Improving data analysis on the measurement of Gaussian laser beam radius using the knife-edge technique

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In this Letter we revisited the well known Khosrofian and Garetz's inversion algorithm, developed to analyze data obtained by the application of traveling knife-edge technique. We have analyzed their approximated fitting function, used for adjusting the experimental data, and found that it is not optimized to work with the full range of the experimentally measured data. We have numerically calculated a new set of coefficients, which turn the approximated function suitable for the full experimental range, considerably improving the accuracy in the measurement of the radius of a focused Gaussian laser beam.

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1. Introduction

The accurate measurement of the waist of a laser beam near the focus of a lens is very important in many applications [1], for instance in the Z-scan [2] and thermal lens spectrometry [3]. Many techniques were developed with this purpose, such as the slit scan technique [4, 5], the pinhole technique [6], but among the most used is the knife-edge technique [7–9]. The knife-edge technique is a beam profiling method that allows for quick, inexpensive, and accurate determination of the beam parameters. The knife-edge technique is being widely used for decades and is considered a standard technique for Gaussian laser beam characterization [10]. In this technique a knife edge moves perpendicular to the direction of propagation of the laser beam and the total transmitted power is measured as a function of the knife edge position. A typical experimental setup is shown in Fig.1. The knife-edge technique requires a sharp edge (typically a razor blade), a translation stage with a micrometer and a power meter or an energy meter when working with pulses.

In our discussion we will consider a radially symmetric Gaussian laser beam with intensity described by

$$I(x,y) = I_0 \exp\left[-\frac{(x-x_0)^2 + (y-y_0)^2}{w^2}\right], \tag{1}$$

where I_0 is the peak intensity at the center of the beam, located at (x_0, y_0), x and y are the transverse Cartesian coordinates of any point with respect to an origin conveniently chosen at the beginning of an experiment, and w is the beam radius, measured at a position where the intensity decreases to $1/e$ times its maximum value I_0. Eq. (1) is not the only way to express the intensity of a Gaussian laser beam. Some authors prefer to define the beam radius at a position where the electric field amplitude drops to $1/e$, while the intensity drops to $1/e^2$ times the maximum value. Our choice in the definition of the intensity follows the choice made by Khosrofian and Garetz [9].

With the knife-edge initially blocking the laser beam, the micrometer can be adjusted in appropriate increments, and the normalized transmitted power is obtained by the integral:

$$P_N = \frac{\int_{-\infty}^{x}\int_{-\infty}^{\infty} I(x',y)dydx'}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x',y)dydx'}, \tag{2}$$

which gives,

$$P_N(x) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{x-x_0}{w}\right)\right], \tag{3}$$

where erf is the error function.

The area of the photodiode is considered to be larger than the laser beam cross-section area at the detection position, so that diffraction effects may be neglected. The large area photodiode may be substituted by an small area photodiode coupled to an integrating sphere [8].

2. The data analysis

The error function in Eq. (3) is not an analytical function and its use in fitting experimental data is not a practical procedure. One approach in data analysis is to work with the derivative of Eq. (3) [7, 11, 12], that is analytical and is given by

$$\frac{dP_N(x)}{dx} = \frac{1}{\sqrt{\pi}w} exp\left[-\frac{(x-x_0)^2}{w^2}\right]. \tag{4}$$

But the process of taking derivatives of experimental data with fluctuations result in amplification of the fluctuations, and consequently an increase on the errors. In order to overcome this problem, Khosrofian and Garetz [9] suggested a substitution of $P_N(x)$ by an analytical function, that approximately represents $P_N(x)$, to fit the experimental data. This fitting function is given by

$$f(s) = \frac{1}{1+\exp\left[p\left(s\right)\right]}, \tag{5}$$

where

$$p(s) = \sum_{i=0}^{m} a_i s^i, \tag{6}$$

and

$$s = \frac{\sqrt{2}(x-x_0)}{w}. \tag{7}$$

For practical reasons Khosrofian and Garetz limited the polynomial $p(s)$ to the third order term, so that

$$f(s) = \frac{1}{1+\exp\left(a_0+a_1 s+a_2 s^2+a_3 s^3\right)}. \tag{8}$$

Using data from tabulated normal distribution function and least-square analysis, the polynomial coefficients were determined as,

$$\begin{aligned} a_0 &= -6.71387 \times 10^{-3}, \\ a_1 &= -1.55115, \end{aligned}$$

$$
\begin{aligned}
a_2 &= -5.13306 \times 10^{-2}, \\
a_3 &= -5.49164 \times 10^{-2}.
\end{aligned}
$$

Although this fitting function is being used for decades, and referenced by many authors [13, 14], we decided to compare it with the exact function, given by Eq. (3). The first step in the comparison process was to plot the equations in the same graphics. The result is shown in Fig. 2. We verified that the fitting function presents a very good adjustment for $f(s) > 0.5$, but fails to adjust for $f(s) < 0.5$. This result is a consequence of the procedure that has been employed to fit $f(s)$ to the data points, because the parameters that define $f(s)$ have been determined from tabulated normal data with positive arguments only. To extend the procedure to include negative arguments of $f(s)$, Khosrofian and Garetz assumed that $f(-s) = 1 - f(s)$. But since $f(s)$ contains $p(s)$, that is a polynomial that includes terms of even powers of s, this assumption is not valid. Considering the symmetry of error function, the fitting function $f(s)$ must contains only terms of odd powers of s. In fact, a fitting of $f(s)$ to the exact data, given by Eq. (3), shows that a_0 and a_2 numerically converge to zero and the new non null adjusted coefficients, up to the third order, are given by

$$
\begin{aligned}
a_1 &= -1.597106847, \\
a_3 &= -7.0924013 \times 10^{-2}.
\end{aligned}
$$

We thus may write Eq. (8) as

$$
f(s) = \frac{1}{1 + \exp\left(a_1 s + a_3 s^3\right)}. \tag{9}
$$

To arrive at these new coefficients we have generated a set of points, directly from Eq. (3) with $x_0 = 0$ and $w = 1$, by using Maple 10, and with the help of Origin 7.5 we fitted the data set with the Eq. (8). The fitting procedure was to keep x_0 and w fixed, letting the coefficients to vary.

By fitting the same simulated data set with $f(s)$ given by Eq. (8), with the old coefficients, the obtained values for x_0 and w were 0.0132 and 0.9612, respectively. This corresponds to a difference of about 3.9% in the laser beam radius, and the error in the center position, relative to the beam radius, of about 1.3%. These differences may represent a serious problem in high accuracy experiments. For example, since the laser intensity is inversely proportional to the square of the radius, an overestimation of about 7.6% on the laser intensity will result if Eq. (8) is used as the fitting function. On the other hand, an estimation of the error in w and x_0 give values in the range $10^{-7} - 10^{-8}$ when fitting Eq. (9) to the exact function, given by Eq. (3). With these results we may say that Eq. (9) is not only a good approximation for our particular problem, but it may also be useful in many numerical problems in different fields of science involving the error function. As an example of the use of analytical expressions for the error function in another physical problem we may refer to the work of P. Van Halen [15], which was used to calculate the electric field and potential distribution in semiconductor junctions with a Gaussian doping profile.

The inclusion of the fifth order term in the polynomial $p(s)$ will further improve the accuracy, but it is not worth doing this in the analysis of the knife-edge technique data, where the experimental fluctuations dominates the errors in the data analysis. But since the focus of our discussion is the improvement of the data analysis, and a possible use of this fitting function in different kind of problems we will extend our discussion to analyze the behavior of $f(s)$ when the fifth order term is included. The first annotation about the inclusion of the fifth order term a_5 in the polynomial $p(s)$ is that it will require a recalculation of all the coefficients, so that a_1 and a_3 will change. So the new calculated coefficients are given by

$$\begin{aligned} a_1 &= -1.5954086, \\ a_3 &= -7.3638857 \times 10^{-2}, \\ a_5 &= +6.4121343 \times 10^{-4}. \end{aligned}$$

In order to verify how close the approximated functions are from the exact function $P_N(x)$ we have plotted the differences between $f(s)$ and $P_N(x)$ for $(x - x_0)/w$ ranging from -4.0 to 4.0, covering the full range of interest. In Fig. 4a., $f(s)$ given by Eq. (8), was used in two different ways: with the parameters $w = 1.0$ and $x_0 = 0.0$ (solid line), and $w = 0.9612$ and $x_0 = 0.0132$ (dashed line) obtained when one tries to fit $P_N(x)$ with $f(s)$. In Fig. 4b. the differences are calculated with $f(s)$ given by Eq. 9, in a situation where only the coefficients a_1 and a_3 are considered (solid line), and when the new set of coefficients that includes a_5 is considered.

By analyzing the curves shown in Fig. 4 we may conclude that the approximated function

$f(s)$ defined by Eq. 9 is, on the average, two orders of magnitude closer to the exact function $P_N(x)$ than that defined by Eq. 8. When the fifth order term is included in the polynomial $p(s)$ the approximation is even better, making the biggest difference to be about 2×10^{-5} in the full range of interest.

3. Analysis of experimental data

In order to verify how the choice of the fitting function interferes in the true experimental data analysis we performed a simple experiment using the setup shown in Fig. 1. In our experiment a He:Ne laser with an output power of 10 mW was focused by a 25 cm focal length lens. A razor blade was mounted on top of a motorized translation stage made by Newport (model M-UTM150PP.1), which resolution was 0.1 μm. The translation stage position was controlled by a computer while the total transmitted laser power was measured by an Ophir NOVA power meter. The analog output signal of the power meter was sent to the computer through a National Instruments USB-6000 acquisition card. We set the speed of the translation stage at 0.5 mm/s and the acquisition rate at 100 samples/s. The experimental data, taken at a position near the focus of the lens, is shown in Fig. 5, where we also show a fitting of the experimental data with Eq. (9). The same fitting was done with Eq. (8), and although both equations give rise to curves that apparently are representative of the experimental data, they result in different values for the laser beam radius. After analyzing 10 scans, fitting each data set with Eq. (9), we arrived at the mean value $w = 36.60 \pm 0.06$ μm. A result 3.8% lower than this is obtained if one tries to fit the same experimental data with Eq. (8). This confirms the necessity of using the correct fitting function to analyze the experimental data. If we now compare the position of the beam center, given by the two fitting functions, we find a difference, relative to the radius, of 1.2% between the results. Since the type of errors introduced by the use of Eq. (8) is systematic, past results on laser beam radius may be corrected by using a multiplying factor of 1.04.

If one defines the radius of the laser beam at a position where the intensity drops to $1/e^2$ times the maximum value, one needs to multiply w by $\sqrt{2}$ to arrive at the desired value.

4. Conclusions

We have shown that a modified sigmoidal function, based on the Khosrofian and Garetz‘s function, with new coefficients are needed for correct laser beam characterization in the knife-edge technique. We have found these new coefficients and showed that the new function fits very well the experimental data and improves the accuracy of the results.

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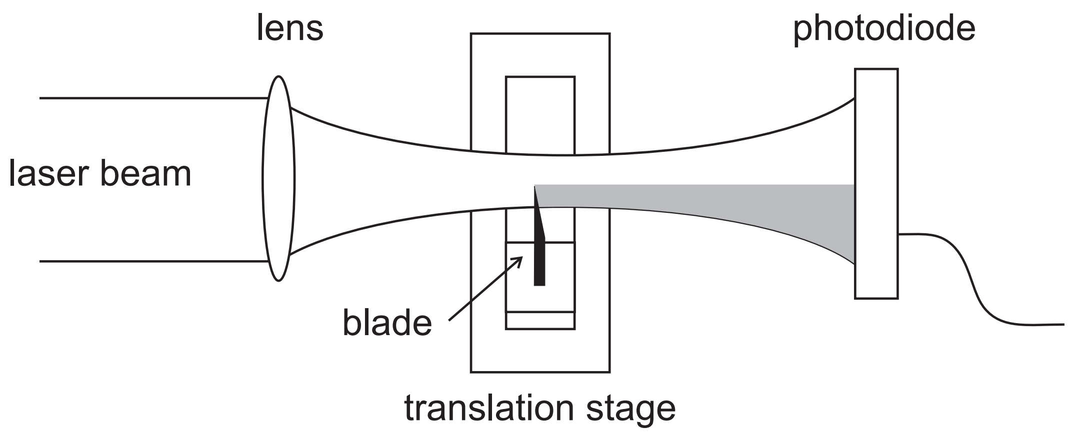


Figure 1

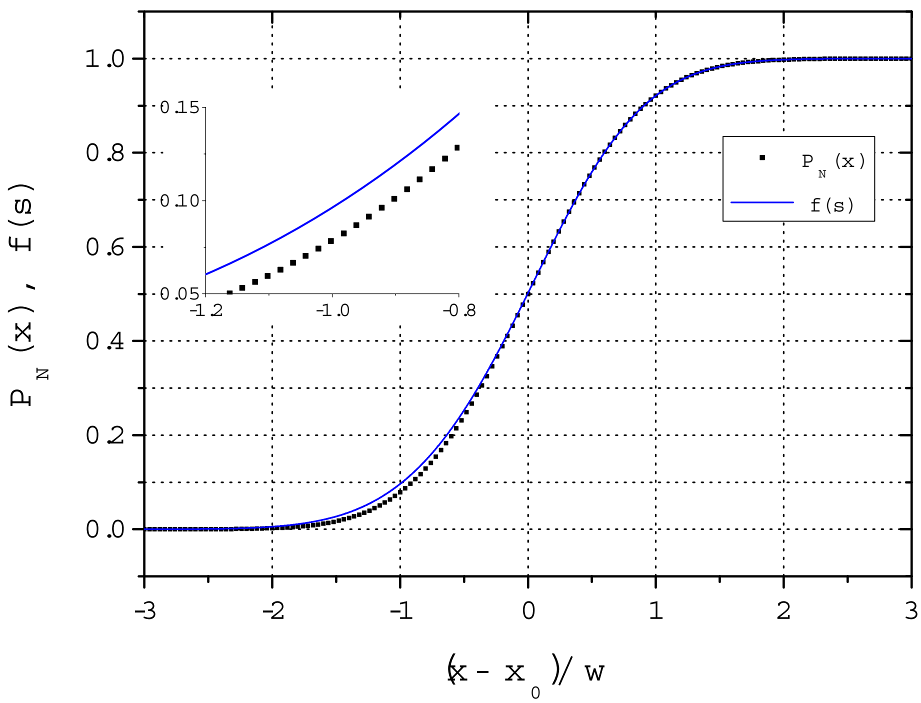


Figure 2

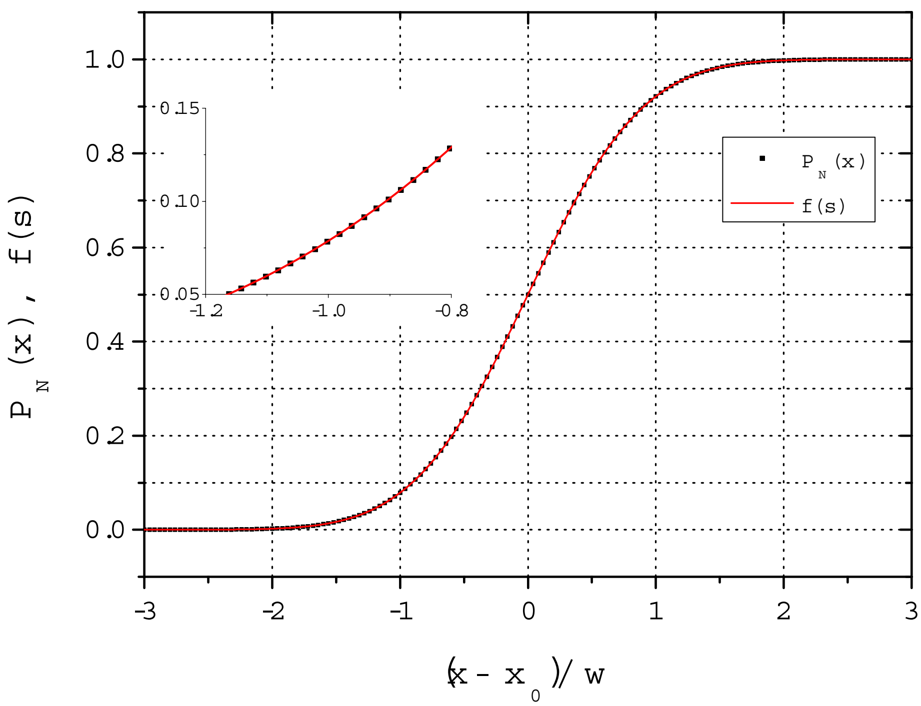
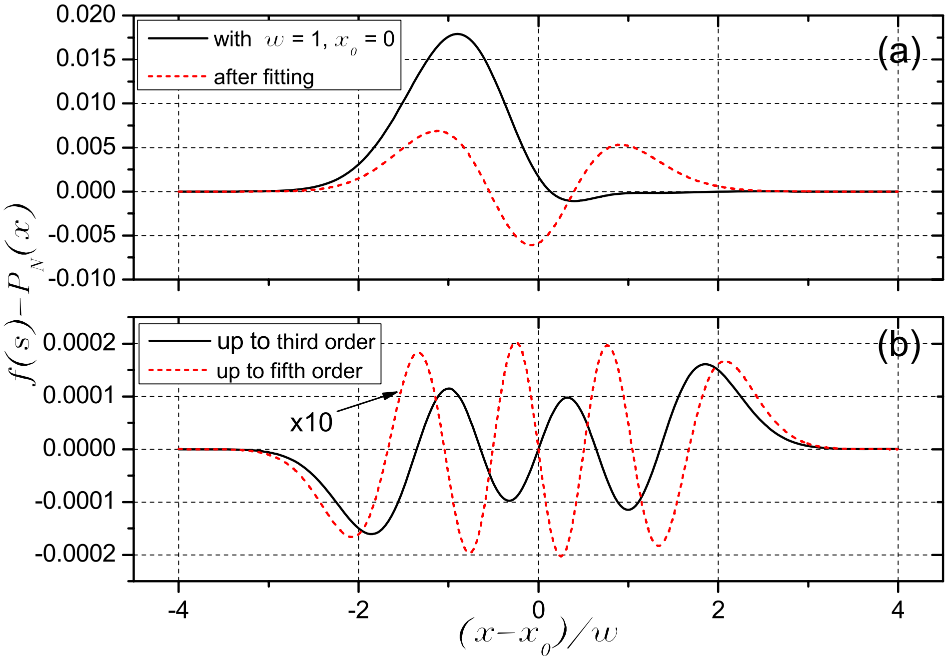


Figure 3



0.020
0.015
0.010
0.005
0.000
-0.005
-0.010
with w = 1, x_o = 0
after fitting
(a)
$f(s)-P_N(x)$
0.0002
0.0001
0.0000
-0.0001
-0.0002
up to third order
up to fifth order
x10
(b)
-4
-2
0
2
4
$(x-x_o)/w$

Figure 4

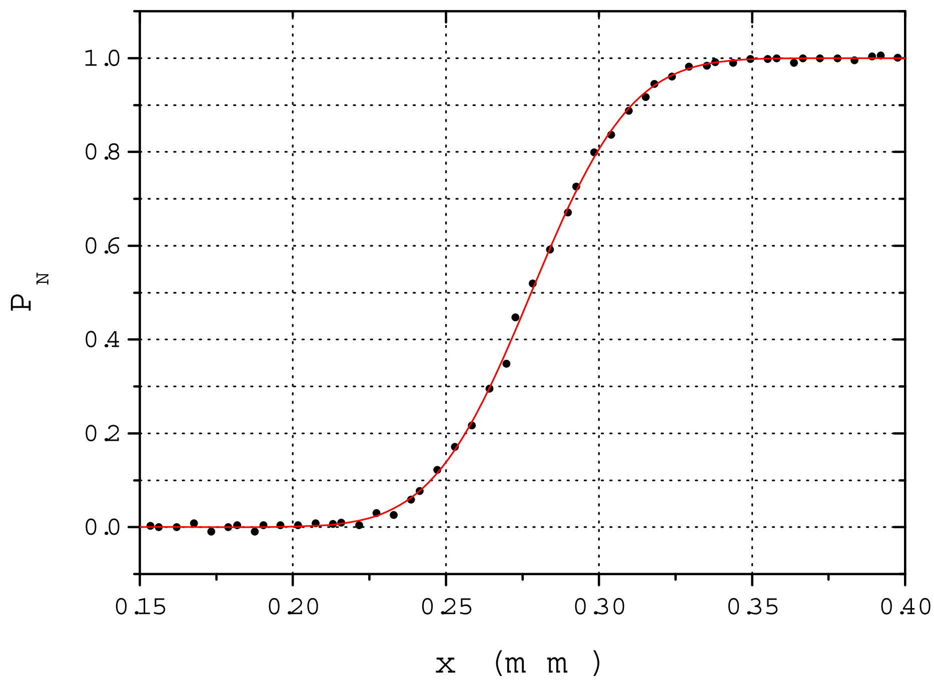


Figure 5

Figure Captions

Fig. 1. Simplified scheme for the measurement of the laser beam radius using the knife-edge technique. The gray color area represents the shadow caused by the knife-edge.

Fig. 2. Comparison of the data obtained from Eq. (3) with $f(s)$ defined by Eq. (8).

Fig. 3. Fitting the data obtained from Eq. (3) with $f(s)$ defined by Eq. (9).

Fig. 4. Differences between $f(s)$ and $P_N(x)$. In (a) $f(s)$ is given by Eq. (8), with the parameters $w = 1.0$ and $x_0 = 0.0$ (solid line), and $w = 0.9612$ and $x_0 = 0.0132$ (dashed line). In (b) $f(s)$ is given by Eq. 9, when only the coefficients a_1 and a_3 are considered (solid line), and when the new set of coefficients that includes a_5 is considered.

Fig. 5. Fitting of the experimental data using Eq. (9). A similar curve is obtained by using Eq. (8), but with the adjusted laser beam radius 3.8% lower.