Characterization of a High-Resolution Analog-to-Digital Converter With a Josephson AC Voltage Source

Waldemar G. Kürten Ihlenfeld, Enrico Mohns, Ralf Behr, Jonathan M. Williams, Pravin Patel, Günther Ramm, and Hans Bachmair

***Abstract*—A Josephson ac voltage source was employed to characterize the dynamic behavior of a high-resolution 28-bit (8.5-digit) integrating analog-to-digital converter of a digital sampling voltmeter (DSV), which is widely used in ac metrology at the Physikalisch-Technische Bundesanstalt. Extensive measurements were carried out to validate previous mathematical models of the DSV when sampling ac signals. The characterization method is based on the discrete Fourier transform applied on sampled data of the ADC and on the known Josephson plateau values for quantifying the ratio of ac quantities and the nonlinearities of the DSV. The method shown allows considerable improvement of the accuracy of sampling techniques at low frequencies (dc up to some kilohertz) to be attained.**

***Index Terms*—Analog-to-digital conversion (ADC), discrete Fourier transform (DFT), electric variables measurement, Josephson arrays, synchronous detection.**

I. Introduction

Analog-to-digital converters (ADCs) became common in ac metrology more than 30 years ago. Despite many developments in sampling techniques, a significant and rather recent proposal was made [1], suggesting the use of regularly spaced samples [2] of two ac voltage signals generated synchronously with the internal clock of a high-resolution ADC (DSV). Since then, much effort at the Physikalisch–Technische Bundesanstalt (PTB) has been concentrated on validating uncertainty evaluation models [3] based mainly on comparisons with PTB primary thermal converters (the standards for ac/dc transfer measurements).

Although the theoretical predictions hitherto agreed with experimental results under sinusoidal conditions, further investigations on the dynamic behavior of the high-resolution 28-bit ADC in the PTB system [3], [4] remained essential for assuring the traceability of ac power in the nonsinusoidal regime by considering nonlinearities of the ADC. Thorough investigations were carried out with an ac voltage source employing a digital-to-analog converter (DAC) based on Josephson arrays [5], [6]. This source was operating synchronously with the ADC under test (DSV) as described next.

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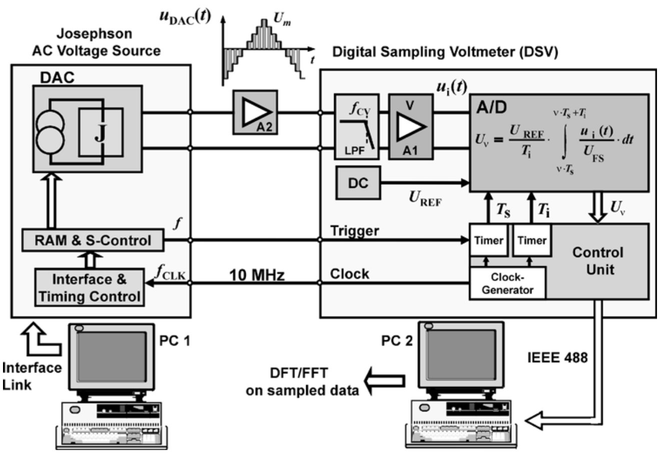

Fig. 1. Block diagram of the system with the ac source represented by a Josephson array biased by a dc current source at the left [5] (controlled by its own computer, PC 1), and the DSV at the right. The quantized ac signal $u_{\mathrm{DAC}}(t)$ is coherent with the clock frequency f_{CLK} of the DSV, which is triggered by the f signal at each period of $u_{\mathrm{DAC}}(t)$. U_{REF} and U_{FS} represent the internal reference and full-scale values, respectively (see text for further details).

II. Description of the Measurement System

As depicted in Fig. 1, a Josephson ac source (on the left) synthesizes an ac signal $u_{\mathrm{DAC}}(t)$ (in this case a step-approximated triangle waveform coherent with the DSV clock f_{CLK}), which is optionally amplified (by A2) and sampled on its quantized plateaus over a time-frame that encompasses an integer number of periods of $u_{\mathrm{DAC}}(t)$.

Leakage on the spectral lines after a discrete Fourier transform (DFT) or fast Fourier transform (FFT) on the sampled data U_{v} (collected and processed by PC2) is thus prevented [2]–[4]. The DSV contains the ADC with its control unit for timing (for setting the sampling and aperture times T_{S} and T_{i} respectively), the analog low-pass input filter (LPF with cutoff frequency f_{CV}) and the signal conditioning amplifier A1 that embodies the ADC's imperfections [gain errors with T_{i} and its nonlinearities resulting the ADC input signal $u_{\mathrm{i}}(t)$] [2]–[4].

Many methods and procedures for characterizing ADCs have been reported [7], [8]. Those based on the DFT/FFT on sampled data are more compatible with the PTB system and were thus preferred [9], [10].

The step-approximated triangle waveform with quantized steps contains multitones [11] (odd harmonics). Since the PTB sampling system is primarily used for the determination of effective values (root-mean-square values) and ratio of DFT/FFT spectral lines (for the measurement of impedances and ac power), the evaluations of the experimental data were directed toward these needs.

III. Measurement Method

The Josephson ac source [5] is capable of generating signals with peak values $U_{\rm m}$ of about 1 V with a resolution of 13 bits. In order to characterize the DSV (a digital multimeter model HP3458A operating in the direct dc sampling mode) not only on the 1 V but also on the 10 V range, the Josephson ac voltage signal was amplified by a dc coupled amplifier with a closed-loop gain of 11. The amplifier (A2 in Fig. 1) is a battery operated highly linear double-stage dc amplifier with 0.1% settling time smaller than 2 μs and output noise voltage density about $50\ {\rm nV/Hz^{1/2}}$. Triangle and sinusoidal waveforms of different frequencies (from 11 Hz up to about 400 Hz) were synthesized with 28 quantized steps that were considered to be "absolutely" accurate (i.e., with negligible uncertainty) and "noise-free" for this purpose. To avoid transients and errors due to the DSV internal analog low-pass filter (with $f_{\rm CV} = 150\ {\rm kHz}$), each quantized step was sampled 40 μs after it was set. Each sample (of a quantized step) corresponds to the mean of 12 measurements (taken over 12 periods of the ac voltage signal) repeated 10 times totalizing 120 single measurements, enabling a noise reduction of about 20 dB. This procedure was repeated for DSV integration times (or aperture time $T_{\rm i}$) varying from 10 μs to 3 ms.

The DSV's errors can be classified in two categories: amplitude and timing errors. Amplitude errors originate, for example, from quantization errors, missing codes, nonlinearity, noise and finite bandwidth. Timing errors arise from time base jitter, trigger uncertainty and latency, aperture-time width and jitter. The use of the technique described in Section II allows timing errors to be nearly neglected. This means that only dynamic errors related to the amplitude of large signals up to the full-scale value of the ADC, i.e., those related to the integral nonlinearity like harmonic distortion will be included with the proposed technique.

The DSV is a highly linear integrating device operating according to the dual-slope principle. From Fig. 1, with $U_{\rm REF} = U_{\rm FS}$ each sampled value $U_{\rm v}$ is impaired by imperfections, which are taken into account by multiplying the integral describing the readout $U_{\rm v}$ (see Fig. 1) by

$$\left(1 + \delta_{\rm REF} + \delta_{\rm G} + \frac{\delta_{\rm LIN} + \delta_{\rm RES}}{|U_{\rm v}|} \cdot U_{\rm FS}\right) \tag{1}$$

where $|U_{\rm v}|$ represents the absolute value of $U_{\rm v}$ under the constraint that all δs are zero. $\delta_{\rm REF}$, $\delta_{\rm G}$, $\delta_{\rm LIN}$, and $\delta_{\rm RES}$ represent deviations from the ideal and are considered as random variables with respect to the internal reference voltage, gain, linearity, and resolution of the sampler [3], [4] respectively. Noise of the sampler is strongly dependent on the aperture (or integration) time $T_{\rm i}$ and is added algebraically to the "noise-free" sample $U_{\rm v}$ (defined by the Josephson plateau). All samples are somewhat correlated among themselves, mainly in respect to gain deviations $\delta_{\rm G}$, which are dependent on $T_{\rm i}$ and embody the integral nonlinearities of the DSV under test. Up to the present time, upper limits describing maximum $\delta_{\rm G}$ values were estimated by other experiments (thermal converters [7] and through calibrations against stable dc sources). Although the statistical model for evaluating uncertainties for the sampled data and derived quantities (rms and ratio of DFT spectral lines, among others) is strongly dependent on the sampling parameters $T_{\rm s}$ and $T_{\rm i}$, it remains thoroughly linear.

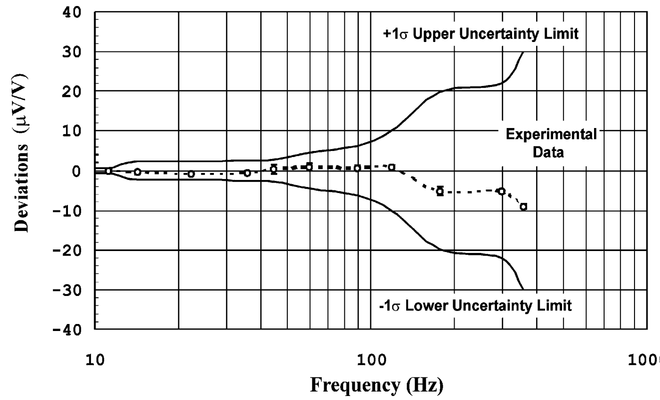

Fig. 2. Deviations in the 10 V range of the computed effective value determined from samples with respect to the true root-mean-square value determined from the quantized steps in the time domain. The measurement standard deviations are also depicted in the figure, although not readily apparent.

IV. Experimental Results

A. Effective or RMS Values

True rms values were calculated from the spectral lines of the DFT with all 28 samples (or 28 steps) at many frequencies of interest. The rms value of the triangle signal synthesized by the Josephson source was calculated in the time domain from the quantized plateaus (determined by $K_{\rm J-90}$ and microwave frequency). The effective values determined from the DSV samples (or from the DFT spectral lines from samples) display deviations between the 1σ-limits predicted by previous modeling [3] for maximum attainable aperture times at all frequencies of interest. This is demonstrated in Fig. 2. The deviations become negative indicating a falloff in gain at the input stage of the ADC, which can be as much as 30 $\mu{\rm V/V}$ for small apertures (10 μs $< T_{\rm i} <$ 100 μs) according to specifications of the manufacturer.

B. Gain Deviations With Aperture Time

Deviations of gain are estimated by the difference between the sampled plateau value and its true value related to the peak value of the input signal. The nonlinearity of amplifier A2 (powered by lead-acid batteries) is much smaller than 10^{-7} so that its influence when measuring the 10 V DSV range can be considered negligible. Fig. 3 shows the variation of gain with aperture times in reference to the gain at $T_{\rm i} = 3\ {\rm ms}$, since only relative deviations are of interest.

Despite the fact that the sampling noise increases as the aperture time is reduced, a strong variability of gain with the input voltage occurs at small apertures near to or around 10 μs. Gain variations may have many causes especially at

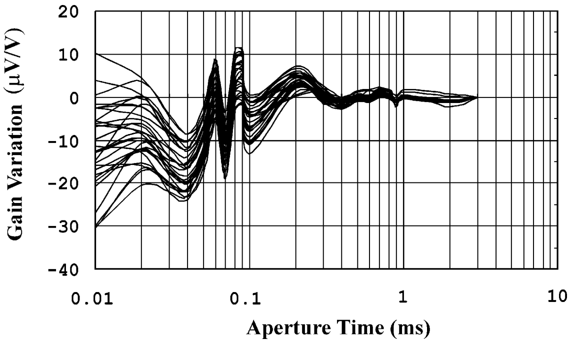

Fig. 3. Relative variation of gain with aperture time for the DSV at the 10 V range sampling Josephson plateaus at a rate of 28 samples of a triangle wave of 11 Hz. The 28 curves display gain variations for each sampled Josephson plateau referenced to the value at the 3 ms aperture time (zero line). The figure shows that strong readout variations may occur at small aperture times, which are also due to a hysteretic behavior. At some integration times (in the range of 40 μs to 100 μs in this figure) it seems that power line coupling and/or some intermodulation with the internal clock signal occur. Many other measurements of this kind were conducted indicating that this pattern changes with frequency, sampling parameters, and DSV range.

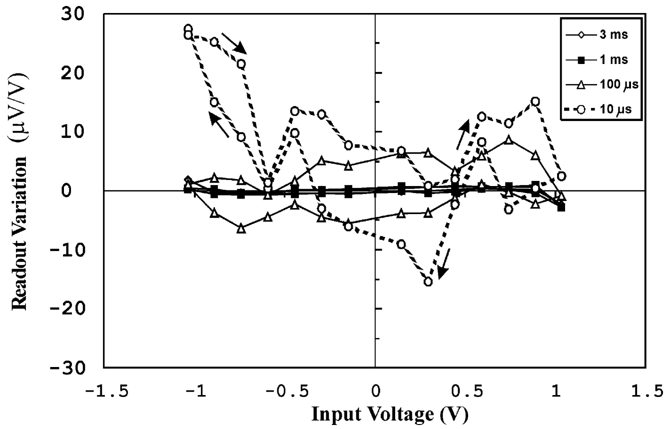

Fig. 4. Hysteretic behavior of the DSV, dependent upon the input voltage and aperture times. The smaller the aperture time, the higher is the hysteretic effect, which is believed to be mainly due to remaining charges in the integrating ADC that are not completely discharged before a new cycle begins. These curves do not resemble the known sigmoid curves of magnetic materials, except those for aperture times higher than 100 μs. Hysteretic behavior causes systematic deviations of phase on the DFT and is an important issue in ac metrology when phase is of concern, for instance, in impedance and power measurement.

high sampling rates, among them are dielectric absorption, polarization of dipoles in the integrator and other effects at the input stages of the ADC. A skeptical speculation about other effects indicates (see Fig. 3) a significant cross talk and intermodulation of internal clock signals in the measurement path that would explain the strong gain variations near some preferential T_{i} values. The same graphic indicates, although somewhat inconspicuously, a hysteretic behavior.

C. Hysteresis

While the Josephson plateaus are time invariant, the readouts (or sampled values U_{v}) do not recover to their original values after a cycle of the input voltage. This phenomenon depends on the sampling parameters, slope, amplitude, and frequency of the input signal. Hysteresis is frequently associated with settling time problems, which however, were not taken into account in these investigations, as previously explained. The hysteretic behavior of the DSV readout is promptly noticeable when the input signal varies from $-U_{\mathrm{m}}$ up to $+U_{\mathrm{m}}$ (see Fig. 4 for an input signal form as shown in Fig. 1). The hysteresis cannot be represented by perfect sigmoid curves. However, hysteresis does not influence amplitudes of the DFT spectral lines but induces phase offsets of the fundamental frequency to appear. Hysteresis of those magnitudes (e.g., for $T_{\mathrm{i}} = 10\ \mu\mathrm{s}$) as shown in the same figure might cause systematic phase deviations as high as some tens of μrads— an important issue when phase measurement is of concern. Fortunately, hysteretic effects can be fully eliminated by using samples that are averages of themselves with their mirrored-values in respect to the peak value of the signal. This previous data treatment, before applying the DFT, is mandatory under stringent accuracy demands rather than using only the original sampled set of raw values.

D. Ratio Measurements and the Determination of Nonlinearities

The ratios of spectral lines (in this case up to the 13th harmonic) were determined from a DFT of both the quantized steps and the DSV samples. The deviations of the DFT from samples with respect to the DFT of the quantized steps were smaller than the predicted 1σ-uncertainty-limits by modeling [4]. The uncertainties of the PTB system are presently determined by a C++ software program that computes all covariances between samples including noise effects. For those uncertainty evaluations, the Josephson uncertainties were considered negligible. The model for evaluating uncertainties is based on the constraint that, once knowing maximum limits of gain errors and nonlinearities of the DSV, all samples are correlated with respect to the gain deviations δ_{G}. This implies that uncertainty estimations were no better than the estimations for maximum gain deviations limits (or nonlinearities) obtained from the manufacturer's specifications of the DSV. The Josephson ac source allows the determination of these limits with much better accuracy.

Assuming a functional $y = TF(x)$ for the DSV that describes its large signal response (readout U_{out}) up to the third order for an input U_{in}, then

$$U_{\mathrm{out}} = TF(U_{\mathrm{in}}) = a \cdot U_{\mathrm{in}} + b \cdot U_{\mathrm{in}}^2 + c \cdot U_{\mathrm{in}}^3. \tag{2}$$

The output of the Josephson ac source (with a resolution of 13 bits) may be approximated by a perfectly linear triangle waveform, since for sampling purposes, one can not infer (just from the set of sampled data) whether the synthesized voltage signal is or is not continuous. In this sense, the sampled Josephson voltage (normalized by its peak value) can be approximated by

$$U_{\mathrm{in}} = \sin(A) + \frac{1}{9} \cdot \sin(3A) + \frac{1}{25} \cdot \sin(5A) + \ \ldots \tag{3}$$

where A represents an immaterial time-dependent phase relationship. Therefore, the DSV output contains even and odd harmonics due to nonlinearities of the DSV transfer function [11].

TABLE I
EXAMPLE OF APPLICATION OF THE PROPOSED METHOD FOR DETERMINING THE ADC'S INTEGRAL NONLINEARITY USING RATIOS OF THE DFT SPECTRAL LINES

Frequency	DFT ratio	Estimated Coefficients
0	NA	a = 0,999 955 9 ± 13E-6
59,5238	0,999967514	b = (9 ± 5) E-6
119,047619	NA	c = (17 ± 15) E-6
178,5714286	0,999939723	
238,0952381	NA	
297,6190476	1,000027892	
357,1428571	NA	
416,6666667	1,000268975	
476,1904762	NA	
535,7142857	0,999907832	
595,2380952	NA	
654,7619048	0,99972421	
714,2857143	NA	
773,8095238	0,999964741	
	NA : Not Available	
	Real DFT 2nd*	
	3,33E-06	

* Real DFT 2nd stands for the real part of the second harmonic component of the DFT.
The results are shown not truncated only for illustration purposes;
significant digits beyond the sixth digit after the comma do not have any physical meaning.

Inserting (3) (input up to the fifth harmonic) into (2) and by expanding and grouping the harmonic components

$$U_{\text{out}} = \frac{51\,331}{101\,250}b - \frac{173}{450}b \cdot \cos(2A) + \text{other even harmonics} + \left(a + \frac{15\,329}{22\,500}c\right) \cdot \sin(A) + \left(\frac{a}{9} - \frac{64\,013}{607\,500}c\right) \cdot \sin(3A) + \text{other odd harmonics}. \tag{4}$$

Dividing every coefficient by the corresponding harmonic coefficient of (3), a system of equations depending only on ratios of spectral components is obtained, which allows the coefficients of TF to be calculated

$$\left\{\begin{array}{l} a + \frac{15\,329}{22\,500}c = r_1; \\ \frac{173}{450}b = \mathrm{Re}\left\{DFT_2\left[U_{\text{out}}\right]\right\}; \\ a - \frac{64\,013}{67\,500}c = r_3; \end{array}\right\} \tag{5}$$

where the r_{i} represent the ratios of the spectral component of the DFT of samples to that of the Josephson steps at the first and third harmonic, respectively, and $\mathrm{Re}\{DFT_2[U_{\text{out}}]\}$ represents the real part of the second harmonic of the DFT of samples from the DSV. By solving these simple equations, the large signal nonlinearities of the DSV are readily determined. Gain deviations at the fundamental frequency are thus associated with coefficient a. Table I illustrates the use of (5) for estimating the nonlinearity (NL) coefficients from DFT ratio measurements with the Josephson ac source synthesizing a triangle waveform of 59.52 Hz. Samples were taken with T_{s} equal to 600 μs, T_{i} equal to 10 μs, and using the 1 V DSV range. The expected uncertainties for the coefficients a, b, and c (for one cycle) are also indicated.

V. CONCLUSION

The Josephson ac source allows the dynamic characterization of a high-resolution ADC. The experiments conducted with this source yielded valuable information about many dynamic ADC error sources and corroborate theoretical models for uncertainty evaluations of the PTB ac sampling technique, which can be improved even further. Once nonlinearities are determined, they can be taken into account to correct for systematic deviations of each sample, improving measurement accuracy.

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