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# Charge tunneling across strongly inhomogeneous potential barriers in metallic heterostructures: A simplified theoretical analysis and possible experimental tests

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ABSTRACT

Universal aspects of the charge transport through strongly disordered potential barriers in metallic heterojunctions are analyzed. A simple theoretical formalism for two kinds of transmission probability distribution functions valid for smooth tunneling barriers and those with abrupt boundaries is presented. We argue that their universality has simple mathematical origin and can arise in totally different physical contexts. Finally, we analyze possible applications of superconducting junctions to test the universality of transport characteristics in structurally disordered insulating films *without any fitting parameters* and point out that the proposed approach can be useful in understanding the dynamics of surface screening currents in superconductors with an inhomogeneous near-surface region.



## 1. Introduction

Phenomenon of electron tunneling through a thin insulating layer underlies the work of numerous solid-state devices, such as tunnel diodes, Josephson trilayers, memory elements based on magnetic tunnel junctions, etc., and is one of the most fundamental research topics in condensed matter physics [1]. The functionality of a tunnel device is very sensitive to the quality and reliability of the nonconductive interlayer. Standard tunneling theories consider the potential barrier within two extreme approaches: (i) a semiclassical WKB approximation with a slowly varying potential and (ii) that with sharply falling down boundaries (see the insets in Fig. 1). A typical example of the first type of tunneling devices is a triangular potential well, usually encountered at the semiconductor heterojunction interfaces. Metal–insulator–metal (MIM) heterostructures belong to the second type of the junctions. In both cases the main potential parameters are frequently assumed to be constant along the barrier plane.

Miniaturization of electronic devices, the principal driving force behind the modern microelectronics industry, has led to the fact that precise control of the material properties of thin insulating interlayers in MIM junctions as well as those of a Schottky potential-energy barrier at a metal–semiconductor interface becomes more and more difficult. As a result, a significant amount of different defects appears within the potential barrier. Due to the exponential dependence of the electron transmission probability on the barrier height and width, their presence considerably complicates the implementation of the claim for maximum junction reproducibility. In particular, such a problem arises in amorphous oxides where the lack of long-range order allows for local rearrangements of atoms. This effect becomes strongly pronounced in ultra-thin oxide films due to the deficit oxygen [2]. Another example discussed in the paper is the presence of a self-organized percolative filamentary structure in conducting marginally stable (anomalously soft) materials which can be especially pronounced at their surfaces [3]. We expect that our theory presented below can be applied to different types of highly disordered interfaces as well as to surface sheaths in metallic inhomogeneous structures.

From the first sight, in such systems it is very difficult to achieve a balance between the two generally opposed features,

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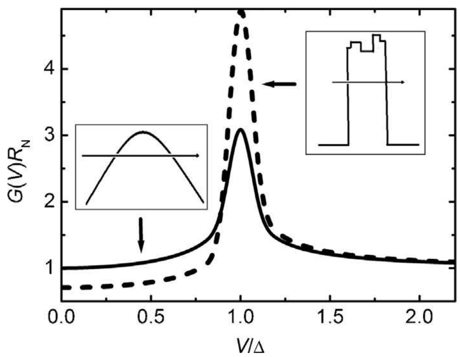


**Fig. 1.** Normalized conductance spectra of NIS junctions with a strongly disordered insulating interlayer, $R_N$ = const is the junction resistance in a normal state. The solid and dashed lines were calculated with distribution functions (1) and (2), respectively. The insets show schematically shapes of the potential barriers that were assumed in deriving the relations (1) and (2).

reproducibility and disorder. At the same time, in the interfaces with a strong disorder the situation is simplified by the inherent complexity of the problem which manifests itself in a large number of degrees of freedom. Indeed, in this case, we can apply statistical tools for quantifying transport properties of the films and, on the base of the knowledge, we are able to seek new strategies for controlling and improving their material characteristics. In this work, we discuss two generic features of the defect-related charge transport across strongly inhomogeneous potential barriers, the mathematical simplicity of underlying physical models and the emergence of universality.

Characteristics of transmission through random media are determined by the statistics of eigenvalues of the scattering matrix **S** which encodes fully multiple scatterings within the medium and thus forms the basis of a powerful approach to quantum and classical wave propagation [4]. For a scattering sample with $N$ propagating channels, the elements $S_{ba}$ of the N × N scattering matrix are the flux transmission coefficients between $N$ inputs and $N$ outputs, $a$ and $b$. In general, description of the wave propagation in strongly scattering media is a fundamental challenge in disordered systems theory. But some important points of this problem have been reliably established. In particular, thirty years ago it was found that in quasi-one-dimensional samples transmission of waves in some eigenchannels can be strongly enhanced and that the eigenvalues $D_n$ of the Hermitian matrix $S^{\dagger}S$ have a bimodal distribution consisting of a large number of strongly reflected "closed" eigenchannels and a number of "open" eigenchannels with $D_n \simeq 1$ [5,6]. The same bimodal result was derived later by Nazarov [7] for the distribution of transmission eigenvalues in higher dimensional diffusive samples.

In the literature there are two analytical formulas for the probability distribution function of local barrier transmission coefficients (transparencies) $D$ in disordered potential barriers. The first one

$$P_1(D) = \frac{\pi\hbar\bar{G}}{2e^2}\frac{1}{D(1-D)^{1/2}} \tag{1}$$

was derived by Dorokhov for diffusive conductors [5] but recently was successfully applied also to classical waves such as light, sound, and microwave radiation [8–10]. The second analytical expression

$$P_2(D) = \frac{\hbar\bar{G}}{e^2}\frac{1}{D^{3/2}(1-D)^{1/2}} \tag{2}$$

was got by Schep and Bauer [11] who considered extremely high and infinitely thin barriers between metallic electrodes. In the next section of the paper we shall show that the two formulas for a strongly inhomogeneous set of potential barriers can be easily derived using a standard scattering approach to elastic tunneling events [4] wherein Eqs. (1) and (2) correspond to a semiclassical WKB approximation with slow spatial dependence of the potential barrier in the transition region and that with abrupt barrier walls, respectively.

Note that in both limiting cases shown schematically in the insets in Fig. 1 system's transport characteristics are controlled by the only macroscopic parameter $\bar{G} = \int_0^1 P(D)G(D)dD$, the disorder-averaged macroscopic conductance, where $G(D) = (2e^2/h)D$. Hence, the distribution functions look very universal. In this context, the notion of universality means that quantitative features of the charge transport across a heterostructure with a locally disordered potential barrier (such as asymptotic behavior, etc.) can be deduced from a single global parameter, without requiring knowledge of the system details. In the third section we attract attention to the fact that comparatively simple dependence of the barrier transparency $D$ on a single governing parameter obtained for smooth and sharp barriers is not limited to these cases but is of a more general character. We provide examples of such relationships which are identical mathematically (and, thus, have the same distribution functions) but come from entirely different physical origins.

Possible means to test the reliability of relations (1) and (2) are discussed in the fourth section. It is argued that the best way to verify them is to perform low-temperature transport measurements in trilayered MIM samples where one or two electrodes are in the superconducting state. It is so due to a very high sensitivity of the shape of measured characteristics to the barrier transmission coefficient $D$ [12]. Moreover, normalization of experimental curves measured in the superconducting state on relevant normal-state characteristics [1] or determination of the ratio of two principally different quantities in a superconducting state (see below) permits to eliminate the last adjustable parameter in formulas (1) and (2), the average macroscopic conductance of the insulating interlayer. This makes it possible to perform test experiments *without any fitting parameters*. Some new experimental data are analyzed from this perspective. At the end of this section, we discuss SQUID-magnetometry measurements of superconducting surface characteristics of borides [13,14] to demonstrate the applicability of the universal function (1) to non-transport experiments as well. In the last section the results of this work are summarized.

## 2. Two universal distribution functions for local transparencies of microscopically disordered potential barriers

In order not to complicate the calculations and to restrict ourselves to principal aspects of the problem, let us consider for simplicity a planar MIM junction. Components $\mathbf{k}_{||}$ of charge wave vectors parallel to the barrier plane are real quantities which do not change when the charge crosses the metal/insulator interface [15]. At the interface, charge Bloch states with energies $E = \hbar^2k^2/(2m)$ ($m$ is the electron mass) which are below the Fermi level $E_F$ and at the same time within the dielectric forbidden gap do not vanish right at the metallic surface but rather penetrate into the insulator decaying as $\exp(-\kappa x)$ at the distance of a few atomic layers. The eigenstates of the Hamiltonian for the trilayered MIM structure can be obtained by matching at the $M/I$ interface ordinary bulk Bloch states in the $M$ electrodes and a wave function within the barrier which decays in the direction $x$ perpendicular to interfaces with the decay length given by $\kappa^{-1}$. If so, the wave functions in the I interlayer can be actually considered as Bloch functions of the bulk insulator with an associated complex wave vector $\tilde{k} = k_{||} + i\kappa$ and we can regard on the transport problem across the MIM junction as one-dimensional one for a $M_1M_2M_3$ contact of three conductors with wave-vector components $k_{1x} = k_{3x}$ for the left and right electrodes and $k_{2x}$ for the interlayer. In the following, in all three conducting layers we approximate the spectrum of electrons by parabolic bands and are dealing with wave functions carrying unit flux. The latter condition means that in a non-superconducting metallic layer the wave function of a quasi-particle excitation $\psi(x,\boldsymbol{\rho}) = \frac{m}{\hbar\sqrt{k_x}}\exp(ik_x x + i\mathbf{k}_{||}\boldsymbol{\rho})$, where $\boldsymbol{\rho}$ is a two-dimensional vector; $\mathbf{k} = (k_x, \mathbf{k}_{||})$ is the wave vector of an electron with the energy $\varepsilon = E - E_F$; $k_x = \sqrt{2m(E_F \pm \varepsilon)/\hbar^2 - \mathbf{k}_{||}^2}$, the sign $\pm$ corresponds to electron (e) and hole (h) excitations, respectively At first, let $k_{2x}$ be real and in the final expression we shall set it equal to an imaginary quantity $i\kappa$. Note that as $k_x$, as well as $\kappa$ depend not only on the total energy but also on its transversal component.

The goal of our calculations in this section is to compute the eigenvalue of the transmission matrix for one transverse channel, i.e., the probability amplitude $T$, and then to find corresponding probability $D = |T|^2$. Let us start with a contact of two metals in proximity. Due to the mismatch of Fermi wave vectors in dissimilar metallic films 1 and 2, their interfaces (even if they are ideal) are acting as scattering planes. By matching the wave functions for two half-spaces, we obtain that an electron wave function incident on the $M_1/M_2$ interface from the metal with an electron wave vector $k_1$ is partly transmitted to another metal with an amplitude $t_{12} = 2\sqrt{k_{1x}k_{2x}}/(k_{1x}+k_{2x})$ and partly reflected back from the electrode 2 with an amplitude $r_{12} = (k_{1x} - k_{2x})/(k_{1x} + k_{2x})$. Respectively, $t_{21} = t_{12} = 2\sqrt{k_{1x}k_{2x}}/(k_{1x}+k_{2x})$ and the reflection amplitude for an electron with an electron wave vector $k_2$ to be scattered back from the electrode 1 $r_{21} = (k_{2x} - k_{1x})/(k_{1x} + k_{2x})$. Next we interpret the charge transmission across an $M_1M_2M_3$ heterostructure as a sequence of an infinite number of interface scattering events. Summing up the probability amplitudes for all the trajectories (the first term $t_{12}\exp(i\chi)t_{23}$ describes the direct transmission, the second term $t_{12}\exp(i\chi)r_{23}\exp(i\chi)r_{21}\exp(i\chi)t_{23}$ represents one back and forth bouncing, and so on), the amplitude of the total probability for an electron to be transmitted across the trilayered $M_1M_2M_3$ structure with the real-value wave vectors reads

$$T = \frac{t_{12}\exp(i\chi)t_{23}}{1 - r_{21}\exp(2i\chi)r_{23}} \tag{3}$$

with $\chi$, the phase shift acquired by an electron traveling between two interlayer boundaries (note that the capital letter $T$ corresponds to the total probability amplitude whereas the total probability will be denoted as $D$).

If the interlayer $M_2$ is insulating, the component $k_{2x}$ should be replaced by $i\kappa$ and after some algebra, we derive the following result:

$$T = \frac{i}{i\cosh(\phi) + (k_x^2 - \kappa^2)\sinh(\phi)/(2k_x\kappa)} \tag{4}$$

with $\phi = \chi(k_2 \to i\kappa)$. Note that as $k_x$, as $\kappa$ are spatially dependent in the case of disorder. In Eq. (4) these quantities have values taken just at the M/I interface.

Now we discuss two limiting cases for a given transverse channel: (i) a slowly varying barrier $U(x)$ when a WKB approximation is applicable to the transition region between the two metallic electrodes and (ii) a barrier with abrupt boundaries (see the insets in Fig. 1). In the first case at classically turning points $x_L$ and $x_R$ $k_x = \kappa$ and Eq. (4) is greatly simplified to the relation $T = \cosh^{-1}(Y)$ where $Y = \int_{x_L}^{x_R}\kappa(x)dx$, $\kappa(x) = \sqrt{2m(U(x) \mp \varepsilon)/\hbar^2 - k_{||}^2}$, the sign $\pm$ corresponds to electron (e) and hole (h) excitations, respectively. Note that this formula leads to the well-known WKB results for different shapes of energy barriers, in particular, to the Fowler–Nordheim formulas for a triangular barrier [1].

Next, we assume that the total transmission coefficient across the disordered barrier area is a sum of independent local transparencies for concerned eigenchannels. For planar junctions with a comparatively large area, strong and uncorrelated spatial changes of the two quantities $U(x,\boldsymbol{\rho})$ and the distance $x_R(\boldsymbol{\rho}) - x_L(\boldsymbol{\rho})$, we may suppose that the parameter $Y$ is a very smooth random variable with a constant distribution function $\rho(Y) = \pi\hbar\bar{G}/e^2 = \text{const}$. With the parametrization $D = T^2 = \cosh^{-2}(Y)$ we can transfer from $\rho(Y) = \text{const}$ to the distribution of a local transparency $D$ and the result is just Eq. (1).

Let us now move to the opposite case of sharp boundaries when a nanometer-thin potential barrier of an average height $U_0$ extends from $x = 0$ to $x = d$. First, we assume that $E_F \gg U_0$ and, hence, $k_x >> \kappa = \sqrt{2m(U_0 \mp \varepsilon)/\hbar^2 - \mathbf{k}_{||}^2}$ in all eigenchannels (a realistic situation for junctions with ordinary metallic electrodes). Moreover, we assume that the decaying length $\kappa^{-1}$ (now it is a constant) is so large that the strong inequality $d \ll \kappa^{-1}$ is valid (junctions with ultrathin barriers). In this case $T = i/(i + Z)$ with $Z = k_x d/2$ and the transparency of the tunnel junction $D = |T|^2$ is given by a Lorentzian $D = 1/(1 + Z^2)$. It is interesting that the same functional form of the transmission coefficient follows for ultrathin barriers $\kappa d \ll 1$ from the inverse inequality $k_x \ll \kappa$. Indeed, in this case we have $T = i/(i - \tilde{Z})$ and again get a Lorentzian $D = 1/(1 + \tilde{Z}^2)$ with a new controlling parameter $\tilde{Z} = \kappa_0^2 d/(2k_x)$. Note that, first, for $k_x \gg \kappa$ small $Z$ values correspond to large incident angles whereas for $k_x \ll \kappa$ we obtain an inverse situation and, second, in both cases we suppose the presence of a very large spread of transmission coefficients over a set of channels. If it is so, we expect that the parameter $Z$ is a uniform random variable distributed with $\rho(Z) = 2\hbar\bar{G}/e^2 = \text{const}$. This supposition results in the probability distribution function of local transparencies $D$ (2) which originally was obtained for a delta-functional barrier, i.e., $\kappa d \ll 1$, with $\kappa \gg k_x$ [11].

## 3. Universality aspects of transmission through random condensed media

It is important to notice that the two distribution functions (1) and (2) which were derived earlier for some particular cases [5,11], in fact, are not limited to the assumptions made there and, hence, it is not unreasonable to expect implementations of the universal behavior in other physical situations. To realize it, we should emphasize that the derivation of the analytical formulas (1) and (2) in the previous section was based on two hypotheses: (i) the transmission coefficient in all transport channels has the same functional form with the only controlling parameter (for a slowly varying barrier it is an inverse squared hyperbolic cosine with the parameter $Y$; for a barrier with sharp boundaries it is a Lorentzian with the parameter $Z$), and (ii) the governing parameter is almost uniformly distributed from very small to very large values (see the related discussion about spatial distribution of barrier defects in the paper by Il'ichev et al. [16]). These suppositions can be realized in other physical situations than those in the works [5,11]. One of them concerning low barriers with $U_0 \ll E_F$ (in contrast to extremely high barriers in [11]) has been discussed above. Some other examples are given below.

A precondition for the realization of universal distribution functions is a relatively complex structure of the tunneling barrier. For instance, it can be formed by two identical insulating films with a metal M interlayer between them. In the case of an $M_1IMIM_2$ heterostructure, probability of charge transmission across the double-barrier $I_1MI_2$ trilayer can be also expressed as a Lorentzian of a complex controlling parameter which is a rapidly oscillating function of the incident angle and changes periodically from zero in the resonance state to very high values [17]. That is why the formula (2) is valid for an ideal two-barrier tunneling structure as well (as was found in [18]).

The Lorenzian-like dependence of the tunneling junction transparency on a single controlling parameter can be related to the resonance charge transmission across a heterostructure with two or more potential barriers and localized quantum states between them. In this case, to describe elastic transport properties of eigenchannels we can use the Breit–Wigner formalism [19] which states that, for a time scale $\hbar/\Gamma_e$ spent in the resonance state, the elements of the scattering matrix have a denominator of the form $\varepsilon - \varepsilon_{res} + i\Gamma_e/2$ ($\varepsilon$ and $\varepsilon_{res}$ are the incident electron energy and that of the resonance state). If so, the transparency amplitude for tunneling through such a state reads as $D_e = (\Gamma_e/2)^2/[(\varepsilon - \varepsilon_{res})^2 + (\Gamma_e/2)^2]$. The spread of two parameters $\varepsilon_{res}$ and $\Gamma_e$ will lead to the probability distribution (2). Note that elastic tunneling through a single localized site within an insulating layer of comparatively thick barrier $d \gg \kappa^{-1}$ depends also on the localized state position relating metallic electrodes. The maximum of the probability is achieved when the resonance state is located near the central point of the insulating layer. When $\varepsilon \approx \varepsilon_{res}$, the transmission coefficient depends on the only parameter, the spatial deviation $x$ of the localized site from the barrier center $D \sim \cosh^{-2}(2\kappa x)$ [20,21]. In the case of chaotic but homogeneous distribution of this parameter it leads to Eq. (1).

An additional factor which can be important in resonance tunneling is an electron-phonon interaction. In this case electrons in a coherent channel are scattered inelastically and it appears as a part of the incident flux in a specified channel is absorbed. To describe such process, we should include in the scattering formalism all relevant interactions and processes. However, the problem can be simplified by modeling the apparent absorption of electrons with a phenomenological approach based on a small constant imaginary part $-i\Gamma_i$ which absorbs a part of the incident flux and causes a breakdown of unitarity [22]. The added imaginary potential in the Schrödinger equation can be eliminated by introducing into the wave function an additional factor $\exp[-\Gamma_i(\boldsymbol{\rho})t/\hbar]$ which clearly describes the intrinsic decay in time. If so, the previous formula for the forward elastic-scattering probability $D_e$ in a certain scattering eigenchannel can be easily generalized to include inelastic scattering events which would be an additional source of the randomization in the problem. In particular, the total elastic transmission probability $D = (\Gamma_e/2)^2/[(\varepsilon - \varepsilon_{res})^2 + (\Gamma/2)^2]$ with the total decay width $\Gamma = \Gamma_e + \Gamma_i$.

Resuming, the universality of the tunneling probability distributions (1) and (2) is based on simple mathematical $D$-versus-single parameter dependencies which arise in different areas of the charge transport through microscopically disordered potential barriers.

## 4. Possible experimental tests without fitting parameters

We have shown above that charge transport across ideally disordered potential barriers can be described by two universal analytical functions which are bimodal with a significant amount of "open" channels for which $D \leq 1$. It means the emergence of novel functionalities, allowing for a comparison with the experimental data. The best way to test the reliability of the relations (1) and (2) is to perform low-temperature transport measurements on trilayered MIM samples where one or two electrodes are in the superconducting ($S$) state. The reason is a very high sensitivity of the shape of measured characteristics on the barrier transmission coefficient $D$ [12]. Moreover, normalization of experimental curves measured in the superconducting state on relevant normal-state characteristics (see [1]) or determination of the ratio of two principally different quantities in a superconducting state (see below) permits to eliminate the only adjustable parameter, the average macroscopic conductance of the insulating interlayer. This makes it possible to perform test experiments *without any fitting parameters*. Below we limit ourselves to zero temperatures.

To calculate the charge current $I$ across an NIS junction with an amorphous interlayer whose local-transparency distribution is characterized by Eqs. (1) or (2), we need to account specific elastic-scattering processes at the N/S interfaces known as Andreev effects [23] when an electron (hole) incident on the interface from the normal side is retroreflected into a hole (electron) with the same energy and nearly the same momentum travelling in the opposite direction to the incoming charge. In a superconductor $k_x^{(S)} = \sqrt{2m(E_F^{(S)} \pm \sqrt{\varepsilon^2 - \Delta^2})/\hbar^2 - \mathbf{k}_{\parallel}^2}$ where $\Delta$ is the energy gap value. For the sake of simplicity, we assume that Fermi energies in the conducting layers, including the superconducting one, are the same. Andreev electron–hole and vice versa transformations at N/S boundaries without ordinary reflections are described by the $r_{eh}$ (an electron is retroreflected into a hole) and $r_{he}$ (a hole–electron transformation) scattering characteristics [12]:

$$r_{eh}(\varepsilon) = r_{he}(\varepsilon) = r(\varepsilon) = \frac{(\varepsilon + i\delta) - \sqrt{(\varepsilon + i\delta)^2 - \Delta^2}}{\Delta} \tag{5}$$

$\delta$ is a positive infinitesimal number, and $r(E) = \exp(-i \cdot \arccos(\varepsilon/\Delta))$ for $\varepsilon < \Delta$. The calculations based on the scattering formalism can be considerably simplified if to introduce a normal ($n$) auxiliary interlayer of a vanishing thickness between I and S layers and to deal with a four-layered structure $N_1N_2n/S$ structure where finally we shall replace $k_{2x}$ by $i\kappa$. At last, the probability of transition from an $N_1$ injector to the S electrode reads as [24]

$$P(\varepsilon, D) = \left|T(\varepsilon, D)\right|^2 = 1 - \left|R(\varepsilon, D)\right|^2$$
$$= 1 - \left| r_{1I}(D) + \frac{t_I^e(D) r_{eh}(\varepsilon) r_{nI}^h(D) r_{he}(\varepsilon) t_I^e(D)}{1 - r_{eh}(\varepsilon) r_{I2}^h(D) r_{he}(\varepsilon) r_{nI}^e(D)} \right|^2 \tag{6}$$

The current-vs-voltage curves are given by

$$I_{1(2)}(V) = \frac{2e^2}{h} \int_0^1 \rho_{1(2)}(D) \int_0^V P(\varepsilon, D) d\varepsilon \tag{7}$$

Fig. 1 demonstrates normalized (to the normal-state resistance $R_N$) differential conductance spectra $G(V) = dI/dV$-vs-$V$ of NIS junctions with a disordered I interlayer. We would like to emphasize that formally the shape of the dependencies is similar to that which can be obtained within a standard delta-functional approximation $U(x) \sim \delta(x)$ [12] for the parameter $Z \simeq 0.6$–$0.7$, i.e., for a very low potential barrier. It can be that it is an origin of some unexpected statements about extremely low barriers, especially, during the earliest tunneling experiments on high-$T_c$ cuprates [25].

The presence of conductance spectra similar to those in Fig. 1 can prove the existence of "open" channels in the charge transport across the insulating interlayer. But the difference between two curves is not so prominent to make a definite decision concerning the nature of the potential barrier. To do it, we propose to deal with a Josephson SIS junction and to extract two quantities from its current-voltage characteristic, the critical supercurrent $I_s$ at $V = 0$ and a constant shift of the superconducting $I$–$V$ curve towards that measured in the normal state at $V$ exceeding $\Delta/e$ which is known as an excess current $I_{exc}$. According to Ref. [12], its value for an SIS sandwich can be found by doubling the corresponding result for an NIS junction $I_{exc} = \int_0^V G_{SIS}(V') - G_{NIN}(V')dV' = 2\int_0^V [G_{NIS}(V') - G_{NIN}(V')]dV'$. The supercurrent for a certain channel with a transparency $D$ depends on the phase difference $\varphi$ between the two superconducting electrodes and is given by $I_s(\varphi) = (\pi e\Delta/h) D \sin\varphi/\sqrt{1 - D\sin^2(\varphi/2)}$ for $T \ll \Delta/k_B$ [26]. In the absence of the barrier $I_s = \pi\Delta/(eR_N)$ [26] and $I_{exc} = 8\Delta/(3eR_N)$ for $V \gg \Delta$ [12]; hence, $I_s/I_{exc} \approx 1.2$. In the tunneling limit $D \ll 1$, $I_s = \pi\Delta\sin\varphi/(2eR_N)$ [26], $I_{exc} \to 0$ and is negative at very high voltages, thus, $I_s/|I_{exc}| \to \infty$. As was stated above, in a realistic disordered layer of a dielectric a part of channels is opened and the expected result for the ratio $I_s/I_{exc}$ is more than 1.2 but not infinitely large. Averaging related

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formulas, we obtain that $I_s/I_{exc} \approx 1.4$ for the distribution function $P_1(D)$ whereas for $P_2(D) I_s/I_{exc} \approx 1.7$.

Let us now analyze related experiment [27]. The authors studied experimentally trilayered junctions made of superconducting MoRe alloy electrodes with a Si interlayer doped by W. For ultrathin ($d < 10$ nm) pure Si barriers as well as those with a low degree of doping ($c_W < 5$ at.%), normal-state $I$–$V$ characteristics have not exhibited any nonlinearities which appeared for increased dopant concentrations $c_W$. This finding was, in our opinion, an indication of the formation of metal nanoclusters with individual localized energies inside the Si layer. As a result, in a superconducting state critical and excess currents at 4.2 K were well pronounced for the $W$ concentration about 10 at.%. The parameter $I_{exc}$ was found by extrapolation back to the axis $V = 0$. The voltage where the extrapolation starts from was taken near the value of $V = \Delta/e$ because of significant nonlinearities in normal-state $I$–$V$ curves even at small voltages for inelastic, charge-hopping transport inside insulating layers [28]. The ratio value $I_s/I_{exc} \approx 1.7$–$1.8$ extracted from these measurements is in the excellent agreement with our theoretical predictions for the $P_2(D)$ distribution. According to the results of the previous section, it means that single localized states were located near the central point of a semiconducting layer and a very large spread of atomic localization energies was the main factor determining appearance of "open" channels in the transport characteristics.

Now we want to show that the probability distributions (1) and (2) can arise not only for a transport problem but in the low-temperature dynamics of other disordered systems which exhibit no obvious long-range order as well. SQUID-magnetometry experiments [13,14] have showed interesting quantum dynamics arisen in metallic single-crystal samples with a normal bulk and a disordered superconducting near-surface region (the inhomogeneous structure of the near-surface region was proved by corresponding tunneling measurements). It is well known that the emergence of the very thin superconducting sheath is favorable in magnetic fields $H$ above the second critical field $H_{c2} = 1.41\kappa H_c$ but below the third critical field $H_{c3} = 2.38\kappa H_c$ ($\kappa$ is the Ginzburg–Landau parameter, $H_c$ is the thermodynamic critical field). In such fields it is able to separate the dynamics of the surface layer which was disordered in boride crystals $ZrB_{12}$ and $YB_6$ studied in [13,14] from that of a perfect bulk. To do it, the sheath with survived superconductivity was driven out of equilibrium by small ac magnetic fields. In this case the temporal dynamics should reflect a random walk on the configuration space, which was expected to be rather complicated due to the presence of a disordered set of potential barriers in the near-surface nanoscaled region. In the papers [13,14] dc magnetic fields from $H_{c2} \approx 180$ Oe to $H_{c3} \approx 300$ Oe for $ZrB_{12}$ and from $H_{c2} \approx 1500$ Oe to $H_{c3} \approx 2500$ Oe for $YB_6$ and ac excitation fields of the amplitude about 0.05 Oe were applied to the boride single crystals and the magnetic susceptibility $\chi(f)$ was measured as a function of frequency $f$ varying from 1 to 1000 Hz at 4.5 K. We suppose that the measured characteristic is affected by an ensemble of surface screening currents which relax to its own equilibrium state after a slight perturbation by tunneling through potential barriers. The corresponding relaxation rate is proportional to the barrier transparency $\lambda = \lambda_0 D$, $\lambda_0$ is an attempt frequency. Each relaxation mode generates a Lorentzian noise spectrum $\sim\lambda/(f^2 + \lambda^2)$ (see [29]). In a strongly disordered system like the boride surfaces the distribution $P(\lambda)$ yields a power spectral density of the response noise which in the case of smooth barriers reads as $S(f) \sim \int d\lambda P_1(\lambda)\frac{\lambda}{f^2+\lambda^2} = \int dD P_1(D)\frac{D}{(f/\lambda_0)^2+D^2}$. For very low frequencies $f \ll \lambda_0$ and $hf \ll k_B T$ the power spectral density $S(f) \sim 1/f$ and the fluctuation-dissipation theorem gives $\mathrm{Im}\,\chi(f) = \pi f S(f)/k_B T$. Thus, $\mathrm{Im}\,\chi(f)$ should be a constant at a fixed temperature and $\mathrm{Re}\,\chi(f)$ should be proportional to $-\ln f$ according to the Kramers–Kronig relation. And it is just what was observed in the borides in the low-frequency limit [13,14], see Fig. 2. Extension of such measurements to high frequencies would allow validation of the assumption about the barrier distribution function.

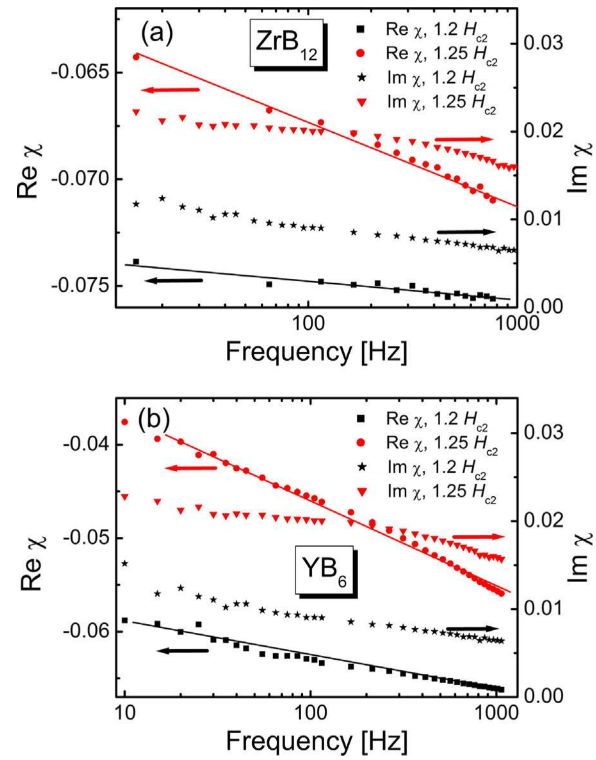


**Fig. 2.** Frequency dependence of real and imaginary parts of the ac magnetic susceptibility measured for $ZrB_{12}$ (a) and $YB_6$ (b) single crystals in a surface superconducting state for two dc magnetic field values between critical magnetic fields $H_{c2}$ and $H_{c3}$ at 4.5 K [13,14].

## 5. Conclusions

In summary, in the present work we have shown that the distribution of local transparencies across a strongly disordered potential barrier can be universal. This question has a long history and has been discussed by several authors [4,10]. In this paper, starting with a scattering approach that describes charge transferring across a single potential barrier of an arbitrary shape, we have revealed what assumptions underlie the derivation of the universal functions (1) and (2). From their application to superconducting junctions, we have demonstrated that experimental study of charge transport across SIS junctions is the best way to test the reliability of the relations (1) and (2) and to distinguish between them *without any fitting parameters.*

Although the proposed two approaches are plausible, their microscopic justification has been lacking in a strict sense. In particular, it is questionable whether the disorder effect is fully described by the spread of a single parameter or not. Novel well-designed experiments are needed to answer this question.

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