

Spiking statistics in noisy hippocampal interneurons [☆]

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Abstract

We have studied the effects of two types of noise in simple model neurons with Hodgkin–Huxley voltage-gated channels. We have considered Gaussian noise currents, and Poisson-distributed excitatory and inhibitory postsynaptic potentials. We determined the distribution of interspike intervals, and calculated its average, τ_{ISI} , and the coefficient of variation (CV). The neuronal dynamics is analyzed in the CV– τ_{ISI} diagram in terms of equi-noise strength D , and equi-drive I_{app} curves. For the Gaussian case one can distinguish four regions of neuronal dynamics. An equivalent analysis is carried out in the Poisson case. In the latter case the behavior is more complicated. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Noise appears to be ubiquitous in neurons of the nervous system. Noise may derive from multiple sources, including the stochastic nature of the opening of voltage-gated channels and the quantal release of vesicles with neurotransmitters in synaptic transmission. Functionally, an important source has been the incoherent activity of presynaptic neurons that impinge on each individual neuron. The spontaneous neuronal activity due to this synaptic noise has been measured in a number of different neurons (see e.g. Ref. [12]). Synaptic noise has been included in some models

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in a wide variety of different ways [2–4,11]. Sometimes important qualitative aspects of the noise are not considered, and in most cases a rigorous justification for the amount and type of noise that is being used, is absent. Because of this the measured characteristics of the noise often bear no clear connection to the corresponding quantities in the theoretical model calculations. It is important to have a simple, but physiologically realistic, model for neurons so as to allow for a systematic study of collective effects in large neuronal networks. The statistics of the action potential onset times are in that respect as important, or sometimes even more important, than the precise shape of the action potential. The functional behavior of neurons is affected by noise, heterogeneities, and cortical architecture. As an example, the synchronization of neuronal firing can be eliminated as well as re-established when increasing the noise strength [9,10]. In some parts of the brain robust synchronization may be needed, for it is purported to be a possible mechanism for binding [7], whereas in other parts it is detrimental, for it may lead to epileptic seizures. There are specific mechanisms by which networks of neurons themselves can change the noise strength, and there are also external means for changing it. It is essential to have a quantitative understanding of these mechanisms and the effect of noise when studying fundamental issues, such as binding, as well as for clinical applications in understanding epilepsy. In this paper we carefully compare two different relevant models for noise in a single neuron.

2. Methods

The prototypical neuron considered here is a hippocampal interneuron. It is modeled as a single compartment with Hodgkin–Huxley type of voltage-gated sodium and potassium currents, with the rate functions and values for the maximum conductances given in Ref. [13]. The equation for an individual neuron is then

$$C_m \frac{dV}{dt} = -I_{Na} - I_K - I_L - I_{syn} + I_{app} + C_m \xi. \quad (1)$$

Here I_{Na} , I_K , I_L , I_{syn} , I_{app} and $C_m \xi$ are the sodium, potassium, leak, synaptic, externally applied and noise currents in $\mu A/cm^2$, respectively. $C_m = 1 \mu F/cm^2$ is the membrane capacitance. Our numerical implementation has been discussed in Ref. [9]. We study two types of noise models here:

Gaussian. The noise is a Gaussian distributed, delta correlated, current, with $\langle \xi(t) \rangle = 0$, and $\langle \xi(t) \xi(t') \rangle = 2D \delta(t - t')$ (the noise strength D is expressed in mV^2/ms). The synaptic current I_{syn} is set to zero.

Poisson. Here the synaptic current I_{syn} in Eq. (1) is the sum of an inhibitory and excitatory channel of the form $I_{syn} = g_{syn} s(t)(V - E_{syn})$. The maximum conductance is $g_{syn} = 0.1 mS/cm^2$, and the reversal potential is $E_{syn} = 0 mV$, and $E_{syn} = -75 mV$, for the excitatory and inhibitory synapses, respectively. Postsynaptic potentials (PSP) are modeled as quantal increases, Δs , in the synaptic kinetic variable $s(t)$. The PSPs are independent and Poisson distributed with frequency f ; $s(t)$ decays exponentially in time with a time constant τ_{syn} ; $D = 0$, and $I_{app} = 0.10$.

The raw data obtained from the simulations are the voltage $V(t)$ traces at discrete times $t = n\tau$ (we take $\tau = 0.2$ ms), which give the i th spike-time t_i , and the i th interspike interval (ISI) $\tau_i = t_{i+1} - t_i$. We calculated the histogram of interspike intervals (ISIH), the average ISI, τ_{ISI} , its standard deviation σ_{ISI} , and the coefficient of variation (CV) equal to $\sigma_{\text{ISI}}/\tau_{\text{ISI}}$. We have also calculated standard autocorrelation functions: $g_x(t) = \langle x(t)x(0) \rangle - \langle x \rangle^2$, where x can be the membrane voltage $V(t)$, or the ISI τ_i , and g_x is normalized by the equal time variance. The $\langle \cdot \rangle$ denotes the time average over all discrete times $n\tau$ in the first case, and an average over all ISI in the latter. Another useful measure is the joint distribution of ranked (or ordered in magnitude) ISIs (JRIS) [5]. The JRIS is a set of points plotted in a plane, of which the x -coordinate is given by the rank of τ_i , and the y -coordinate by the rank of the previous value τ_{i-1} .

3. Results

In our stochastic Gaussian model we can identify as relevant variables the noise-strength (D) and driving force (I_{app}). The intuitive notion of the effect of weak and strong noise, and below and above threshold driving forces, leads us to the schematic phase diagram with four regions shown in Fig. 1. In the same figure we also show representative voltage traces, g_V , and ISIHs for each of these regions. The ISIHs in this

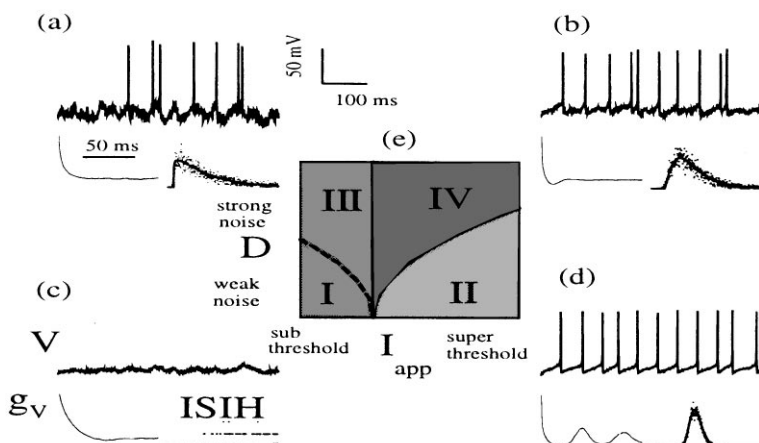


Fig. 1. Different regimes of single neuron behavior with Gaussian noise. In (a)–(d) we plot voltage traces V , the voltage autocorrelations g_V , the ISIHs, and a fit to the ISIH by a gamma distribution, $P(t) = (\mu r)^\mu (t - \tau_d)^{\mu-1} e^{-\mu r(t - \tau_d)} / \Gamma(r)$ when $t > \tau_d$, and equal to 0 when $t \leq \tau_d$. The fitting parameters (μ , r , τ_d) are (a) (0.032, 1.14, 5.60); (b) (0.039, 2.37, 1.4); (c) (0.0025, 1.27, 32.2); (d) (0.044, 15.5, 18.7). The scale bar for the voltage traces is shown to the right of (a). The time scale for g_V and ISIH is shown in (a). We used (a) $I_{\text{app}} = 0$, $D = 7.6$, (b) $I_{\text{app}} = 0.38$, $D = 0.8$, (c) $I_{\text{app}} = 0.0$, $D = 0.4$, and (d) $I_{\text{app}} = 0.38$, $D = 0.04$. The correlation functions and the histograms were calculated using an averaging time of 200 s. In (e) we give a qualitative phase diagram denoting the different regimes.

figure are fitted to a gamma probability distribution function [1]. To study these regions quantitatively we plot the equi- D and equi- I_{app} curves in a $\text{CV}-\tau_{\text{ISI}}$ diagram. The distinction between regions I and III is not that clear. For the Poisson case we found it useful to parametrize the curves in terms of the analytic average $\eta = \langle I_{\text{syn}} \rangle$, and the variance $\eta_2 = \langle I_{\text{syn}}^2 \rangle - \eta^2$. The resulting $\text{CV}-\tau_{\text{ISI}}$ diagram looks quite different. The equi- η are directed to the right for increasing CV. In addition the CV has a maximum value on these curves, due to the saturation of the inhibitory channel. The theoretical average value for s is close to its maximum value of one, and as a result the actual average will be reduced, and the variance decreased. One should note that we have fixed the Δs and τ_{syn} values, since for different values one may obtain qualitatively different $\text{CV}-\tau_{\text{ISI}}$ diagrams. An important difference between the Poisson and Gaussian noises, that is not clear from the ISIHs, are the time correlations. We find that for $\tau_{\text{syn}} \gg 1$, consecutive ISIs are correlated. This can clearly be seen in g_{ISI} and the JRISs (not shown here). In addition, the autocorrelations g_V of the voltage fluctuations are also different. We have derived analytic expressions for g_V that are useful to determine the statistics of the EPSP and IPSP inputs (not shown here).

4. Conclusions

We have compared the ISI distributions obtained from Gaussian and Poisson noise calculations. We can, given a proper value of τ_{ISI} and CV, in both noise cases find a set of model parameters that yield the same statistics. In the Gaussian case this set of parameters would be unique. The model parameters of Poisson noise, however, have a direct physiological meaning: the size Δs and time scale τ_{syn} can be directly obtained from experimental measurements of individual EPSPs and IPSPs. This obviously constrains the ranges of CV and τ_{ISI} values one can obtain, but these observed values can be directly translated into the presynaptic firing rates f_{EPSP} and f_{IPSP} (Fig. 2). In addition, the effects of correlated ISIs and synaptic saturation are included. In summary, Gaussian noise offers a simple way of evaluating the effect of a certain CV and τ_{ISI} value on for example the dynamics of a large network of model neurons. But Poisson noise is necessary to directly compare to experiments.

Our results can also be used to compare the Shannon entropy of the presynaptic input to the Shannon entropy of the ISI output. We find that synaptic saturation significantly reduces the maximal information rate of the neuron. For a few cases we have determined the mutual information between the inputs and outputs. The question of how the brain encodes information ranks amongst the most important in cognitive neuroscience. A major controversy has arisen as to whether the brain uses a rate coding or a temporal coding strategy [6,8]. For this issue to be resolved it is important to know how much information a neuron can encode as part of a large network, or what is the range of CV under physiological conditions. Here we have determined the CV values for simple models that are nonetheless more realistic than integrate and fire models [1]. We plan to study the information processing capabilities of the networks of two compartment pyramidal neurons using the same methods. A more extensive discussion of the results mentioned here will appear elsewhere [14].

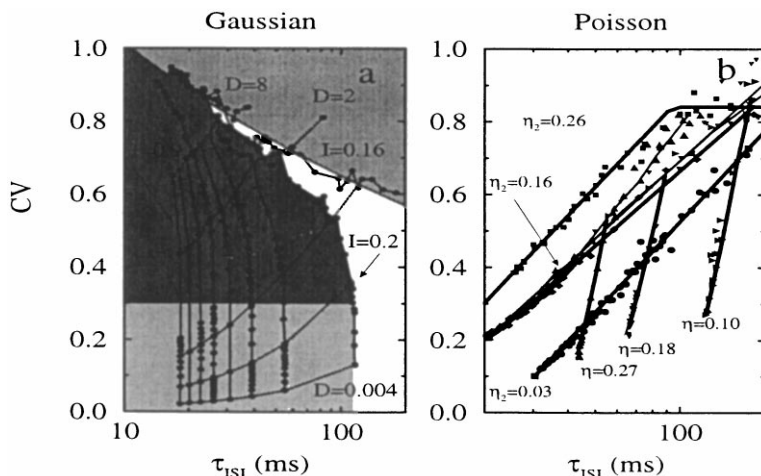
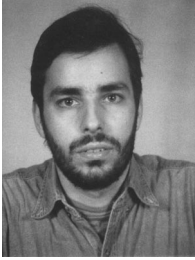


Fig. 2. We plot the results from our calculation with (a) equi- D and equi- I_{app} , and (b) equi- η and equi- η_2 curves. Selected curves are labeled by their value. For the Poisson case we used $(\Delta s, \tau_{syn}) = (0.02, 2 \text{ ms})$ and $(0.10, 10 \text{ ms})$ for excitatory and inhibitory channels, respectively. For these values the increments in f_{EPSP} and f_{IPSP} are linearly related with ratio 5.09 on equi- η , and ratio -5.19 on equi- η_2 curves. The patterns in (a) correspond to those in Fig. 1(a).

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