}\right\} \tag{7}$$

Solution (7) enables us to obtain the parameters of the vector voltmeter:

$$\tan\Delta\varphi = \frac{U_2U_3 - U_1U_4}{U_1U_2 + U_3U_4}; \quad K = \frac{U_1}{U_2\sqrt{1+\tan^2\Delta\varphi}}\left(1+\frac{U_3}{U_1}\tan\Delta\varphi\right). \tag{8}$$

The values of the components of the error of the vector voltmeter, defined in (8), are later used to correct the results of actual measurements using the formulas

$$U_{s\text{cor}} = U_s / K; \quad U_{c\text{cor}} = U_{c\text{cor}}\tan\Delta\varphi + U_c\sqrt{1+\tan^2\Delta\varphi}, \tag{9}$$

where U_s, U_c, $U_{s\text{cor}}$, and $U_{c\text{cor}}$ are the measured and corrected orthogonal components of the input signal of the vector voltmeter, respectively.

The apparatus described above is functionally capable of reproducing the units of the parameters of impedance and transferring their dimensions, and can also provide a metrological backup for all the parameters of impedance with support, in principle, of only one of the standards of impedance parameters. The apparatus was subjected to metrological investigations at GUM (Poland), NIST (USA), and PTB (Germany). It has been delivered to GUM, NIST, and Belarus, and has been used to construct the State Standard of inductance of the Ukraine. As apparatus for reproducing the unit of inductance, the system has participated in comparisons of inductance, carried out at EUROMET in 2006, where the differences between the results of measurements in the GUM and PTB laboratories amounted to less than $4\cdot10^{-6}$.

Brief Characteristics of the Comparator

Zero shift at the main limits, not greater than	$0.5\cdot10^{-6}$
Error of decimal transfer at the main limits, not greater than	$0.5\cdot10^{-6}$
Sensitivity threshold of the comparator (without averaging), less than	$0.01\cdot10^{-6}$
$C \to L$ transfer error, not greater than	$10\cdot10^{-6}$
Sensitivity threshold of $C \to L$ transfer (without averaging), less than	$0.1\cdot10^{-6}$
Zero shift of the $R \leftrightarrow C$ comparator, not greater than	$1\cdot10^{-6}$
Sensitivity threshold of the $R \leftrightarrow C$ comparator, less than	$0.05\cdot10^{-6}$
Error of direct transmission of the signal of the quantized Hall standard to the 10 kΩ measure-carrier (during the experimental investigations)	$0.2\cdot10^{-6}$
Operating frequencies	1.0 and 1.59 kHz

Theoretical and experimental investigations have shown that the problem mentioned at the beginning of this paper of constructing automatic apparatus to reproduce and transfer the dimensions of the units of the parameters of impedance over a range of values, has been successfully solved.

REFERENCES

1. A. M. Thompson and D. G. Lampard, "A new theorem in electrostatic and its application to a calculable standard of capacitance," *Nature*, **177**, 888–896 (1956).
2. W. K. Clothier, "A calculable standard of capacitance," *Metrologia*, **1**, No. 2, 36–55 (1965).
3. H. Bachmair, "Design criteria for investigation on the PTB calculable cross capacitor," *BNM-LCIE*, 1–27 (1998).
4. G. H. Rainer, "NPL calculable capacitor," *IEEE Trans. Instrum. Meas.*, **1M-21**, 361–365 (1972).
5. A. M. Jeffery et al., "New multi-frequency bridge at NIST," *BNM-LCIE*, G1–G37 (1998).
6. B. D. Josephson, "The discovery of tunnelling supercurrents," *Rev. Mod. Phys.*, **46**, No. 2, 251–254 (1974).
7. A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect*, John Wiley and Sons, New York (1982).

8. K. V. Klitzing, G. Dorda, and M. Pepper, "New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance," *Phys. Rev. Lett.*, **45**, 494–501 (1980).
9. N. E. Fletcher, J. M. Williams, and T. J. B. M. Janssen, "A cryogenic current comparator resistance ratio bridge for the range 10 kΩ to 1GΩ," *Precision Electromagnetic Measurements: Digest Conf.*, Sydney (2000), pp. 482–483.
10. M. E. Bartolome et al., "Resistance bridge based on the cryogenic current comparator in a transport dewar," *IEEE Trans. Appl. Supercond.*, **11**, No. 1, 867–870 (2001).
11. A. M. Thompson, "An absolute determination of resistance based on calculable standard of capacitance," *Metrologiya*, **4**, No. 1, 1–7 (1969).
12. G. Trapon et al., "Determination of the von Klitzing constant R_K in terms of the BNM calculable capacitor – fifteen years of investigations," *Metrologia*, **40**, Iss. 4, 159–171 (2003).
13. S. W. Chua, B. P. Kibble, and A. Hartland, "Comparison of capacitance with AC quantized Hall resistance," *Precision Electromagnetic Measurements: Digest Conf.*, Washington, DC, USA (1998), pp. 418–419.
14. K. B. Karandeev (ed.), *Transformer Measuring Bridges* [in Russian], Energiya, Moscow (1970).
15. B. P. Kibble and G. H. Rayner, *Coaxial AC Bridges*, Adam Hilger Ltd., Bristol (1984).
16. V. S. Guryanov, "A method of obtaining quadrature voltages with equal amplitudes," *Metrologiya*, No. 4, 54–58 (1986).
17. M. N. Surdu et al., "Theoretical principles of the construction of variational quadrature ac bridges," *Izmer. Tekhn.*, No. 10, 58–64 (2006); *Measur. Techn.*, **49**, No. 10, 1043–1052 (2006).
18. F. V. Grinevich and M. N. Surdu, *High-Accuracy Variational AC Measuring Systems* [in Russian], Naukova Dumka, Kiev (1989).