Nonlinear Circuit Model for Discontinuity of Step in Width in Superconducting Microstrip Structures and Its Impact on Nonlinear Effects

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***Abstract*—Superconducting materials are known to exhibit nonlinear effects and to produce harmonic generation and intermodulation distortion in superconductive circuits. In planar structures, these nonlinearities depend on the current distribution on the strip which is mainly determined by the structure of the device. This paper investigates the current distribution at the step-in-width discontinuity in superconducting microstrip transmission lines, which is computed by a numerical approach based on a 3-D finite-element method. This current distribution is used to obtain the parameters of the nonlinear circuit model for the superconducting microstrip step-in-width discontinuity. The proposed equivalent nonlinear circuit can be solved using the harmonic balance method. Examples of two superconducting structures which contain the steps in width are given and validated by comparison with electromagnetic full-wave results. The proposed model can be used for effective optimization of the superconducting microwave filter resonators in order to minimize their nonlinear distortions.**

***Index Terms*—Circuit modeling, current distribution, nonlinearity, step-in-width discontinuity, superconducting microstrip.**

I. Introduction

Superconducting microwave devices have found a niche in communication systems. Microwave filters [1], [2], duplexers [3], delay lines, antennas [4], and couplers [5] are the most important applications of the superconductor materials in high-performance microwave devices. These modules are implemented in microstrip form usually involving a number of discontinuities like the step in width.

The nonlinear behavior of superconducting microwave devices, due to the dependence of the surface impedance on the applied field [6], [7], limits the power handling capability of these devices. This leads to restriction of possible applications of the superconductors in microwave modules. For example, intermodulation distortion (IMD) is a serious limitation in the use of superconducting microwave filters in communication systems [8], [9]. These limitations might be eased if engineers could reliably predict the nonlinear effects in their designs. A number of nonlinear models of superconducting microwave transmission lines (TLs) [10]–[12], superconducting bends [13], and also superconducting coupled lines [14] have been introduced. For the analysis of nonlinearities in superconducting microstrip discontinuities, mainly, numerical methods are used. Therefore, a simple and accurate-enough nonlinear circuit model of discontinuities in superconducting microwave structures can decrease time and memory usage of numerical techniques for the nonlinear analysis. One of the most often used discontinuities in the microstrip structures is the step in width. An accurate nonlinear model of this discontinuity, particularly the current distribution in the cross section, is thus needed to predict the nonlinear behavior of superconducting microwave devices. There are two theories for the modeling of nonlinearities in superconductors: the local [6] and the nonlocal theories [15], [16]. Both theories model nonlinearities in superconductors in the different ways of the dependence on the current. In this paper, we apply the local approach to the modeling of the superconductor materials.

In this paper, the current distribution for the discontinuity of step in width in the superconducting microstrip TLs (MTLs), based on a 3-D finite-element method (FEM) (3D-FEM), is computed. This approach is used to obtain the nonlinear circuit model of the superconducting microstrip step-in-width discontinuity. The developed model can be used for the prediction of nonlinear behaviors of the superconducting microstrip structures through nonlinear analysis of the superconducting MTLs using the harmonic balance (HB) method. As an example, a superconducting MTL containing one step in width is analyzed, and then, its IMD and third-order harmonic (H3) generation are calculated at two different temperatures versus the width ratio of the step. Additionally, a microstrip superconducting fifth-order low-pass filter (LPF) made of yttrium barium copper oxide (YBCO) on a $LaAlO_3$ substrate with six steps in width is analyzed. Then, linear and nonlinear contributions of the discontinuities in its overall linear and nonlinear performance are determined. To validate the accuracy of the proposed approach, we compare some calculated results with results of the analysis by a full-wave electromagnetic method based on 3D-FEM. The proposed model is found to be very useful for optimizing the resonators of the superconducting microwave filters in order to minimize their nonlinear distortions.

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The organization of this paper is as follows. In Section II, the nonlinearity in the superconducting MTLs is introduced. Section III presents the proposed nonlinear circuit model of the step-in-width discontinuity. Section IV is dedicated to numerical computations of the current distribution and calculations of the values of the circuit model elements. In Section V, the proposed circuit model is discussed and used for the prediction of nonlinearity in a sample LPF. Finally, conclusions are presented in Section VI.

II. Nonlinearity in the Superconducting MTLs

A superconductor is intrinsically nonlinear due to the dependence of its superfluid density (n_s) on the current density (j) at finite temperatures below the critical temperature T_c. The relative variations of n_s caused by j can be introduced by a nonlinearity function $f(T,j)$ which is defined as [6]

$$f(T,j) = \frac{n_s(T,0) - n_s(T,j)}{n_s(T,0)}. \tag{1}$$

According to [6], $f(T,j)$ can be described for medium current levels by

$$f(T,j) = b_\theta(T)\left(\frac{j}{j_C}\right)^2 \tag{2}$$

where j_C is the critical current of the superconducting thin film and $b_\theta(T)$ is a coefficient which depends on the direction of the superfluid flow and the temperature. Hence, the dependence of σ_1 (the real part of the conductivity in the superconducting state) and the penetration depth (λ) on the current density can be given by [17]

$$\sigma_1(T,j) = \sigma_1(T,0)\left[1 + a(T)f(T,j)\right] \tag{3}$$

$$\lambda^2(T,j) = \lambda^2(T,0)\left[1 - f(T,j)\right]^{-1} \tag{4}$$

where the function $a(T)$ can be calculated by

$$a(T) = \left[\left(\lambda(T,0)/\lambda(0,0)\right)^2 - 1\right]^{-1}. \tag{5}$$

The presented equations are used to define the nonlinear circuit model of the superconducting MTLs. The general configuration of the superconducting MTL is shown in Fig. 1, for which we use the nonlinear distributed circuit model given in Fig. 2 [11]. In fact, the nonlinearity is embedded in the inductance and resistance per unit length. In this model, we have

$$L(T,i) = L_0(T) + \Delta L(T,i) \qquad R(T,i) = R_0(T) + \Delta R(T,i) \tag{6}$$

where i is the total current in the cross section of the MTL and R_0 and L_0 are current-independent terms. For the quadratic case of nonlinearity, which occurs for relatively small currents, based on that in [18], we have

$$\Delta L(T,i) = \Delta L_q(T)\cdot i^2 \qquad \Delta R(T,i) = \Delta R_q(T)\cdot i^2 \tag{7}$$

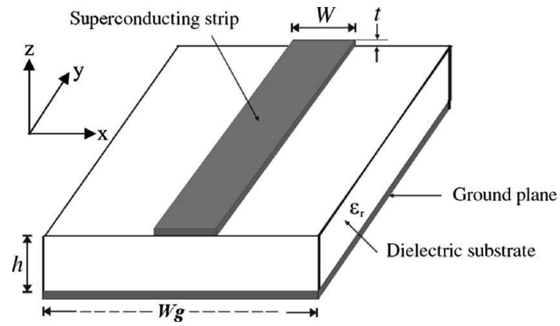

Fig. 1. General configuration of the superconducting MTL.

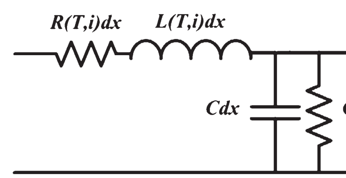

Fig. 2. Nonlinear circuit model for an elemental cell of a superconducting MTL.

in which

$$\Delta L_q(T) = \frac{\mu_0\lambda^2(T,0)}{j^2_{\mathrm{IMD}}(T)}\Lambda(T) \tag{8}$$

$$\Delta R_q(T) = \sigma_1(T,0)\omega^2\mu_0^2\lambda^4(T,0)\frac{2+a(T)}{j^2_{\mathrm{IMD}}(T)}\Lambda(T) \tag{9}$$

where

$$j_{\mathrm{IMD}}(T) = \frac{j_C}{\sqrt{b_\theta(T)}}. \tag{10}$$

The parameter $\Lambda(T)$ in (8) is the geometrical nonlinear factor (GNF) and can be written as follows [18]:

$$\Lambda(T) = \frac{\int j^4 dS}{\left(\int j dS\right)^4} \tag{11}$$

where the integration is made over the MTL's cross section. It is well known that the current distribution in the thin-film superconducting MTLs can be written as in the following [6]:

$$j(x,z) = \begin{cases} \frac{j(0)}{\sqrt{1-(2x/W)^2}}, & |x| < \frac{W}{2} - \frac{\lambda^2}{t} \\ \frac{j(0)\sqrt{Wt}}{2\lambda}, & \frac{W}{2} - \frac{\lambda^2}{t} < |x| \le \frac{W}{2}. \end{cases} \tag{12}$$

We propose a closed-form expression of the GNF for any superconducting MTL with uniform configuration shown in Fig. 1. The formulation can be written in an analytical form as

$$\Lambda(T,W) = \frac{\left[\frac{W^2 t}{8\lambda^2} + \frac{\frac{W^3}{2} - \frac{\lambda^2 W^2}{t}}{W^2 - 4\left(\frac{W}{2} - \frac{\lambda^2}{t}\right)^2} + \frac{W}{2}\operatorname{arctanh}\left(1 - \frac{2\lambda^2}{Wt}\right)\right]}{t^3\left[W\arcsin\left(1 - \frac{2\lambda^2}{Wt}\right) + \lambda\sqrt{\frac{W}{t}}\right]^4} \tag{13}$$

which depends on the temperature and the linewidth. Therefore, all parameters of this nonlinear lumped-element model for any simple superconducting MTL are obtained analytically.

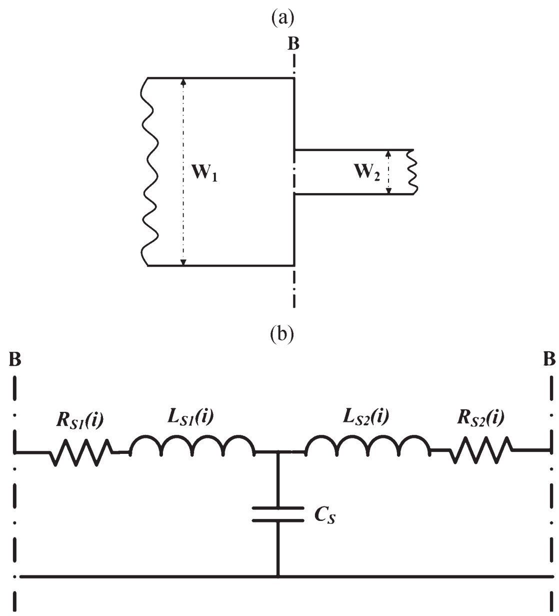

Fig. 3. (a) Step in width. (b) Proposed nonlinear circuit model.

However, if the MTL consists of some discontinuities like the step in width, those discontinuities must be further modeled as well.

III. Nonlinear Circuit Model for Step in Width

We propose a nonlinear circuit model for the step-in-width discontinuity in superconducting MTLs. The model is inferred by an analogy to the linear model of step in width in the microstrip structures, in which a step in width is modeled by two series inductances and a shunt capacitance [19], [20]. Using the nonlinear model of a superconducting MTL, as descended in Section II, we propose a nonlinear circuit model as shown in Fig. 3, where

$$L_{Sk}(i) = L_{Sk0} + \Delta L_{Sk}(i) \qquad R_{Sk}(i) = \Delta R_{Sk}(i) \quad (14)$$

in which

$$\Delta L_{Sk}(T,i) = \Delta L_{Skq}(T)\cdot i^2 \qquad \Delta R_{Sk}(T,i) = \Delta R_{Skq}(T)\cdot i^2 \quad (15)$$

where

$$\Delta L_{Skq}(T) = \frac{W_k \mu_0 \lambda^2(T,0)}{2 j_{\mathrm{IMD}}^2(T)} \Lambda_{Sk}(T, W_{2N}, W_R) \quad (16)$$

$$\Delta R_{Skq}(T) = \sigma_1(T,0)\omega^2 \mu_0^2 \lambda^4(T,0) W_k \frac{2 + a(T)}{2 j_{\mathrm{IMD}}^2(T)} \times \Lambda_{Sk}(T, W_{2N}, W_R) \quad (17)$$

where $k = 1$ and $k = 2$ are for the wide and narrow sides of the step, respectively, W_R is the width ratio ($W_R = W_1/W_2$), and W_{2N} is the normalized width of the narrow side of the step ($W_{2N} = W_2/W_0$), where W_0 is the width of a 50-Ω TL.

The parameters Λ_{S1} and Λ_{S2} are GNFs for both sides of the step in width and can be written as

$$\Lambda_{Sk}(T, W_{2N}, W_R) = \left\langle \frac{\int j_k^4 dS}{\left(\int j_k dS\right)^4} \right\rangle_{W_k/2} - \Lambda(T, W_k) \quad (18)$$

in which

$$\langle f(x) \rangle_T = \frac{1}{T} \int_T f(x) dx \quad (19)$$

and j_1 and j_2 are the current distributions in the wide and narrow sides of the step in width, respectively. j_1 and j_2 should be computed through averaging on $W_k/2$ ($k = 1, 2$) from the step boundary in order to calculate the GNFs in the two sides of the discontinuity, which will be given in the next section. Additionally, the linear terms of L_{S10}, L_{S20}, and C_S are extracted from the linear circuit model of the microstrip step-in-width discontinuity from [19], which are modified for the superconducting case accordingly as shown in the following:

$$L_{S10} = \frac{L_{w1}}{L_{w1} + L_{w2}} L \qquad L_{S20} = \frac{L_{w2}}{L_{w1} + L_{w2}} L. \quad (20)$$

Magnetic and kinetic inductances can be written as

$$L_{wi} = \frac{\mu_0 h}{W_i K(W_i)} + \frac{X_s}{\omega W_i K(W_i)} = \frac{\mu_0}{W_i K(W_i)} \left\{ h + \lambda \coth(t/\lambda) \right\}, \qquad i = 1, 2 \quad (21)$$

$$L = 0.000987 h \left(1 - \frac{L_{w1}}{L_{w2}}\right)^2 \quad (\mathrm{nH}). \quad (22)$$

The shunt capacitance can be calculated as in the following:

$$C_S = 0.00137 h \frac{K(W_1)\sqrt{\varepsilon_{re1}}}{Z_{C1}} \left(1 - \frac{1}{W_R}\right) \times \left(\frac{\varepsilon_{re1} + 0.3}{\varepsilon_{re1} - 0.26}\right) \left(\frac{W_1/h + 0.26}{W_1/h + 0.8}\right) \quad (\mathrm{pF}) \quad (23)$$

where Z_{C1} and ε_{re1} are the characteristic impedance and effective dielectric constant in the wider side of the step, respectively, and $K(W)$ is the fringing factor [21] given in

$$K(W) = \begin{cases} \left(\frac{1}{2\pi} \ln\left(\frac{8h}{W} + \frac{W}{4h}\right)\right)^{-1} \frac{h}{W}, & W \le h \\ \left(\frac{W}{h} + 2.42 - 0.44\frac{h}{W} + \left(1 - \frac{h}{W}\right)^6\right) \frac{h}{W}, & W \ge h. \end{cases} \quad (24)$$

Therefore, all the elements of the proposed model can be analytically calculated except the GNFs, for which we propose a closed-form expression in Section IV.

IV. Numerical Computations

A. Current Distribution

To compute the current distribution in the superconducting microstrip structures, we solve the London equations using 3D-FEM based on a commercial FEM simulator [22]. In fact, we

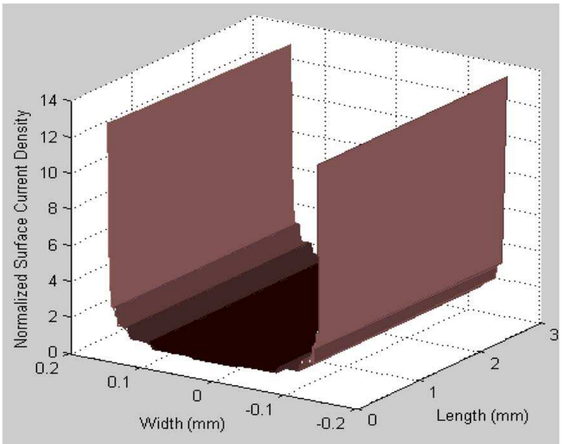

Fig. 4. Normalized current distribution in a simple superconducting MTL for $W = 0.34$ mm.

solve the Maxwell equations in all regions of the structure and consider the superconductor region as an environment with a complex permittivity as follows [23]:

$$\varepsilon_{Sc} = \varepsilon_0 - \frac{1}{\omega^2 \mu_0 \lambda^2} - j\frac{\sigma_1}{\omega}. \tag{25}$$

The computed normalized current distribution (i.e., $j(0, z) = 1$) in a simple superconducting MTL is displayed in Fig. 4, and it shows good agreement with the well-known formulation in (12). We assume that the current distribution is uniform in the z-direction, which is nearly accurate for thin-film superconducting MTLs with a thickness of λ. The superconducting microstrip structures in this paper are assumed to be made of a 400-nm YBCO thin film deposited on a 1-mm-thick crystalline $LaAlO_3$ substrate.

The computed current distribution of the step-in-width discontinuity with different width ratios and $W_2 = 0.3$ mm is shown in Fig. 5. The distribution of the current density close to the step position is definitely different from that of an ordinary TL. Normalization means that $j(0, z)$ is set to one at the end of the narrow line ($x = 3$ mm). Numerical computations of the current density were performed at a frequency of 1 GHz. However, this is not an important factor because the current distribution is approximately frequency independent. As can be observed in Fig. 5, the current density has peaks at two internal corners of the step in width, significantly increasing the nonlinear behavior of the superconducting structure. In Fig. 6, the computed peaks of the current density in the cross section for different width ratios are presented. The peaks of the current density increase with increasing of the width ratios. Moreover, the level of these peaks also depends on the normalized width of the narrow side of the step (W_{2N}). Similarly, for W_R, the level of the current peaks, in the two mentioned corners, is decreased by the reduction of W_{2N}.

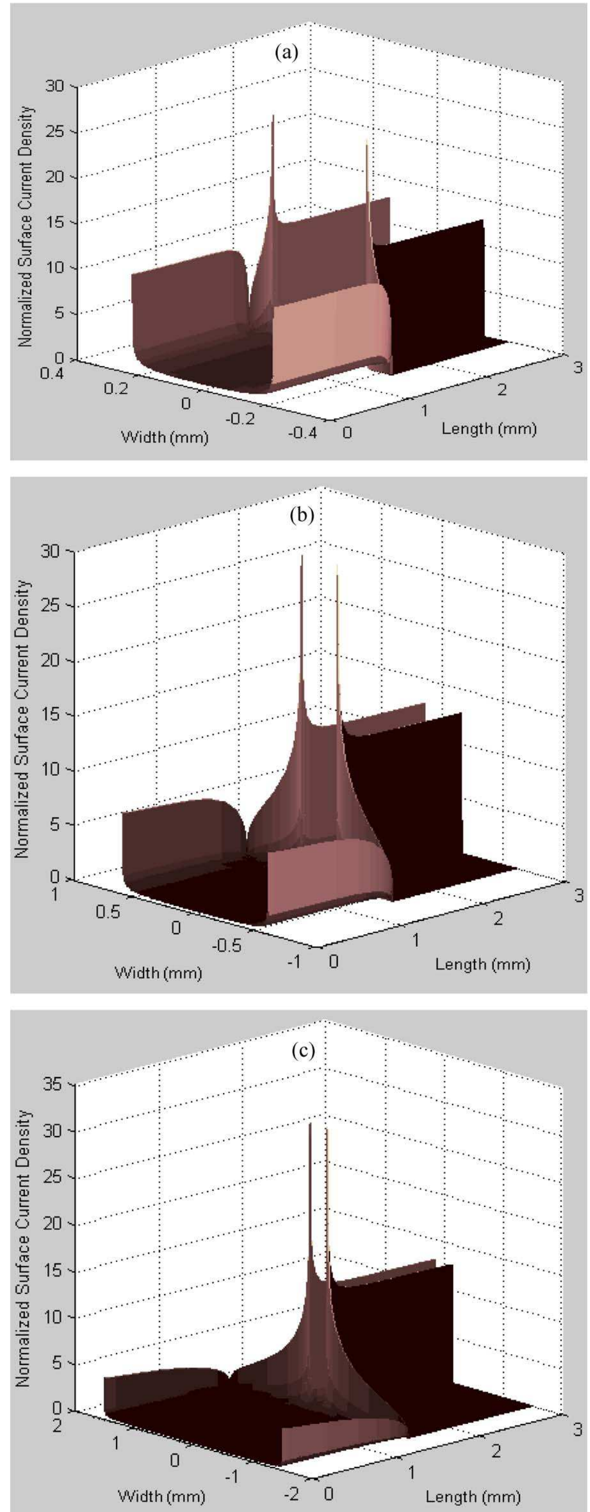

Fig. 5. Normalized current distribution in a step in width with $W_2 = 0.3$ mm and (a) $W_R = 1.5$, (b) $W_R = 4$, and (c) $W_R = 10$.

B. Calculation of GNF of Step in Width

To calculate the GNFs of the step, we used the numerically computed current distributions and calculated GNFs for different cases of the step in width in (18). We consider the current distribution in the area with lengths of $W_1/2$ toward the W_1 side and $W_2/2$ toward the W_2 side, as taken in (18). Two closed-form expressions for the GNFs in both sides of the discontinuity are obtained by curve fitting. These analytical relations can be used for fast and efficient computation of nonlinearity of superconducting structures.

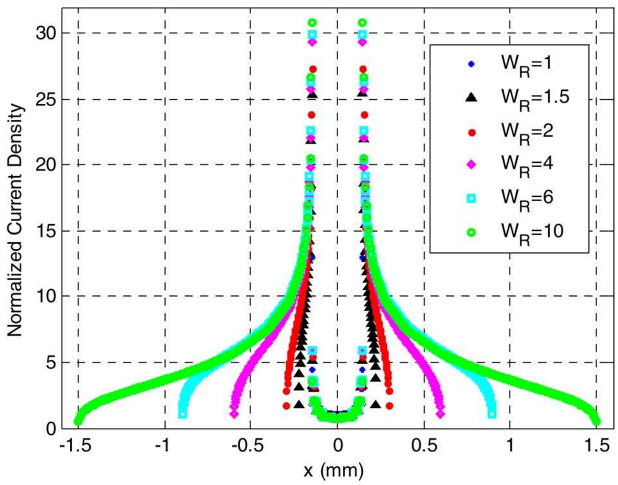

Fig. 6. Normalized current distribution in the cross section of the step in width for variation of W_1 and $W_2 = 0.3$ mm. The center of the linewidth is assumed at $x = 0$.

For the GNF of the wide side of the discontinuity, i.e., W_1 in Fig. 3, the closed-form formula can be written as in the following:

$$\tilde{\Lambda}_{S1}(W_{2N}, W_R) = P_1 \exp(-0.36 W_R) W_R^{0.25} + P_2 W_R^{1.2} - P_3 \tag{26}$$

where

$$\tilde{\Lambda}_{S1}(W_{2N}, W_R) = \frac{\Lambda_{S1}(T, W_{2N}, W_R)}{\Lambda(T, W_1)}. \tag{27}$$

Other parameters in the proposed closed-form expression are given as follows:

$$\begin{aligned} P_1 &= 185.8 W_{2N}^{4.125} + 35.36 \\ P_2 &= 17.25 W_{2N}^{4.09} + 5.132 \\ P_3 &= 147 W_{2N}^{4.153} + 29.84. \end{aligned} \tag{28}$$

Fig. 7 shows the variation of the GNF for the wide side of the step-in-width discontinuity (Λ_{S1}) relative to the GNF of a uniform TL with a width of W_1 (Λ) as a function of W_R and W_2 in both numerically computed and closed-form cases. As shown in the figure, Λ_{S1} is relatively higher than the GNF of an ordinary TL, and therefore, it will have considerable impact on the nonlinear behavior of the structure. Additionally, for the other side of the step, the closed-form equation can be written as

$$\tilde{\Lambda}_{S2}(W_{2N}, W_R) = Q_1 \log(Q_2 W_R) W_R^{Q_3} - Q_4 W_R^{-3.4} - 1 \tag{29}$$

where

$$\tilde{\Lambda}_{S2}(W_{2N}, W_R) = \frac{\Lambda_{S2}(T, W_{2N}, W_R)}{\Lambda(T, W_2)} \tag{30}$$

$$\begin{aligned} Q_1 &= -5.395 W_{2N}^{0.489} + 7.574 \\ Q_2 &= 25.57 W_{2N}^{3.521} + 2.803 \\ Q_3 &= 0.1577 W_{2N}^2 + 0.2201 W_{2N} - 0.299 \\ Q_4 &= 2.306 W_{2N}^{0.5093} + 0.0587. \end{aligned} \tag{31}$$

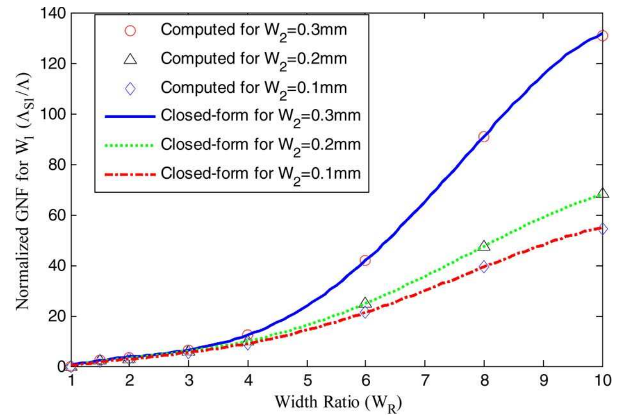

Fig. 7. Variation of Λ_{S1} for the step-in-width discontinuity relative to the GNF of a TL with a width of W_1 as a function of W_R and W_2 and $W_0 = 0.344$ mm.

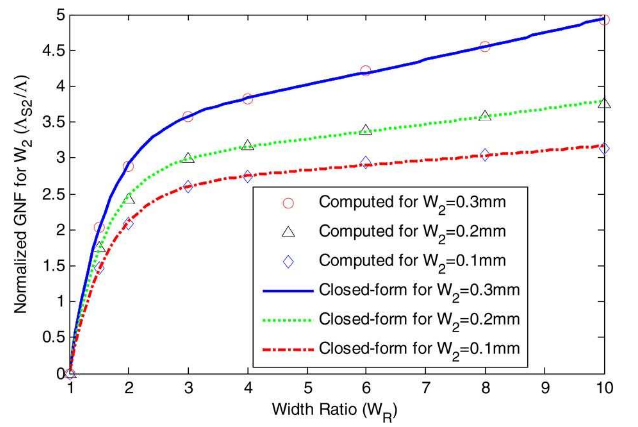

Fig. 8. Variation of Λ_{S2} for the step-in-width discontinuity relative to the GNF of a TL with a width of W_2 as a function of W_R and W_2 and $W_0 = 0.344$ mm.

Fig. 8 shows the relative computed GNF for the narrow side of the step in width for different cases resulting from numerical computation and proposed closed-form expressions. Both Λ_{S1} and Λ_{S2} are functions of W_R and the normalized narrow width (W_{2N}).

V. Results and Discussion

We have analyzed the step-in-width discontinuity in the superconducting microstrip structures. A nonlinear lumped-element model for the prediction of nonlinearity has been proposed, to which all model elements can be calculated analytically. Therefore, using the HB method with this model, the nonlinear effects of the superconducting structure can be predicted. At first, we analyzed a simple step in width as shown in Fig. 9(a). Fig. 9(b) displays the value of the relative power of the third-order IMD at the upper sideband and for $\Delta f = 500$ kHz and the H3 at two different temperatures of $T = 20$ K and $T = 77$ K versus the width ratio for $W_2 = 0.3$ mm, $f_0 = 4$ GHz, and $P_{\text{in}} = 20$ dBm. According to [6], in (2), we set $b_\theta(T) = 6$ at $T = 77$ K and $b_\theta(T) = 1.6$ at $T = 20$ K, because, based on that in [24], we regard YBCO predominately having $d_{x^2-y^2}$ gap. As it can be seen in Fig. 9(b), the signal levels of both the IMD and H3 are considerably decreased by the reduction of the temperature from 77 K to 20 K.

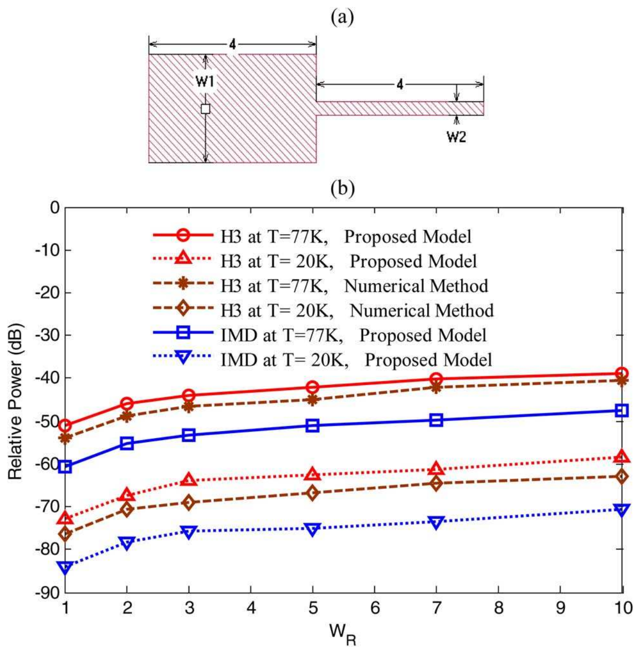

Fig. 9. (a) Step-in-width structure (dimensions in millimeters). (b) Power of the IMD (at the upper sideband and for $\Delta f = 500$ kHz) and H3 relative to the input power for $T = 20$ K and $T = 77$ K versus the width ratio for $W_2 = 0.3$ mm, $f_0 = 4$ GHz, and $P_{\rm in} = 20$ dBm. For validation, the amounts of H3 for both temperatures are also displayed with full-wave numerical analysis based on 3D-FEM.

Additionally, we used the numerical full-wave electromagnetic analysis based on 3D-FEM using a commercial FEM simulator [22] for the validation of the proposed approach. We numerically compute the H3 generation in the structure in Fig. 9(a). To achieve this goal, the superconducting material is defined, described in (25), with both parameters λ and σ_1, which are the functions of the current density according to (3) and (4). Then, in a time-domain simulator environment, a sinusoidal wave is stimulated in the input port of the considered structure of the superconducting MTL in Fig. 9(a). The output signal can be obtained in the time-domain regime with an iterative solution of the Maxwell equations. Finally, the power level of the H3 can be calculated with fast Fourier transform of the output signal. Both of the proposed model and the numerically calculated power levels of the H3 in the superconducting MTL with one step in width at two temperatures of 20 K and 77 K are displayed in Fig. 9(b). Good agreement between numerical and model results confirms the accuracy of the here-proposed model.

As another example, a superconducting LPF, as shown in Fig. 10(a), is considered for nonlinear analysis. All dimensions given in the figure are in millimeters. This LPF contains seven TLs and six discontinuities of step in width. Therefore, same as the previous structure, the equivalent circuit of this LPF can be nonlinearly analyzed by the HB approach. Fig. 10(b) displays both the frequency response of the mentioned filter and the calculated amount of the upper sideband third-order IMD for $\Delta f = 500$ kHz and $P_{\rm in} = 10$ dBm for each signal at $T = 77$ K. In fact, in each input frequency of f_0, we activate two signals with frequencies of $f_0 - \Delta f/2$ and $f_0 + \Delta f/2$, and we calculate the level of the signal at a frequency of "$f_0 + 3/2\Delta f$." Additionally, the relative amount of this calculated IMD, i.e.,

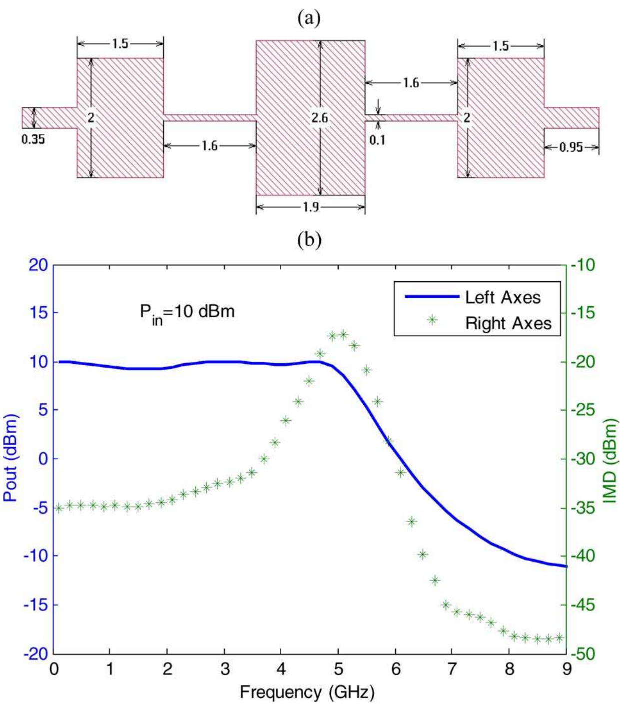

Fig. 10. (a) Designed LPF. (b) (Left axis) Filter response and (right axis) calculated values for IMD at the upper sideband in the LPF for $T = 77$ K, $\Delta f = 500$ kHz, and $P_{\rm in} = 10$ dBm.

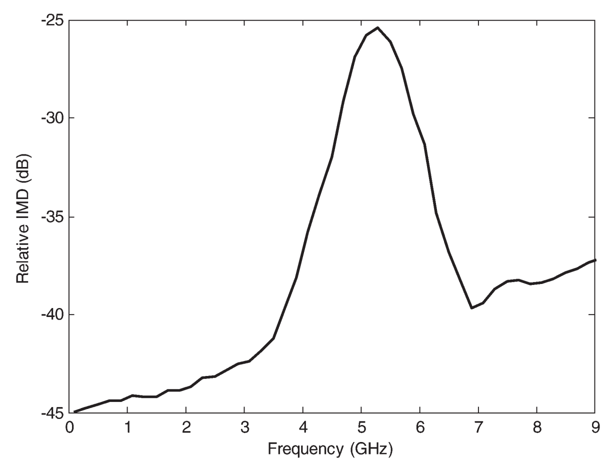

Fig. 11. Relative IMD (in decibels) of the LPF (IMD power to power of the main signal) for IMD at the upper sideband in the LPF for $\Delta f = 500$ kHz and $P_{\rm in} = 10$ dBm.

the IMD relation to the fundamental signal, is shown in Fig. 11, in which it can be seen that IMD increases with frequency and, in the cutoff frequency of the LPF, has a considerable peak. The input-power dependence of the amplitude of the fundamental signal and the third-order upper sideband intermodulation signal in the mentioned LPF at its cutoff frequency, i.e., $f_0 = 5.2$ GHz, is shown in Fig. 12. As expected, the fundamental signal has a slope of one and the IMD has a slope of three, and the intercept point is at about $P_{\rm in} = 23$ dBm.

VI. Conclusion

In this paper, an analytical nonlinear lumped-element model of step-in-width discontinuity in superconducting microstrip

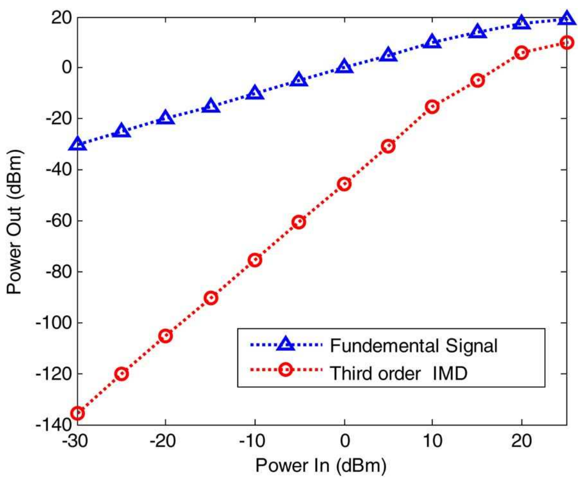

Fig. 12. Input-power dependence of the amplitude of the (triangles) fundamental signal and the (circles) third-order IMD signal in the mentioned LPF at $f = 5.2$ GHz and for $T = 77$ K and $\Delta f = 500$ kHz.

structures has been proposed. To calculate the nonlinear part of the inductance and resistance of the model, we used 3D-FEM to compute the current distribution in step in width. Then, by curve fitting, two closed-form equations of GNFs for both sides of the discontinuity have been proposed. The circuit model was used for HB analysis of the step-in-width discontinuity of an LPF to predict nonlinear distortions. Results show that the discontinuity of step in width has considerable impact on the nonlinear behavior. We inferred that the nonlinearity impacts of a step in width depend on both its width ratio and the width of the lines, and for similar width ratios, the nonlinearity is increased in the step with the wider lines. The accuracy of the proposed model was validated by comparison with full-wave numerical nonlinear analysis. The proposed nonlinear analysis method can be very useful for minimizing the nonlinear distortions of high temperature superconductors resonators and superconducting microstrip filters.

References

[1] G. Tsuzuki, S. Ye, and S. Berkowitz, "Ultra-selective 22-pole, 10 transmission zero superconducting bandpass filter surpasses 50-pole Chebyshev filter," *IEEE Trans. Microw. Theory Tech.*, vol. 50, no. 12, pp. 2924–2928, Dec. 2002.

[2] L.-M. Wang, W.-C. Lin, M.-L. Chang, C.-Y. Shiau, and C.-T. Wu, "Characteristics of ultra-wideband bandpass YBCO filter with impendence stub," *IEEE Trans. Appl. Supercond.*, vol. 21, no. 3, pp. 551–554, Jun. 2011.

[3] Q. Zhang, Y. Bian, J. Guo, B. Cui, J. Wang, T. Yu, L. Gao, Y. Wang, C. Li, X. Zhang, H. Li, C. Gao, and Y. He, "A compact HTS duplexer for communication application," *IEEE Trans. Appl. Supercond.*, vol. 20, no. 1, pp. 2–7, Feb. 2010.

[4] H. Y. Wang and M. J. Lancaster, "Aperture-coupled thin-film superconducting meander antennas," *IEEE Trans. Antennas Propag.*, vol. 47, no. 5, pp. 829–836, May 1999.

[5] E. Hoffmann, F. Deppe, T. Niemczyk, T. Wirth, E. P. Menzel, G. Wild, H. Huebl, M. Mariantoni, T. WeiBl, A. Lukashenko, A. P. Zhuravel, A. V. Ustinov, A. Marx, and R. Gross, "A superconducting 180° hybrid ring coupler for circuit quantum electrodynamics," *Appl. Phys. Lett.*, vol. 97, no. 22, pp. 222508-1–222508-3, Nov. 2010.

[6] T. Dahm and D. Scalapino, "Theory of intermodulation in superconducting microstrip resonator," *J. Appl. Phys.*, vol. 81, no. 4, pp. 2002–2009, Feb. 1997.

[7] O. Vendik, I. Vendik, and T. Samoilova, "Nonlinearity of superconducting transmission line and microstrip resonator," *IEEE Trans. Microw. Theory Tech.*, vol. 45, no. 2, pp. 173–178, Feb. 1997.

[8] C. Collado, J. Mateu, R. Ferrús, and J. OCallaghan, "Prediction of nonlinear distortion in HTS filters for CDMA communication systems," *IEEE Trans. Appl. Supercond.*, vol. 13, no. 2, pp. 328–331, Jun. 2003.

[9] C. Collado, J. Mateu, O. Menendez, and J. M. O'Callaghan, "Nonlinear distortion in a 8-pole quasi-elliptic bandpass HTS filter for CDMA system," *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pp. 992–995, Jun. 2005.

[10] C. Lam, D. M. Sheen, S. M. Ali, and D. E. Oates, "Modeling the nonlinearity of superconducting strip transmission lines," *IEEE Trans. Appl. Supercond.*, vol. 2, no. 2, pp. 58–66, Jun. 1992.

[11] C. Collado, J. Mateu, and J. OCallaghan, "Nonlinear simulation and characterization of devices with HTS transmission lines using harmonic balance algorithms," *IEEE Trans. Appl. Supercond.*, vol. 11, no. 1, pp. 1396–1399, Mar. 2001.

[12] J. Booth, J. Beall, D. Rudman, L. Vale, R. Ono, C. Holloway, S. Qadri, M. Osofsky, E. Skelton, J. Claassen, G. Gibson, J. MacManus-Driscoll, N. Malde, and L. Cohen, "Simultaneous optimization of the linear and nonlinear microwave response of YBCO films and devices," *IEEE Trans. Appl. Supercond.*, vol. 9, no. 2, pp. 4176–4180, Jun. 1999.

[13] J. Mateu, C. Collado, and J. M. OCallaghan, "Modeling superconducting transmission line bends and their impact on nonlinear effects," *IEEE Trans. Microw. Theory Tech.*, vol. 55, no. 5, pp. 822–828, May 2007.

[14] J. Mateu, J. C. Booth, C. Collado, and J. M. OCallaghan, "Intermodulation distortion in coupled-resonator filters with nonuniformly distributed nonlinear properties: Use in HTS IMD compensation," *IEEE Trans. Microw. Theory Tech.*, vol. 55, no. 4, pp. 616–624, Apr. 2007.

[15] D. Agassi and D. E. Oates, "Nonlinear Meissner effect in a high-temperature superconductor," *Phys. Rev. B, Condens. Matter*, vol. 72, no. 1, pp. 014538-1–014538-15, Jul. 2005.

[16] D. E. Oates, D. Agassi, E. Wong, A. Leese de Escobar, and K. Irgmaier, "Nonlinear Meissner effect in a high-temperature superconductor: Local versus nonlocal electrodynamics," *Phys. Rev. B, Condens. Matter*, vol. 77, no. 21, pp. 214 521–214 529, Jun. 2008.

[17] C. Collado, J. Mateu, and J. M. OCallaghan, "Analysis and simulation of the effects of distributed nonlinearities in microwave superconducting devices," *IEEE Trans. Appl. Supercond.*, vol. 15, no. 1, pp. 26–39, Mar. 2005.

[18] J. C. Booth, K. Leong, S. A. Schima, C. Collado, J. Mateu, and J. M. OCallaghan, "Unified description of nonlinear effects in high temperature superconductor microwave devices," *J. Supercond. Novel Magn.*, vol. 19, no. 7/8, pp. 531–540, Nov. 2006.

[19] K. C. Gupta, R. Garg, I. Bahl, and P. Bhartia, *Microstrip Lines and Slotlines*., 2nd ed. Norwood, MA: Artech House, 1996.

[20] J.-S. Hong and M. J. Lancaster, *Microstrip Filters for RF/Microwave Applications*. Hoboken, NJ: Wiley, 2001.

[21] M. S. Boutboul, H. Kokabi, and M. Pyee, "Modeling of microstrip quasi-TEM superconducting transmission lines, comparison with experimental results," *Phys. C, Supercond.*, vol. 309, no. 1/2, pp. 71–78, Dec. 1998.

[22] COMSOL Inc., v4.0 Burlington, MA, 2012.

[23] O. G. Vendik, I. B. Vendick, and D. I. Kaparkov, "Empirical model of the microwave properties of high-temperature superconductors," *IEEE Trans. Microwave Theory Tech.*, vol. 46, no. 5, pp. 469–478, May 1998.

[24] Kirtley, J. R. Tsuei, C. C. Ariando, A. Verwijs, C. J. Harkema, S. Hilgenkamp, and H., "Angle-resolved phase-sensitive determination of the in-plane gap symmetry in $YBa_2Cu_3O_{7-\delta}$," *Nat. Phys.*, vol. 2, no. 3, pp. 190–194, Mar. 2006.

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