A Novel Phase-Locked State in Discrete Josephson Oscillators

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***Abstract*—We have measured a novel phase-locked state in discrete parallel arrays of Josephson junctions which can be used for oscillator applications. Previous Josephson junction oscillators have been based on the Eck step, where a large-amplitude wave of nearly a single harmonic travels through the system. Multi-row systems biased on the Eck step could improve the output power, but their in-phase oscillations are difficult to stabilize. A new in-phase state which is very stable has been measured as a step in the dc I-V characteritic of one and two row systems. Simulations show that large-amplitude oscillations of two harmonics characterize the state. The rows are phase-locked and in-phase for the higher harmonic. We present an analytic expression for the oscillation frequencies and their magnetic tunability.**

I. Introduction

Long Josephson junctions and Josephson junction arrays have potential for microwave oscillator applications. Underdamped long Josephson junctions biased on the Eck step produce oscillations at frequencies proportional to the step voltage. Radiation from these devices has been measured, at power levels of $5\,\mu$W at 440 GHz [1] and linewidths of about 1 MHz. Discrete parallel arrays of short junctions also produce an Eck step and are expected to radiate at frequencies proportional to the step voltage [2]. Although the discrete version is not expected to significantly improve upon the power level or linewith of long junction oscillators, the output impedence more closely matches typical microwave circuit loads. Various configurations have been proposed to improve the oscillator power and linewidth, including series arrays of short junctions, two-dimensional arrays of short junctions, stacked long junctions, and shorted arrays of short junctions. The proposed geometries increase the number of junctions across which the output voltage is measured. If the output junctions can be phase-locked in-phase, then the power and possibly the linewidth of the oscillator are improved. In addition, these devices have an increased output impedence which can be controlled.

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Recent studies of underdamped discrete parallel arrays and stacked long junctions have shown that the output junctions can be phase-locked in a state with very large amplitude oscillations [3], [4]. Measurements of two discrete rows and of two stacked junctions show that, in both systems, the Eck step splits into two states. The lower voltage state corresponds to oscillations in the two rows which are exactly out-of-phase, while the higher voltage state corresponds to in-phase oscillations. Biased in the in-phase state, the oscillator power is expected to double (for two rows). For an arbitrary number N of rows oscillating in-phase, the power should be N times the power of a single row [5]. However, the in-phase state of the Eck step is difficult to stabilize experimentally, and the out-of-phase state can produce at most the power of a single row oscillator (if N is odd).

In contrast, we have found a new state in discrete arrays which is stable experimentally and, according to simulations, produces voltage oscillations which are in-phase across all the output junctions. This state is manifested as a step in the dc I-V curve at a voltage which is below the Eck step. Simulations indictate that oscillations occur at a frequency corresponding to the step voltage and at twice this frequency. It is this second harmonic which is in-phase between the rows. We have measured this state in single and double rows of 54 junctions per row. We find that it is analogous to the sub-structure observed in discrete ring arrays [6] and is possible only in discrete systems. Simulations of arrays with up to 5 rows show that all rows are phase-locked, and that the higher harmonic adds in-phase. Thus, the output power to a matched load should scale as the number of rows. In addition, the linewidth is still expected to scale as $1/N$ [5]. Although ac measurements must be made to verify these characteristics, our studies indicate that this device has potential for oscillator applications.

II. Experiments

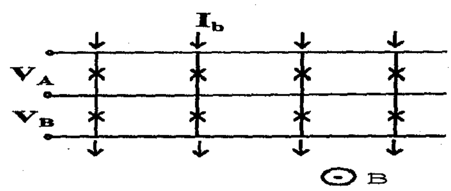

Fig. 1. Schematic of array. 54 junctions per row.

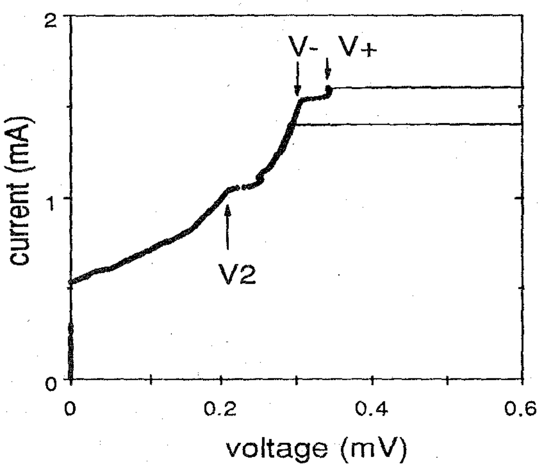

Fig. 2. Current-Voltage characteristic of 54x2 array. $\beta = 14$ and $\Lambda_J^2 = 0.68$. $V-$ and $V+$ are the split Eck peak, while $V2$ is the second harmonic state.

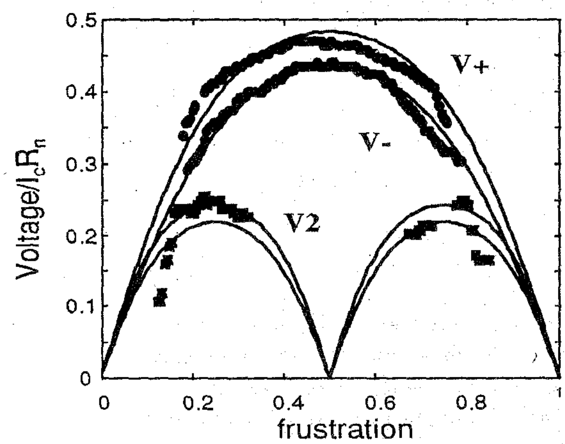

Fig. 3. Magnetic tunability of step voltages. The solid lines are Eq. 5 for $m = 1$ and $m = 2$, with $\beta = 14$ and $\Lambda_J^2 = 0.9$. The symbols are data points.

We have measured arrays of two rows ($N = 2$), with 54 junctions per row. Fig. 1 shows a schematic of our device. Resistors are used at positions indicated by the arrows in Fig. 1 to make the applied bias current as uniform as possible. The voltage of each row can be measured separately at the array edge. The array is placed above a superconducting ground plane, and a separate control wire is used to apply a magnetic field. Because the system is discrete, we expect its properties to be periodic with magnetic field. Thus we will discuss the applied field in terms of frustration, which is the flux applied to a single loop of the array, normalized to the flux quantum, $f = \Phi_{\text{app}}/\Phi_o$. The applied flux is proportional to the control current.

Samples were fabricated using a Nb trilayer process [7]. The junctions are $3 \times 3\,\mu\text{m}^2$ with a critical current density of $j_c(T = 0) = 1400\,\text{A/cm}^2$. Device parameters have been determined using the diagnostic procedures described by van der Zant *et al.* [8]. We find that the normal-state resistance $R_n = 15.3\,\Omega$, the self-inductance of a single loop in the array $L_s = 8.5\,\text{pH}$, the nearest-neighbor coupling ratios (to L_s) $M_h = M_v = 0.13$, the capacitance $C = 342\,\text{fF}$, and the Josephson inductance, $L_J = \Phi_o/(2\pi I_c(T = 0)) = 2.8\,\text{pH}$. Our measurements were taken at $T = 7.4\,\text{K}$. At this temperature, the Stewart-McCumber damping parameter $\beta = R_n^2 C/L_J = 14$ and the discreteness parameter $\Lambda_J^2 = L_J/L_s = 0.68$.

Previous measurements of this particular device at a higher temperature ($\beta = 8.1$ and $\Lambda_J^2 = 1.2$) have already been reported [3]. We now lower the temperature so that Λ_J^2 is less than one, and additional resonances appear in the I-V. The most prominent of these is marked as $V2$ in the I-V of Fig. 2. The split Eck step is also marked as $V-$ and $V+$. Fig. 2 shows the voltage across a single row of the array. Measurements of the second row are identical except at very low voltages. All of the resonances respond periodically to a changing magnetic field. In Fig. 3 we plot this behavior. The $V2$ step has twice the period and about half the amplitude of the Eck step. It can be tuned in a voltage range of about $0.12 - 0.25\,\text{mV}$. Since it is the second harmonic being considered, this corresponds to a bandwidth of $120 - 240\,\text{GHz}$.

III. Simulations

The governing equations for the system are derived by applying Kirchoff's current laws and using the RSJ model for the current through a single junction. We normalize the current to I_c, the voltage to I_cR_n, and time to $\sqrt{L_JC}$. With $\Gamma = \beta^{-1/2}$, our equations become

$$\mathcal{N}[\phi_j] - M_h(\mathcal{N}[\phi_{j-1}] + \mathcal{N}[\phi_{j+1}]) - M_v\mathcal{N}[\psi_j] = I_b/I_c + \Lambda_J^2(\phi_{j+1} - 2\phi_j + \phi_{j-1}) \qquad (1)$$

and

$$\mathcal{N}[\psi_j] - M_h(\mathcal{N}[\psi_{j-1}] + \mathcal{N}[\psi_{j+1}]) - M_v\mathcal{N}[\phi_j] = I_b/I_c + \Lambda_J^2(\psi_{j+1} - 2\psi_j + \psi_{j-1}) \qquad (2)$$

where ϕ_j and ψ_j represent the gauge-invariant phase differences across the upper row and lower row junctions respectively. The functional $\mathcal{N}[\phi_j(t)] \equiv \ddot{\phi}_j(t) + \Gamma\dot{\phi}_j(t) + \sin\phi_j(t)$ returns the total current through junction j. For consistency in comparing with theory, we include only nearest-neighbor inductances, M_h and M_v. We integrate using a fourth-order Runge-Kutta scheme. Our method is described in more detail in [3].

Figure 4 shows a simulated I-V curve with the same features found in the experimental measurements. In this simulation, frustration$= 0.3$, $\beta = 14$, $\Lambda_J^2 = 0.9$. The discreteness parameter used is slightly larger than that calculated for experiments, since smaller values of Λ_J^2 yield more fine-structure than that which appears in experiments. The Eck step ($V+$ and $V-$) and the $V2$ step are marked. These steps are tunable with frustration, as in our experiments. In Fig. 5 we plot the phase vs. time of the middle junction in each row while the system is biased at the top of the $V2$ step. The inset shows the

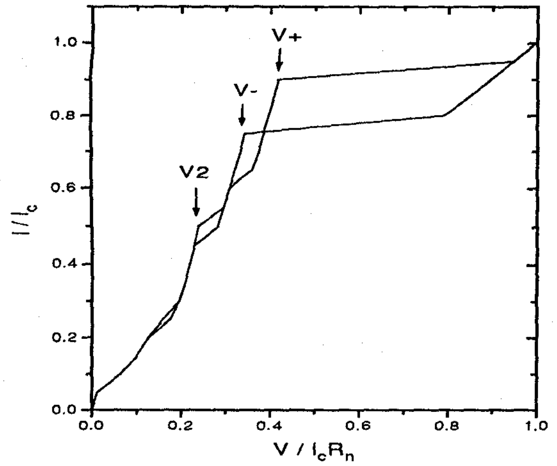

Fig. 4. Simulated Current-Voltage characteristic which shows the same features as experiments. $\beta = 14$ and $\Lambda_J^2 = 0.9$.

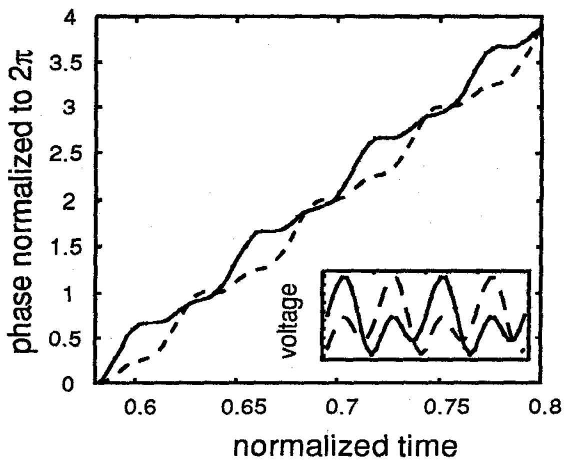

Fig. 5. Phase vs. time of vertically adjacent junctions in a 2x54 array biased at $V2$. Inset shows voltage vs. time of the two junctions.

voltage vs. time. Two harmonics dominate the time evolution of the junctions. The lower harmonic, which has a frequency corresponding to the step dc voltage, is out-of-phase between the two rows. The second harmonic is in-phase. This relationship becomes clearer when we take the Fourier transform of the voltage signals. Fig. 6 compares the Fast Fourier Transform of the ac voltage from a single row to the transform of the sum of both rows. When the voltage across the two rows is added, the amplitude of the second harmonic doubles, while the lower harmonic disappears. Simulations for up to 5 rows of 54 junctions show that the voltage across N rows scales as N. However, preliminary results on a 10-row array suggest that there is a slight phase-shift between rows which causes a saturation of the second harmonic voltage amplitude across larger arrays. Simulations also indicate that the oscillation amplitude in the $V2$ state is about three times smaller than the oscillation amplitude on the Eck step of a single row. Thus, it is necessary to bias three rows on the $V2$ step to obtain equivalent voltage output.

IV. Discussion

Substructures on the flux-flow part of the I–V characteristics have already been observed in single-row discrete rings [6]. In this system, a vortex was trapped and driven with a dc current, forcing it to travel with a constant velocity around the ring. The resonance frequencies in the system were first presented by Ustinov *et al.*[9] by linearizing the governing equation and calculating the dispersion relation. Petgralia *et al.* extended the analysis to include an arbitrary number of inductively coupled rows by deriving such a relation for the linearized system of coupled equations [11]. For two rows, the leading order term reads:

$$\omega_r^2 = \frac{4\Lambda_J^2 \sin^2(\kappa_r p/2)}{1 \mp M_v - 2M_h \cos(\kappa_r p)} \tag{3}$$

The parameter κ_r is the wavenumber and p is the lattice spacing. In an annular system, this linear wave was phase-matched to the kink by a condition

$$\omega_r/\kappa_r = \omega/\kappa, \tag{4}$$

where ω and κ are the kink rotation frequency and its wavenumber, respectively. The kink frequency ω is proportional to the dc voltage while κ is determined by the periodic geometry. (For M vortices trapped in an n-junction ring, $\kappa = 2\pi M/(np)$.) Therefore, the condition predicts a discrete set of special voltages, that were identified with the observed resonant steps.

When the phase-matching condition (4) is fulfilled, the solutions would appear as a traveling wave, with a waveform being a superposition of a kink and a small amplitude oscillation. Such a traveling wave was found to be the dominant type of solutions not only on the resonant steps but generally in the flux-flow region [10]. Then, the functional form of the solutions is tightly restricted and can be written in the form: $\phi_j(t) = \xi + \Phi(\xi)$ where $\xi = \omega t + \kappa p j$ is a moving coordinate with a kink, and Φ is the 2π-periodic part of the waveform function. Starting from this traveling wave assumption, the interaction between the kink and the "linear" wave can be quantitatively taken into account. The function Φ is first expanded into Fourier modes, and the system can be rewritten in terms of these modes. The modal equations then appear as a coupled system driven by a kink [12].

These traveling wave solutions are not sensitive to the boundary conditions, and the same resonance mechanism is expected also in the arrays with open boundaries [3]. Instead of a periodically circulating kink, an evenly-spaced train of vortices propagates through the system in this case. The solution is still of the above form, but the wavelength $\kappa = 2\pi f$ can now be continuously tuned. (This continuous dependence on f distinguishes this type of step from Fiske steps, which could occur in a similar voltage

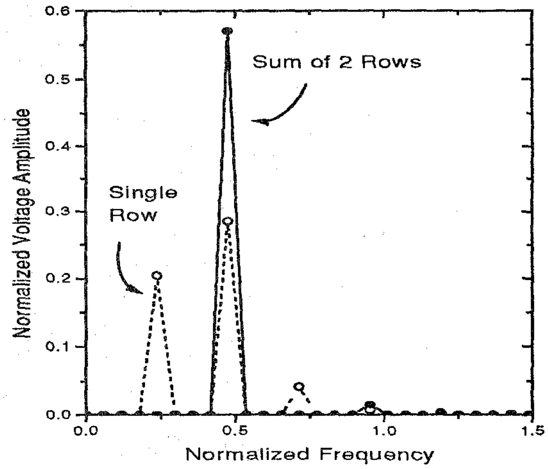

Fig. 6. Fast Fourier Transform of AC voltage signals. The dotted line represents the output of a single row. The solid line is obtained by adding the ac voltage across two rows and then computing the transform. The voltage is normalized to I_cR_n and the frequency is normalized to $\Phi_o/(2\pi I_cR_n)$.

region of the I–V for shorter arrays but at fixed resonant voltages.) We have indeed observed the splitting of the Eck peak in a system of two coupled arrays, and explained the resonance from this approach [3]. The Eck peak in a discrete array corresponds to the resonance between the kink and the first harmonic of the waveform function Φ. Therefore, the higher harmonics were neglected and the harmonic balance approximation was carried out.

The new step we have found in Fig. 2 corresponds to when the second harmonic also has a non-negligible amplitude. The analysis of the modal equations is still possible in principle, but generally becomes complicated when higher harmonics are involved. Assuming that the coupling between modes are negligible and that the amplitudes of the modes are small, the resonance of the m-th Fourier mode is expected to occur at:

$$V_m = \frac{\Phi_o}{m\pi\sqrt{L_sC}} \left| \frac{\sin(m\pi f)}{1 \mp M_v - 2M_h \cos(2m\pi f)} \right|^{1/2} \tag{5}$$

in terms of voltage [12]. To first order, the linear analysis [11] would also yield this result. We compare the expression to our data in Fig. 3. By using a slightly larger value of $\Lambda_J^2 = 0.9$ we obtain a reasonable fit for both Eck peak and the V2 steps. We do not know if this discrepancy between the parameters can be explained by taking the coupling and the amplitudes of the modal equations into account. In addition, no splitting of the $m = 2$ step is observed; the reason is also to be studied further.

V. Conclusions

We have found a new phase-locked state in parallel arrays of Josephson junctions. This state has been measured as a step in the I-V characteristic of two row arrays with 54 junctions per row. The step is tunable with magnetic field and has half the period of the Eck step. Simulations for up to 5 row arrays indicate that, in this state, the junctions undergo large amplitude oscillations with two harmonics. The higher harmonic is in-phase for all the rows, and the ac voltage of the system is N times the ac voltage of a single row biased on this step. Unfortunately, simulations show that the oscillation amplitude of a single row biased in this state is three times smaller than when the row is biased on the Eck step. Thus the array must be relatively large to improve upon the power output of single row oscillators. However, additional rows are expected to narrow the linewidth by a factor of $1/N$. The higher output impedence of a larger system is also important for coupling to typical microwave circuit loads.

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