# Observation of a New Fluxon Resonant Mechanism in Annular Josephson Tunnel Structures

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A novel dynamical state has been observed in the dynamics of a perdurbed sine-Gordon system. This resonant state, has been experimentally observed as a singularity in the dc current voltage characteristic of an annular Josephson tunnel junction, excited in the presence of a magnetic field. With this respect, it can be assimilated to self-resonances known as Fiske steps. Differently from these, however, we demonstrate, on the basis of numerical simulations, that its detailed dynamics involves rotating fluxon pairs, a mechanism associated, so far, to self-resonances known as zero-field steps.

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Singularities, or resonances, in the current - voltage characteristic of a Josephson tunnel junction, reflect the underlying dynamics of the phase difference between the two superconducting junction electrode order parameters. Two kinds of singularities are usually observed: resonances, known as Fiske steps (FS), that arise when a magnetic field is externally applied in the junction plane and the so-called zero field steps (ZFS), excited even in the absence of an external magnetic field, exclusively in large junctions (a Josephson junction is said to be large when one of its dimensions is wider than the Josephson penetration depth $\lambda_J$ [1]). Two different resonance mechanisms have been proposed to explain the existence of these current-voltage singularities: interaction of cavity modes with the ac Josephson effect and fluxon oscillations. For small junctions, the theory of Fiske steps developed by Kulik [2], based on the excitation of e.m. standing waves, accounts for the experimental observations. In large junctions, the fluxon based picture, first proposed by Fulton and Dynes [3], constitutes the convincing explanation frame of reference of all ZFS phenomenology. In the last approach, fluxons, or particle-like magnetic flux quanta (solitons), shuttle back and forth along the extended dimension of the junction. Here the relevant equation is the one-dimensional sine-Gordon equation with appropriate perturbing terms and b.c. [4]. There have been attempts to extend the Kulik theory to long junctions such that a single type of analysis could work for both FS and ZFS. Alternatively, the idea that fluxon propagation was responsible also for existence of FS, besides ZFS, in long junctions was put forward. This hypothesis, sometimes referred as "Samuelsen hypothesis" [5, 6], is based on the simple observation that an applied magnetic field renders the junction dynamics asymmetric through the boundary conditions, i.e., the fluxon propagation becomes unidirectional: fluxons enter one of the junction edge and annihilate on the opposite one.

The situation has been partially redefined by a number of experiments performed by Cirillo et al. [7]. In these experiments there is evidence of a possibility which can account for both descriptions. There would exist separate regimes in long junctions, depending on the intensity of applied magnetic field, in which the two mechanisms are well separated and active. At small fields the fluxon picture applies, for larger fields, i.e. beyond the threshold represented by $H_0 = 2\lambda_J j_c$ [8], where $j_c$ is the maximum critical Josephson current density, the field penetrates stably the junction which starts to behave like a short junction as far as its magnetic properties are concerned. In other words the cavity mode mechanism described by Kulik operates beyond $H_0$ while the Samuelsen hypothesis acts at low field values, when the external magnetic field localizes at the edges of the junction.

Hereafter we report on observations made on an annular Josephson junction , i.e., one in which the two electrodes are stacked superconducting rings coupled through a thin dielectric tunnel barrier. In annular Josephson junctions, the above described phenomena have a peculiar character due to the absence of fluxon reflections at the boundaries. Much of the present knownledge about the classical fluxon dynamics has been derived by experiments perfomed on annular Josephson junctions [9, 10]. Recently Kulik theory has been succefully extended to small annular junctions including 2d effects and the possibility of trapped fluxons [11, 12]. Moreover quantum effects involving trapped fluxons and fluxon-antifluxon (F-AF) pair generation have been shown in these devices [13, 14], so that fluxon dynamics in annular junctions remains of topmost interest in view of their possible use as quantum controllable devices [15].

In long annular junctions FS and ZFS appear at the asymptotic voltage positions $V_n = n\bar{c}\Phi_0/L$, where n is an integer number, $L$ is the length of the circumference of the junction, $\bar{c}$ is the velocity of light in the junc-

tifluxons trapped into the junction, existance of stable F-AF pairs requires that the junction be sufficiently extended to permit the pair dynamics. In particular it appears problematic here, for $\bar{l} = 2\pi\bar{r} < 2\pi$, the occurrence of resonances involving more than two pairs [19]. So that the above described Samuelsen mechanism requires some modifications induced by the junction having such a critical dimension.

In order to investigate deeper this possibility, we solved numerically the following 2d perturbed sine-Gordon equation which governs the dynamics of the phase $\varphi(r, \vartheta, t)$ in an annular junction:

$$\frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \vartheta^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varphi}{\partial r}\right) - \frac{\partial^2 \varphi}{\partial t^2} - \alpha\frac{\partial \varphi}{\partial t} = \sin(\varphi) \quad (1)$$

$$\left.\frac{\partial \varphi}{\partial r}\right|_{r_e} = \zeta(\vartheta) - \eta_e\cos(\vartheta) \quad (2)$$

$$\left.\frac{\partial \varphi}{\partial r}\right|_{r_i} = -\eta_i\cos(\vartheta) \quad (3)$$

$$\varphi(\vartheta + 2\pi) = \varphi(\vartheta) \quad (4)$$

In the above equation lenghts are normalized to the Josephson penetration depth $\lambda_J$ and times to the Josephson plasma frequency $\omega_J = \bar{c}/\lambda_J$. $\eta_e, \eta_i$ are the external and internal magnetic field normalized with respect to $\lambda_J j_c$ respectively. We assume a $\theta$ dependence of normalized bias current which is closer to the experiment, i.e., two thin leads feed the bias current to the junction. In this case $\zeta = (j_b/j_c)\gamma$, with $j_b$ the bias current density and $\gamma = (\pi/\Delta\theta)(1 + r_i/2r_e)\Delta r$, with $\Delta\theta$ the angular width of current lead, $\Delta r$ the normalized junction width, $r_e$ ($r_i$) the normalized external (internal) radius. Fig. 1b shows the result of simulation of the dependence of third resonance current on the external magnetic field $\eta_e$. The internal magnetic field was set to $0.85\eta_e$ to optimize the agreement with the experimental data, taking into account the screening effects. The simulation reproduces fairly well the result of Kulik theory for fields higher than $H_0$. The peaks at small fields are also well reproduced. From Fig. 1b it is seen that these lobes are coexisting with the beginning of the subsequent lobes as two separate branches. In fact the low field resonant peaks numerical solution branch is accessed by FS2 not from the McCumber branch [1].
The dynamics involved in the low field peaks is illustrated by the numerical simulation results reported in Fig. 2. The phase difference evolution between the two electrodes can be followed during a full period $T = 3\Phi_0/V_3\omega_j \sim \bar{l}$. The phase distribution appearing in Fig.2 represents only a rough approximation to single fluxon propagation, but the overall picture is sufficiently clear to draw the following description. A F-AF pair (identified with the two points of steepest phase slope) is enucleated in the point A and it propagates up the
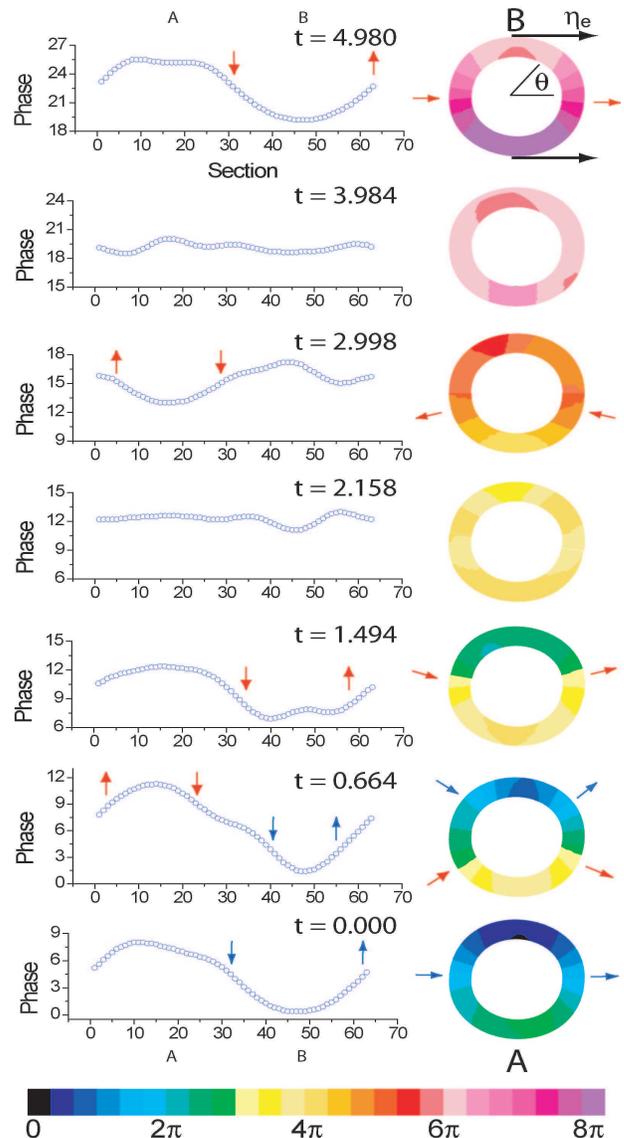


FIG. 2: (Color online) Time plots of the phase at the external border of the junction with the corresponding 2d phase snapshots for the annular junctions considered in Fig. 1. $\zeta = 1.104$ and $\eta_t = 1.55$ just at the top of the lobe. Eq. (1) is discretized in $63 \times 10$ spatial sections each long about $0.1\lambda_J$. Section 63 is followed by section 1. The bias current drive is located just at sections 1-5 and 32-36. Point A and B are the maximum tangential field locations where pair nucleation (annihilation) occurs. Blue arrows indicate the first F-AF pair, red arrows the second one. Directions of arrows indicate the fluxon polarity

positions indicated by the blue arrows as is seen at $t = 0.000$ of Fig. 2. While it moves towards the opposite edge ($t = 0.664$ in Fig. 2 blue arrows) a second pair enucleates in the point A and move towards the former (see again $t = 0.664$ in Fig. 2, red arrows). After the first pair has reached the point B ($t = 1.494$ in Fig. 2) it has an oscillation mode there because energy is

subtracted and the pair prepares to annihilate. Fig. 2 (red arrows) at $t = 2.998$ shows that after the complex fourfold collision ($t = 2.158$), the second pair survives. In fact, it has still enough energy to turn back and complete the entire rotation at $t = 3.984$, there the phase flatten and a new pair prepares to enuclate at point A (this last is shown at $t = 4.980$). Things can be described as during the collision the second incoming pair gains energy at the expenses of the former oscillating pair, avoids annihilation, emerges from the collision and turns back. From the above analysis we see that the mechanism is hybrid: during a single period, it involves both a full rotating F-AF pair and a half propagating F-AF pair. From this point of view this dynamical state shares the nature of a Fiske resonance (the second Fiske resonance at $V_2$ in which the two pairs both annihilate) with that of a zero field resonance (the second zero field resonance at $V_4$ in which the two pairs both propagate around the junction making a complete turn). This shows that the fluxon picture in the presence of magnetic field is not limited to the half propagation mode, on the contrary it can be of complex (hybrid) nature. Numerical simulations show that hybrid dynamics on third resonance is stable in all range of normalized length $\bar{l}$ between about 4.5 and $2\pi$. The hybrid lobes for shorter length tend to become smaller with respect to Kulik subsequent lobes until they disappear when the junction length is less than about 4.5. On the other hand for $\bar{l} > 2\pi$ the junction dynamics switches to Samuelsen mechanism, i.e., three F-AF pairs are enucleated on one side and annihilated on the other one. In principle hybrid dynamics could exist whenever the length of the junction is not sufficient to accomodate the required number of pairs to achieve the correct voltage. The possibility of hybrid dynamics on other resonances, like fourth or fifth, is more questionable and maybe depends on the stability of these solutions against perturbations induced by other annihilating pairs on the ring.

In conclusion, we have observed a novel fluxon dynamical resonant state. In this both half propagating motion of a F-AF pair, due to external magnetic field, and propagation around the whole junction of an other F-AF pair coexist. This is probably a further striking observation of how nonlinear dynamics described by the sine-Gordon equation can be very complex even in a relatively simple structure such as an annular Josephson junction. It is easy to imagine how hybrid dynamics could be generalized to involve both asymmetric propagating and back and forth motion of fluxons also in standard long linear junctions. Beyond stability the ultimate existence of these solutions is tightly related to the dimension of their attraction basin which can be difficult to access. The possibility of ascribing the observed resonances to an hybrid complex dynamics requires a detailed analysis of the actual underlaying fluxon propagation mode. This should be taken into account specially in connection with the potential implementation of vortex qubit circuits based on multiply-connected Josephson structures [15], where a severe control of all possible "states" is mandatory.

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[19] In fact it was impossible both in experiment and simulations to access ZFS6 (three pairs) and the observation of ZFS4 (two pairs) was critical.