Instructor Solutions Manual
for
Physics
by
Halliday, Resnick, and Krane

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Volume 2

A Note To The Instructor...

The solutions here are somewhat brief, as they are designed for the instructor, not for the student. Check with the publishers before electronically posting any part of these solutions; website, ftp, or server access *must* be restricted to your students.

I have been somewhat casual about subscripts whenever it is obvious that a problem is one dimensional, or that the choice of the coordinate system is irrelevant to the *numerical* solution. Although this does not change the validity of the answer, it will sometimes obfuscate the approach if viewed by a novice.

There are some *traditional* formula, such as

$$v_x^2 = v_{0x}^2 + 2a_x x,$$

which are not used in the text. The worked solutions use only material from the text, so there may be times when the solution here seems unnecessarily convoluted and drawn out. Yes, I know an easier approach existed. But if it was not in the text, I did not use it here.

I also tried to avoid reinventing the wheel. There are some exercises and problems in the text which build upon previous exercises and problems. Instead of rederiving expressions, I simply refer you to the previous solution.

I adopt a different approach for rounding of significant figures than previous authors; in particular, I usually round intermediate answers. As such, some of my answers will differ from those in the back of the book.

Exercises and Problems which are enclosed in a box also appear in the Student's Solution Manual with considerably more detail and, when appropriate, include discussion on any physical implications of the answer. These student solutions carefully discuss the steps required for solving problems, point out the relevant equation numbers, or even specify where in the text additional information can be found. When two almost equivalent methods of solution exist, often both are presented. You are encouraged to refer students to the Student's Solution Manual for these exercises and problems. However, the material from the Student's Solution Manual must *not* be copied.

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$\boxed{\textbf{E25-1}}$ The charge transferred is

$$Q = (2.5 \times 10^4\,\mathrm{C/s})(20 \times 10^{-6}\,\mathrm{s}) = 5.0 \times 10^{-1}\,\mathrm{C}.$$

E25-2 Use Eq. 25-4:

$$r = \sqrt{\frac{(8.99\times10^9\mathrm{N\cdot m^2/C^2})(26.3\times10^{-6}\mathrm{C})(47.1\times10^{-6}\mathrm{C})}{(5.66\,\mathrm{N})}} = 1.40\,\mathrm{m}$$

E25-3 Use Eq. 25-4:

$$F = \frac{(8.99\times10^9\mathrm{N\cdot m^2/C^2})(3.12\times10^{-6}\mathrm{C})(1.48\times10^{-6}\mathrm{C})}{(0.123\,\mathrm{m})^2} = 2.74\,\mathrm{N}.$$

E25-4 (a) The forces are equal, so $m_1a_1 = m_2a_2$, or

$$m_2 = (6.31\times10^{-7}\mathrm{kg})(7.22\,\mathrm{m/s^2})/(9.16\,\mathrm{m/s^2}) = 4.97\times10^{-7}\mathrm{kg}.$$

(b) Use Eq. 25-4:

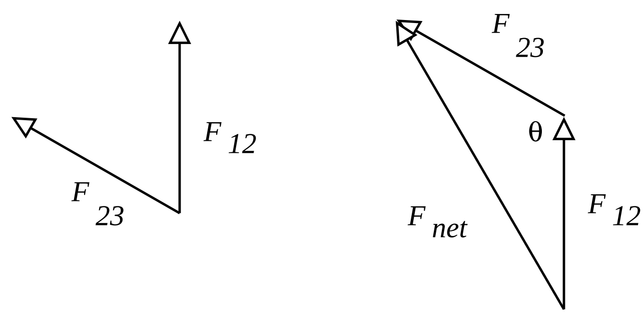
$$q = \sqrt{\frac{(6.31\times10^{-7}\mathrm{kg})(7.22\,\mathrm{m/s^2})(3.20\times10^{-3}\mathrm{m})^2}{(8.99\times10^9\mathrm{N\cdot m^2/C^2})}} = 7.20\times10^{-11}\mathrm{C}$$

$\boxed{\textbf{E25-5}}$ (a) Use Eq. 25-4,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2} = \frac{1}{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})}\frac{(21.3\,\mu\mathrm{C})(21.3\,\mu\mathrm{C})}{(1.52\,\mathrm{m})^2} = 1.77\,\mathrm{N}$$

(b) In part (a) we found F_{12}; to solve part (b) we need to first find F_{13}. Since $q_3 = q_2$ and $r_{13} = r_{12}$, we can immediately conclude that $F_{13} = F_{12}$.

We must assess the direction of the force of q_3 on q_1; it will be directed along the line which connects the two charges, and will be directed away from q_3. The diagram below shows the directions.



From this diagram we want to find the magnitude of the *net* force on q_1. The cosine law is appropriate here:

$$\begin{aligned}
{F_\mathrm{net}}^2 &= F_{12}^2 + F_{13}^2 - 2F_{12}F_{13}\cos\theta,\\
&= (1.77\,\mathrm{N})^2 + (1.77\,\mathrm{N})^2 - 2(1.77\,\mathrm{N})(1.77\,\mathrm{N})\cos(120^\circ),\\
&= 9.40\,\mathrm{N}^2,\\
F_\mathrm{net} &= 3.07\,\mathrm{N}.
\end{aligned}$$

E25-6 Originally $F_0 = CQ_0^2 = 0.088\,\mathrm{N}$, where C is a constant. When sphere 3 touches 1 the charge on both becomes $Q_0/2$. When sphere 3 the touches sphere 2 the charge on each becomes $(Q_0 + Q_0/2)/2 = 3Q_0/4$. The force between sphere 1 and 2 is then

$$F = C(Q_0/2)(3Q_0/4) = (3/8)CQ_0^2 = (3/8)F_0 = 0.033\,\mathrm{N}.$$

$\boxed{\textbf{E25-7}}$ The forces on q_3 are $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$. These forces are given by the vector form of Coulomb's Law, Eq. 25-5,

$$\begin{aligned}
\vec{\mathbf{F}}_{31} &= \frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{r_{31}^2}\hat{\mathbf{r}}_{31} = \frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{(2d)^2}\hat{\mathbf{r}}_{31}, \\
\vec{\mathbf{F}}_{32} &= \frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{r_{32}^2}\hat{\mathbf{r}}_{32} = \frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{(d)^2}\hat{\mathbf{r}}_{32}.
\end{aligned}$$

These two forces are the only forces which act on q_3, so in order to have q_3 in equilibrium the forces must be equal in magnitude, but opposite in direction. In short,

$$\begin{aligned}
\vec{\mathbf{F}}_{31} &= -\vec{\mathbf{F}}_{32}, \\
\frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{(2d)^2}\hat{\mathbf{r}}_{31} &= -\frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{(d)^2}\hat{\mathbf{r}}_{32}, \\
\frac{q_1}{4}\hat{\mathbf{r}}_{31} &= -\frac{q_2}{1}\hat{\mathbf{r}}_{32}.
\end{aligned}$$

Note that $\hat{\mathbf{r}}_{31}$ and $\hat{\mathbf{r}}_{32}$ both point in the same direction and are both of unit length. We then get

$$q_1 = -4q_2.$$

E25-8 The horizontal and vertical contributions from the upper left charge and lower right charge are straightforward to find. The contributions from the upper left charge require slightly more work. The diagonal distance is $\sqrt{2}a$; the components will be weighted by $\cos 45^\circ = \sqrt{2}/2$. The diagonal charge will contribute

$$\begin{aligned}
F_x &= \frac{1}{4\pi\epsilon_0}\frac{(q)(2q)}{(\sqrt{2}a)^2}\frac{\sqrt{2}}{2}\hat{\mathbf{i}} = \frac{\sqrt{2}}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{i}}, \\
F_y &= \frac{1}{4\pi\epsilon_0}\frac{(q)(2q)}{(\sqrt{2}a)^2}\frac{\sqrt{2}}{2}\hat{\mathbf{j}} = \frac{\sqrt{2}}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{j}}.
\end{aligned}$$

(a) The horizontal component of the net force is then

$$\begin{aligned}
F_x &= \frac{1}{4\pi\epsilon_0}\frac{(2q)(2q)}{a^2}\hat{\mathbf{i}} + \frac{\sqrt{2}}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{i}}, \\
&= \frac{4+\sqrt{2}/2}{4\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{i}}, \\
&= (4.707)(8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(1.13\times10^{-6}\mathrm{C})^2/(0.152\,m)^2\hat{\mathbf{i}} = 2.34\,\mathrm{N}\,\hat{\mathbf{i}}.
\end{aligned}$$

(b) The vertical component of the net force is then

$$\begin{aligned}
F_y &= -\frac{1}{4\pi\epsilon_0}\frac{(q)(2q)}{a^2}\hat{\mathbf{j}} + \frac{\sqrt{2}}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{j}}, \\
&= \frac{-2+\sqrt{2}/2}{8\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{j}}, \\
&= (-1.293)(8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(1.13\times10^{-6}\mathrm{C})^2/(0.152\,m)^2\hat{\mathbf{j}} = -0.642\,\mathrm{N}\,\hat{\mathbf{j}}.
\end{aligned}$$

E25-9 The magnitude of the force on the negative charge from each positive charge is

$$F = (8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(4.18\times10^{-6}\text{C})(6.36\times10^{-6}\text{C})/(0.13\,\text{m})^2 = 14.1\,\text{N}.$$

The force from each positive charge is directed along the side of the triangle; but from symmetry only the component along the bisector is of interest. This means that we need to weight the above answer by a factor of $2\cos(30^\circ) = 1.73$. The net force is then 24.5 N.

E25-10 Let the charge on one sphere be q, then the charge on the other sphere is $Q = (52.6\times 10^{-6}\text{C}) - q$. Then

$$\begin{aligned}
\frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2} &= F,\\
(8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)q(52.6\times10^{-6}\text{C} - q) &= (1.19\,\text{N})(1.94\,\text{m})^2.
\end{aligned}$$

Solve this quadratic expression for q and get answers $q_1 = 4.02\times10^{-5}\text{C}$ and $q_2 = 1.24\times10^{-6}\text{N}$.

$\boxed{\textbf{E25-11}}$ This problem is similar to Ex. 25-7. There are some additional issues, however. It is easy enough to write expressions for the forces on the third charge

$$\begin{aligned}
\vec{\mathbf{F}}_{31} &= \frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{r_{31}^2}\hat{\mathbf{r}}_{31},\\
\vec{\mathbf{F}}_{32} &= \frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{r_{32}^2}\hat{\mathbf{r}}_{32}.
\end{aligned}$$

Then

$$\begin{aligned}
\vec{\mathbf{F}}_{31} &= -\vec{\mathbf{F}}_{32},\\
\frac{1}{4\pi\epsilon_0}\frac{q_3q_1}{r_{31}^2}\hat{\mathbf{r}}_{31} &= -\frac{1}{4\pi\epsilon_0}\frac{q_3q_2}{r_{32}^2}\hat{\mathbf{r}}_{32},\\
\frac{q_1}{r_{31}^2}\hat{\mathbf{r}}_{31} &= -\frac{q_2}{r_{32}^2}\hat{\mathbf{r}}_{32}.
\end{aligned}$$

The only way to satisfy the *vector* nature of the above expression is to have $\hat{\mathbf{r}}_{31} = \pm\hat{\mathbf{r}}_{32}$; this means that q_3 must be collinear with q_1 and q_2. q_3 could be between q_1 and q_2, or it could be on either side. Let's resolve this issue now by putting the values for q_1 and q_2 into the expression:

$$\begin{aligned}
\frac{(1.07\,\mu\text{C})}{r_{31}^2}\hat{\mathbf{r}}_{31} &= -\frac{(-3.28\,\mu\text{C})}{r_{32}^2}\hat{\mathbf{r}}_{32},\\
r_{32}^2\hat{\mathbf{r}}_{31} &= (3.07)r_{31}^2\hat{\mathbf{r}}_{32}.
\end{aligned}$$

Since squared quantities are positive, we can only get this to work if $\hat{\mathbf{r}}_{31} = \hat{\mathbf{r}}_{32}$, so q_3 is *not* between q_1 and q_2. We are then left with

$$r_{32}^2 = (3.07)r_{31}^2,$$

so that q_3 is closer to q_1 than it is to q_2. Then $r_{32} = r_{31} + r_{12} = r_{31} + 0.618\,\text{m}$, and if we take the square root of both sides of the above expression,

$$\begin{aligned}
r_{31} + (0.618\,\text{m}) &= \sqrt{(3.07)}r_{31},\\
(0.618\,\text{m}) &= \sqrt{(3.07)}r_{31} - r_{31},\\
(0.618\,\text{m}) &= 0.752r_{31},\\
0.822\,\text{m} &= r_{31}
\end{aligned}$$

E25-12 The magnitude of the magnetic force between any two charges is kq^2/a^2, where $a = 0.153\,\text{m}$. The force between each charge is directed along the side of the triangle; but from symmetry only the component along the bisector is of interest. This means that we need to weight the above answer by a factor of $2\cos(30°) = 1.73$. The net force on any charge is then $1.73kq^2/a^2$.

The length of the angle bisector, d, is given by $d = a\cos(30°)$.

The distance from any charge to the center of the equilateral triangle is x, given by $x^2 = (a/2)^2 + (d - x)^2$. Then

$$x = a^2/8d + d/2 = 0.644a.$$

The angle between the strings and the plane of the charges is θ, given by

$$\sin\theta = x/(1.17\,\text{m}) = (0.644)(0.153\,\text{m})/(1.17\,\text{m}) = 0.0842,$$

or $\theta = 4.83°$.

The force of gravity on each ball is directed vertically and the electric force is directed horizontally. The two must then be related by

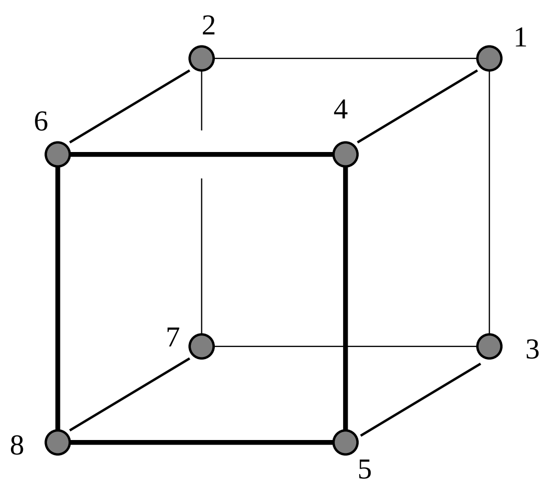
$$\tan\theta = F_\text{E}/F_\text{G},$$

so

$$1.73(8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)q^2/(0.153\,\text{m})^2 = (0.0133\,\text{kg})(9.81\,\text{m/s}^2)\tan(4.83°),$$

or $q = 1.29\times10^{-7}\text{C}$.

E25-13 On any corner charge there are seven forces; one from each of the other seven charges. The net force will be the sum. Since all eight charges are the same all of the forces will be repulsive. We need to sketch a diagram to show how the charges are labeled.



The magnitude of the force of charge 2 on charge 1 is

$$F_{12} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r_{12}^2},$$

where $r_{12} = a$, the length of a side. Since both charges are the same we wrote q^2. By symmetry we expect that the magnitudes of F_{12}, F_{13}, and F_{14} will all be the same and they will all be at right angles to each other directed along the edges of the cube. Written in terms of vectors the forces would be

$$
\begin{aligned}
\vec{\mathbf{F}}_{12} &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{i}},\\
\vec{\mathbf{F}}_{13} &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{j}},\\
\vec{\mathbf{F}}_{14} &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\hat{\mathbf{k}}.
\end{aligned}
$$

The force from charge 5 is

$$F_{15} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r_{15}^2},$$

and is directed along the side diagonal away from charge 5. The distance r_{15} is also the side diagonal distance, and can be found from

$$r_{15}^2 = a^2 + a^2 = 2a^2,$$

then

$$F_{15} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2a^2}.$$

By symmetry we expect that the magnitudes of F_{15}, F_{16}, and F_{17} will all be the same and they will all be directed along the diagonals of the faces of the cube. In terms of components we would have

$$
\begin{aligned}
\vec{\mathbf{F}}_{15} &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{2a^2}\left(\hat{\mathbf{j}}/\sqrt{2}+\hat{\mathbf{k}}/\sqrt{2}\right),\\
\vec{\mathbf{F}}_{16} &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{2a^2}\left(\hat{\mathbf{i}}/\sqrt{2}+\hat{\mathbf{k}}/\sqrt{2}\right),\\
\vec{\mathbf{F}}_{17} &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{2a^2}\left(\hat{\mathbf{i}}/\sqrt{2}+\hat{\mathbf{j}}/\sqrt{2}\right).
\end{aligned}
$$

The last force is the force from charge 8 on charge 1, and is given by

$$F_{18} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r_{18}^2},$$

and is directed along the cube diagonal away from charge 8. The distance r_{18} is also the cube diagonal distance, and can be found from

$$r_{18}^2 = a^2 + a^2 + a^2 = 3a^2,$$

then in term of components

$$\vec{\mathbf{F}}_{18} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{3a^2}\left(\hat{\mathbf{i}}/\sqrt{3}+\hat{\mathbf{j}}/\sqrt{3}+\hat{\mathbf{k}}/\sqrt{3}\right).$$

We can add the components together. By symmetry we expect the same answer for each components, so we'll just do one. How about $\hat{\mathbf{i}}$. This component has contributions from charge 2, 6, 7, and 8:

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\left(\frac{1}{1}+\frac{2}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right),$$

or

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}(1.90)$$

The three components add according to Pythagoras to pick up a final factor of $\sqrt{3}$, so

$$F_{\text{net}} = (0.262)\frac{q^2}{\epsilon_0 a^2}.$$

E25-14 (a) Yes. Changing the sign of y will change the sign of F_y; since this is equivalent to putting the charge q_0 on the "other" side, we would expect the force to also push in the "other" direction.

(b) The equation should look Eq. 25-15, except all y's should be replaced by x's. Then

$$F_x = \frac{1}{4\pi\epsilon_0} \frac{q_0\, q}{x\sqrt{x^2 + L^2/4}}.$$

(c) Setting the particle a distance d away should give a force with the same magnitude as

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_0\, q}{d\sqrt{d^2 + L^2/4}}.$$

This force is directed along the 45° line, so $F_x = F\cos 45°$ and $F_y = F\sin 45°$.

(d) Let the distance be $d = \sqrt{x^2 + y^2}$, and then use the fact that $F_x/F = \cos\theta = x/d$. Then

$$F_x = F\frac{x}{d} = \frac{1}{4\pi\epsilon_0} \frac{x\, q_0\, q}{(x^2 + y^2 + L^2/4)^{3/2}}.$$

and

$$F_y = F\frac{y}{d} = \frac{1}{4\pi\epsilon_0} \frac{y\, q_0\, q}{(x^2 + y^2 + L^2/4)^{3/2}}.$$

E25-15 (a) The equation *is* valid for both positive and negative z, so in vector form it would read

$$\vec{\mathbf{F}} = F_z\hat{\mathbf{k}} = \frac{1}{4\pi\epsilon_0} \frac{q_0\, q\, z}{(z^2 + R^2)^{3/2}}\hat{\mathbf{k}}.$$

(b) The equation *is not* valid for both positive and negative z. Reversing the sign of z should reverse the sign of F_z, and one way to fix this is to write $1 = z/\sqrt{z^2}$. Then

$$\vec{\mathbf{F}} = F_z\hat{\mathbf{k}} = \frac{1}{4\pi\epsilon_0} \frac{2q_0\, qz}{R^2}\left(\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2}}\right)\hat{\mathbf{k}}.$$

E25-16 Divide the rod into small differential lengths dr, each with charge $dQ = (Q/L)dr$. Each differential length contributes a differential force

$$dF = \frac{1}{4\pi\epsilon_0} \frac{q\, dQ}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2 L} dr.$$

Integrate:

$$\begin{aligned} F &= \int dF = \int_x^{x+L} \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2 L} dr, \\ &= \frac{1}{4\pi\epsilon_0} \frac{qQ}{L}\left(\frac{1}{x} - \frac{1}{x+L}\right) \end{aligned}$$

E25-17 You must solve Ex. 16 before solving this problem! q_0 refers to the charge that had been called q in that problem. In either case the distance from q_0 will be the same regardless of the sign of q; if $q = Q$ then q will be on the right, while if $q = -Q$ then q will be on the left.

Setting the forces equal to each other one gets

$$\frac{1}{4\pi\epsilon_0} \frac{qQ}{L}\left(\frac{1}{x} - \frac{1}{x+L}\right) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2},$$

or

$$r = \sqrt{x(x+L)}.$$

E25-18 You must solve Ex. 16 and Ex. 17 before solving this problem.

If all charges are positive then moving q_0 off axis will result in a net force away from the axis. That's unstable.

If $q = -Q$ then both q and Q are on the same side of q_0. Moving q_0 closer to q will result in the attractive force growing faster than the repulsive force, so q_0 will move away from equilibrium.

$\boxed{\textbf{E25-19}}$ We can start with the work that was done for us on Page 577, except since we are concerned with $\sin\theta = z/r$ we would have

$$dF_x = dF\sin\theta = \frac{1}{4\pi\epsilon_0}\frac{q_0\lambda\,dz}{(y^2+z^2)}\frac{z}{\sqrt{y^2+z^2}}.$$

We will need to take into consideration that λ changes sign for the two halves of the rod. Then

$$\begin{aligned}
F_x &= \frac{q_0\lambda}{4\pi\epsilon_0}\left(\int_{-L/2}^{0}\frac{-z\,dz}{(y^2+z^2)^{3/2}} + \int_0^{L/2}\frac{+z\,dz}{(y^2+z^2)^{3/2}}\right),\\
&= \frac{q_0\lambda}{2\pi\epsilon_0}\int_0^{L/2}\frac{z\,dz}{(y^2+z^2)^{3/2}},\\
&= \frac{q_0\lambda}{2\pi\epsilon_0}\left.\frac{-1}{\sqrt{y^2+z^2}}\right|_0^{L/2},\\
&= \frac{q_0\lambda}{2\pi\epsilon_0}\left(\frac{1}{y}-\frac{1}{\sqrt{y^2+(L/2)^2}}\right).
\end{aligned}$$

E25-20 Use Eq. 25-15 to find the magnitude of the force from any one rod, but write it as

$$F = \frac{1}{4\pi\epsilon_0}\frac{q\,Q}{r\sqrt{r^2+L^2/4}},$$

where $r^2 = z^2 + L^2/4$. The component of this along the z axis is $F_z = Fz/r$. Since there are 4 rods, we have

$$F = \frac{1}{\pi\epsilon_0}\frac{q\,Q\,z}{r^2\sqrt{r^2+L^2/4}}, = \frac{1}{\pi\epsilon_0}\frac{q\,Q\,z}{(z^2+L^2/4)\sqrt{z^2+L^2/2}},$$

Equating the electric force with the force of gravity and solving for Q,

$$Q = \frac{\pi\epsilon_0 mg}{qz}(z^2+L^2/4)\sqrt{z^2+L^2/2};$$

putting in the numbers,

$$\frac{\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N\cdot m}^2)(3.46\times10^{-7}\mathrm{kg})(9.8\mathrm{m/s}^2)}{(2.45\times10^{-12}\mathrm{C})(0.214\,\mathrm{m})}((0.214\mathrm{m})^2+(0.25\mathrm{m})^2/4)\sqrt{(0.214\mathrm{m})^2+(0.25\mathrm{m})^2/2}$$

so $Q = 3.07\times10^{-6}$C.

$\boxed{\textbf{E25-21}}$ In each case we conserve charge by making sure that the total number of protons is the same on both sides of the expression. We also need to conserve the number of neutrons.

(a) Hydrogen has one proton, Beryllium has four, so X must have five protons. Then X must be Boron, B.

(b) Carbon has six protons, Hydrogen has one, so X must have seven. Then X is Nitrogen, N.

(c) Nitrogen has seven protons, Hydrogen has one, but Helium has two, so X has $7 + 1 - 2 = 6$ protons. This means X is Carbon, C.

E25-22 (a) Use Eq. 25-4:

$$F = \frac{(8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(2)(90)(1.60\times10^{-19}\text{C})^2}{(12\times10^{-15}\text{m})^2} = 290\,\text{N}.$$

(b) $a = (290\,\text{N})/(4)(1.66\times10^{-27}\text{kg}) = 4.4\times10^{28}\text{m/s}^2$.

E25-23 Use Eq. 25-4:

$$F = \frac{(8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})^2}{(282\times10^{-12}\text{m})^2} = 2.89\times10^{-9}\text{N}.$$

E25-24 (a) Use Eq. 25-4:

$$q = \sqrt{\frac{(3.7\times10^{-9}\text{N})(5.0\times10^{-10}\text{m})^2}{(8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)}} = 3.20\times10^{-19}\text{C}.$$

(b) $N = (3.20\times10^{-19}\text{C})/(1.60\times10^{-19}\text{C}) = 2$.

$\boxed{\textbf{E25-25}}$ Use Eq. 25-4,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2} = \frac{(\frac{1}{3}1.6\times10^{-19}\,\text{C})(\frac{1}{3}1.6\times10^{-19}\,\text{C})}{4\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.6\times10^{-15}\,\text{m})^2} = 3.8\,\text{N}.$$

E25-26 (a) $N = (1.15\times10^{-7}\text{C})/(1.60\times10^{-19}\text{C}) = 7.19\times10^{11}$.

(b) The penny has enough electrons to make a total charge of -1.37×10^5C. The fraction is then

$$(1.15\times10^{-7}\text{C})/(1.37\times10^5\text{C}) = 8.40\times10^{-13}.$$

E25-27 Equate the magnitudes of the forces:

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = mg,$$

so

$$r = \sqrt{\frac{(8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})^2}{(9.11\times10^{-31}\text{kg})(9.81\,\text{m/s}^2)}} = 5.07\,\text{m}$$

E25-28 $Q = (75.0\,\text{kg})(-1.60\times10^{-19}\text{C})/(9.11\times10^{-31}\text{kg}) = -1.3\times10^{13}\text{C}$.

E25-29 The mass of water is $(250\ \text{cm}^3)(1.00\ \text{g/cm}^3) = 250$ g. The number of moles of water is $(250\ \text{g})/(18.0\ \text{g/mol}) = 13.9$ mol. The number of water molecules is $(13.9\ \text{mol})(6.02\times10^{23}\text{mol}^{-1}) = 8.37\times10^{24}$. Each molecule has ten protons, so the total positive charge is

$$Q = (8.37\times10^{24})(10)(1.60\times10^{-19}\text{C}) = 1.34\times10^7\text{C}.$$

E25-30 The total positive charge in 0.250 kg of water is 1.34×10^7C. Mary's imbalance is then

$$q_1 = (52.0)(4)(1.34\times10^7\text{C})(0.0001) = 2.79\times10^5\text{C},$$

while John's imbalance is

$$q_2 = (90.7)(4)(1.34\times10^7\text{C})(0.0001) = 4.86\times10^5\text{C},$$

The electrostatic force of attraction is then

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2} = (8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.79\times10^5)(4.86\times10^5)}{(28.0\,\text{m})^2} = 1.6\times10^{18}\text{N}.$$

E25-31 (a) The gravitational force of attraction between the Moon and the Earth is

$$F_{\rm G} = \frac{GM_{\rm E}M_{\rm M}}{R^2},$$

where R is the distance between them. If both the Earth and the moon are provided a charge q, then the electrostatic repulsion would be

$$F_{\rm E} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{R^2}.$$

Setting these two expression equal to each other,

$$\frac{q^2}{4\pi\epsilon_0} = GM_{\rm E}M_{\rm M},$$

which has solution

$$\begin{aligned} q &= \sqrt{4\pi\epsilon_0 GM_{\rm E}M_{\rm M}}, \\ &= \sqrt{4\pi(8.85\times10^{-12}{\rm C}^2/{\rm Nm}^2)(6.67\times10^{-11}{\rm Nm}^2/{\rm kg}^2)(5.98\times10^{24}{\rm kg})(7.36\times10^{22}{\rm kg})}, \\ &= 5.71\times10^{13}\,{\rm C}. \end{aligned}$$

(b) We need

$$(5.71\times10^{13}\,{\rm C})/(1.60\times10^{-19}\,{\rm C}) = 3.57\times10^{32}$$

protons on each body. The mass of protons needed is then

$$(3.57\times10^{32})(1.67\times10^{-27}\,{\rm kg}) = 5.97\times10^{65}\,{\rm kg}.$$

Ignoring the mass of the electron (why not?) we can assume that hydrogen is all protons, so we need that much hydrogen.

P25-1 Assume that the spheres initially have charges q_1 and q_2. The force of attraction between them is

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2} = -0.108\,{\rm N},$$

where $r_{12} = 0.500\,{\rm m}$. The net charge is q_1+q_2, and after the conducting wire is connected each sphere will get *half* of the total. The spheres will have the same charge, and repel with a force of

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{\frac{1}{2}(q_1+q_2)\frac{1}{2}(q_1+q_2)}{r_{12}^2} = 0.0360\,{\rm N}.$$

Since we know the separation of the spheres we can find q_1+q_2 quickly,

$$q_1+q_2 = 2\sqrt{4\pi\epsilon_0 r_{12}^2(0.0360\,{\rm N})} = 2.00\,\mu{\rm C}$$

We'll put this back into the first expression and solve for q_2.

$$\begin{aligned} -0.108\,{\rm N} &= \frac{1}{4\pi\epsilon_0}\frac{(2.00\,\mu{\rm C}-q_2)q_2}{r_{12}^2}, \\ -3.00\times10^{-12}\,{\rm C}^2 &= (2.00\,\mu{\rm C}-q_2)q_2, \\ 0 &= -q_2^2+(2.00\,\mu{\rm C})q_2+(1.73\,\mu{\rm C})^2. \end{aligned}$$

The solution is $q_2 = 3.0\,\mu{\rm C}$ or $q_2 = -1.0\,\mu{\rm C}$. Then $q_1 = -1.0\,\mu{\rm C}$ or $q_1 = 3.0\,\mu{\rm C}$.

P25-2 The electrostatic force on Q from each q has magnitude $qQ/4\pi\epsilon_0 a^2$, where a is the length of the side of the square. The magnitude of the vertical (horizontal) component of the force of Q on Q is $\sqrt{2}Q^2/16\pi\epsilon_0 a^2$.

(a) In order to have a zero net force on Q the magnitudes of the two contributions must balance, so

$$\frac{\sqrt{2}Q^2}{16\pi\epsilon_0 a^2} = \frac{qQ}{4\pi\epsilon_0 a^2},$$

or $q = \sqrt{2}Q/4$. The charges must actually have opposite charge.

(b) No.

$\boxed{\textbf{P25-3}}$ (a) The third charge, q_3, will be between the first two. The net force on the third charge will be zero if

$$\frac{1}{4\pi\epsilon_0}\frac{q\,q_3}{r_{31}^2} = \frac{1}{4\pi\epsilon_0}\frac{4q\,q_3}{r_{32}^2},$$

which will occur if

$$\frac{1}{r_{31}} = \frac{2}{r_{32}}$$

The total distance is L, so $r_{31} + r32 = L$, or $r_{31} = L/3$ and $r_{32} = 2L/3$.

Now that we have found the position of the third charge we need to find the magnitude. The second and third charges both exert a force on the first charge; we want this net force on the first charge to be zero, so

$$\frac{1}{4\pi\epsilon_0}\frac{q\,q_3}{r_{13}^2} = \frac{1}{4\pi\epsilon_0}\frac{q\,4q}{r_{12}^2},$$

or

$$\frac{q_3}{(L/3)^2} = \frac{4q}{L^2},$$

which has solution $q_3 = -4q/9$. The negative sign is because the force between the first and second charge must be in the opposite direction to the force between the first and third charge.

(b) Consider what happens to the net force on the middle charge if is is displaced a small distance z. If the charge 3 is moved toward charge 1 then the force of attraction with charge 1 will increase. But moving charge 3 closer to charge 1 means moving charge 3 away from charge 2, so the force of attraction between charge 3 and charge 2 will decrease. So charge 3 experiences more attraction to ward the charge that it moves toward, and less attraction to the charge it moves away from. Sounds unstable to me.

P25-4 (a) The electrostatic force on the charge on the right has magnitude

$$F = \frac{q^2}{4\pi\epsilon_0 x^2},$$

The weight of the ball is $W = mg$, and the two forces are related by

$$F/W = \tan\theta \approx \sin\theta = x/2L.$$

Combining, $2Lq^2 = 4\pi\epsilon_0 mgx^3$, so

$$x = \left(\frac{q^2 L}{2\pi\epsilon_0}\right)^{1/3}.$$

(b) Rearrange and solve for q,

$$q = \sqrt{\frac{2\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(0.0112\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(4.70\times10^{-2}\mathrm{m})^3}{(1.22\,\mathrm{m})}} = 2.28\times10^{-8}\mathrm{C}.$$

P25-5 (a) Originally the balls would not repel, so they would move together and touch; after touching the balls would "split" the charge ending up with $q/2$ each. They would then repel again.

(b) The new equilibrium separation is

$$x' = \left(\frac{(q/2)^2 L}{2\pi\epsilon_0 mg}\right)^{1/3} = \left(\frac{1}{4}\right)^{1/3} x = 2.96 \text{ cm}.$$

P25-6 Take the time derivative of the expression in Problem 25-4. Then

$$\frac{dx}{dt} = \frac{2}{3}\frac{x}{q}\frac{dq}{dt} = \frac{2}{3}\frac{(4.70\times10^{-2}\text{m})}{(2.28\times10^{-8}\text{C})}(-1.20\times10^{-9}\text{C/s}) = 1.65\times10^{-3}\text{m/s}.$$

$\boxed{\textbf{P25-7}}$ The force between the two charges is

$$F = \frac{1}{4\pi\epsilon_0}\frac{(Q-q)q}{r_{12}^2}.$$

We want to maximize this force with respect to variation in q, this means finding dF/dq and setting it equal to 0. Then

$$\frac{dF}{dq} = \frac{d}{dq}\left(\frac{1}{4\pi\epsilon_0}\frac{(Q-q)q}{r_{12}^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q-2q}{r_{12}^2}.$$

This will vanish if $Q - 2q = 0$, or $q = \frac{1}{2}Q$.

P25-8 Displace the charge q a distance y. The net restoring force on q will be approximately

$$F \approx 2\frac{qQ}{4\pi\epsilon_0}\frac{1}{(d/2)^2}\frac{y}{(d/2)} = \frac{qQ}{4\pi\epsilon_0}\frac{16}{d^3}y.$$

Since F/y is effectively a force constant, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = \left(\frac{\epsilon_0 m\pi^3 d^3}{qQ}\right)^{1/2}.$$

P25-9 Displace the charge q a distance x toward one of the positive charges Q. The net restoring force on q will be

$$\begin{aligned} F &= \frac{qQ}{4\pi\epsilon_0}\left(\frac{1}{(d/2-x)^2} - \frac{1}{(d/2+x)^2}\right), \\ &\approx \frac{qQ}{4\pi\epsilon_0}\frac{32}{d^3}x. \end{aligned}$$

Since F/x is effectively a force constant, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = \left(\frac{\epsilon_0 m\pi^3 d^3}{2qQ}\right)^{1/2}.$$

P25-10 (a) Zero, by symmetry.

(b) Removing a positive Cesium ion is equivalent to adding a singly charged negative ion at that same location. The net force is then

$$F = e^2/4\pi\epsilon_0 r^2,$$

where r is the distance between the Chloride ion and the newly placed negative ion, or

$$r = \sqrt{3(0.20\times10^{-9}\text{m})^2}$$

The force is then

$$F = \frac{(1.6\times10^{-19}\text{C})^2}{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)3(0.20\times10^{-9}\text{m})^2} = 1.92\times10^{-9}\text{N}.$$

$\boxed{\textbf{P25-11}}$ We can pretend that this problem is in a single plane containing all three charges. The magnitude of the force on the test charge q_0 from the charge q on the left is

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q\,q_0}{(a^2+R^2)}.$$

A force of identical magnitude exists from the charge on the right. we need to add these two forces as vectors. Only the components along R will survive, and each force will contribute an amount

$$F_1\sin\theta = F_1\frac{R}{\sqrt{R^2+a^2}},$$

so the net force on the test particle will be

$$\frac{2}{4\pi\epsilon_0}\frac{q\,q_0}{(a^2+R^2)}\frac{R}{\sqrt{R^2+a^2}}.$$

We want to find the maximum value as a function of R. This means take the derivative, and set it equal to zero. The derivative is

$$\frac{2q\,q_0}{4\pi\epsilon_0}\left(\frac{1}{(a^2+R^2)^{3/2}} - \frac{3R^2}{(a^2+R^2)^{5/2}}\right),$$

which will vanish when

$$a^2+R^2 = 3R^2,$$

a *simple* quadratic equation with solutions $R = \pm a/\sqrt{2}$.

E26-1 $E = F/q = ma/q$. Then

$$E = (9.11\times10^{-31}\text{kg})(1.84\times10^{9}\text{m/s}^2)/(1.60\times10^{-19}\text{C}) = 1.05\times10^{-2}\text{N/C}.$$

E26-2 The answers to (a) and (b) are the same!
$F = Eq = (3.0\times10^{6}\text{N/C})(1.60\times10^{-19}\text{C}) = 4.8\times10^{-13}\text{N}.$

$\boxed{\textbf{E26-3}}$ $F = W$, or $Eq = mg$, so

$$E = \frac{mg}{q} = \frac{(6.64\times10^{-27}\,\text{kg})(9.81\,\text{m/s}^2)}{2(1.60\times10^{-19}\,\text{C})} = 2.03\times10^{-7}\,\text{N/C}.$$

The alpha particle has a positive charge, this means that it will experience an electric force which is in the same direction as the electric field. Since the gravitational force is down, the electric force, and consequently the electric field, must be directed up.

E26-4 (a) $E = F/q = (3.0\times10^{-6}\text{N})/(2.0\times10^{-9}\text{C}) = 1.5\times10^{3}\text{N/C}.$
(b) $F = Eq = (1.5\times10^{3}\text{N/C})(1.60\times10^{-19}\text{C}) = 2.4\times10^{-16}\text{N}.$
(c) $F = mg = (1.67\times10^{-27}\text{kg})(9.81\,\text{m/s}^2) = 1.6\times10^{-26}\text{N}.$
(d) $(2.4\times10^{-16}\text{N})/(1.6\times10^{-26}\text{N}) = 1.5\times10^{10}.$

E26-5 Rearrange $E = q/4\pi\epsilon_0 r^2$,

$$q = 4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.750\,\text{m})^2(2.30\,\text{N/C}) = 1.44\times10^{-10}\text{C}.$$

E26-6 $p = qd = (1.60\times10^{-19}\text{C})(4.30\times10^{-9}) = 6.88\times10^{-28}\text{C}\cdot\text{m}.$

$\boxed{\textbf{E26-7}}$ Use Eq. 26-12 for points along the perpendicular bisector. Then

$$E = \frac{1}{4\pi\epsilon_0}\frac{p}{x^3} = (8.99\times10^{9}\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.56\times10^{-29}\,\text{C}\cdot\text{m})}{(25.4\times10^{-9}\,\text{m})^3} = 1.95\times10^{4}\text{N/C}.$$

E26-8 If the charges on the line $x = a$ where $+q$ and $-q$ instead of $+2q$ and $-2q$ then at the center of the square $E = 0$ by symmetry. This simplifies the problem into finding E for a charge $+q$ at $(a, 0)$ and $-q$ at (a, a). This is a dipole, and the field is given by Eq. 26-11. For this exercise we have $x = a/2$ and $d = a$, so

$$E = \frac{1}{4\pi\epsilon_0}\frac{qa}{[2(a/2)^2]^{3/2}},$$

or, putting in the numbers, $E = 1.11\times10^{5}\text{N/C}.$

E26-9 The charges at 1 and 7 are opposite and can be effectively replaced with a single charge of $-6q$ at 7. The same is true for 2 and 8, 3 and 9, on up to 6 and 12. By symmetry we expect the field to point along a line so that three charges are above and three below. That would mean 9:30.

E26-10 If both charges are positive then Eq. 26-10 would read $E = 2E_+\sin\theta$, and Eq. 26-11 would look like

$$\begin{aligned} E &= 2\frac{1}{4\pi\epsilon_0}\frac{q}{x^2+(d/2)^2}\frac{x}{\sqrt{x^2+(d/2)^2}}, \\ &\approx 2\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\frac{x}{\sqrt{x^2}} \end{aligned}$$

when $x \gg d$. This can be simplified to $E = 2q/4\pi\epsilon_0 x^2$.

$\boxed{\textbf{E26-11}}$ Treat the two charges on the left as one dipole and treat the two charges on the right as a second dipole. Point P is on the perpendicular bisector of both dipoles, so we can use Eq. 26-12 to find the two fields.

For the dipole on the left $p = 2aq$ and the electric field due to this dipole at P has magnitude

$$E_\mathrm{l} = \frac{1}{4\pi\epsilon_0}\frac{2aq}{(x+a)^3}$$

and is directed *up*.

For the dipole on the right $p = 2aq$ and the electric field due to this dipole at P has magnitude

$$E_\mathrm{r} = \frac{1}{4\pi\epsilon_0}\frac{2aq}{(x-a)^3}$$

and is directed *down*.

The net electric field at P is the sum of these two fields, but since the two component fields point in opposite directions we must actually subtract these values,

$$\begin{aligned}
E &= E_\mathrm{r} - E_\mathrm{l}, \\
&= \frac{2aq}{4\pi\epsilon_0}\left(\frac{1}{(x-a)^3} - \frac{1}{(x+a)^3}\right), \\
&= \frac{aq}{2\pi\epsilon_0}\frac{1}{x^3}\left(\frac{1}{(1-a/x)^3} - \frac{1}{(1+a/x)^3}\right).
\end{aligned}$$

We can use the binomial expansion on the terms containing $1 \pm a/x$,

$$\begin{aligned}
E &\approx \frac{aq}{2\pi\epsilon_0}\frac{1}{x^3}\left((1+3a/x) - (1-3a/x)\right), \\
&= \frac{aq}{2\pi\epsilon_0}\frac{1}{x^3}\left(6a/x\right), \\
&= \frac{3(2qa^2)}{2\pi\epsilon_0 x^4}.
\end{aligned}$$

E26-12 Do a series expansion on the part in the parentheses

$$1 - \frac{1}{\sqrt{1+R^2/z^2}} \approx 1 - \left(1 - \frac{1}{2}\frac{R^2}{z^2}\right) = \frac{R^2}{2z^2}.$$

Substitute this in,

$$E_z \approx \frac{\sigma}{2\epsilon_0}\frac{R^2}{2z^2}\frac{\pi}{\pi} = \frac{Q}{4\pi\epsilon_0 z^2}.$$

E26-13 At the surface $z = 0$ and $E_z = \sigma/2\epsilon_0$. Half of this value occurs when z is given by

$$\frac{1}{2} = 1 - \frac{z}{\sqrt{z^2+R^2}},$$

which can be written as $z^2 + R^2 = (2z)^2$. Solve this, and $z = R/\sqrt{3}$.

E26-14 Look at Eq. 26-18. The electric field will be a maximum when $z/(z^2+R^2)^{3/2}$ is a maximum. Take the derivative of this with respect to z, and get

$$\frac{1}{(z^2+R^2)^{3/2}} - \frac{3}{2}\frac{2z^2}{(z^2+R^2)^{5/2}} = \frac{z^2+R^2-3z^2}{(z^2+R^2)^{5/2}}.$$

This will vanish when the numerator vanishes, or when $z = R/\sqrt{2}$.

$\boxed{\textbf{E26-15}}$ (a) The electric field strength just above the center surface of a charged disk is given by Eq. 26-19, but with $z = 0$,

$$E = \frac{\sigma}{2\epsilon_0}$$

The surface charge density is $\sigma = q/A = q/(\pi R^2)$. Combining,

$$q = 2\epsilon_0 \pi R^2 E = 2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})\pi(2.5\times10^{-2}\mathrm{m})^2(3\times10^6\,\mathrm{N/C}) = 1.04\times10^{-7}\mathrm{C}.$$

Notice we used an electric field strength of $E = 3\times10^6$ N/C, which is the field at air breaks down and sparks happen.

(b) We want to find out how many atoms are on the surface; if a is the cross sectional area of one atom, and N the number of atoms, then $A = Na$ is the surface area of the disk. The number of atoms is

$$N = \frac{A}{a} = \frac{\pi(0.0250\,\mathrm{m})^2}{(0.015\times10^{-18}\,\mathrm{m}^2)} = 1.31\times10^{17}$$

(c) The total charge on the disk is 1.04×10^{-7}C, this corresponds to

$$(1.04\times10^{-7}\mathrm{C})/(1.6\times10^{-19}\mathrm{C}) = 6.5\times10^{11}$$

electrons. (We are ignoring the sign of the charge here.) If each surface atom can have at most one excess electron, then the fraction of atoms which are charged is

$$(6.5\times10^{11})/(1.31\times10^{17}) = 4.96\times10^{-6},$$

which isn't very many.

E26-16 Imagine switching the positive and negative charges. The electric field would also need to switch directions. By symmetry, then, the electric field can only point vertically down. Keeping only that component,

$$\begin{aligned} E &= 2\int_0^{\pi/2} \frac{1}{4\pi\epsilon_0}\frac{\lambda d\theta}{r^2}\sin\theta, \\ &= \frac{2}{4\pi\epsilon_0}\frac{\lambda}{r^2}. \end{aligned}$$

But $\lambda = q/(\pi/2)$, so $E = q/\pi^2\epsilon_0 r^2$.

$\boxed{\textbf{E26-17}}$ We want to fit the data to Eq. 26-19,

$$E_z = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right).$$

There are only two variables, R and q, with $q = \sigma\pi R^2$.

We can find σ *very* easily if we assume that the measurements have no error because then at the surface (where $z = 0$), the expression for the electric field simplifies to

$$E = \frac{\sigma}{2\epsilon_0}.$$

Then $\sigma = 2\epsilon_0 E = 2(8.854\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(2.043\times10^7\,\mathrm{N/C}) = 3.618\times10^{-4}\,\mathrm{C/m^2}$.

Finding the radius will take a little more work. We can choose one point, and make that the reference point, and then solve for R. Starting with

$$E_z = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right),$$

and then rearranging,

$$\begin{aligned}
\frac{2\epsilon_0 E_z}{\sigma} &= 1 - \frac{z}{\sqrt{z^2+R^2}}, \\
\frac{2\epsilon_0 E_z}{\sigma} &= 1 - \frac{1}{\sqrt{1+(R/z)^2}}, \\
\frac{1}{\sqrt{1+(R/z)^2}} &= 1 - \frac{2\epsilon_0 E_z}{\sigma}, \\
1+(R/z)^2 &= \frac{1}{(1-2\epsilon_0 E_z/\sigma)^2}, \\
\frac{R}{z} &= \sqrt{\frac{1}{(1-2\epsilon_0 E_z/\sigma)^2} - 1}.
\end{aligned}$$

Using $z = 0.03\,\mathrm{m}$ and $E_z = 1.187 \times 10^7\,\mathrm{N/C}$, along with our value of $\sigma = 3.618 \times 10^{-4}\,\mathrm{C/m^2}$, we find

$$\begin{aligned}
\frac{R}{z} &= \sqrt{\frac{1}{(1-2(8.854\times10^{-12}\mathrm{C^2/Nm^2})(1.187\times10^{7}\mathrm{N/C})/(3.618\times10^{-4}\mathrm{C/m^2}))^2} - 1}, \\
R &= 2.167(0.03\,\mathrm{m}) = 0.065\,\mathrm{m}.
\end{aligned}$$

(b) And now find the charge from the charge density and the radius,

$$q = \pi R^2 \sigma = \pi(0.065\,\mathrm{m})^2(3.618 \times 10^{-4}\,\mathrm{C/m^2}) = 4.80\,\mu\mathrm{C}.$$

E26-18 (a) $\lambda = -q/L$.

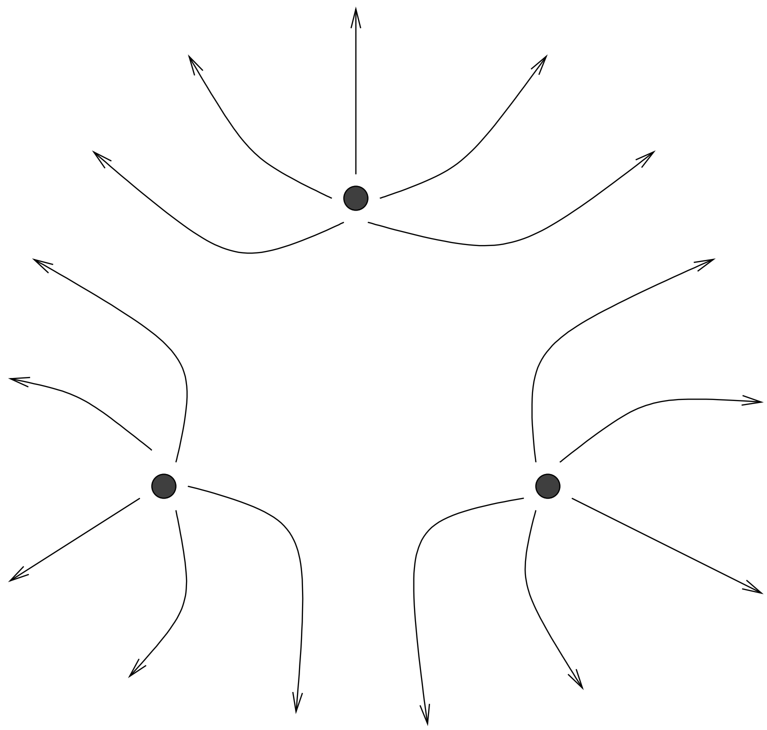
(b) Integrate:

$$\begin{aligned}
E &= \int_a^{L+a} \frac{1}{4\pi\epsilon_0} \lambda\, dx x^2, \\
&= \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{L+a}\right), \\
&= \frac{q}{4\pi\epsilon_0}\frac{1}{a(L+a)},
\end{aligned}$$

since $\lambda = q/L$.

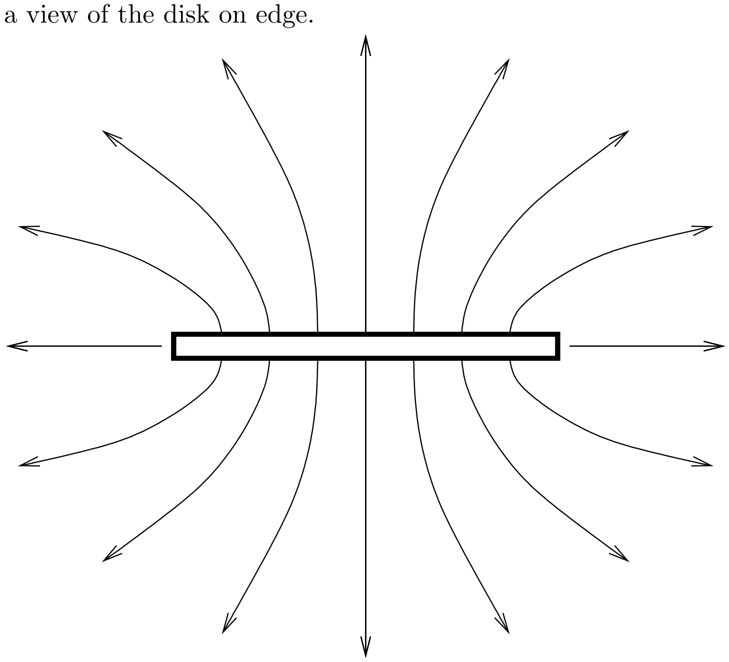
(c) If $a \gg L$ then L can be replaced with 0 in the above expression.

E26-19 A sketch of the field looks like this.

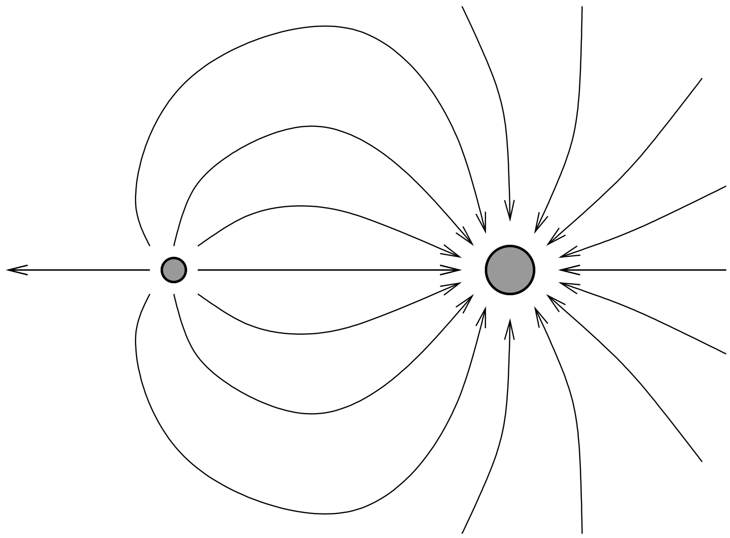


E26-20 (a) $F = Eq = (40\,\mathrm{N/C})(1.60\times10^{-19}\mathrm{C}) = 6.4\times10^{-18}\mathrm{N}$
(b) Lines are twice as far apart, so the field is half as large, or $E = 20\mathrm{N/C}$.

E26-21 Consider a view of the disk on edge.



E26-22 A sketch of the field looks like this.



E26-23 To the right.

E26-24 (a) The electric field is zero nearer to the smaller charge; since the charges have opposite signs it must be to the right of the $+2q$ charge. Equating the magnitudes of the two fields,

$$\frac{2q}{4\pi\epsilon_0 x^2} = \frac{5q}{4\pi\epsilon_0 (x+a)^2},$$

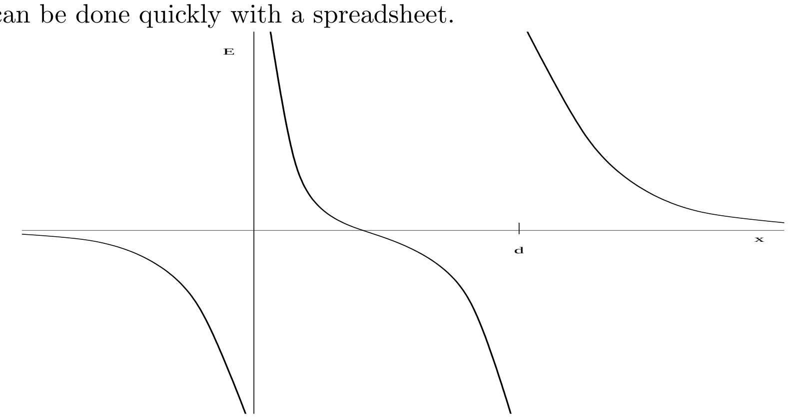
or

$$\sqrt{5}x = \sqrt{2}(x+a),$$

which has solution

$$x = \frac{\sqrt{2}\,a}{\sqrt{5}-\sqrt{2}} = 2.72a.$$

E26-25 This can be done quickly with a spreadsheet.



E26-26 (a) At point A,

$$E = \frac{1}{4\pi\epsilon_0}\left(-\frac{q}{d^2} - \frac{-2q}{(2d)^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{-q}{2d^2},$$

where the negative sign indicates that $\vec{\mathbf{E}}$ is directed to the left.

At point B,

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(d/2)^2} - \frac{-2q}{(d/2)^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{6q}{d^2},$$

where the positive sign indicates that $\vec{\mathbf{E}}$ is directed to the right.

At point C,

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(2d)^2} + \frac{-2q}{d^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{-7q}{4d^2},$$

where the negative sign indicates that $\vec{\mathbf{E}}$ is directed to the left.

E26-27 (a) The electric field does (negative) work on the electron. The magnitude of this work is $W = Fd$, where $F = Eq$ is the magnitude of the electric force on the electron and d is the distance through which the electron moves. Combining,

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = q\vec{\mathbf{E}} \cdot \vec{\mathbf{d}},$$

which gives the work done by the electric field on the electron. The electron originally possessed a kinetic energy of $K = \frac{1}{2}mv^2$, since we want to bring the electron to a rest, the work done must be negative. The charge q of the electron is negative, so $\vec{\mathbf{E}}$ and $\vec{\mathbf{d}}$ are pointing in the same direction, and $\vec{\mathbf{E}} \cdot \vec{\mathbf{d}} = Ed$.

By the work energy theorem,

$$W = \Delta K = 0 - \frac{1}{2}mv^2.$$

We put all of this together and find d,

$$d = \frac{W}{qE} = \frac{-mv^2}{2qE} = \frac{-(9.11\times10^{-31}\text{kg})(4.86\times10^6\text{ m/s})^2}{2(-1.60\times10^{-19}\text{C})(1030\text{ N/C})} = 0.0653\text{ m}.$$

(b) $Eq = ma$ gives the magnitude of the acceleration, and $v_\text{f} = v_\text{i} + at$ gives the time. But $v_\text{f} = 0$. Combining these expressions,

$$t = -\frac{mv_\text{i}}{Eq} = -\frac{(9.11\times10^{-31}\text{kg})(4.86\times10^6\text{ m/s})}{(1030\text{ N/C})(-1.60\times10^{-19}\text{C})} = 2.69\times10^{-8}\text{ s}.$$

(c) We will apply the work energy theorem again, except now we don't assume the final kinetic energy is zero. Instead,

$$W = \Delta K = K_\text{f} - K_\text{i},$$

and dividing through by the initial kinetic energy to get the fraction lost,

$$\frac{W}{K_\text{i}} = \frac{K_\text{f} - K_\text{i}}{K_\text{i}} = \text{ fractional change of kinetic energy.}$$

But $K_\text{i} = \frac{1}{2}mv^2$, and $W = qEd$, so the fractional change is

$$\frac{W}{K_\text{i}} = \frac{qEd}{\frac{1}{2}mv^2} = \frac{(-1.60\times10^{-19}\text{C})(1030\text{ N/C})(7.88\times10^{-3}\text{m})}{\frac{1}{2}(9.11\times10^{-31}\text{kg})(4.86\times10^6\text{ m/s})^2} = -12.1\%.$$

E26-28 (a) $a = Eq/m = (2.16\times10^4\text{N/C})(1.60\times10^{-19}\text{C})/(1.67\times10^{-27}\text{kg}) = 2.07\times10^{12}\text{m/s}^2$.

(b) $v = \sqrt{2ax} = \sqrt{2(2.07\times10^{12}\text{m/s}^2)(1.22\times10^{-2}\text{m})} = 2.25\times10^5\text{m/s}$.

E26-29 (a) $E = 2q/4\pi\epsilon_0 r^2$, or

$$E = \frac{(1.88\times10^{-7}\text{C})}{2\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.152\,\text{m}/2)^2} = 5.85\times10^5\text{N/C}.$$

(b) $F = Eq = (5.85\times10^5\text{N/C})(1.60\times10^{-19}\text{C}) = 9.36\times10^{-14}\text{N}.$

E26-30 (a) The average speed between the plates is $(1.95\times10^{-2}\text{m})/(14.7\times10^{-9}\text{s}) = 1.33\times10^6\text{m/s}$. The speed with which the electron hits the plate is twice this, or $2.65\times10^6\text{m/s}$.

(b) The acceleration is $a = (2.65\times10^6\text{m/s})/(14.7\times10^{-9}\text{s}) = 1.80\times10^{14}\text{m/s}^2$. The electric field then has magnitude $E = ma/q$, or

$$E = (9.11\times10^{-31}\text{kg})(1.80\times10^{14}\text{m/s}^2)/(1.60\times10^{-19}\text{C}) = 1.03\times10^3\text{N/C}.$$

$\boxed{\textbf{E26-31}}$ The drop is balanced if the electric force is equal to the force of gravity, or $Eq = mg$. The mass of the drop is given in terms of the density by

$$m = \rho V = \rho\frac{4}{3}\pi r^3.$$

Combining,

$$q = \frac{mg}{E} = \frac{4\pi\rho r^3 g}{3E} = \frac{4\pi(851\,\text{kg/m}^3)(1.64\times10^{-6}\text{m})^3(9.81\,\text{m/s}^2)}{3(1.92\times10^5\text{N/C})} = 8.11\times10^{-19}\text{C}.$$

We want the charge in terms of e, so we divide, and get

$$\frac{q}{e} = \frac{(8.11\times10^{-19}\text{C})}{(1.60\times10^{-19}\text{C})} = 5.07 \approx 5.$$

E26-32 (b) $F = (8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(2.16\times10^{-6}\text{C})(85.3\times10^{-9}\text{C})/(0.117\text{m})^2 = 0.121\,\text{N}.$

(a) $E_2 = F/q_1 = (0.121\,\text{N})/(2.16\times10^{-6}\text{C}) = 5.60\times10^4\text{N/C}.$

$E_1 = F/q_2 = (0.121\,\text{N})/(85.3\times10^{-9}\text{C}) = 1.42\times10^6\text{N/C}.$

$\boxed{\textbf{E26-33}}$ If each value of q measured by Millikan was a multiple of e, then the difference between any two values of q must also be a multiple of q. The smallest difference would be the smallest multiple, and this multiple might be unity. The differences are 1.641, 1.63, 1.60, 1.63, 3.30, 3.35, 3.18, 3.24, all times 10^{-19} C. This is a pretty clear indication that the fundamental charge is on the order of 1.6×10^{-19} C. If so, the likely number of fundamental charges on each of the drops is shown below in a table arranged like the one in the book:

4	8	12
5	10	14
7	11	16

The total number of charges is 87, while the total charge is 142.69×10^{-19} C, so the average charge per quanta is 1.64×10^{-19} C.

E26-34 Because of the electric field the acceleration toward the ground of a charged particle is not g, but $g \pm Eq/m$, where the sign depends on the direction of the electric field.

(a) If the lower plate is positively charged then $a = g - Eq/m$. Replace g in the pendulum period expression by this, and then

$$T = 2\pi\sqrt{\frac{L}{g - Eq/m}}.$$

(b) If the lower plate is negatively charged then $a = g + Eq/m$. Replace g in the pendulum period expression by this, and then

$$T = 2\pi\sqrt{\frac{L}{g + Eq/m}}.$$

E26-35 The ink drop travels an additional time $t' = d/v_x$, where d is the additional horizontal distance between the plates and the paper. During this time it travels an additional vertical distance $y' = v_y t'$, where $v_y = at = 2y/t = 2yv_x/L$. Combining,

$$y' = \frac{2yv_x t'}{L} = \frac{2yd}{L} = \frac{2(6.4\times10^{-4}\text{m})(6.8\times10^{-3}\text{m})}{(1.6\times10^{-2}\text{m})} = 5.44\times10^{-4}\text{m},$$

so the total deflection is $y + y' = 1.18\times10^{-3}$m.

E26-36 (a) $p = (1.48\times10^{-9}\text{C})(6.23\times10^{-6}\text{m}) = 9.22\times10^{-15}\text{C}\cdot\text{m}$.

(b) $\Delta U = 2pE = 2(9.22\times10^{-15}\text{C}\cdot\text{m})(1100\,\text{N/C}) = 2.03\times10^{-11}\text{J}$.

$\boxed{\textbf{E26-37}}$ Use $\tau = pE\sin\theta$, where θ is the angle between $\vec{\mathbf{p}}$ and $\vec{\mathbf{E}}$. For this dipole $p = qd = 2ed$ or $p = 2(1.6\times10^{-19}\,\text{C})(0.78\times10^{-9}\,\text{m}) = 2.5\times10^{-28}\,\text{C}\cdot\text{m}$. For all three cases

$$pE = (2.5\times10^{-28}\,\text{C}\cdot\text{m})(3.4\times10^{6}\text{N/C}) = 8.5\times10^{-22}\,\text{N}\cdot\text{m}.$$

The only thing we care about is the angle.

(a) For the parallel case $\theta = 0$, so $\sin\theta = 0$, and $\tau = 0$.

(b) For the perpendicular case $\theta = 90°$, so $\sin\theta = 1$, and $\tau = 8.5\times10^{-22}\,\text{N}\cdot\text{m}$..

(c) For the anti-parallel case $\theta = 180°$, so $\sin\theta = 0$, and $\tau = 0$.

E26-38 (c) Equal and opposite, or 5.22×10^{-16}N.

(d) Use Eq. 26-12 and $F = Eq$. Then

$$\begin{aligned} p &= \frac{4\pi\epsilon_0 x^3 F}{q}, \\ &= \frac{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.285\text{m})^3(5.22\times10^{-16}\text{N})}{(3.16\times10^{-6}\text{C})}, \\ &= 4.25\times10^{-22}\text{C}\cdot\text{m}. \end{aligned}$$

$\boxed{\textbf{E26-39}}$ The point-like nucleus contributes an electric field

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{Ze}{r^2},$$

while the uniform sphere of negatively charged electron cloud of radius R contributes an electric field given by Eq. 26-24,

$$E_- = \frac{1}{4\pi\epsilon_0}\frac{-Zer}{R^3}.$$

The net electric field is just the sum,

$$E = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

E26-40 The shell theorem first described for gravitation in chapter 14 is applicable here since both electric forces and gravitational forces fall off as $1/r^2$. The net positive charge inside the sphere of radius $d/2$ is given by $Q = 2e(d/2)^3/R^3 = ed^3/4R^3$.

The net force on either electron will be zero when

$$\frac{e^2}{d^2} = \frac{eQ}{(d/2)^2} = \frac{4e^2}{d^2}\frac{d^3}{4R^3} = \frac{e^2 d}{R^3},$$

which has solution $d = R$.

$\boxed{\textbf{P26-1}}$ (a) Let the positive charge be located *closer* to the point in question, then the electric field from the positive charge is

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{(x-d/2)^2}$$

and is directed *away from* the dipole.

The negative charge is located farther from the point in question, so

$$E_- = \frac{1}{4\pi\epsilon_0}\frac{q}{(x+d/2)^2}$$

and is directed *toward* the dipole.

The net electric field is the sum of these two fields, but since the two component fields point in opposite direction we must actually subtract these values,

$$\begin{aligned} E &= E_+ - E_-, \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{(z-d/2)^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{(z+d/2)^2}, \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\left(\frac{1}{(1-d/2z)^2} - \frac{1}{(1+d/2z)^2}\right) \end{aligned}$$

We can use the binomial expansion on the terms containing $1 \pm d/2z$,

$$\begin{aligned} E &\approx \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\left((1+d/z) - (1-d/z)\right), \\ &= \frac{1}{2\pi\epsilon_0}\frac{qd}{z^3} \end{aligned}$$

(b) The electric field is directed away from the positive charge when you are closer to the positive charge; the electric field is directed toward the negative charge when you are closer to the negative charge. In short, along the axis the electric field is directed in the same direction as the dipole moment.

P26-2 The key to this problem will be the expansion of

$$\frac{1}{(x^2 + (z \pm d/2)^2)^{3/2}} \approx \frac{1}{(x^2+z^2)^{3/2}}\left(1 \mp \frac{3}{2}\frac{zd}{x^2+z^2}\right).$$

for $d \ll \sqrt{x^2+z^2}$. Far from the charges the electric field of the positive charge has magnitude

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2+(z-d/2)^2},$$

the components of this are

$$\begin{aligned}
E_{x,+} &= \frac{1}{4\pi\epsilon_0}\frac{q}{x^2+z^2}\frac{x}{\sqrt{x^2+(z-d/2)^2}},\\
E_{z,+} &= \frac{1}{4\pi\epsilon_0}\frac{q}{x^2+z^2}\frac{(z-d/2)}{\sqrt{x^2+(z-d/2)^2}}.
\end{aligned}$$

Expand both according to the first sentence, then

$$\begin{aligned}
E_{x,+} &\approx \frac{1}{4\pi\epsilon_0}\frac{xq}{(x^2+z^2)^{3/2}}\left(1+\frac{3}{2}\frac{zd}{x^2+z^2}\right),\\
E_{z,+} &= \frac{1}{4\pi\epsilon_0}\frac{(z-d/2)q}{(x^2+z^2)^{3/2}}\left(1+\frac{3}{2}\frac{zd}{x^2+z^2}\right).
\end{aligned}$$

Similar expression exist for the negative charge, except we must replace q with $-q$ and the $+$ in the parentheses with a $-$, and $z-d/2$ with $z+d/2$ in the E_z expression. All that is left is to add the expressions. Then

$$\begin{aligned}
E_x &= \frac{1}{4\pi\epsilon_0}\frac{xq}{(x^2+z^2)^{3/2}}\left(1+\frac{3}{2}\frac{zd}{x^2+z^2}\right)+\frac{1}{4\pi\epsilon_0}\frac{-xq}{(x^2+z^2)^{3/2}}\left(1-\frac{3}{2}\frac{zd}{x^2+z^2}\right),\\
&= \frac{1}{4\pi\epsilon_0}\frac{3xqzd}{(x^2+z^2)^{5/2}},\\
E_z &= \frac{1}{4\pi\epsilon_0}\frac{(z-d/2)q}{(x^2+z^2)^{3/2}}\left(1+\frac{3}{2}\frac{zd}{x^2+z^2}\right)+\frac{1}{4\pi\epsilon_0}\frac{-(z+d/2)q}{(x^2+z^2)^{3/2}}\left(1-\frac{3}{2}\frac{zd}{x^2+z^2}\right),\\
&= \frac{1}{4\pi\epsilon_0}\frac{3z^2dq}{(x^2+z^2)^{5/2}}-\frac{1}{4\pi\epsilon_0}\frac{dq}{(x^2+z^2)^{3/2}},\\
&= \frac{1}{4\pi\epsilon_0}\frac{(2z^2-x^2)dq}{(x^2+z^2)^{5/2}}.
\end{aligned}$$

P26-3 (a) Each point on the ring is a distance $\sqrt{z^2+R^2}$ from the point on the axis in question. Since all points are equal distant and subtend the same angle from the axis then the top half of the ring contributes

$$E_{1z} = \frac{q_1}{4\pi\epsilon_0(x^2+R^2)}\frac{z}{\sqrt{z^2+R^2}},$$

while the bottom half contributes a similar expression. Add, and

$$E_z = \frac{q_1+q_2}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}} = \frac{q}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}},$$

which is identical to Eq. 26-18.

(b) The perpendicular component would be zero if $q_1 = q_2$. It isn't, so it must be the difference $q_1 - q_2$ which is of interest. Assume this charge difference is evenly distributed on the *top* half of the ring. If it is a positive difference, then $E_\perp$ must point down. We are only interested then in the vertical component as we integrate around the top half of the ring. Then

$$\begin{aligned}
E_\perp &= \int_0^\pi \frac{1}{4\pi\epsilon_0}\frac{(q_1-q_2)/\pi}{z^2+R^2}\cos\theta\,d\theta,\\
&= \frac{q_1-q_2}{2\pi^2\epsilon_0}\frac{1}{z^2+R^2}.
\end{aligned}$$

P26-4 Use the approximation $1/(z \pm d)^2 \approx (1/z^2)(1 \mp 2d/z + 3d^2/z^2)$.

Add the contributions:

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(z+d)^2} - \frac{2q}{z^2} + \frac{q}{(z-d)^2}\right), \\ &\approx \frac{q}{4\pi\epsilon_0 z^2}\left(1 - \frac{2d}{z} + \frac{3d^2}{z^2} - 2 + 1 + \frac{2d}{z} + \frac{3d^2}{z^2}\right), \\ &= \frac{q}{4\pi\epsilon_0 z^2}\frac{6d^2}{z^2} = \frac{3Q}{4\pi\epsilon_0 z^4}, \end{aligned}$$

where $Q = 2qd^2$.

P26-5 A monopole field falls off as $1/r^2$. A dipole field falls off as $1/r^3$, and consists of two oppositely charge monopoles close together. A quadrupole field (see Exercise 11 above or read Problem 4) falls off as $1/r^4$ and (can) consist of two otherwise identical dipoles arranged with anti-parallel dipole moments. Just taking a leap of faith it seems as if we can construct a $1/r^6$ field behavior by extending the reasoning.

First we need an *octopole* which is constructed from a quadrupole. We want to keep things as simple as possible, so the construction steps are

1. The monopole is a charge $+q$ at $x = 0$.
2. The dipole is a charge $+q$ at $x = 0$ and a charge $-q$ at $x = a$. We'll call this a dipole at $x = a/2$
3. The quadrupole is the dipole at $x = a/2$, and a second dipole pointing the other way at $x = -a/2$. The charges are then $-q$ at $x = -a$, $+2q$ at $x = 0$, and $-q$ at $x = a$.
4. The octopole will be two stacked, offset quadrupoles. There will be $-q$ at $x = -a$, $+3q$ at $x = 0$, $-3q$ at $x = a$, and $+q$ at $x = 2a$.
5. Finally, our distribution with a far field behavior of $1/r^6$. There will be $+q$ at $x = 2a$, $-4q$ at $x = -a$, $+6q$ at $x = 0$, $-4q$ at $x = a$, and $+q$ at $x = 2a$.

P26-6 The vertical component of $\vec{\mathbf{E}}$ is simply half of Eq. 26-17. The horizontal component is given by a variation of the work required to derive Eq. 26-16,

$$dE_z = dE\,\sin\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dz}{y^2+z^2}\frac{z}{\sqrt{y^2+z^2}},$$

which integrates to zero if the limits are $-\infty$ to $+\infty$, but in this case,

$$E_z = \int_0^\infty dE_z = \frac{\lambda}{4\pi\epsilon_0}\frac{1}{z}.$$

Since the vertical and horizontal components are equal then $\vec{\mathbf{E}}$ makes an angle of 45°.

P26-7 (a) Swap all positive and negative charges in the problem and the electric field must reverse direction. But this is the same as flipping the problem over; consequently, the electric field must point parallel to the rod. This only holds true at point P, because point P doesn't move when you flip the rod.

(b) We are only interested in the vertical component of the field as contributed from each point on the rod. We can integrate only half of the rod and double the answer, so we want to evaluate

$$\begin{aligned} E_z &= 2\int_0^{L/2} \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dz}{y^2+z^2}\frac{z}{\sqrt{y^2+z^2}}, \\ &= \frac{2\lambda}{4\pi\epsilon_0}\frac{\sqrt{(L/2)^2+y^2}-y}{y\sqrt{(L/2)^2+y^2}}. \end{aligned}$$

(c) The previous expression is exact. If $y \gg L$, then the expression simplifies with a Taylor expansion to

$$E_z = \frac{\lambda}{4\pi\epsilon_0}\frac{L^2}{y^3},$$

which looks similar to a dipole.

P26-8 Evaluate

$$E = \int_0^R \frac{1}{4\pi\epsilon_0}\frac{z\,dq}{(z^2+r^2)^{3/2}},$$

where r is the radius of the ring, z the distance to the plane of the ring, and dq the differential charge on the ring. But $r^2 + z^2 = R^2$, and $dq = \sigma(2\pi r\,dr)$, where $\sigma = q/2\pi R^2$. Then

$$\begin{aligned} E &= \int_0^R \frac{q}{4\pi\epsilon_0}\frac{\sqrt{R^2-r^2}\,r\,dr}{R^5}, \\ &= \frac{q}{4\pi\epsilon_0}\frac{1}{3R^2}. \end{aligned}$$

P26-9 The key statement is the second italicized paragraph on page 595; the number of field lines through a unit cross-sectional area is proportional to the electric field strength. If the exponent is n, then the electric field strength a distance r from a point charge is

$$E = \frac{kq}{r^n},$$

and the *total* cross sectional area at a distance r is the area of a spherical shell, $4\pi r^2$. Then the number of field lines through the shell is proportional to

$$EA = \frac{kq}{r^n}4\pi r^2 = 4\pi kqr^{2-n}.$$

Note that the number of field lines varies with r if $n \neq 2$. This means that as we go farther from the point charge we need more and more field lines (or fewer and fewer). But the field lines can only start on charges, and we don't have any except for the point charge. We have a problem; we really do need $n = 2$ if we want workable field lines.

P26-10 The distance traveled by the electron will be $d_1 = a_1t^2/2$; the distance traveled by the proton will be $d_2 = a_2t^2/2$. a_1 and a_2 are related by $m_1a_1 = m_2a_2$, since the electric force is the same (same charge magnitude). Then $d_1 + d_2 = (a_1 + a_2)t^2/2$ is the 5.00 cm distance. Divide by the proton distance, and then

$$\frac{d_1+d_2}{d_2} = \frac{a_1+a_2}{a_2} = \frac{m_2}{m_1} + 1.$$

Then

$$d_2 = (5.00\times10^{-2}\text{m})/(1.67\times10^{-27}/9.11\times10^{-31} + 1) = 2.73\times10^{-5}\text{m}.$$

P26-11 This is merely a fancy projectile motion problem. $v_x = v_0 \cos\theta$ while $v_{y,0} = v_0 \sin\theta$. The x and y positions are $x = v_x t$ and

$$y = \frac{1}{2}at^2 + v_{y,0}t = \frac{ax^2}{2v_0^2\cos^2\theta} + x\tan\theta.$$

The acceleration of the electron is vertically down and has a magnitude of

$$a = \frac{F}{m} = \frac{Eq}{m} = \frac{(1870\,\text{N/C})(1.6\times10^{-19}\text{C})}{(9.11\times10^{-31}\text{kg})} = 3.284\times10^{14}\text{m/s}^2.$$

We want to find out how the vertical velocity of the electron at the location of the top plate. If we get an imaginary answer, then the electron doesn't get as high as the top plate.

$$\begin{aligned} v_y &= \sqrt{{v_{y,0}}^2 + 2a\Delta y}, \\ &= \sqrt{(5.83\times10^6\text{m/s})^2\sin(39^\circ)^2 + 2(-3.284\times10^{14}\text{m/s}^2)(1.97\times10^{-2}\text{m})}, \\ &= 7.226\times10^5\text{m/s}. \end{aligned}$$

This is a real answer, so this means the electron either hits the top plate, or it misses both plates. The time taken to reach the height of the top plate is

$$t = \frac{\Delta v_y}{a} = \frac{(7.226\times10^5\text{m/s}) - (5.83\times10^6\text{m/s})\sin(39^\circ)}{(-3.284\times10^{14}\text{m/s}^2)} = 8.972\times10^{-9}\text{s}.$$

In this time the electron has moved a horizontal distance of

$$x = (5.83\times10^6\text{m/s})\cos(39^\circ)(8.972\times10^{-9}\text{s}) = 4.065\times10^{-2}\text{m}.$$

This is clearly on the upper plate.

P26-12 Near the center of the ring $z \ll R$, so a Taylor expansion yields

$$E = \frac{\lambda}{2\epsilon_0}\frac{z}{R^2}.$$

The force on the electron is $F = Ee$, so the effective "spring" constant is $k = e\lambda/2\epsilon_0 R^2$. This means

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{e\lambda}{2\epsilon_0 m R^2}} = \sqrt{\frac{eq}{4\pi\epsilon_0 m R^3}}.$$

P26-13 $U = -pE\cos\theta$, so the work required to flip the dipole is

$$W = -pE\left[\cos(\theta_0 + \pi) - \cos\theta_0\right] = 2pE\cos\theta_0.$$

P26-14 If the torque on a system is given by $|\tau| = \kappa\theta$, where κ is a constant, then the frequency of oscillation of the system is $f = \sqrt{\kappa/I}/2\pi$. In this case $\tau = pE\sin\theta \approx pE\theta$, so

$$f = \sqrt{pE/I}/2\pi.$$

$\boxed{\textbf{P26-15}}$ Use the a variation of the *exact* result from Problem 26-1. The two charge are positive, but since we will eventually focus on the area between the charges we must *subtract* the two field contributions, since they point in opposite directions. Then

$$E_z = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(z-a/2)^2} - \frac{1}{(z+a/2)^2}\right)$$

and then take the derivative,

$$\frac{dE_z}{dz} = -\frac{q}{2\pi\epsilon_0}\left(\frac{1}{(z-a/2)^3} - \frac{1}{(z+a/2)^3}\right).$$

Applying the binomial expansion for points $z \ll a$,

$$\begin{aligned}
\frac{dE_z}{dz} &= -\frac{8q}{2\pi\epsilon_0}\frac{1}{a^3}\left(\frac{1}{(2z/a-1)^3} - \frac{1}{(2z/a+1)^3}\right),\\
&\approx -\frac{8q}{2\pi\epsilon_0}\frac{1}{a^3}\left(-(1+6z/a) - (1-6z/a)\right),\\
&= \frac{8q}{\pi\epsilon_0}\frac{1}{a^3}.
\end{aligned}$$

There were some fancy sign flips in the second line, so review those steps carefully!

(b) The electrostatic force on a dipole is the difference in the magnitudes of the electrostatic forces on the two charges that make up the dipole. Near the center of the above charge arrangement the electric field behaves as

$$E_z \approx E_z(0) + \left.\frac{dE_z}{dz}\right|_{z=0} z + \text{ higher ordered terms.}$$

The net force on a dipole is

$$F_+ - F_- = q(E_+ - E_-) = q\left(E_z(0) + \left.\frac{dE_z}{dz}\right|_{z=0} z_+ - E_z(0) - \left.\frac{dE_z}{dz}\right|_{z=0} z_-\right)$$

where the "+" and "-" subscripts refer to the locations of the positive and negative charges. This last line can be simplified to yield

$$q\left.\frac{dE_z}{dz}\right|_{z=0}(z_+ - z_-) = qd\left.\frac{dE_z}{dz}\right|_{z=0}.$$

E27-1 $\Phi_E = (1800\,\text{N/C})(3.2\times10^{-3}\text{m})^2\cos(145°) = -7.8\times10^{-3}\text{N}\cdot\text{m}^2/\text{C}.$

E27-2 The right face has an area element given by $\vec{\mathbf{A}} = (1.4\,\text{m})^2\hat{\mathbf{j}}$.

(a) $\Phi_E = \vec{\mathbf{A}}\cdot\vec{\mathbf{E}} = (2.0\,\text{m}^2)\hat{\mathbf{j}}\cdot(6\,\text{N/C})\hat{\mathbf{i}} = 0.$

(b) $\Phi_E = (2.0\,\text{m}^2)\hat{\mathbf{j}}\cdot(-2\,\text{N/C})\hat{\mathbf{j}} = -4\text{N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi_E = (2.0\,\text{m}^2)\hat{\mathbf{j}}\cdot[(-3\,\text{N/C})\hat{\mathbf{i}} + (4\,\text{N/C})\hat{\mathbf{k}}] = 0.$

(d) In each case the field is uniform so we can simply evaluate $\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}}$, where $\vec{\mathbf{A}}$ has six parts, one for every face. The faces, however, have the same size but are organized in pairs with opposite directions. These will cancel, so the total flux is zero in all three cases.

$\boxed{\textbf{E27-3}}$ (a) The flat base is easy enough, since according to Eq. 27-7,

$$\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}.$$

There are two important facts to consider in order to integrate this expression. $\vec{\mathbf{E}}$ is parallel to the axis of the hemisphere, $\vec{\mathbf{E}}$ points inward while $\vec{\mathbf{dA}}$ points outward on the flat base. $\vec{\mathbf{E}}$ is uniform, so it is everywhere the same on the flat base. Since $\vec{\mathbf{E}}$ is anti-parallel to $d\vec{\mathbf{A}}$, $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = -E\,dA$, then

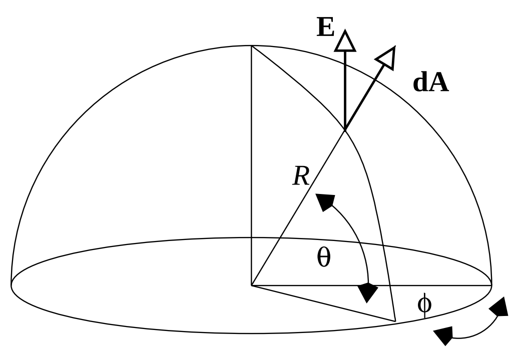
$$\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = -\int E\,dA.$$

Since $\vec{\mathbf{E}}$ is uniform we can simplify this as

$$\Phi_E = -\int E\,dA = -E\int dA = -EA = -\pi R^2 E.$$

The last steps are just substituting the area of a circle for the flat side of the hemisphere.

(b) We must first sort out the dot product



We can simplify the vector part of the problem with $\vec{\mathbf{E}}\cdot\, d\vec{\mathbf{A}} = \cos\theta E\,dA$, so

$$\Phi_E = \int \vec{\mathbf{E}}\cdot\, d\vec{\mathbf{A}} = \int \cos\theta E\,dA$$

Once again, $\vec{\mathbf{E}}$ is uniform, so we can take it out of the integral,

$$\Phi_E = \int \cos\theta E\,dA = E\int \cos\theta\,dA$$

Finally, $dA = (R\,d\theta)(R\sin\theta\,d\phi)$ on the surface of a sphere centered on $R = 0$.

We'll integrate ϕ around the axis, from 0 to 2π. We'll integrate θ from the axis to the equator, from 0 to $\pi/2$. Then

$$\Phi_E = E \int \cos\theta \, dA = E \int_0^{2\pi} \int_0^{\pi/2} R^2 \cos\theta \sin\theta \, d\theta \, d\phi.$$

Pulling out the constants, doing the ϕ integration, and then writing $2\cos\theta\sin\theta$ as $\sin(2\theta)$,

$$\Phi_E = 2\pi R^2 E \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = \pi R^2 E \int_0^{\pi/2} \sin(2\theta) \, d\theta,$$

Change variables and let $\beta = 2\theta$, then we have

$$\Phi_E = \pi R^2 E \int_0^{\pi} \sin\beta \frac{1}{2} d\beta = \pi R^2 E.$$

E27-4 Through S_1, $\Phi_E = q/\epsilon_0$. Through S_2, $\Phi_E = -q/\epsilon_0$. Through S_3, $\Phi_E = q/\epsilon_0$. Through S_4, $\Phi_E = 0$. Through S_5, $\Phi_E = q/\epsilon_0$.

$\boxed{\textbf{E27-5}}$ By Eq. 27-8,

$$\Phi_E = \frac{q}{\epsilon_0} = \frac{(1.84\,\mu\text{C})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} = 2.08\times10^5\,\text{N}\cdot\text{m}^2/\text{C}.$$

E27-6 The total flux through the sphere is

$$\Phi_E = (-1 + 2 - 3 + 4 - 5 + 6)(\times10^3\text{N}\cdot\text{m}^2/\text{C}) = 3\times10^3\text{N}\cdot\text{m}^2/\text{C}.$$

The charge inside the die is $(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(3\times10^3\text{N}\cdot\text{m}^2/\text{C}) = 2.66\times10^{-8}\text{C}$.

E27-7 The total flux through a cube would be q/ϵ_0. Since the charge is in the center of the cube we expect that the flux through any side would be the same, or 1/6 of the total flux. Hence the flux through the square surface is $q/6\epsilon_0$.

E27-8 If the electric field is uniform then there are no free charges near (or inside) the net. The flux through the netting must be equal to, but opposite in sign, from the flux through the opening. The flux through the opening is $E\pi a^2$, so the flux through the netting is $-E\pi a^2$.

E27-9 There is no flux through the sides of the cube. The flux through the top of the cube is $(-58\,\text{N/C})(100\,\text{m})^2 = -5.8\times10^5\text{N}\cdot\text{m}^2/\text{C}$. The flux through the bottom of the cube is

$$(110\,\text{N/C})(100\,\text{m})^2 = 1.1\times10^6\text{N}\cdot\text{m}^2/\text{C}.$$

The total flux is the sum, so the charge contained in the cube is

$$q = (8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(5.2\times10^5\text{N}\cdot\text{m}^2/\text{C}) = 4.60\times10^{-6}\text{C}.$$

E27-10 (a) There is only a flux through the right and left faces. Through the right face

$$\Phi_\text{R} = (2.0\,\text{m}^2)\hat{\mathbf{j}}\cdot(3\,\text{N/C}\cdot\text{m})(1.4\,\text{m})\hat{\mathbf{j}} = 8.4\,\text{N}\cdot\text{m}^2/\text{C}.$$

The flux through the left face is zero because $y = 0$.

$\boxed{\textbf{E27-11}}$ There are *eight* cubes which can be "wrapped" around the charge. Each cube has three external faces that are indistinguishable for a total of twenty-four faces, each with the same flux Φ_E. The total flux is q/ϵ_0, so the flux through one face is $\Phi_E = q/24\epsilon_0$. Note that this is the flux through faces opposite the charge; for faces which touch the charge the electric field is parallel to the surface, so the flux would be zero.

E27-12 Use Eq. 27-11,

$$\lambda = 2\pi\epsilon_0 rE = 2\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(1.96\,\mathrm{m})(4.52\times10^4\mathrm{N/C}) = 4.93\times10^{-6}\mathrm{C/m}.$$

E27-13 (a) $q = \sigma A = (2.0\times10^{-6}\mathrm{C/m}^2)\pi(0.12\,\mathrm{m})(0.42\,\mathrm{m}) = 3.17\times10^{-7}\mathrm{C}$.

(b) The charge density will be the same! $q = \sigma A = (2.0\times10^{-6}\mathrm{C/m}^2)\pi(0.08\,\mathrm{m})(0.28\,\mathrm{m}) = 1.41\times10^{-7}\mathrm{C}$.

E27-14 The electric field from the sheet on the left is of magnitude $E_l = \sigma/2\epsilon_0$, and points directly away from the sheet. The magnitude of the electric field from the sheet on the right is the same, but it points directly away from the sheet on the right.

(a) To the left of the sheets the two fields add since they point in the same direction. This means that the electric field is $\vec{\mathbf{E}} = -(\sigma/\epsilon_0)\hat{\mathbf{i}}$.

(b) Between the sheets the two electric fields cancel, so $\vec{\mathbf{E}} = 0$.

(c) To the right of the sheets the two fields add since they point in the same direction. This means that the electric field is $\vec{\mathbf{E}} = (\sigma/\epsilon_0)\hat{\mathbf{i}}$.

E27-15 The electric field from the plate on the left is of magnitude $E_l = \sigma/2\epsilon_0$, and points directly toward the plate. The magnitude of the electric field from the plate on the right is the same, but it points directly away from the plate on the right.

(a) To the left of the plates the two fields cancel since they point in the opposite directions. This means that the electric field is $\vec{\mathbf{E}} = 0$.

(b) Between the plates the two electric fields add since they point in the same direction. This means that the electric field is $\vec{\mathbf{E}} = -(\sigma/\epsilon_0)\hat{\mathbf{i}}$.

(c) To the right of the plates the two fields cancel since they point in the opposite directions. This means that the electric field is $\vec{\mathbf{E}} = 0$.

E27-16 The magnitude of the electric field is $E = mg/q$. The surface charge density on the plates is $\sigma = \epsilon_0 E = \epsilon_0 mg/q$, or

$$\sigma = \frac{(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(9.11\times10^{-31}\mathrm{kg})(9.81\,\mathrm{m/s}^2)}{(1.60\times10^{-19}\mathrm{C})} = 4.94\times10^{-22}\mathrm{C/m}^2.$$

$\boxed{\textbf{E27-17}}$ We don't really need to write an integral, we just need the charge per unit length in the cylinder to be equal to zero. This means that the positive charge in cylinder must be $+3.60n\mathrm{C/m}$. This positive charge is uniformly distributed in a circle of radius $R = 1.50$ cm, so

$$\rho = \frac{3.60n\mathrm{C/m}}{\pi R^2} = \frac{3.60n\mathrm{C/m}}{\pi(0.0150\,\mathrm{m})^2} = 5.09\mu\mathrm{C/m}^3.$$

E27-18 The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\mathrm{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\,dA = E \oint dA = 4\pi r^2 E.$$

(a) For point P_1 the charge enclosed is $q_{\mathrm{enc}} = 1.26\times10^{-7}\mathrm{C}$, so

$$E = \frac{(1.26\times10^{-7}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(1.83\times10^{-2}\mathrm{m})^2} = 3.38\times10^6\mathrm{N/C}.$$

(b) Inside a conductor $E = 0$.

$\boxed{\textbf{E27-19}}$ The proton orbits with a speed v, so the centripetal force on the proton is $F_C = mv^2/r$. This centripetal force is from the electrostatic attraction with the sphere; so long as the proton is outside the sphere the electric field is equivalent to that of a point charge Q (Eq. 27-15),

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}.$$

If q is the charge on the proton we can write $F = Eq$, or

$$\frac{mv^2}{r} = q\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

Solving for Q,

$$\begin{aligned} Q &= \frac{4\pi\epsilon_0 mv^2 r}{q}, \\ &= \frac{4\pi(8.85\times10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(1.67\times10^{-27}\mathrm{kg})(294\times10^3\mathrm{m/s})^2(0.0113\,\mathrm{m})}{(1.60\times10^{-19}\mathrm{C})}, \\ &= -1.13\times10^{-9}\mathrm{C}. \end{aligned}$$

E27-20 The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\mathrm{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\,dA = E \oint dA = 4\pi r^2 E.$$

(a) At $r = 0.120\,\mathrm{m}$ $q_{\mathrm{enc}} = 4.06\times10^{-8}\mathrm{C}$. Then

$$E = \frac{(4.06\times10^{-8}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(1.20\times10^{-1}\mathrm{m})^2} = 2.54\times10^4\mathrm{N/C}.$$

(b) At $r = 0.220\,\mathrm{m}$ $q_{\mathrm{enc}} = 5.99\times10^{-8}\mathrm{C}$. Then

$$E = \frac{(5.99\times10^{-8}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(2.20\times10^{-1}\mathrm{m})^2} = 1.11\times10^4\mathrm{N/C}.$$

(c) At $r = 0.0818\,\mathrm{m}$ $q_{\mathrm{enc}} = 0\,\mathrm{C}$. Then $E = 0$.

E27-21 The problem has cylindrical symmetry, so use a Gaussian surface which is a cylindrical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the curved surface and parallel to the end surfaces, so Gauss' law will simplify to

$$q_{\rm enc}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int E\,dA = E\int dA = 2\pi rLE,$$

where L is the length of the cylinder. Note that $\sigma = q/2\pi rL$ represents a surface charge density.

(a) $r = 0.0410\,\mathrm{m}$ is between the two cylinders. Then

$$E = \frac{(24.1\times10^{-6}\mathrm{C/m^2})(0.0322\,\mathrm{m})}{(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(0.0410\,\mathrm{m})} = 2.14\times10^{6}\mathrm{N/C}.$$

It points outward.

(b) $r = 0.0820\,\mathrm{m}$ is outside the two cylinders. Then

$$E = \frac{(24.1\times10^{-6}\mathrm{C/m^2})(0.0322\,\mathrm{m}) + (-18.0\times10^{-6}\mathrm{C/m^2})(0.0618\,\mathrm{m})}{(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(0.0820\,\mathrm{m})} = -4.64\times10^{5}\mathrm{N/C}.$$

The negative sign is because it is pointing inward.

E27-22 The problem has cylindrical symmetry, so use a Gaussian surface which is a cylindrical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the curved surface and parallel to the end surfaces, so Gauss' law will simplify to

$$q_{\rm enc}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int E\,dA = E\int dA = 2\pi rLE,$$

where L is the length of the cylinder. The charge enclosed is

$$q_{\rm enc} = \int \rho dV = \rho\pi L\left(r^2 - R^2\right)$$

The electric field is given by

$$E = \frac{\rho\pi L\left(r^2 - R^2\right)}{2\pi\epsilon_0 rL} = \frac{\rho\left(r^2 - R^2\right)}{2\epsilon_0 r}.$$

At the surface,

$$E_{\rm s} = \frac{\rho\left((2R)^2 - R^2\right)}{2\epsilon_0 2R} = \frac{3\rho R}{4\epsilon_0}.$$

Solve for r when E is half of this:

$$\begin{aligned} \frac{3R}{8} &= \frac{r^2 - R^2}{2r}, \\ 3rR &= 4r^2 - 4R^2, \\ 0 &= 4r^2 - 3rR - 4R^2. \end{aligned}$$

The solution is $r = 1.443R$. That's $(2R - 1.443R) = 0.557R$ beneath the surface.

E27-23 The electric field must do work on the electron to stop it. The electric field is given by $E = \sigma/2\epsilon_0$. The work done is $W = Fd = Eqd$. d is the distance in question, so

$$d = \frac{2\epsilon_0 K}{\sigma q} = \frac{2(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(1.15\times10^{5}\ \mathrm{eV})}{(2.08\times10^{-6}\mathrm{C/m^2})e} = 0.979\,\mathrm{m}$$

E27-24 Let the spherical Gaussian surface have a radius of R and be centered on the origin. Choose the orientation of the axis so that the infinite line of charge is along the z axis. The electric field is then directed radially outward from the z axis with magnitude $E = \lambda/2\pi\epsilon_0\rho$, where ρ is the perpendicular distance from the z axis. Now we want to evaluate

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}},$$

over the surface of the sphere. In spherical coordinates, $dA = R^2 \sin\theta \, d\theta \, d\phi$, $\rho = R\sin\theta$, and $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA\sin\theta$. Then

$$\Phi_E = \oint \frac{\lambda}{2\pi\epsilon_0} \sin\theta R \, d\theta \, d\phi = \frac{2\lambda R}{\epsilon_0}.$$

$\boxed{\textbf{E27-25}}$ (a) The problem has cylindrical symmetry, so use a Gaussian surface which is a cylindrical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the curved surface and parallel to the end surfaces, so Gauss' law will simplify to

$$q_{\mathrm{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int E \, dA = E \int dA = 2\pi r L E,$$

where L is the length of the cylinder. Now for the q_{enc} part. If the (uniform) volume charge density is ρ, then the charge enclosed in the Gaussian cylinder is

$$q_{\mathrm{enc}} = \int \rho dV = \rho \int dV = \rho V = \pi r^2 L \rho.$$

Combining, $\pi r^2 L\rho/\epsilon_0 = E2\pi r L$ or $E = \rho r/2\epsilon_0$.

(b) Outside the charged cylinder the charge enclosed in the Gaussian surface is just the charge in the cylinder. Then

$$q_{\mathrm{enc}} = \int \rho dV = \rho \int dV = \rho V = \pi R^2 L \rho.$$

and

$$\pi R^2 L\rho/\epsilon_0 = E2\pi r L,$$

and then finally

$$E = \frac{R^2\rho}{2\epsilon_0 r}.$$

E27-26 (a) $q = 4\pi(1.22\,\mathrm{m})^2(8.13\times10^{-6}\mathrm{C/m^2}) = 1.52\times10^{-4}\mathrm{C}$.

(b) $\Phi_E = q/\epsilon_0 = (1.52\times10^{-4}\mathrm{C})/(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2}) = 1.72\times10^{7}\mathrm{N\cdot m^2/C}$.

(c) $E = \sigma/\epsilon_0 = (8.13\times10^{-6}\mathrm{C/m^2})/(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2}) = 9.19\times10^{5}\mathrm{N/C}$

E27-27 (a) $\sigma = (2.4\times10^{-6}\mathrm{C})/4\pi(0.65\,\mathrm{m})^2 = 4.52\times10^{-7}\mathrm{C/m^2}$.

(b) $E = \sigma/\epsilon_0 = (4.52\times10^{-7}\mathrm{C/m^2})/(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2}) = 5.11\times10^{4}\mathrm{N/C}$.

E27-28 $E = \sigma/\epsilon_0 = q/4\pi r^2\epsilon_0$.

$\boxed{\textbf{E27-29}}$ (a) The near field is given by Eq. 27-12, $E = \sigma/2\epsilon_0$, so

$$E \approx \frac{(6.0\times10^{-6}\mathrm{C})/(8.0\times10^{-2}\,\mathrm{m})^2}{2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})} = 5.3\times10^{7}\mathrm{N/C}.$$

(b) Very far from *any* object a point charge approximation is valid. Then

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})}\frac{(6.0\times10^{-6}\mathrm{C})}{(30\,\mathrm{m})^2} = 60\mathrm{N/C}.$$

P27-1 For a spherically symmetric mass distribution choose a spherical Gaussian shell. Then

$$\oint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = \oint g\,dA = g \oint dA = 4\pi r^2 g.$$

Then

$$\frac{\Phi_g}{4\pi G} = \frac{gr^2}{G} = -m,$$

or

$$g = -\frac{Gm}{r^2}.$$

The negative sign indicates the direction; $\vec{\mathbf{g}}$ point toward the mass center.

P27-2 (a) The flux through all surfaces *except* the right and left faces will be zero. Through the left face,

$$\Phi_\mathrm{l} = -E_y A = -b\sqrt{a}a^2.$$

Through the right face,

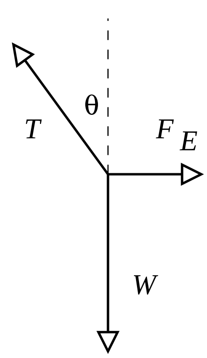
$$\Phi_\mathrm{r} = E_y A = b\sqrt{2a}a^2.$$

The net flux is then

$$\Phi = ba^{5/2}(\sqrt{2} - 1) = (8830\,\mathrm{N/C\cdot m^{1/2}})(0.130\,\mathrm{m})^{5/2}(\sqrt{2} - 1) = 22.3\,\mathrm{N\cdot m^2/C}.$$

(b) The charge enclosed is $q = (8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(22.3\,\mathrm{N\cdot m^2/C}) = 1.97\times10^{-10}\mathrm{C}$.

$\boxed{\textbf{P27-3}}$ The net force on the small sphere is zero; this force is the vector sum of the force of gravity W, the electric force F_E, and the tension T.



These forces are related by $Eq = mg\tan\theta$. We also have $E = \sigma/2\epsilon_0$, so

$$\begin{aligned}
\sigma &= \frac{2\epsilon_0 mg\tan\theta}{q}, \\
&= \frac{2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(1.12\times10^{-6}\mathrm{kg})(9.81\,\mathrm{m/s^2})\tan(27.4°)}{(19.7\times10^{-9}\mathrm{C})}, \\
&= 5.11\times10^{-9}\mathrm{C/m^2}.
\end{aligned}$$

P27-4 The materials are conducting, so *all* charge will reside on the surfaces. The electric field inside any conductor is zero. The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\text{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\,dA = E \oint dA = 4\pi r^2 E.$$

Consequently, $E = q_{\text{enc}}/4\pi\epsilon_0 r^2$.

(a) Within the sphere $E = 0$.

(b) Between the sphere and the shell $q_{\text{enc}} = q$. Then $E = q/4\pi\epsilon_0 r^2$.

(c) Within the shell $E = 0$.

(d) Outside the shell $q_{\text{enc}} = +q - q = 0$. Then $E = 0$.

(e) Since $E = 0$ inside the shell, $q_{\text{enc}} = 0$, this requires that $-q$ reside on the inside surface. This is no charge on the outside surface.

P27-5 The problem has cylindrical symmetry, so use a Gaussian surface which is a cylindrical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the curved surface and parallel to the end surfaces, so Gauss' law will simplify to

$$q_{\text{enc}}/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int E\,dA = E \int dA = 2\pi r L E,$$

where L is the length of the cylinder. Consequently, $E = q_{\text{enc}}/2\pi\epsilon_0 r L$.

(a) Outside the conducting shell $q_{\text{enc}} = +q - 2q = -q$. Then $E = -q/2\pi\epsilon_0 r L$. The negative sign indicates that the field is pointing inward toward the axis of the cylinder.

(b) Since $E = 0$ inside the conducting shell, $q_{\text{enc}} = 0$, which means a charge of $-q$ is on the inside surface of the shell. The remaining $-q$ must reside on the outside surface of the shell.

(c) In the region between the cylinders $q_{\text{enc}} = +q$. Then $E = +q/2\pi\epsilon_0 r L$. The positive sign indicates that the field is pointing outward from the axis of the cylinder.

P27-6 Subtract Eq. 26-19 from Eq. 26-20. Then

$$E = \frac{\sigma}{2\epsilon_0} \frac{z}{\sqrt{z^2 + R^2}}.$$

$\boxed{\textbf{P27-7}}$ This problem is closely related to Ex. 27-25, except for the part concerning q_{enc}. We'll set up the problem the same way: the Gaussian surface will be a (imaginary) cylinder centered on the axis of the physical cylinder. For Gaussian surfaces of radius $r < R$, there is *no* charge enclosed while for Gaussian surfaces of radius $r > R$, $q_{\text{enc}} = \lambda l$.

We've already worked out the integral

$$\int_{\text{tube}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi r l E,$$

for the cylinder, and then from Gauss' law,

$$q_{\text{enc}} = \epsilon_0 \int_{\text{tube}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi\epsilon_0 r l E.$$

(a) When $r < R$ there is no enclosed charge, so the left hand vanishes and consequently $E = 0$ inside the physical cylinder.

(b) When $r > R$ there is a charge λl enclosed, so

$$E = \frac{\lambda}{2\pi\epsilon_0 r}.$$

P27-8 This problem is closely related to Ex. 27-25, except for the part concerning q_{enc}. We'll set up the problem the same way: the Gaussian surface will be a (imaginary) cylinder centered on the axis of the physical cylinders. For Gaussian surfaces of radius $r < a$, there is *no* charge enclosed while for Gaussian surfaces of radius $b > r > a$, $q_{\text{enc}} = \lambda l$.

We've already worked out the integral

$$\int_{\text{tube}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi r l E,$$

for the cylinder, and then from Gauss' law,

$$q_{\text{enc}} = \epsilon_0 \int_{\text{tube}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi\epsilon_0 r l E.$$

(a) When $r < a$ there is no enclosed charge, so the left hand vanishes and consequently $E = 0$ inside the inner cylinder.

(b) When $b > r > a$ there is a charge λl enclosed, so

$$E = \frac{\lambda}{2\pi\epsilon_0 r}.$$

P27-9 Uniform circular orbits require a constant net force towards the center, so $F = Eq = \lambda q/2\pi\epsilon_0 r$. The speed of the positron is given by $F = mv^2/r$; the kinetic energy is $K = mv^2/2 = Fr/2$. Combining,

$$\begin{aligned}
K &= \frac{\lambda q}{4\pi\epsilon_0}, \\
&= \frac{(30\times10^{-9}\text{C/m})(1.6\times10^{-19}\text{C})}{4\pi((8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}, \\
&= 4.31\times10^{-17}\,\text{J} = 270 \text{ eV}.
\end{aligned}$$

P27-10 $\lambda = 2\pi\epsilon_0 r E$, so

$$q = 2\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.014\,\text{m})(0.16\,\text{m})(2.9\times10^4\text{N/C}) = 3.6\times10^{-9}\text{C}.$$

$\boxed{\textbf{P27-11}}$ (a) Put a spherical Gaussian surface inside the shell centered on the point charge. Gauss' law states

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\epsilon_0}.$$

Since there is spherical symmetry the electric field is normal to the spherical Gaussian surface, and it is everywhere the same on this surface. The dot product simplifies to $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\,dA$, while since E is a constant on the surface we can pull it out of the integral, and we end up with

$$E \oint dA = \frac{q}{\epsilon_0},$$

where q is the point charge in the center. Now $\oint dA = 4\pi r^2$, where r is the radius of the Gaussian surface, so

$$E = \frac{q}{4\pi\epsilon_0 r^2}.$$

(b) Repeat the above steps, except put the Gaussian surface outside the conducting shell. Keep it centered on the charge. Two things are different from the above derivation: (1) r is bigger, and

(2) there is an uncharged spherical conducting shell inside the Gaussian surface. Neither change will affect the surface integral or $q_{\rm enc}$, so the electric field outside the shell is still

$$E = \frac{q}{4\pi\epsilon_0 r^2},$$

(c) This is a subtle question. With all the symmetry here it appears as if the shell has no effect; the field just looks like a point charge field. If, however, the charge were moved off center the field inside the shell would become distorted, and we wouldn't be able to use Gauss' law to find it. So the shell does make a difference.

Outside the shell, however, we can't tell what is going on inside the shell. So the electric field outside the shell looks like a point charge field originating from the center of the shell *regardless of where inside the shell the point charge is placed!*

(d) Yes, q induces surface charges on the shell. There will be a charge $-q$ on the inside surface and a charge q on the outside surface.

(e) Yes, as there is an electric field from the shell, isn't there?

(f) No, as the electric field from the outside charge won't make it through a conducting shell. The conductor acts as a shield.

(g) No, this is not a contradiction, because the outside charge never experienced any electrostatic attraction or repulsion from the inside charge. The force is between the shell and the outside charge.

P27-12 The repulsive electrostatic forces must exactly balance the attractive gravitational forces. Then

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = G\frac{m^2}{r^2},$$

or $m = q/\sqrt{4\pi\epsilon_0 G}$. Numerically,

$$m = \frac{(1.60\times10^{-19}\mathrm{C})}{\sqrt{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)}} = 1.86\times10^{-9}\mathrm{kg}.$$

P27-13 The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\rm enc}/\epsilon_0 = \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \oint E\,dA = E\oint dA = 4\pi r^2 E.$$

Consequently, $E = q_{\rm enc}/4\pi\epsilon_0 r^2$.

$q_{\rm enc} = q + 4\pi\int \rho\, r^2 dr$, or

$$q_{\rm enc} = q + 4\pi\int_a^r Ar\,dr = q + 2\pi A(r^2 - a^2).$$

The electric field will be constant if $q_{\rm enc}$ behaves as r^2, which requires $q = 2\pi A a^2$, or $A = q/2\pi a^2$.

P27-14 (a) The problem has spherical symmetry, so use a Gaussian surface which is a spherical shell. The $\vec{\mathbf{E}}$ field will be perpendicular to the surface, so Gauss' law will simplify to

$$q_{\rm enc}/\epsilon_0 = \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \oint E\,dA = E\oint dA = 4\pi r^2 E.$$

Consequently, $E = q_{\rm enc}/4\pi\epsilon_0 r^2$.

$q_{\rm enc} = 4\pi\int \rho\, r^2 dr = 4\pi\rho r^3/3$, so

$$E = \rho r/3\epsilon_0$$

and is directed radially out from the center. Then $\vec{\mathbf{E}} = \rho\vec{\mathbf{r}}/3\epsilon_0$.

(b) The electric field in the hole is given by $\vec{\mathbf{E}}_h = \vec{\mathbf{E}} - \vec{\mathbf{E}}_b$, where $\vec{\mathbf{E}}$ is the field from part (a) and $\vec{\mathbf{E}}_b$ is the field that would be produced by the matter that would have been in the hole had the hole not been there. Then

$$\vec{\mathbf{E}}_b = \rho\vec{\mathbf{b}}/3\epsilon_0,$$

where $\vec{\mathbf{b}}$ is a vector pointing from the center of the hole. Then

$$\vec{\mathbf{E}}_h = \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0} - \frac{\rho\vec{\mathbf{b}}}{3\epsilon_0} = \frac{\rho}{3\epsilon_0}(\vec{\mathbf{r}} - \vec{\mathbf{b}}).$$

But $\vec{\mathbf{r}} - \vec{\mathbf{b}} = \vec{\mathbf{a}}$, so $\vec{\mathbf{E}}_h = \rho\vec{\mathbf{a}}/3\epsilon_0$.

$\boxed{\textbf{P27-15}}$ If a point is an equilibrium point then the electric field at that point should be zero. If it is a stable point then moving the test charge (assumed positive) a small distance from the equilibrium point should result in a restoring force directed back toward the equilibrium point. In other words, there will be a point where the electric field is zero, and around this point there will be an electric field pointing inward. Applying Gauss' law to a small surface surrounding our point P, we have a net inward flux, so there must be a negative charge *inside* the surface. But there should be nothing inside the surface except an empty point P, so we have a contradiction.

P27-16 (a) Follow the example on Page 618. By symmetry $E = 0$ along the median plane. The charge enclosed between the median plane and a surface a distance x from the plane is $q = \rho Ax$. Then

$$E = \rho Ax/\epsilon_0 A = \rho A/\epsilon_0.$$

(b) Outside the slab the charge enclosed between the median plane and a surface a distance x from the plane is is $q = \rho Ad/2$, regardless of x. The

$$E = \rho Ad/2/\epsilon_0 A = \rho d/2\epsilon_0.$$

$\boxed{\textbf{P27-17}}$ (a) The total charge is the volume integral over the whole sphere,

$$Q = \int \rho\, dV.$$

This is actually a three dimensional integral, and $dV = A\,dr$, where $A = 4\pi r^2$. Then

$$\begin{aligned} Q &= \int \rho\, dV, \\ &= \int_0^R \left(\frac{\rho_S r}{R}\right) 4\pi r^2\, dr, \\ &= \frac{4\pi\rho_S}{R}\frac{1}{4}R^4, \\ &= \pi\rho_S R^3. \end{aligned}$$

(b) Put a spherical Gaussian surface inside the sphere centered on the center. We can use Gauss' law here because there is spherical symmetry in the entire problem, both inside and outside the Gaussian surface. Gauss' law states

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\epsilon_0}.$$

Since there is spherical symmetry the electric field is normal to the spherical Gaussian surface, and it is everywhere the same on this surface. The dot product simplifies to $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\,dA$, while since E is a constant on the surface we can pull it out of the integral, and we end up with

$$E \oint dA = \frac{q_{\rm enc}}{\epsilon_0},$$

Now $\oint dA = 4\pi r^2$, where r is the radius of the Gaussian surface, so

$$E = \frac{q_{\rm enc}}{4\pi\epsilon_0 r^2}.$$

We aren't done yet, because the charge enclosed depends on the radius of the Gaussian surface. We need to do part (a) again, except this time we don't want to do the whole volume of the sphere, we only want to go out as far as the Gaussian surface. Then

$$\begin{aligned} q_{\rm enc} &= \int \rho\, dV, \\ &= \int_0^r \left(\frac{\rho_S r}{R}\right) 4\pi r^2\, dr, \\ &= \frac{4\pi\rho_S}{R}\frac{1}{4}r^4, \\ &= \pi\rho_S \frac{r^4}{R}. \end{aligned}$$

Combine these last two results and

$$\begin{aligned} E &= \frac{\pi\rho_S}{4\pi\epsilon_0 r^2}\frac{r^4}{R}, \\ &= \frac{\pi\rho_S}{4\pi\epsilon_0}\frac{r^2}{R}, \\ &= \frac{Q}{4\pi\epsilon_0}\frac{r^2}{R^4}. \end{aligned}$$

In the last line we used the results of part (a) to eliminate ρ_S from the expression.

P27-18 (a) Inside the conductor $E = 0$, so a Gaussian surface which is embedded in the conductor but containing the hole must have a net enclosed charge of zero. The cavity wall must then have a charge of $-3.0\,\mu$C.

(b) The net charge on the conductor is $+10.0\,\mu$C; the charge on the outer surface must then be $+13.0\,\mu$C.

P27-19 (a) Inside the shell $E = 0$, so the net charge inside a Gaussian surface embedded in the shell must be zero, so the inside surface has a charge $-Q$.

(b) Still $-Q$; the outside has nothing to do with the inside.

(c) $-(Q + q)$; see reason (a).

(d) Yes.

Throughout this chapter we will use the convention that $V(\infty) = 0$ unless explicitly stated otherwise. Then the potential in the vicinity of a point charge will be given by Eq. 28-18, $V = q/4\pi\epsilon_0 r$.

E28-1 (a) Let U_{12} be the potential energy of the interaction between the two "up" quarks. Then

$$U_{12} = (8.99\times 10^9 \mathrm{N\cdot m^2/C^2})\frac{(2/3)^2 e(1.60\times 10^{-19}\mathrm{C})}{(1.32\times 10^{-15}\mathrm{m})} = 4.84\times 10^5 \mathrm{eV}.$$

(b) Let U_{13} be the potential energy of the interaction between an "up" quark and a "down" quark. Then

$$U_{13} = (8.99\times 10^9 \mathrm{N\cdot m^2/C^2})\frac{(-1/3)(2/3) e(1.60\times 10^{-19}\mathrm{C})}{(1.32\times 10^{-15}\mathrm{m})} = -2.42\times 10^5 \mathrm{eV}$$

Note that $U_{13} = U_{23}$. The total electric potential energy is the sum of these three terms, or zero.

E28-2 There are six interaction terms, one for every charge pair. Number the charges clockwise from the upper left hand corner. Then

$$\begin{aligned}
U_{12} &= -q^2/4\pi\epsilon_0 a, \\
U_{23} &= -q^2/4\pi\epsilon_0 a, \\
U_{34} &= -q^2/4\pi\epsilon_0 a, \\
U_{41} &= -q^2/4\pi\epsilon_0 a, \\
U_{13} &= (-q)^2/4\pi\epsilon_0(\sqrt{2}a), \\
U_{24} &= q^2/4\pi\epsilon_0(\sqrt{2}a).
\end{aligned}$$

Add these terms and get

$$U = \left(\frac{2}{\sqrt{2}} - 4\right)\frac{q^2}{4\pi\epsilon_0 a} = -0.206\frac{q^2}{\epsilon_0 a}$$

The amount of work required is $W = U$.

$\boxed{\textbf{E28-3}}$ (a) We build the electron one part at a time; each part has a charge $q = e/3$. Moving the first part from infinity to the location where we want to construct the electron is easy and takes no work at all. Moving the second part in requires work to change the potential energy to

$$U_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r},$$

which is basically Eq. 28-7. The separation $r = 2.82 \times 10^{-15}$ m.

Bringing in the third part requires work against the force of repulsion between the third charge and both of the other two charges. Potential energy then exists in the form U_{13} and U_{23}, where all three charges are the same, and all three separations are the same. Then $U_{12} = U_{13} = U_{12}$, so the total potential energy of the system is

$$U = 3\frac{1}{4\pi\epsilon_0}\frac{(e/3)^2}{r} = \frac{3}{4\pi(8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2})}\frac{(1.60\times 10^{-19}\,\mathrm{C}/3)^2}{(2.82\times 10^{-15}\,\mathrm{m})} = 2.72\times 10^{-14}\,\mathrm{J}$$

(b) Dividing our answer by the speed of light squared to find the mass,

$$m = \frac{2.72\times 10^{-14}\,\mathrm{J}}{(3.00\times 10^8\,\mathrm{m/s})^2} = 3.02\times 10^{-31}\,\mathrm{kg}.$$

E28-4 There are three interaction terms, one for every charge pair. Number the charges from the left; let $a = 0.146\,\text{m}$. Then

$$\begin{aligned} U_{12} &= \frac{(25.5\times10^{-9}\text{C})(17.2\times10^{-9}\text{C})}{4\pi\epsilon_0 a}, \\ U_{13} &= \frac{(25.5\times10^{-9}\text{C})(-19.2\times10^{-9}\text{C})}{4\pi\epsilon_0 (a+x)}, \\ U_{23} &= \frac{(17.2\times10^{-9}\text{C})(-19.2\times10^{-9}\text{C})}{4\pi\epsilon_0 x}. \end{aligned}$$

Add these and set it equal to zero. Then

$$\frac{(25.5)(17.2)}{a} = \frac{(25.5)(19.2)}{a+x} + \frac{(17.2)(19.2)}{x},$$

which has solution $x = 1.405a = 0.205\,\text{m}$.

E28-5 The volume of the nuclear material is $4\pi a^3/3$, where $a = 8.0\times10^{-15}\text{m}$. Upon dividing in half each part will have a radius r where $4\pi r^3/3 = 4\pi a^3/6$. Consequently, $r = a/\sqrt[3]{2} = 6.35\times10^{-15}\text{m}$. Each fragment will have a charge of $+46e$.

(a) The force of repulsion is

$$F = \frac{(46)^2(1.60\times10^{-19}\text{C})^2}{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)[2(6.35\times10^{-15}\text{m})]^2} = 3000\,\text{N}$$

(b) The potential energy is

$$U = \frac{(46)^2 e(1.60\times10^{-19}\text{C})}{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)2(6.35\times10^{-15}\text{m})} = 2.4\times10^8\text{eV}$$

E28-6 This is a work/kinetic energy problem: $\frac{1}{2}mv_0^2 = q\Delta V$. Then

$$v_0 = \sqrt{\frac{2(1.60\times10^{-19}\text{C})(10.3\times10^3\text{V})}{(9.11\times10^{-31}\text{kg})}} = 6.0\times10^7\text{m/s}.$$

$\boxed{\textbf{E28-7}}$ (a) The energy released is equal to the charges times the potential through which the charge was moved. Then

$$\Delta U = q\Delta V = (30\,\text{C})(1.0\times10^9\,\text{V}) = 3.0\times10^{10}\,\text{J}.$$

(b) Although the problem mentions acceleration, we want to focus on energy. The energy will change the kinetic energy of the car from 0 to $K_\text{f} = 3.0\times10^{10}$ J. The speed of the car is then

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.0\times10^{10}\,\text{J})}{(1200\,\text{kg})}} = 7100\,\text{m/s}.$$

(c) The energy required to melt ice is given by $Q = mL$, where L is the latent heat of fusion. Then

$$m = \frac{Q}{L} = \frac{(3.0\times10^{10}\,\text{J})}{(3.33\times10^5\text{J/kg})} = 90,100\text{kg}.$$

E28-8 (a) $\Delta U = (1.60\times10^{-19}\mathrm{C})(1.23\times10^{9}\mathrm{V}) = 1.97\times10^{-10}\mathrm{J}$.
(b) $\Delta U = e(1.23\times10^{9}\mathrm{V}) = 1.23\times10^{9}\mathrm{eV}$.

E28-9 This is an energy conservation problem: $\frac{1}{2}mv^2 = q\Delta V$; $\Delta V = q/4\pi\epsilon_0(1/r_1 - 1/r_2)$. Combining,

$$\begin{aligned} v &= \sqrt{\frac{q^2}{2\pi\epsilon_0 m}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}, \\ &= \sqrt{\frac{(3.1\times10^{-6}\mathrm{C})^2}{2\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(18\times10^{-6}\mathrm{kg})}\left(\frac{1}{(0.90\times10^{-3}\mathrm{m})}-\frac{1}{(2.5\times10^{-3}\mathrm{m})}\right)}, \\ &= 2600\,\mathrm{m/s}. \end{aligned}$$

E28-10 This is an energy conservation problem:

$$\frac{1}{2}m(2v)^2 - \frac{q^2}{4\pi\epsilon_0 r} = \frac{1}{2}mv^2.$$

Rearrange,

$$\begin{aligned} r &= \frac{q^2}{6\pi\epsilon_0 mv^2}, \\ &= \frac{(1.60\times10^{-19}\mathrm{C})^2}{6\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(9.11\times10^{-31}\mathrm{kg})(3.44\times10^{5}\mathrm{m/s})^2)} = 1.42\times10^{-9}\mathrm{m}. \end{aligned}$$

E28-11 (a) $V = (1.60\times10^{-19}\mathrm{C})/4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(5.29\times10^{-11}\mathrm{m}) = 27.2\,\mathrm{V}$.
(b) $U = qV = (-e)(27.2\,\mathrm{V}) = -27.2$ eV.
(c) For uniform circular orbits $F = mv^2/r$; the force is electrical, or $F = e^2/4\pi\epsilon_0 r^2$. Kinetic energy is $K = mv^2/2 = Fr/2$, so

$$K = \frac{e^2}{8\pi\epsilon_0 r} = \frac{(1.60\times10^{-19}\mathrm{C})}{8\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(5.29\times10^{-11}\mathrm{m})} = 13.6\ \mathrm{eV}.$$

(d) The ionization energy is $-(K+U)$, or

$$E_{\mathrm{ion}} = -[(13.6\ \mathrm{eV}) + (-27.2\ \mathrm{eV})] = 13.6\ \mathrm{eV}.$$

E28-12 (a) The electric potential at A is

$$V_A = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right) = (8.99\times10^{9}\mathrm{N\cdot m^2/C})\left(\frac{(-5.0\times10^{-6}\mathrm{C})}{(0.15\,\mathrm{m})}+\frac{(2.0\times10^{-6}\mathrm{C})}{(0.05\,\mathrm{m})}\right) = 6.0\times10^{4}\mathrm{V}.$$

The electric potential at B is

$$V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_2}+\frac{q_2}{r_1}\right) = (8.99\times10^{9}\mathrm{N\cdot m^2/C})\left(\frac{(-5.0\times10^{-6}\mathrm{C})}{(0.05\,\mathrm{m})}+\frac{(2.0\times10^{-6}\mathrm{C})}{(0.15\,\mathrm{m})}\right) = -7.8\times10^{5}\mathrm{V}.$$

(b) $W = q\Delta V = (3.0\times10^{-6}\mathrm{C})(6.0\times10^{4}\mathrm{V} - -7.8\times10^{5}\mathrm{V}) = 2.5\,\mathrm{J}$.
(c) Since work is positive then external work is converted to electrostatic potential energy.

$\boxed{\textbf{E28-13}}$ (a) The magnitude of the electric field would be found from

$$E = \frac{F}{q} = \frac{(3.90 \times 10^{-15}\,\mathrm{N})}{(1.60 \times 10^{-19}\,\mathrm{C})} = 2.44 \times 10^4\,\mathrm{N/C}.$$

(b) The potential difference between the plates is found by evaluating Eq. 28-15,

$$\Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}.$$

The electric field between two parallel plates is uniform and perpendicular to the plates. Then $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E\,ds$ along this path, and since E is uniform,

$$\Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_a^b E\,ds = -E\int_a^b ds = E\Delta x,$$

where Δx is the separation between the plates. Finally, $\Delta V = (2.44 \times 10^4\,\mathrm{N/C})(0.120\,\mathrm{m}) = 2930\,\mathrm{V}$.

E28-14 $\Delta V = E\Delta x$, so

$$\Delta x = \frac{2\epsilon_0}{\sigma}\Delta V = \frac{2(8.85\times 10^{-12}\mathrm{C^2/N\cdot m^2})}{(0.12\times 10^{-6}\mathrm{C/m^2})}(48\,\mathrm{V}) = 7.1\times 10^{-3}\mathrm{m}$$

E28-15 The electric field around an infinitely long straight wire is given by $E = \lambda/2\pi\epsilon_0 r$. The potential difference between the inner wire and the outer cylinder is given by

$$\Delta V = -\int_a^b (\lambda/2\pi\epsilon_0 r)\,dr = (\lambda/2\pi\epsilon_0)\ln(a/b).$$

The electric field near the surface of the wire is then given by

$$E = \frac{\lambda}{2\pi\epsilon_0 a} = \frac{\Delta V}{a\ln(a/b)} = \frac{(-855\,\mathrm{V})}{(6.70\times 10^{-7}\mathrm{m})\ln(6.70\times 10^{-7}\,\mathrm{m}/1.05\times 10^{-2}\,\mathrm{m})} = 1.32\times 10^8\mathrm{V/m}.$$

The electric field near the surface of the cylinder is then given by

$$E = \frac{\lambda}{2\pi\epsilon_0 a} = \frac{\Delta V}{a\ln(a/b)} = \frac{(-855\,\mathrm{V})}{(1.05\times 10^{-2}\mathrm{m})\ln(6.70\times 10^{-7}\,\mathrm{m}/1.05\times 10^{-2}\,\mathrm{m})} = 8.43\times 10^3\mathrm{V/m}.$$

E28-16 $\Delta V = E\Delta x = (1.92\times 10^5\mathrm{N/C})(1.50\times 10^{-2}\mathrm{m}) = 2.88\times 10^3\mathrm{V}$.

E28-17 (a) This is an energy conservation problem:

$$K = \frac{1}{4\pi\epsilon_0}\frac{(2)(79)e^2}{r} = (8.99\times 10^9\mathrm{N\cdot m^2/C})\frac{(2)(79)e(1.60\times 10^{-19}\mathrm{C})}{(7.0\times 10^{-15}\mathrm{m})} = 3.2\times 10^7\ \mathrm{eV}$$

(b) The alpha particles used by Rutherford never came close to hitting the gold nuclei.

E28-18 This is an energy conservation problem: $mv^2/2 = eq/4\pi\epsilon_0 r$, or

$$v = \sqrt{\frac{(1.60\times 10^{-19}\mathrm{C})(1.76\times 10^{-15}\mathrm{C})}{2\pi(8.85\times 10^{-12}\mathrm{C^2/N\cdot m^2})(1.22\times 10^{-2}\mathrm{m})(9.11\times 10^{-31}\mathrm{kg})}} = 2.13\times 10^4\mathrm{m/s}$$

$\boxed{\textbf{E28-19}}$ (a) We evaluate V_A and V_B individually, and then find the difference.

$$V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{1}{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})}\frac{(1.16\mu\mathrm{C})}{(2.06\,\mathrm{m})} = 5060\,\mathrm{V},$$

and

$$V_B = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{1}{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})}\frac{(1.16\mu\mathrm{C})}{(1.17\,\mathrm{m})} = 8910\,\mathrm{V},$$

The difference is then $V_A - V_B = -3850\,\mathrm{V}$.

(b) The answer is the same, since when concerning ourselves with electric potential we only care about distances, and not directions.

E28-20 The number of "excess" electrons on each grain is

$$n = \frac{4\pi\epsilon_0 rV}{e} = \frac{4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m})(1.0\times10^{-6}\mathrm{m})(-400\,\mathrm{V})}{(-1.60\times10^{-19}\mathrm{C})} = 2.8\times10^{5}$$

E28-21 The excess charge on the shuttle is

$$q = 4\pi\epsilon_0 rV = 4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m})(10\,\mathrm{m})(-1.0\,\mathrm{V}) = -1.1\times10^{-9}\mathrm{C}$$

E28-22 $q = 1.37\times10^{5}\mathrm{C}$, so

$$V = (8.99\times10^{9}\mathrm{N\cdot m^2/C^2})\frac{(1.37\times10^{5}\mathrm{C})}{(6.37\times10^{6}\mathrm{m})} = 1.93\times10^{8}\mathrm{V}.$$

$\boxed{\textbf{E28-23}}$ The ratio of the electric potential to the electric field strength is

$$\frac{V}{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r}\right) / \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right) = r.$$

In this problem r is the radius of the Earth, so at the surface of the Earth the potential is

$$V = Er = (100\,\mathrm{V/m})(6.38\times10^{6}\mathrm{m}) = 6.38\times10^{8}\,\mathrm{V}.$$

E28-24 Use Eq. 28-22:

$$V = (8.99\times10^{9}\mathrm{N\cdot m^2/C^2})\frac{(1.47)(3.34\times10^{-30}\mathrm{C\cdot m})}{(52.0\times10^{-9}\mathrm{m})^2} = 1.63\times10^{-5}\mathrm{V}.$$

$\boxed{\textbf{E28-25}}$ (a) When finding V_A we need to consider the contribution from both the positive and the negative charge, so

$$V_A = \frac{1}{4\pi\epsilon_0}\left(qa + \frac{-q}{a+d}\right)$$

There will be a similar expression for V_B,

$$V_B = \frac{1}{4\pi\epsilon_0}\left(-qa + \frac{q}{a+d}\right).$$

Now to evaluate the difference.

$$
\begin{aligned}
V_A - V_B &= \frac{1}{4\pi\epsilon_0}\left(qa + \frac{-q}{a+d}\right) - \frac{1}{4\pi\epsilon_0}\left(-qa + \frac{q}{a+d}\right), \\
&= \frac{q}{2\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{a+d}\right), \\
&= \frac{q}{2\pi\epsilon_0}\left(\frac{a+d}{a(a+d)} - \frac{a}{a(a+d)}\right), \\
&= \frac{q}{2\pi\epsilon_0}\frac{d}{a(a+d)}.
\end{aligned}
$$

(b) Does it do what we expect when $d = 0$? I expect it the difference to go to zero as the two points A and B get closer together. The numerator will go to zero as d gets smaller. The denominator, however, stays finite, which is a good thing. So yes, $V_a - V_B \to 0$ as $d \to 0$.

E28-26 (a) Since both charges are positive the electric potential from both charges will be positive. There will be no *finite* points where $V = 0$, since two positives can't add to zero.

(b) Between the charges the electric field from each charge points toward the other, so $\vec{\mathbf{E}}$ will vanish when $q/x^2 = 2q/(d-x)^2$. This happens when $d - x = \sqrt{2}x$, or $x = d/(1+\sqrt{2})$.

E28-27 The distance from C to either charge is $\sqrt{2}d/2 = 1.39\times10^{-2}\mathrm{m}$.

(a) V at C is

$$
V = (8.99\times10^{9}\mathrm{N\cdot m^2/C^2})\frac{2(2.13\times10^{-6}\mathrm{C})}{(1.39\times10^{-2}\mathrm{m})} = 2.76\times10^{6}\mathrm{V}
$$

(b) $W = q\delta V = (1.91\times10^{-6}\mathrm{C})(2.76\times10^{6}\mathrm{V}) = 5.27\,\mathrm{J}$.

(c) Don't forget about the potential energy of the original two charges!

$$
U_0 = (8.99\times10^{9}\mathrm{N\cdot m^2/C^2})\frac{(2.13\times10^{-6}\mathrm{C})^2}{(1.96\times10^{-2}\mathrm{m})} = 2.08\,\mathrm{J}
$$

Add this to the answer from part (b) to get 7.35 J.

E28-28 The potential is given by Eq. 28-32; at the surface $V_s = \sigma R/2\epsilon_0$, half of this occurs when

$$
\begin{aligned}
\sqrt{R^2+z^2} - z &= R/2, \\
R^2 + z^2 &= R^2/4 + Rz + z^2, \\
3R/4 &= z.
\end{aligned}
$$

$\boxed{\textbf{E28-29}}$ We can find the linear charge density by dividing the charge by the circumference,

$$
\lambda = \frac{Q}{2\pi R},
$$

where Q refers to the charge on the ring. The work done to move a charge q from a point x to the origin will be given by

$$
\begin{aligned}
W &= q\Delta V, \\
W &= q\left(V(0) - V(x)\right), \\
&= q\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2}} - \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+x^2}}\right), \\
&= \frac{qQ}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+x^2}}\right).
\end{aligned}
$$

Putting in the numbers,

$$\frac{(-5.93\times10^{-12}\text{C})(-9.12\times10^{-9}\text{C})}{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{1}{1.48\text{m}}-\frac{1}{\sqrt{(1.48\text{m})^2+(3.07\text{m})^2}}\right)=1.86\times10^{-10}\text{J}.$$

E28-30 (a) The electric field strength is greatest where the gradient of V is greatest. That is between d and e.

(b) The least absolute value occurs where the gradient is zero, which is between b and c and again between e and f.

E28-31 The potential on the positive plate is $2(5.52\,\text{V}) = 11.0\,\text{V}$; the electric field between the plates is $E = (11.0\,\text{V})/(1.48\times10^{-2}\text{m}) = 743\,\text{V/m}$.

E28-32 Take the derivative: $E = -\partial V/\partial z$.

E28-33 The radial potential gradient is just the magnitude of the radial component of the electric field,

$$E_r = -\frac{\partial V}{\partial r}$$

Then

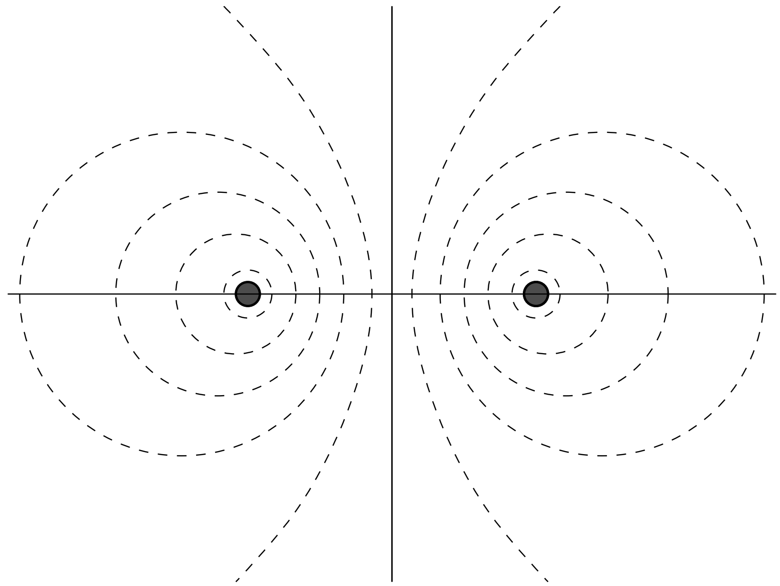
$$\begin{aligned}\frac{\partial V}{\partial r} &= -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2},\\ &= \frac{1}{4\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\frac{79(1.60\times10^{-19}\text{C})}{(7.0\times10^{-15}\text{m})^2},\\ &= -2.32\times10^{21}\,\text{V/m}.\end{aligned}$$

E28-34 Evaluate $\partial V/\partial r$, and

$$E = -\frac{Ze}{4\pi\epsilon_0}\left(\frac{-1}{r^2}+2\frac{r}{2R^3}\right).$$

E28-35 $E_x = -\partial V/\partial x = -2(1530\,\text{V/m}^2)x$. At the point in question, $E = -2(1530\,\text{V/m}^2)(1.28\times10^{-2}\text{m}) = 39.2\,\text{V/m}$.

E28-36 Draw the wires so that they are perpendicular to the plane of the page; they will then "come out of" the page. The equipotential surfaces are then lines where they intersect the page, and they look like



$\boxed{\textbf{E28-37}}$ (a) $|V_B - V_A| = |W/q| = |(3.94 \times 10^{-19}\,\mathrm{J})/(1.60 \times 10^{-19}\,\mathrm{C})| = 2.46\,\mathrm{V}$. The electric field did work on the electron, so the electron was moving from a region of low potential to a region of high potential; or $V_B > V_A$. Consequently, $V_B - V_A = 2.46\,\mathrm{V}$.

(b) V_C is at the same potential as V_B (both points are on the same equipotential line), so $V_C - V_A = V_B - V_A = 2.46\,\mathrm{V}$.

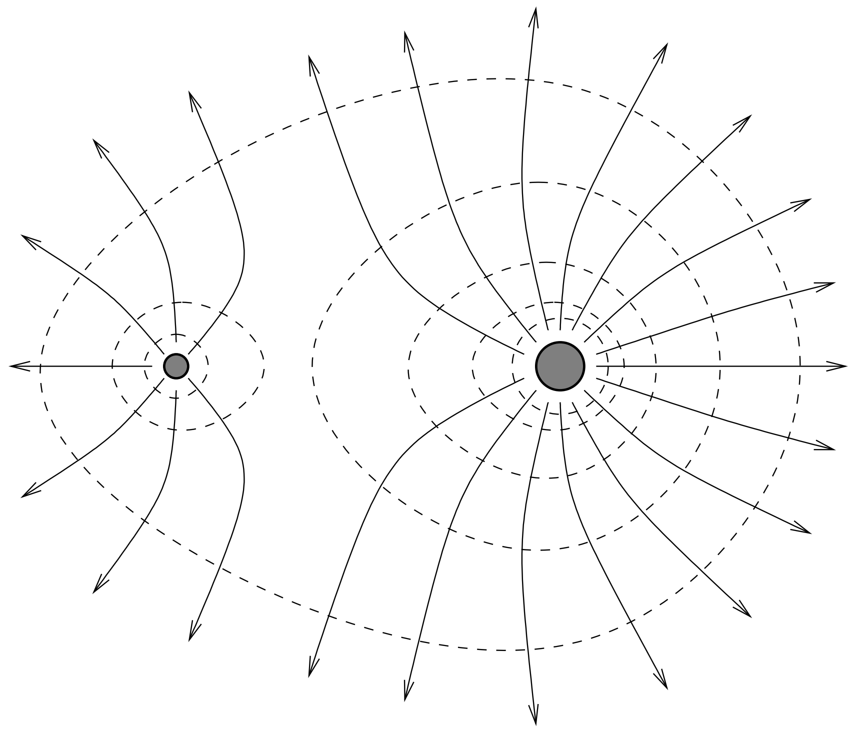
(c) V_C is at the same potential as V_B (both points are on the same equipotential line), so $V_C - V_B = 0\,\mathrm{V}$.

E28-38 (a) For point charges $r = q/4\pi\epsilon_0 V$, so

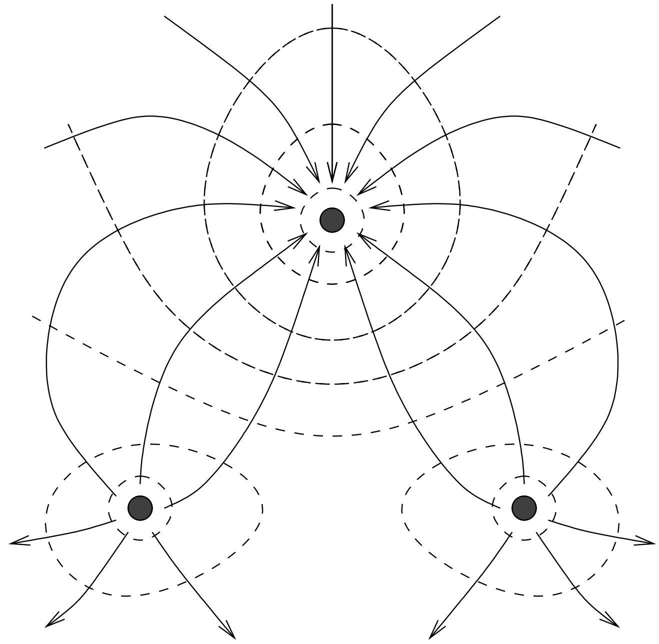
$$r = (8.99\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(1.5\times10^{-8}\mathrm{C})/(30\,\mathrm{V}) = 4.5\,\mathrm{m}$$

(b) No, since $V \propto 1/r$.

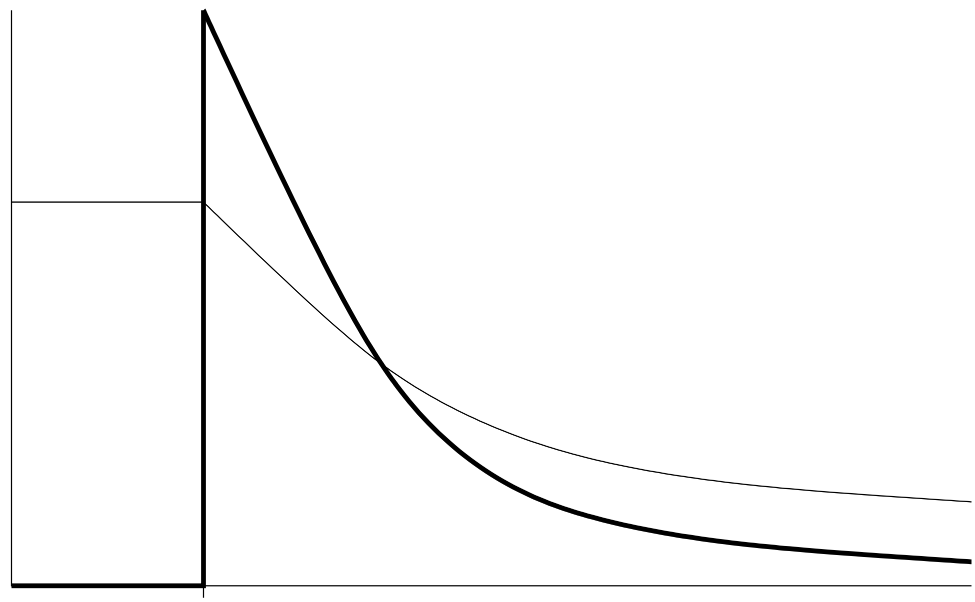
E28-39 The dotted lines are equipotential lines, the solid arrows are electric field lines. Note that there are twice as many electric field lines from the larger charge!



E28-40 The dotted lines are equipotential lines, the solid arrows are electric field lines.



E28-41 This can easily be done with a spreadsheet. The following is a *sketch*; the electric field is the bold curve, the potential is the thin curve.



E28-42 Originally $V = q/4\pi\epsilon_0 r$, where r is the radius of the smaller sphere.

(a) Connecting the spheres will bring them to the same potential, or $V_1 = V_2$.

(b) $q_1 + q_2 = q$; $V_1 = q_1/4\pi\epsilon_0 r$ and $V_2 = q_2/4\pi\epsilon_0 2r$; combining all of the above $q_2 = 2q_1$ and $q_1 = q/3$ and $q_2 = 2q/3$.

E28-43 (a) $q = 4\pi R^2\sigma$, so $V = q/4\pi\epsilon_0 R = \sigma R/\epsilon_0$, or

$$V = (-1.60\times10^{-19}\text{C/m}^2)(6.37\times10^6\text{m})/(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2) = 0.115\,\text{V}$$

(b) Pretend the Earth is a conductor, then $E = \sigma/epsilon_0$, so

$$E = (-1.60\times10^{-19}\text{C/m}^2)/(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2) = 1.81\times10^{-8}\text{V/m}.$$

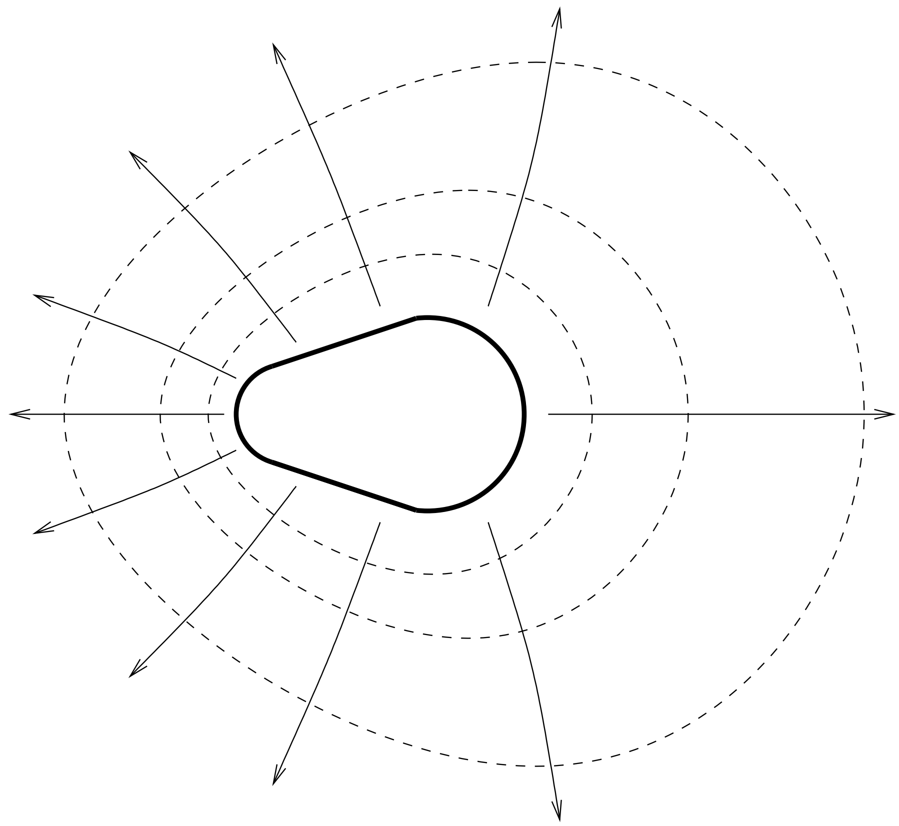
E28-44 $V = q/4\pi\epsilon_0 R$, so

$$V = (8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(15\times10^{-9}\text{C})/(0.16\,\text{m}) = 850\,\text{V}.$$

E28-45 (a) $q = 4\pi\epsilon_0 RV = 4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.152\,\text{m})(215\,\text{V}) = 3.63\times10^{-9}\text{C}$

(b) $\sigma = q/4\pi R^2 = (3.63\times10^{-9}\text{C})/4\pi(0.152\,\text{m})^2 = 1.25\times10^{-8}\text{C/m}^2$.

E28-46 The dotted lines are equipotential lines, the solid arrows are electric field lines.



$\boxed{\textbf{E28-47}}$ (a) The total charge ($Q = 57.2$nC) will be divided up between the two spheres so that they are at the same potential. If q_1 is the charge on one sphere, then $q_2 = Q - q_1$ is the charge on the other. Consequently

$$\begin{aligned}
V_1 &= V_2, \\
\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} &= \frac{1}{4\pi\epsilon_0}\frac{Q-q_1}{r_2}, \\
q_1 r_2 &= (Q-q_1)r_1, \\
q_1 &= \frac{Qr_2}{r_2+r_1}.
\end{aligned}$$

Putting in the numbers, we find

$$q_1 = \frac{Qr_1}{r_2+r_1} = \frac{(57.2\text{ nC})(12.2\text{ cm})}{(5.88\text{ cm}) + (12.2\text{ cm})} = 38.6\text{ nC},$$

and $q_2 = Q - q_1 = (57.2\text{ nC}) - (38.6\text{ nC}) = 18.6\text{ nC}$.

(b) The potential on each sphere should be the same, so we only need to solve one. Then

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} = \frac{1}{4\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\frac{(38.6\text{ nC})}{(12.2\text{ cm})} = 2850\,\text{V}.$$

E28-48 (a) $V = (8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(31.5\times10^{-9}\text{C})/(0.162\,\text{m}) = 1.75\times10^3\text{V}$.

(b) $V = q/4\pi\epsilon_0 r$, so $r = q/4\pi\epsilon_0 V$, and then

$$r = (8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(31.5\times10^{-9}\text{C})/(1.20\times10^3\text{V}) = 0.236\,\text{m}.$$

That is $(0.236\,\text{m}) - (0.162\,\text{m}) = 0.074\,\text{m}$ above the surface.

$\boxed{\textbf{E28-49}}$ (a) Apply the point charge formula, but solve for the charge. Then

$$\begin{aligned}
\frac{1}{4\pi\epsilon_0}\frac{q}{r} &= V, \\
q &= 4\pi\epsilon_0 rV, \\
q &= 4\pi(8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(1\,\mathrm{m})(10^6\,\mathrm{V}) = 0.11\,\mathrm{mC}.
\end{aligned}$$

Now that's a fairly small charge. But if the radius were decreased by a factor of 100, so would the charge ($1.10\,\mu$C). Consequently, smaller metal balls can be raised to higher potentials with less charge.

(b) The electric field near the surface of the ball is a function of the surface charge density, $E = \sigma/\epsilon_0$. But surface charge density depends on the area, and varies as r^{-2}. For a given potential, the electric field near the surface would then be given by

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{4\pi\epsilon_0 r^2} = \frac{V}{r}.$$

Note that the electric field grows as the ball gets smaller. This means that the break down field is more likely to be exceeded with a low voltage small ball; you'll get sparking.

E28-50 A "Volt" is a Joule per Coulomb. The power required by the drive belt is the product $(3.41\times 10^6\mathrm{V})(2.83\times 10^{-3}\mathrm{C/s}) = 9650\,\mathrm{W}$.

$\boxed{\textbf{P28-1}}$ (a) According to Newtonian mechanics we want $K = \frac{1}{2}mv^2$ to be equal to $W = q\Delta V$ which means

$$\Delta V = \frac{mv^2}{2q} = \frac{(0.511\text{ MeV})}{2e} = 256\text{ kV}.$$

mc^2 is the rest mass energy of an electron.

(b) Let's do some rearranging first.

$$\begin{aligned}
K &= mc^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right], \\
\frac{K}{mc^2} &= \frac{1}{\sqrt{1-\beta^2}} - 1, \\
\frac{K}{mc^2} + 1 &= \frac{1}{\sqrt{1-\beta^2}}, \\
\frac{1}{\frac{K}{mc^2}+1} &= \sqrt{1-\beta^2}, \\
\frac{1}{\left(\frac{K}{mc^2}+1\right)^2} &= 1-\beta^2,
\end{aligned}$$

and finally,

$$\beta = \sqrt{1 - \frac{1}{\left(\frac{K}{mc^2}+1\right)^2}}$$

Putting in the numbers,

$$\sqrt{1 - \frac{1}{\left(\frac{(256\text{ keV})}{(511\text{ keV})}+1\right)^2}} = 0.746,$$

so $v = 0.746c$.

P28-2 (a) The potential of the hollow sphere is $V = q/4\pi\epsilon_0 r$. The work required to increase the charge by an amount dq is $dW = V/, dq$. Integrating,

$$W = \int_0^e \frac{q}{4\pi\epsilon_0 r} dq = \frac{e^2}{8\pi\epsilon_0 r}.$$

This corresponds to an electric potential energy of

$$W = \frac{e(1.60\times10^{-19}\mathrm{C})}{8\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(2.82\times10^{-15}\mathrm{m})} = 2.55\times10^5 \text{ eV} = 4.08\times10^{-14}\mathrm{J}.$$

(b) This would be a mass of $m = (4.08\times10^{-14}\mathrm{J})/(3.00\times10^8\mathrm{m/s})^2 = 4.53\times10^{-31}\mathrm{kg}$.

P28-3 The negative charge is held in orbit by electrostatic attraction, or

$$\frac{mv^2}{r} = \frac{qQ}{4\pi\epsilon_0 r^2}.$$

The kinetic energy of the charge is

$$K = \frac{1}{2}mv^2 = \frac{qQ}{8\pi\epsilon_0 r}.$$

The electrostatic potential energy is

$$U = -\frac{qQ}{4\pi\epsilon_0 r},$$

so the total energy is

$$E = -\frac{qQ}{8\pi\epsilon_0 r}.$$

The work required to change orbit is then

$$W = \frac{qQ}{8\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

P28-4 (a) $V = -\int E\, dr$, so

$$V = -\int_0^r \frac{qr}{4\pi\epsilon_0 R^3} dr = -\frac{qr^2}{8\pi\epsilon_0 R^3}.$$

(b) $\Delta V = q/8\pi\epsilon_0 R$.

(c) If instead of $V = 0$ at $r = 0$ as was done in part (a) we take $V = 0$ at $r = \infty$, then $V = q/4\pi\epsilon_0 R$ on the surface of the sphere. The new expression for the potential inside the sphere will look like $V = V' + V_s$, where V' is the answer from part (a) and V_s is a constant so that the surface potential is correct. Then

$$V_s = \frac{q}{4\pi\epsilon_0 R} + \frac{qR^2}{8\pi\epsilon_0 R^3} = \frac{3qR^2}{8\pi\epsilon_0 R^3},$$

and then

$$V = -\frac{qr^2}{8\pi\epsilon_0 R^3} + \frac{3qR^2}{8\pi\epsilon_0 R^3} = \frac{q(3R^2 - r^2)}{8\pi\epsilon_0 R^3}.$$

P28-5 The total electric potential energy of the system is the sum of the three interaction pairs. One of these pairs does not change during the process, so it can be ignored when finding the change in potential energy. The change in electrical potential energy is then

$$\Delta U = 2\frac{q^2}{4\pi\epsilon_0 r_\mathrm{f}} - 2\frac{q^2}{4\pi\epsilon_0 r_\mathrm{i}} = \frac{q^2}{2\pi\epsilon_0}\left(\frac{1}{r_\mathrm{f}} - \frac{1}{r_\mathrm{i}}\right).$$

In this case $r_\mathrm{i} = 1.72\,\mathrm{m}$, while $r_\mathrm{f} = 0.86\,\mathrm{m}$. The change in potential energy is then

$$\Delta U = 2(8.99\times 10^9 \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(0.122\,\mathrm{C})^2\left(\frac{1}{(0.86\,\mathrm{m})} - \frac{1}{(1.72\,\mathrm{m})}\right) = 1.56\times 10^8\mathrm{J}$$

The time required is

$$t = (1.56\times 10^8)/(831\,\mathrm{W}) = 1.87\times 10^5\mathrm{s} = 2.17\text{ days}.$$

P28-6 (a) Apply conservation of energy:

$$K = \frac{qQ}{4\pi\epsilon_0 d},\text{ or } d = \frac{qQ}{4\pi\epsilon_0 K},$$

where d is the distance of closest approach.

(b) Apply conservation of energy:

$$K = \frac{qQ}{4\pi\epsilon_0(2d)} + \frac{1}{2}mv^2,$$

so, combining with the results in part (a), $v = \sqrt{K/m}$.

P28-7 (a) First apply Eq. 28-18, but solve for r. Then

$$r = \frac{q}{4\pi\epsilon_0 V} = \frac{(32.0\times 10^{-12}\,\mathrm{C})}{4\pi(8.85\times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)(512\,\mathrm{V})} = 562\,\mu\mathrm{m}.$$

(b) If two such drops join together the charge doubles, and the volume of water doubles, but the radius of the new drop only increases by a factor of $\sqrt[3]{2} = 1.26$ because volume is proportional to the radius cubed.

The potential on the surface of the new drop will be

$$\begin{aligned} V_\mathrm{new} &= \frac{1}{4\pi\epsilon_0}\frac{q_\mathrm{new}}{r_\mathrm{new}}, \\ &= \frac{1}{4\pi\epsilon_0}\frac{2q_\mathrm{old}}{\sqrt[3]{2}\,r_\mathrm{old}}, \\ &= (2)^{2/3}\frac{1}{4\pi\epsilon_0}\frac{q_\mathrm{old}}{r_\mathrm{old}} = (2)^{2/3}V_\mathrm{old}. \end{aligned}$$

The new potential is 813 V.

P28-8 (a) The work done is $W = -Fz = -Eqz = -q\sigma z/2\epsilon_0$.

(b) Since $W = q\Delta V$, $\Delta V = -\sigma z/2\epsilon_0$, so

$$V = V_0 - (\sigma/2\epsilon_0)z.$$

P28-9 (a) The potential at any point will be the sum of the contribution from each charge,

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2},$$

where r_1 is the distance the point in question from q_1 and r_2 is the distance the point in question from q_2. Pick a point, call it (x, y). Since q_1 is at the origin,

$$r_1 = \sqrt{x^2 + y^2}.$$

Since q_2 is at $(d, 0)$, where $d = 9.60$ nm,

$$r_2 = \sqrt{(x-d)^2 + y^2}.$$

Define the "Stanley Number" as $S = 4\pi\epsilon_0 V$. Equipotential surfaces are also equi-Stanley surfaces. In particular, when $V = 0$, so does S. We can then write the potential expression in a sightly simplified form

$$S = \frac{q_1}{r_1} + \frac{q_2}{r_2}.$$

If $S = 0$ we can rearrange and square this expression.

$$\begin{aligned} \frac{q_1}{r_1} &= -\frac{q_2}{r_2}, \\ \frac{r_1^2}{q_1^2} &= \frac{r_2^2}{q_2^2}, \\ \frac{x^2+y^2}{q_1^2} &= \frac{(x-d)^2+y^2}{q_2^2}, \end{aligned}$$

Let $\alpha = q_2/q_1$, then we can write

$$\begin{aligned} \alpha^2\left(x^2+y^2\right) &= (x-d)^2+y^2, \\ \alpha^2x^2+\alpha^2y^2 &= x^2-2xd+d^2+y^2, \\ (\alpha^2-1)x^2+2xd+(\alpha^2-1)y^2 &= d^2. \end{aligned}$$

We complete the square for the $(\alpha^2-1)x^2+2xd$ term by adding $d^2/(\alpha^2-1)$ to both sides of the equation. Then

$$(\alpha^2-1)\left[\left(x+\frac{d}{\alpha^2-1}\right)^2+y^2\right] = d^2\left(1+\frac{1}{\alpha^2-1}\right).$$

The center of the circle is at

$$-\frac{d}{\alpha^2-1} = \frac{(9.60 \text{ nm})}{(-10/6)^2-1} = -5.4 \text{ nm}.$$

(b) The radius of the circle is

$$\sqrt{d^2\frac{\left(1+\frac{1}{\alpha^2-1}\right)}{\alpha^2-1}},$$

which can be simplified to

$$d\frac{\alpha}{\alpha^2-1} = (9.6 \text{ nm})\frac{|(-10/6)|}{(-10/6)^2-1} = 9.00 \text{ nm}.$$

(c) No.

P28-10 An annulus is composed of differential rings of varying radii r and width dr; the charge on any ring is the product of the area of the ring, $dA = 2\pi r\, dr$, and the surface charge density, or

$$dq = \sigma\, dA = \frac{k}{r^3} 2\pi r\, dr = \frac{2\pi k}{r^2} dr.$$

The potential at the center can be found by adding up the contributions from each ring. Since we are at the center, the contributions will each be $dV = dq/4\pi\epsilon_0 r$. Then

$$V = \int_a^b \frac{k}{2\epsilon_0} \frac{dr}{r^3} = \frac{k}{4\epsilon_0}\left(\frac{1}{a^2} - \frac{1}{b^2}\right). = \frac{k}{4\epsilon_0}\frac{b^2 - a^2}{b^2 a^2}.$$

The total charge on the annulus is

$$Q = \int_a^b \frac{2\pi k}{r^2} dr = 2\pi k \left(\frac{1}{a} - \frac{1}{b}\right) = 2\pi k \frac{b-a}{ba}.$$

Combining,

$$V = \frac{Q}{8\pi\epsilon_0}\frac{a+b}{ab}.$$

P28-11 Add the three contributions, and then do a series expansion for $d \ll r$.

$$\begin{aligned}
V &= \frac{q}{4\pi\epsilon_0}\left(\frac{-1}{r+d} + \frac{1}{r} + \frac{1}{r-d}\right), \\
&= \frac{q}{4\pi\epsilon_0 r}\left(\frac{-1}{1+d/r} + 1 + \frac{1}{1-d/r}\right), \\
&\approx \frac{q}{4\pi\epsilon_0 r}\left(-1 + \frac{d}{r} + 1 + 1 + \frac{d}{r}\right), \\
&\approx \frac{q}{4\pi\epsilon_0 r}\left(1 + \frac{2d}{r}\right).
\end{aligned}$$

P28-12 (a) Add the contributions from each differential charge: $dq = \lambda\, dy$. Then

$$V = \int_y^{y+L} \frac{\lambda}{4\pi\epsilon_0 y} dy = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{y+L}{y}\right).$$

(b) Take the derivative:

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\lambda}{4\pi\epsilon_0}\frac{y}{y+L}\frac{-L}{y^2} = \frac{\lambda}{4\pi\epsilon_0}\frac{L}{y(y+L)}.$$

(c) By symmetry it must be zero, since the system is invariant under rotations about the axis of the rod. Note that we can't determine $E_\perp$ from derivatives because we don't have the functional form of V for points off-axis!

$\boxed{\textbf{P28-13}}$ (a) We follow the work done in Section 28-6 for a uniform line of charge, starting with Eq. 28-26,

$$\begin{aligned}
dV &= \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{\sqrt{x^2+y^2}}, \\
dV &= \frac{1}{4\pi\epsilon_0}\int_0^L \frac{kx\, dx}{\sqrt{x^2+y^2}},
\end{aligned}$$

$$= \frac{k}{4\pi\epsilon_0} \left. \sqrt{x^2+y^2} \right|_0^L ,$$
$$= \frac{k}{4\pi\epsilon_0} \left(\sqrt{L^2+y^2} - y \right).$$

(b) The y component of the electric field can be found from

$$E_y = -\frac{\partial V}{\partial y},$$

which (using a computer-aided math program) is

$$E_y = \frac{k}{4\pi\epsilon_0} \left(1 - \frac{y}{\sqrt{L^2+y^2}} \right).$$

(c) We could find E_x if we knew the x variation of V. But we don't; we only found the values of V along a fixed value of x.

(d) We want to find y such that the ratio

$$\left[\frac{k}{4\pi\epsilon_0} \left(\sqrt{L^2+y^2} - y \right) \right] / \left[\frac{k}{4\pi\epsilon_0} (L) \right]$$

is one-half. Simplifying, $\sqrt{L^2+y^2} - y = L/2$, which can be written as

$$L^2 + y^2 = L^2/4 + Ly + y^2,$$

or $3L^2/4 = Ly$, with solution $y = 3L/4$.

P28-14 The spheres are small compared to the separation distance. Assuming only *one* sphere at a potential of 1500 V, the charge would be

$$q = 4\pi\epsilon_0 rV = 4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m})(0.150\,\mathrm{m})(1500\,\mathrm{V}) = 2.50\times10^{-8}\mathrm{C}.$$

The potential from the sphere at a distance of 10.0 m would be

$$V = (1500\,\mathrm{V})\frac{(0.150\,\mathrm{m})}{(10.0\,\mathrm{m})} = 22.5\,\mathrm{V}.$$

This is small compared to 1500 V, so we will treat it as a perturbation. This means that we can assume that the spheres have charges of

$$q = 4\pi\epsilon_0 rV = 4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m})(0.150\,\mathrm{m})(1500\,\mathrm{V} + 22.5\,\mathrm{V}) = 2.54\times10^{-8}\mathrm{C}.$$

$\boxed{\textbf{P28-15}}$ Calculating the fraction of excess electrons is the same as calculating the fraction of excess charge, so we'll skip counting the electrons. This problem is effectively the same as Exercise 28-47; we have a total charge that is divided between two unequal size spheres which are at the same potential on the surface. Using the result from that exercise we have

$$q_1 = \frac{Qr_1}{r_2 + r_1},$$

where $Q = -6.2$ nC is the total charge available, and q_1 is the charge left on the sphere. r_1 is the radius of the small ball, r_2 is the radius of Earth. Since the fraction of charge remaining is q_1/Q, we can write

$$\frac{q_1}{Q} = \frac{r_1}{r_2 + r_1} \approx \frac{r_1}{r_2} = 2.0 \times 10^{-8}.$$

P28-16 The positive charge on the sphere would be

$$q = 4\pi\epsilon_0 rV = 4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(1.08\times10^{-2}\mathrm{m})(1000\,\mathrm{V}) = 1.20\times10^{-9}\mathrm{C}.$$

The number of decays required to build up this charge is

$$n = 2(1.20\times10^{-9}\mathrm{C})/(1.60\times10^{-19}\mathrm{C}) = 1.50\times10^{10}.$$

The extra factor of two is because only half of the decays result in an increase in charge. The time required is

$$t = (1.50\times10^{10})/(3.70\times10^{8}\mathrm{s^{-1}}) = 40.6\,\mathrm{s}.$$

P28-17 (a) None.
(b) None.
(c) None.
(d) None.
(e) No.

P28-18 (a) Outside of an isolated charged spherical object $E = q/4\pi\epsilon_0 r^2$ and $V = q/4\pi\epsilon_0 r$. Then $E = V/r$. Consequently, the sphere must have a radius larger than $r = (9.15\times10^6\mathrm{V})/(100\times10^6\mathrm{V/m}) = 9.15\times10^{-2}\mathrm{m}$.

(b) The power required is $(320\times10^{-6}\mathrm{C/s})(9.15\times10^6\mathrm{V}) = 2930\,\mathrm{W}$.

(c) $\sigma wv = (320\times10^{-6}\mathrm{C/s})$, so

$$\sigma = \frac{(320\times10^{-6}\mathrm{C/s})}{(0.485\,\mathrm{m})(33.0\,\mathrm{m/s})} = 2.00\times10^{-5}\mathrm{C/m^2}.$$

$\boxed{\textbf{E29-1}}$ (a) The charge which flows through a cross sectional surface area in a time t is given by $q = it$, where i is the current. For this exercise we have

$$q = (4.82\,\mathrm{A})(4.60 \times 60\,\mathrm{s}) = 1330\,\mathrm{C}$$

as the charge which passes through a cross section of this resistor.

(b) The number of electrons is given by $(1330\,\mathrm{C})/(1.60 \times 10^{-19}\,\mathrm{C}) = 8.31 \times 10^{21}$ electrons.

E29-2 $Q/t = (200\times10^{-6}\mathrm{A/s})(60\mathrm{s/min})/(1.60\times10^{-19}\mathrm{C}) = 7.5\times10^{16}$ electrons per minute.

E29-3 (a) $j = nqv = (2.10\times10^{14}/\mathrm{m}^3)2(1.60\times10^{-19}\mathrm{C})(1.40\times10^5\mathrm{m/s}) = 9.41\,\mathrm{A/m}^2$. Since the ions have positive charge then the current density is in the same direction as the velocity.

(b) We need an area to calculate the current.

E29-4 (a) $j = i/A = (123\times10^{-12}\mathrm{A})/\pi(1.23\times10^{-3}\mathrm{m})^2 = 2.59\times10^{-5}\mathrm{A/m}^2$.

(b) $v_\mathrm{d} = j/ne = (2.59\times10^{-5}\mathrm{A/m}^2)/(8.49\times10^{28}/\mathrm{m}^3)(1.60\times10^{-19}\mathrm{C}) = 1.91\times10^{-15}\mathrm{m/s}$.

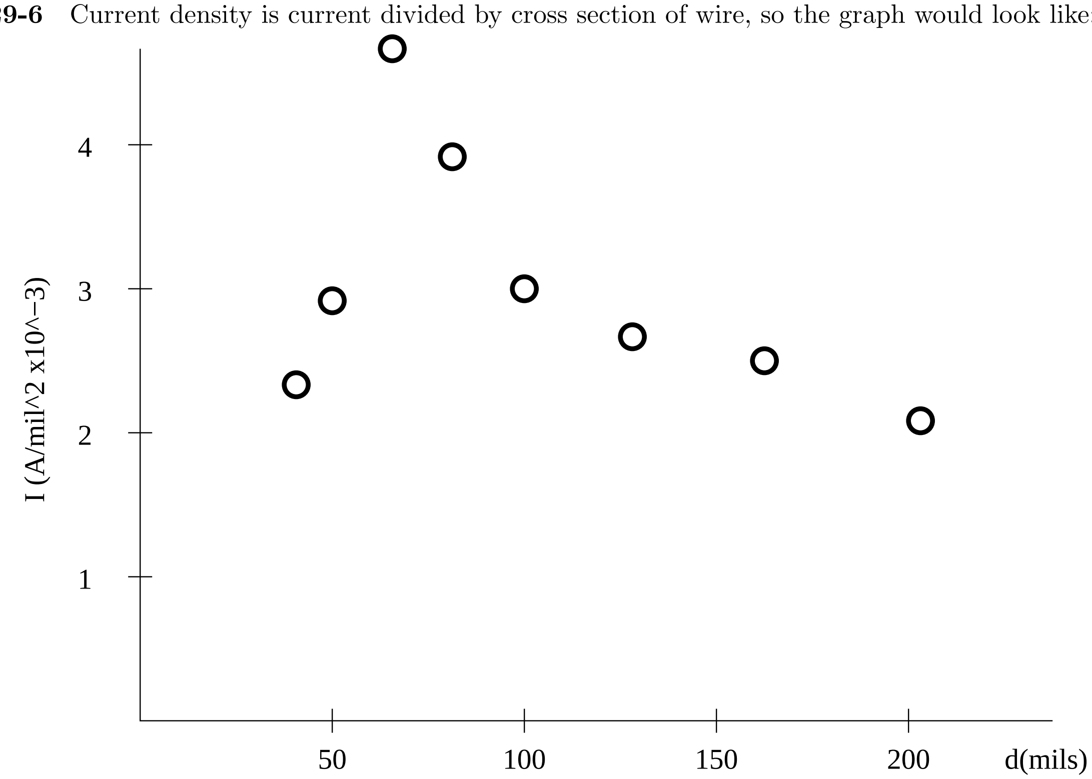
$\boxed{\textbf{E29-5}}$ The current rating of a fuse of cross sectional area A would be

$$i_\mathrm{max} = (440\,\mathrm{A/cm}^2)A,$$

and if the fuse wire is cylindrical $A = \pi d^2/4$. Then

$$d = \sqrt{\frac{4}{\pi}\frac{(0.552\,\mathrm{A})}{(440\,\mathrm{A/m}^2)}} = 4.00\times10^{-2}\ \mathrm{cm}.$$

E29-6 Current density is current divided by cross section of wire, so the graph would look like:



E29-7 The current is in the direction of the motion of the positive charges. The magnitude of the current is

$$i = (3.1\times10^{18}/\text{s} + 1.1\times10^{18}/\text{s})(1.60\times10^{-19}\text{C}) = 0.672\,\text{A}.$$

E29-8 (a) The total current is

$$i = (3.50\times10^{15}/\text{s} + 2.25\times10^{15}/\text{s})(1.60\times10^{-19}\text{C}) = 9.20\times10^{-4}\text{A}.$$

(b) The current density is

$$j = (9.20\times10^{-4}\text{A})/\pi(0.165\times10^{-3}\text{m})^2 = 1.08\times10^4\text{A/m}^2.$$

E29-9 (a) $j = (8.70\times10^6/\text{m}^3)(1.60\times10^{-19}\text{C})(470\times10^3\text{m/s}) = 6.54\times10^{-7}\text{A/m}^2$.
(b) $i = (6.54\times10^{-7}\text{A/m}^2)\pi(6.37\times10^6\text{m})^2 = 8.34\times10^7\text{A}$.

E29-10 $i = \sigma w v$, so

$$\sigma = (95.0\times10^{-6}\text{A})/(0.520\,\text{m})(28.0\,\text{m/s}) = 6.52\times10^{-6}\text{C/m}^2.$$

E29-11 The drift velocity is given by Eq. 29-6,

$$v_\text{d} = \frac{j}{ne} = \frac{i}{Ane} = \frac{(115\,\text{A})}{(31.2\times10^{-6}\text{m}^2)(8.49\times10^{28}/m^3)(1.60\times10^{-19}\text{C})} = 2.71\times10^{-4}\text{m/s}.$$

The time it takes for the electrons to get to the starter motor is

$$t = \frac{x}{v} = \frac{(0.855\,\text{m})}{(2.71\times10^{-4}\text{m/s})} = 3.26\times10^3\text{s}.$$

That's about 54 minutes.

E29-12 $\Delta V = iR = (50\times10^{-3}\text{A})(1800\,\Omega) = 90\,\text{V}$.

E29-13 The resistance of an object with constant cross section is given by Eq. 29-13,

$$R = \rho\frac{L}{A} = (3.0\times10^{-7}\,\Omega\cdot\text{m})\frac{(11,000\,\text{m})}{(0.0056\,\text{m}^2)} = 0.59\,\Omega.$$

E29-14 The slope is approximately $[(8.2 - 1.7)/1000]\mu\Omega\cdot\text{cm}/°\text{C}$, so

$$\alpha = \frac{1}{1.7\mu\Omega\cdot\text{cm}}6.5\times10^{-3}\mu\Omega\cdot\text{cm}/°\text{C} \approx 4\times10^{-3}/\text{C}°$$

E29-15 (a) $i = \Delta V/R = (23\,\text{V})/(15\times10^{-3}\Omega) = 1500\,\text{A}$.
(b) $j = i/A = (1500\,\text{A})/\pi(3.0\times10^{-3}\text{m})^2 = 5.3\times10^7\text{A/m}^2$.
(c) $\rho = RA/L = (15\times10^{-3}\Omega)\pi(3.0\times10^{-3}\text{m})^2/(4.0\,\text{m}) = 1.1\times10^{-7}\Omega\cdot\text{m}$. The material is possibly platinum.

E29-16 Use the equation from Exercise 29-17. $\Delta R = 8\,\Omega$; then

$$\Delta T = (8\,\Omega)/(50\,\Omega)(4.3\times10^{-3}/\text{C}°) = 37\,\text{C}°.$$

The final temperature is then 57°C.

$\boxed{\textbf{E29-17}}$ Start with Eq. 29-16,

$$\rho - \rho_0 = \rho_0 \alpha_{\rm av}(T - T_0),$$

and multiply through by L/A,

$$\frac{L}{A}(\rho - \rho_0) = \frac{L}{A}\rho_0 \alpha_{\rm av}(T - T_0),$$

to get

$$R - R_0 = R_0 \alpha_{\rm av}(T - T_0).$$

E29-18 The wire has a length $L = (250)2\pi(0.122\,\text{m}) = 192\,\text{m}$. The diameter is 0.129 inches; the cross sectional area is then

$$A = \pi(0.129 \times 0.0254\,\text{m})^2/4 = 8.43\times10^{-6}\text{m}^2.$$

The resistance is

$$R = \rho L/A = (1.69\times10^{-8}\Omega\cdot\text{m})(192\,\text{m})/(8.43\times10^{-6}\text{m}^2) = 0.385\,\Omega.$$

$\boxed{\textbf{E29-19}}$ If the length of each conductor is L and has resistivity ρ, then

$$R_A = \rho\frac{L}{\pi D^2/4} = \rho\frac{4L}{\pi D^2}$$

and

$$R_B = \rho\frac{L}{(\pi 4D^2/4 - \pi D^2/4)} = \rho\frac{4L}{3\pi D^2}.$$

The ratio of the resistances is then

$$\frac{R_A}{R_B} = 3.$$

E29-20 $R = R$, so $\rho_1 L_1/\pi(d_1/2)^2 = \rho_2 L_2/\pi(d_2/2)^2$. Simplifying, $\rho_1/d_1^2 = \rho_2/d_2^2$. Then

$$d_2 = (1.19\times10^{-3}\text{m})\sqrt{(9.68\times10^{-8}\Omega\cdot\text{m})/(1.69\times10^{-8}\Omega\cdot\text{m})} = 2.85\times10^{-3}\text{m}.$$

E29-21 (a) $(750\times10^{-3}\text{A})/(125) = 6.00\times10^{-3}\text{A}$.
(b) $\Delta V = iR = (6.00\times10^{-3}\text{A})(2.65\times10^{-6}\Omega) = 1.59\times10^{-8}\text{V}$.
(c) $R = \Delta V/i = (1.59\times10^{-8}\text{V})/(750\times10^{-3}\text{A}) = 2.12\times10^{-8}\Omega$.

E29-22 Since $\Delta V = iR$, then if ΔV and i are the same, then R must be the same.
(a) Since $R = R$, $\rho_1 L_1/\pi r_1^2 = \rho_2 L_2/\pi r_2^2$, or $\rho_1/r_1^2 = \rho_2/r_2^2$. Then

$$r_{\rm iron}/r_{\rm copper} = \sqrt{(9.68\times10^{-8}\Omega\cdot\text{m})(1.69\times10^{-8}\Omega\cdot\text{m})} = 2.39.$$

(b) Start with the definition of current density:

$$j = \frac{i}{A} = \frac{\Delta V}{RA} = \frac{\Delta V}{\rho L}.$$

Since ΔV and L is the same, but ρ is different, then the current densities *will* be different.

$\boxed{\textbf{E29-23}}$ Conductivity is given by Eq. 29-8, $\vec{\mathbf{j}} = \sigma\vec{\mathbf{E}}$. If the wire is long and thin, then the magnitude of the electric field in the wire will be given by

$$E \approx \Delta V/L = (115\,\text{V})/(9.66\,\text{m}) = 11.9\,\text{V/m}.$$

We can now find the conductivity,

$$\sigma = \frac{j}{E} = \frac{(1.42\times 10^4\text{A/m}^2)}{(11.9\,\text{V/m})} = 1.19\times 10^3(\Omega\cdot\text{m})^{-1}.$$

E29-24 (a) $v_\text{d} = j/en = \sigma E/en$. Then

$$v_\text{d} = (2.70\times10^{-14}/\Omega\cdot\text{m})(120\,\text{V/m})/(1.60\times10^{-19}\text{C})(620\times10^6/\text{m}^3 + 550\times10^6/\text{m}^3) = 1.73\times10^{-2}\text{m/s}.$$

(b) $j = \sigma E = (2.70\times10^{-14}/\Omega\cdot\text{m})(120\,\text{V/m}) = 3.24\times10^{-14}\text{A/m}^2$.

E29-25 (a) $R/L = \rho/A$, so $j = i/A = (R/L)i/\rho$. For copper,

$$j = (0.152\times10^{-3}\Omega/\text{m})(62.3\,\text{A})/(1.69\times10^{-8}\Omega\cdot\text{m}) = 5.60\times10^5\text{A/m}^2;$$

for aluminum,

$$j = (0.152\times10^{-3}\Omega/\text{m})(62.3\,\text{A})/(2.75\times10^{-8}\Omega\cdot\text{m}) = 3.44\times10^5\text{A/m}^2.$$

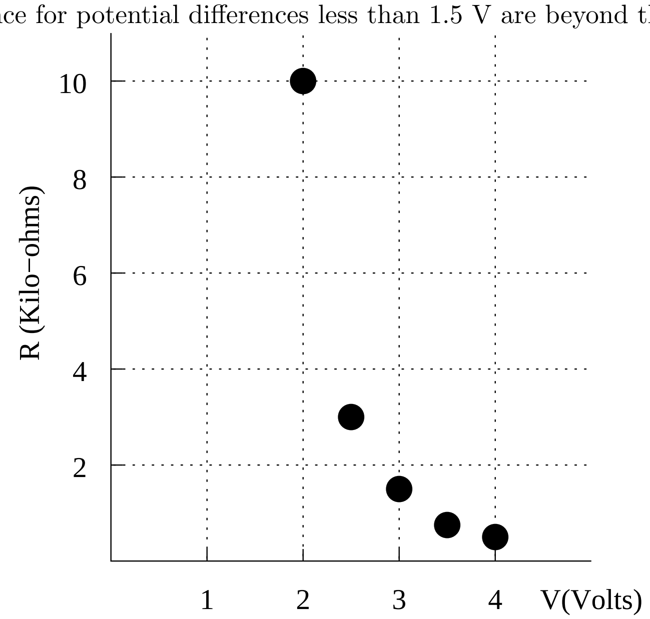
(b) $A = \rho L/R$; if δ is density, then $m = \delta l A = l\delta\rho/(R/L)$. For copper,

$$m = (1.0\,\text{m})(8960\,\text{kg/m}^3)(1.69\times10^{-8}\Omega\cdot\text{m})/(0.152\times10^{-3}\Omega/\text{m}) = 0.996\,\text{kg};$$

for aluminum,

$$m = (1.0\,\text{m})(2700\,\text{kg/m}^3)(2.75\times10^{-8}\Omega\cdot\text{m})/(0.152\times10^{-3}\Omega/\text{m}) = 0.488\,\text{kg}.$$

E29-26 The resistance for potential differences less than 1.5 V are beyond the scale.



$\boxed{\textbf{E29-27}}$ (a) The resistance is defined as

$$R = \frac{\Delta V}{i} = \frac{(3.55\times 10^6\,\mathrm{V/A^2})i^2}{i} = (3.55\times 10^6\,\mathrm{V/A^2})i.$$

When $i = 2.40$ mA the resistance would be

$$R = (3.55\times 10^6\,\mathrm{V/A^2})(2.40\times 10^{-3}\mathrm{A}) = 8.52\,\mathrm{k\Omega}.$$

(b) Invert the above expression, and

$$i = R/(3.55\times 10^6\,\mathrm{V/A^2}) = (16.0\,\Omega)/(3.55\times 10^6\,\mathrm{V/A^2}) = 4.51\,\mu\mathrm{A}.$$

E29-28 First, $n = 3(6.02\times10^{23})(2700\,\mathrm{kg/m^3})(27.0\times10^{-3}\mathrm{kg}) = 1.81\times10^{29}/\mathrm{m^3}$. Then

$$\tau = \frac{m}{ne^2\rho} = \frac{(9.11\times10^{-31}\mathrm{kg})}{(1.81\times10^{29}/\mathrm{m^3})(1.60\times10^{-19}\mathrm{C})^2(2.75\times10^{-8}\Omega\cdot\mathrm{m})} = 7.15\times10^{-15}\mathrm{s}.$$

E29-29 (a) $E = E_0/\kappa_\mathrm{e} = q/4\pi\epsilon_0\kappa_\mathrm{e}R^2$, so

$$E = \frac{(1.00\times10^{-6}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(4.7)(0.10\,\mathrm{m})^2} =$$

(b) $E = E_0 = q/4\pi\epsilon_0 R^2$, so

$$E = \frac{(1.00\times10^{-6}\mathrm{C})}{4\pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(0.10\,\mathrm{m})^2} =$$

(c) $\sigma_\mathrm{ind} = \epsilon_0(E_0 - E) = q(1 - 1/\kappa_\mathrm{e})/4\pi R^2$. Then

$$\sigma_\mathrm{ind} = \frac{(1.00\times10^{-6}\mathrm{C})}{4\pi(0.10\,\mathrm{m})^2}\left(1 - \frac{1}{(4.7)}\right) = 6.23\times10^{-6}\mathrm{C/m^2}.$$

E29-30 Midway between the charges $E = q/\pi\epsilon_0 d$, so

$$q = \pi(8.85\times10^{-12}\mathrm{C^2/N\cdot m^2})(0.10\,\mathrm{m})(3\times10^6\mathrm{V/m}) = 8.3\times10^{-6}\mathrm{C}.$$

$\boxed{\textbf{E29-31}}$ (a) At the surface of a conductor of radius R with charge Q the magnitude of the electric field is given by

$$E = \frac{1}{4\pi\epsilon_0}QR^2,$$

while the potential (assuming $V = 0$ at infinity) is given by

$$V = \frac{1}{4\pi\epsilon_0}QR.$$

The ratio is $V/E = R$.

The potential on the sphere that would result in "sparking" is

$$V = ER = (3\times10^6\mathrm{N/C})R.$$

(b) It is "easier" to get a spark off of a sphere with a smaller radius, because any potential on the sphere will result in a larger electric field.

(c) The points of a lighting rod are like small hemispheres; the electric field will be large near these points so that this will be the likely place for sparks to form and lightning bolts to strike.

$\boxed{\textbf{P29-1}}$ If there is more current flowing into the sphere than is flowing out then there must be a change in the net charge on the sphere. The net current is the difference, or $2\,\mu$A. The potential on the surface of the sphere will be given by the point-charge expression,

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r},$$

and the charge will be related to the current by $q = it$. Combining,

$$V = \frac{1}{4\pi\epsilon_0}\frac{it}{r},$$

or

$$t = \frac{4\pi\epsilon_0 Vr}{i} = \frac{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(980\,\mathrm{V})(0.13\,\mathrm{m})}{(2\,\mu\mathrm{A})} = 7.1\,\mathrm{ms}.$$

P29-2 The net current density is in the direction of the positive charges, which is to the east. There are two electrons for every alpha particle, and each alpha particle has a charge equal in magnitude to two electrons. The current density is then

$$\begin{aligned} j &= q_\mathrm{e}n_\mathrm{e}v_\mathrm{e} + q_\alpha + n_\alpha v_\alpha, \\ &= (-1.6\times10^{-19}\mathrm{C})(5.6\times10^{21}/\mathrm{m}^3)(-88\,\mathrm{m/s}) + (3.2\times10^{-19}\mathrm{C})(2.8\times10^{21}/\mathrm{m}^3)(25\,\mathrm{m/s}), \\ &= 1.0\times10^5\mathrm{C/m^2}. \end{aligned}$$

P29-3 (a) The resistance of the segment of the wire is

$$R = \rho L/A = (1.69\times10^{-8}\Omega\cdot\mathrm{m})(4.0\times10^{-2}\mathrm{m})/\pi(2.6\times10^{-3}\mathrm{m})^2 = 3.18\times10^{-5}\Omega.$$

The potential difference across the segment is

$$\Delta V = iR = (12\,\mathrm{A})(3.18\times10^{-5}\Omega) = 3.8\times10^{-4}\mathrm{V}.$$

(b) The tail is negative.
(c) The drift speed is $v = j/en = i/Aen$, so

$$v = (12\,\mathrm{A})/\pi(2.6\times10^{-3}\mathrm{m})^2(1.6\times10^{-19}\mathrm{C})(8.49\times10^{28}/\mathrm{m}^3) = 4.16\times10^{-5}\mathrm{m/s}.$$

The electrons will move 1 cm in $(1.0\times10^{-2}\mathrm{m})/(4.16\times10^{-5}\mathrm{m/s}) = 240\,\mathrm{s}$.

P29-4 (a) $N = it/q = (250\times10^{-9}\mathrm{A})(2.9\,\mathrm{s})/(3.2\times10^{-19}\mathrm{C}) = 2.27\times10^{12}$.
(b) The speed of the particles in the beam is given by $v = \sqrt{2K/m}$, so

$$v = \sqrt{2(22.4\ \mathrm{MeV})/4(932\ \mathrm{MeV}/c^2)} = 0.110c.$$

It takes $(0.180\,\mathrm{m})/(0.110)(3.00\times10^8\mathrm{m/s}) = 5.45\times10^{-9}\mathrm{s}$ for the beam to travel 18.0 cm. The number of charges is then

$$N = it/q = (250\times10^{-9}\mathrm{A})(5.45\times10^{-9}\mathrm{s})/(3.2\times10^{-19}\mathrm{C}) = 4260.$$

(c) $W = q\Delta V$, so $\Delta V = (22.4\ \mathrm{MeV})/2e = 11.2\ \mathrm{MV}$.

P29-5 (a) The time it takes to complete one turn is $t = (250\,\text{m})/c$. The total charge is

$$q = it = (30.0\,\text{A})(950\,\text{m})/(3.00\times10^{8}\text{m/s}) = 9.50\times10^{-5}\text{C}.$$

(b) The number of charges is $N = q/e$, the total energy absorbed by the block is then

$$\Delta U = (28.0\times10^{9}\ \text{eV})(9.50\times10^{-5}\text{C})/e = 2.66\times10^{6}\text{J}.$$

This will raise the temperature of the block by

$$\Delta T = \Delta U/mC = (2.66\times10^{6}\text{J})/(43.5\,\text{kg})(385\text{J/kgC}^\circ) = 159\,\text{C}^\circ.$$

P29-6 (a) $i = \int j\,dA = 2\pi\int jr\,dr$;

$$i = 2\pi\int -0^{R} j_0(1 - r/R)r\,dr = 2\pi j_0(R^2/2 - R^3/3R) = \pi j_0 R^2/6.$$

(b) Integrate, again:

$$i = 2\pi\int -0^{R} j_0(r/R)r\,dr = 2\pi j_0(R^3/3R) = \pi j_0 R^2/3.$$

P29-7 (a) Solve $2\rho_0 = \rho_0[1 + \alpha(T - 20^\circ\text{C})]$, or

$$T = 20^\circ\text{C} + 1/(4.3\times10^{-3}/\text{C}^\circ) = 250^\circ\text{C}.$$

(b) Yes, ignoring changes in the physical dimensions of the resistor.

P29-8 The resistance when on is $(2.90\,\text{V})/(0.310\,\text{A}) = 9.35\,\Omega$. The temperature is given by

$$T = 20^\circ\text{C} + (9.35\,\Omega - 1.12\,\Omega)/(1.12\,\Omega)(4.5\times10^{-3}/^\circ\text{C}) = 1650^\circ\text{C}.$$

$\boxed{\textbf{P29-9}}$ Originally we have a resistance R_1 made out of a wire of length l_1 and cross sectional area A_1. The volume of this wire is $V_1 = A_1 l_1$. When the wire is drawn out to the new length we have $l_2 = 3l_1$, but the volume of the wire should be constant so

$$\begin{aligned} A_2 l_2 &= A_1 l_1, \\ A_2(3l_1) &= A_1 l_1, \\ A_2 &= A_1/3. \end{aligned}$$

The original resistance is

$$R_1 = \rho\frac{l_1}{A_1}.$$

The new resistance is

$$R_2 = \rho\frac{l_2}{A_2} = \rho\frac{3l_1}{A_1/3} = 9R_1,$$

or $R_2 = 54\,\Omega$.

P29-10 (a) $i = (35.8\,\text{V})/(935\,\Omega) = 3.83\times10^{-2}\text{A}$.
(b) $j = i/A = (3.83\times10^{-2}\text{A})/(3.50\times10^{-4}\text{m}^2) = 109\,\text{A/m}^2$.
(c) $v = (109\,\text{A/m}^2)/(1.6\times10^{-19}\text{C})(5.33\times10^{22}/\text{m}^3) = 1.28\times10^{-2}\text{m/s}$.
(d) $E = (35.8\,\text{V})/(0.158\,\text{m}) = 227\,\text{V/m}$.

P29-11 (a) $\rho = (1.09\times10^{-3}\Omega)\pi(5.5\times10^{-3}\text{m})^2/4(1.6\,\text{m}) = 1.62\times10^{-8}\Omega\cdot\text{m}$. This is possibly silver.
(b) $R = (1.62\times10^{-8}\Omega\cdot\text{m})(1.35\times10^{-3}\text{m})4/\pi(2.14\times10^{-2}\text{m})^2 = 6.08\times10^{-8}\Omega$.

P29-12 (a) $\Delta L/L = 1.7\times10^{-5}$ for a temperature change of $1.0\,\text{C}^\circ$. Area changes are twice this, or $\Delta A/A = 3.4\times10^{-5}$.

Take the differential of $RA = \rho L$: $R\,dA + A\,dR = \rho\,dL + L\,d\rho$, or $dR = \rho\,dL/A + L\,d\rho/A - R\,dA/A$. For finite changes this can be written as

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + \frac{\Delta \rho}{\rho} - \frac{\Delta A}{A}.$$

$\Delta\rho/\rho = 4.3\times10^{-3}$. Since this term is so much larger than the other two it is the only significant effect.

$\boxed{\textbf{P29-13}}$ We will use the results of Exercise 29-17,

$$R - R_0 = R_0\alpha_{\text{av}}(T - T_0).$$

To save on subscripts we will drop the "av" notation, and just specify whether it is carbon "c" or iron "i".

The disks will be effectively in series, so we will add the resistances to get the total. Looking only at *one* disk pair, we have

$$\begin{aligned} R_\text{c} + R_\text{i} &= R_{0,\text{c}}\left(\alpha_\text{c}(T - T_0) + 1\right) + R_{0,\text{i}}\left(\alpha_\text{i}(T - T_0) + 1\right), \\ &= R_{0,\text{c}} + R_{0,\text{i}} + \left(R_{0,\text{c}}\alpha_\text{c} + R_{0,\text{i}}\alpha_\text{i}\right)(T - T_0). \end{aligned}$$

This last equation will only be constant if the coefficient for the term $(T - T_0)$ vanishes. Then

$$R_{0,\text{c}}\alpha_\text{c} + R_{0,\text{i}}\alpha_\text{i} = 0,$$

but $R = \rho L/A$, and the disks have the same cross sectional area, so

$$L_\text{c}\rho_\text{c}\alpha_\text{c} + L_\text{i}\rho_\text{i}\alpha_\text{i} = 0,$$

or

$$\frac{L_\text{c}}{L_\text{i}} = -\frac{\rho_\text{i}\alpha_\text{i}}{\rho_\text{c}\alpha_\text{c}} = -\frac{(9.68\times10^{-8}\Omega\cdot\text{m})(6.5\times10^{-3}/\text{C}^\circ)}{(3500\times10^{-8}\Omega\cdot\text{m})(-0.50\times10^{-3}/\text{C}^\circ)} = 0.036.$$

P29-14 The current entering the cone is i. The current density as a function of distance x from the left end is then

$$j = \frac{i}{\pi[a + x(b - a)/L]^2}.$$

The electric field is given by $E = \rho j$. The potential difference between the ends is then

$$\Delta V = \int_0^L E\,dx = \int_0^L \frac{i\rho}{\pi[a + x(b - a)/L]^2}dx = \frac{i\rho L}{\pi ab}$$

The resistance is $R = \Delta V/i = \rho L/\pi ab$.

$\boxed{\textbf{P29-15}}$ The current is found from Eq. 29-5,

$$i = \int \vec{\mathbf{j}} \cdot d\vec{\mathbf{A}},$$

where the region of integration is over a spherical shell concentric with the two conducting shells but between them. The current density is given by Eq. 29-10,

$$\vec{\mathbf{j}} = \vec{\mathbf{E}}/\rho,$$

and we will have an electric field which is perpendicular to the spherical shell. Consequently,

$$i = \frac{1}{\rho}\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\rho}\int E\,dA$$

By symmetry we expect the electric field to have the same magnitude anywhere on a spherical shell which is concentric with the two conducting shells, so we can bring it out of the integral sign, and then

$$i = \frac{1}{\rho}E\int dA = \frac{4\pi r^2 E}{\rho},$$

where E is the magnitude of the electric field on the shell, which has radius r such that $b > r > a$.

The above expression can be inverted to give the electric field as a function of radial distance, since the current is a constant in the above expression. Then $E = i\rho/4\pi r^2$ The potential is given by

$$\Delta V = -\int_b^a \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}},$$

we will integrate along a radial line, which is parallel to the electric field, so

$$\begin{aligned}
\Delta V &= -\int_b^a E\,dr, \\
&= -\int_b^a \frac{i\rho}{4\pi r^2}\,dr, \\
&= -\frac{i\rho}{4\pi}\int_b^a \frac{dr}{r}, \\
&= \frac{i\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right).
\end{aligned}$$

We divide this expression by the current to get the resistance. Then

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

P29-16 Since $\tau = \lambda/v_\mathrm{d}$, $\rho \propto v_\mathrm{d}$. For an ideal gas the kinetic energy is proportional to the temperature, so $\rho \propto \sqrt{K} \propto \sqrt{T}$.

$\boxed{\textbf{E30-1}}$ We apply Eq. 30-1,

$$q = C\Delta V = (50 \times 10^{-12}\,\mathrm{F})(0.15\,\mathrm{V}) = 7.5 \times 10^{-12}\,\mathrm{C};$$

E30-2 (a) $C = \Delta V/q = (73.0\times10^{-12}\,C)/(19.2\,\mathrm{V}) = 3.80\times10^{-12}\mathrm{F}$.
(b) The capacitance doesn't change!
(c) $\Delta V = q/C = (210\times10^{-12}\mathrm{C})/(3.80\times10^{-12}\mathrm{F}) = 55.3\,\mathrm{V}$.

E30-3 $q = C\Delta V = (26.0\times10^{-6}\mathrm{F})(125\,\mathrm{V}) = 3.25\times10^{-3}\mathrm{C}$.

E30-4 (a) $C = \epsilon_0 A/d = (8.85\times10^{-12}\mathrm{F/m})\pi(8.22\times10^{-2}\mathrm{m})^2/(1.31\times10^{-3}\mathrm{m}) = 1.43\times10^{-10}\mathrm{F}$.
(b) $q = C\Delta V = (1.43\times10^{-10}\mathrm{F})(116\,\mathrm{V}) = 1.66\times10^{-8}\mathrm{C}$.

$\boxed{\textbf{E30-5}}$ Eq. 30-11 gives the capacitance of a cylinder,

$$C = 2\pi\epsilon_0\frac{L}{\ln(b/a)} = 2\pi(8.85\times10^{-12}\,\mathrm{F/m})\frac{(0.0238\,\mathrm{m})}{\ln((9.15\mathrm{mm})/(0.81\mathrm{mm}))} = 5.46\times10^{-13}\mathrm{F}.$$

E30-6 (a) $A = Cd/\epsilon_0 = (9.70\times10^{-12}\mathrm{F})(1.20\times10^{-3}\mathrm{m})/(8.85\times10^{-12}\mathrm{F/m}) = 1.32\times10^{-3}\mathrm{m}^2$.
(b) $C = C_0 d_0/d = (9.70\times10^{-12}\mathrm{F})(1.20\times10^{-3}\mathrm{m})/(1.10\times10^{-3}\mathrm{m}) = 1.06\times10^{-11}\mathrm{F}$.
(c) $\Delta V = q_0/C = [\Delta V]_0 C_0/C = [\Delta V]_0 d/d_0$. Using this formula, the new potential difference would be $[\Delta V]_0 = (13.0\,\mathrm{V})(1.10\times10^{-3}\mathrm{m})/(1.20\times10^{-3}\mathrm{m}) = 11.9\,\mathrm{V}$. The potential energy has *changed* by $(11.9\,\mathrm{V}) - (30.0\,\mathrm{V}) = -1.1\,\mathrm{V}$.

E30-7 (a) From Eq. 30-8,

$$C = 4\pi(8.85\times10^{-12}\mathrm{F/m})\frac{(0.040\,\mathrm{m})(0.038\,\mathrm{m})}{(0.040\,\mathrm{m}) - (0.038\,\mathrm{m})} = 8.45\times10^{-11}\mathrm{F}.$$

(b) $A = Cd/\epsilon_0 = (8.45\times10^{-11}\mathrm{F})(2.00\times10^{-3}\mathrm{m})/(8.85\times10^{-12}\mathrm{F/m}) = 1.91\times10^{-2}\mathrm{m}^2$.

E30-8 Let $a = b + d$, where d is the *small* separation between the shells. Then

$$\begin{aligned} C &= 4\pi\epsilon_0\frac{ab}{a-b} = 4\pi\epsilon_0\frac{(b+d)b}{d}, \\ &\approx 4\pi\epsilon_0\frac{b^2}{d} = \epsilon_0 A/d. \end{aligned}$$

$\boxed{\textbf{E30-9}}$ The potential difference across each capacitor in parallel is the same; it is equal to 110 V. The charge on each of the capacitors is then

$$q = C\Delta V = (1.00 \times 10^{-6}\,\mathrm{F})(110\,\mathrm{V}) = 1.10 \times 10^{-4}\,\mathrm{C}.$$

If there are N capacitors, then the total charge will be Nq, and we want this total charge to be 1.00 C. Then

$$N = \frac{(1.00\,\mathrm{C})}{q} = \frac{(1.00\,\mathrm{C})}{(1.10 \times 10^{-4}\,\mathrm{C})} = 9090.$$

E30-10 First find the equivalent capacitance of the parallel part:

$$C_{\rm eq} = C_1 + C_2 = (10.3\times10^{-6}{\rm F}) + (4.80\times10^{-6}{\rm F}) = 15.1\times10^{-6}{\rm F}.$$

Then find the equivalent capacitance of the series part:

$$\frac{1}{C_{\rm eq}} = \frac{1}{(15.1\times10^{-6}{\rm F})} + \frac{1}{(3.90\times10^{-6}{\rm F})} = 3.23\times10^{5}{\rm F}^{-1}.$$

Then the equivalent capacitance of the entire arrangement is 3.10×10^{-6}F.

E30-11 First find the equivalent capacitance of the series part:

$$\frac{1}{C_{\rm eq}} = \frac{1}{(10.3\times10^{-6}{\rm F})} + \frac{1}{(4.80\times10^{-6}{\rm F})} = 3.05\times10^{5}{\rm F}^{-1}.$$

The equivalent capacitance is 3.28×10^{-6}F. Then find the equivalent capacitance of the parallel part:

$$C_{\rm eq} = C_1 + C_2 = (3.28\times10^{-6}{\rm F}) + (3.90\times10^{-6}{\rm F}) = 7.18\times10^{-6}{\rm F}.$$

This is the equivalent capacitance for the entire arrangement.

E30-12 For one capacitor $q = C\Delta V = (25.0\times10^{-6}{\rm F})(4200\,{\rm V}) = 0.105\,{\rm C}$. There are three capacitors, so the total charge to pass through the ammeter is 0.315 C.

E30-13 (a) The equivalent capacitance is given by Eq. 30-21,

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{(4.0\mu{\rm F})} + \frac{1}{(6.0\mu{\rm F})} = \frac{5}{(12.0\mu{\rm F})}$$

or $C_{\rm eq} = 2.40\mu$F.

(b) The charge on the equivalent capacitor is $q = C\Delta V = (2.40\mu{\rm F})(200\,{\rm V}) = 0.480\,{\rm mC}$. For series capacitors, the charge on the equivalent capacitor is the same as the charge on each of the capacitors. *This statement is wrong in the Student Solutions!*

(c) The potential difference across the equivalent capacitor is *not* the same as the potential difference across each of the individual capacitors. We need to apply $q = C\Delta V$ to each capacitor using the charge from part (b). Then for the 4.0μF capacitor,

$$\Delta V = \frac{q}{C} = \frac{(0.480\,{\rm mC})}{(4.0\mu{\rm F})} = 120\,{\rm V};$$

and for the 6.0μF capacitor,

$$\Delta V = \frac{q}{C} = \frac{(0.480\,{\rm mC})}{(6.0\mu{\rm F})} = 80\,{\rm V}.$$

Note that the sum of the potential differences across each of the capacitors is equal to the potential difference across the equivalent capacitor.

E30-14 (a) The equivalent capacitance is

$$C_{\rm eq} = C_1 + C_2 = (4.0\mu{\rm F}) + (6.0\mu{\rm F}) = (10.0\mu{\rm F}).$$

(c) For parallel capacitors, the potential difference across the equivalent capacitor is the same as the potential difference across either of the capacitors.

(b) For the 4.0μF capacitor,

$$q = C\Delta V = (4.0\mu{\rm F})(200\,{\rm V}) = 8.0\times10^{-4}\,{\rm C};$$

and for the 6.0μF capacitor,

$$q = C\Delta V = (6.0\mu{\rm F})(200\,{\rm V}) = 12.0\times10^{-4}\,{\rm C}.$$

E30-15 (a) $C_{\text{eq}} = C + C + C = 3C$;

$$d_{\text{eq}} = \frac{\epsilon_0 A}{C_{\text{eq}}} = \frac{\epsilon_0 A}{3C} = \frac{d}{3}.$$

(b) $1/C_{\text{eq}} = 1/C + 1/C + 1/C = 3/C$;

$$d_{\text{eq}} = \frac{\epsilon_0 A}{C_{\text{eq}}} = \frac{\epsilon_0 A}{C/3} = 3d.$$

E30-16 (a) The maximum potential across any individual capacitor is 200 V; so there must be at least $(1000\,\text{V})/(200\,\text{V}) = 5$ series capacitors in any parallel branch. This branch would have an equivalent capacitance of $C_{\text{eq}} = C/5 = (2.0\times10^{-6}\text{F})/5 = 0.40\times10^{-6}\text{F}$.

(b) For parallel branches we add, which means we need $(1.2\times10^{-6}\text{F})/(0.40\times10^{-6}\text{F}) = 3$ parallel branches of the combination found in part (a).

E30-17 Look back at the solution to Ex. 30-10. If C_3 breaks down electrically then the circuit is effectively two capacitors in parallel.

(b) $\Delta V = 115\,\text{V}$ after the breakdown.

(a) $q_1 = (10.3\times10^{-6}\text{F})(115\,\text{V}) = 1.18\times10^{-3}\text{C}$.

E30-18 The 108μF capacitor originally has a charge of $q = (108\times10^{-6}\text{F})(52.4\,\text{V}) = 5.66\times10^{-3}\text{C}$. After it is connected to the second capacitor the 108μF capacitor has a charge of $q = (108\times10^{-6}\text{F})(35.8\,\text{V}) = 3.87\times10^{-3}\text{C}$. The difference in charge must reside on the second capacitor, so the capacitance is $C = (1.79\times10^{-3}\text{C})/(35.8\,\text{V}) = 5.00\times10^{-5}\text{F}$.

$\boxed{\textbf{E30-19}}$ Consider any junction other than A or B. Call this junction point 0; label the four nearest junctions to this as points 1, 2, 3, and 4. The charge on the capacitor that links point 0 to point 1 is $q_1 = C\Delta V_{01}$, where ΔV_{01} is the potential difference across the capacitor, so $\Delta V_{01} = V_0 - V_1$, where V_0 is the potential at the junction 0, and V_1 is the potential at the junction 1. Similar expressions exist for the other three capacitors.

For the junction 0 the net charge must be zero; there is no way for charge to cross the plates of the capacitors. Then $q_1 + q_2 + q_3 + q_4 = 0$, and this means

$$C\Delta V_{01} + C\Delta V_{02} + C\Delta V_{03} + C\Delta V_{04} = 0$$

or

$$\Delta V_{01} + \Delta V_{02} + \Delta V_{03} + \Delta V_{04} = 0.$$

Let $\Delta V_{0i} = V_0 - V_i$, and then rearrange,

$$4V_0 = V_1 + V_2 + V_3 + V_4,$$

or

$$V_0 = \frac{1}{4}\left(V_1 + V_2 + V_3 + V_4\right).$$

E30-20 $U = uV = \epsilon_0 E^2 V/2$, where V is the volume. Then

$$U = \frac{1}{2}(8.85\times10^{-12}\text{F/m})(150\,\text{V/m})^2(2.0\,\text{m}^3) = 1.99\times10^{-7}\text{J}.$$

E30-21 The total capacitance is $(2100)(5.0\times10^{-6}\text{F}) = 1.05\times10^{-2}\text{F}$. The total energy stored is

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(1.05\times10^{-2}\text{F})(55\times10^{3}\text{V})^2 = 1.59\times10^{7}\,\text{J}.$$

The cost is

$$(1.59\times10^{7}\text{J})\left(\frac{\$0.03}{3600\times10^{3}\text{J}}\right) = \$0.133.$$

E30-22 (a) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(0.061\,\text{F})(1.0\times10^{4}\text{V})^2 = 3.05\times10^{6}\text{J}$.
(b) $(3.05\times10^{6}\text{J})/(3600\times10^{3}\text{J/kW}\cdot\text{h}) = 0.847\text{kW}\cdot\text{h}$.

$\boxed{\textbf{E30-23}}$ (a) The capacitance of an air filled parallel-plate capacitor is given by Eq. 30-5,

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\text{F/m})(42.0\times10^{-4}\text{m}^2)}{(1.30\times10^{-3}\text{m})} = 2.86\times10^{-11}\,\text{F}.$$

(b) The magnitude of the charge on each plate is given by

$$q = C\Delta V = (2.86\times10^{-11}\,\text{F})(625\,\text{V}) = 1.79\times10^{-8}\,\text{C}.$$

(c) The stored energy in a capacitor is given by Eq. 30-25, regardless of the type or shape of the capacitor, so

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(2.86\times10^{-11}\,\text{F})(625\,\text{V})^2 = 5.59\,\mu\text{J}.$$

(d) Assuming a parallel plate arrangement with *no* fringing effects, the magnitude of the electric field between the plates is given by $Ed = \Delta V$, where d is the separation between the plates. Then

$$E = \Delta V/d = (625\,\text{V})/(0.00130\,\text{m}) = 4.81\times10^{5}\,\text{V/m}.$$

(e) The energy density is Eq. 30-28,

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}((8.85\times10^{-12}\text{F/m}))(4.81\times10^{5}\,\text{V/m})^2 = 1.02\,\text{J/m}^3.$$

E30-24 The equivalent capacitance is given by

$$1/C_{\text{eq}} = 1/(2.12\times10^{-6}\text{F}) + 1/(3.88\times10^{-6}\text{F}) = 1/(1.37\times10^{-6}\text{F}).$$

The energy stored is $U = \frac{1}{2}(1.37\times10^{-6}\text{F})(328\,\text{V})^2 = 7.37\times10^{-2}\text{J}$.

E30-25 $V/r = q/4\pi\epsilon_0 r^2 = E$, so that if V is the potential of the sphere then $E = V/r$ is the electric field on the surface. Then the energy density of the electric field near the surface is

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{(8.85\times10^{-12}\text{F/m})}{2}\left(\frac{(8150\,\text{V})}{(0.063\,\text{m})}\right)^2 = 7.41\times10^{-2}\text{J/m}^3.$$

E30-26 The charge on C_3 can be found from considering the equivalent capacitance. $q_3 = (3.10\times10^{-6}\text{F})(112\,\text{V}) = 3.47\times10^{-4}\text{C}$. The potential across C_3 is given by $[\Delta V]_3 = (3.47\times10^{-4}\text{C})/(3.90\times10^{-6}\text{F}) = 89.0\,\text{V}$.

The potential across the parallel segment is then $(112\,\text{V}) - (89.0\,\text{V}) = 23.0\,\text{V}$. So $[\Delta V]_1 = [\Delta V]_2 = 23.0\,\text{V}$.

Then $q_1 = (10.3\times10^{-6}\text{F})(23.0\,\text{V}) = 2.37\times10^{-4}\text{C}$ and $q_2 = (4.80\times10^{-6}\text{F})(23.0\,\text{V}) = 1.10\times10^{-4}\text{C}$..

$\boxed{\textbf{E30-27}}$ There is enough work on this problem without deriving once again the electric field between charged cylinders. I will instead refer you back to Section 26-4, and state

$$E = \frac{1}{2\pi\epsilon_0}\frac{q}{Lr},$$

where q is the magnitude of the charge on a cylinder and L is the length of the cylinders.

The energy density as a function of radial distance is found from Eq. 30-28,

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{8\pi^2\epsilon_0}\frac{q^2}{L^2r^2}$$

The total energy stored in the electric field is given by Eq. 30-24,

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{q^2}{2}\frac{\ln(b/a)}{2\pi\epsilon_0 L},$$

where we substituted into the last part Eq. 30-11, the capacitance of a cylindrical capacitor.

We want to show that integrating a volume integral from $r = a$ to $r = \sqrt{ab}$ over the energy density function will yield $U/2$. Since we want to do this problem the hard way, we will pretend we don't know the answer, and integrate from $r = a$ to $r = c$, and then find out what c is.

Then

$$\begin{aligned}
\frac{1}{2}U &= \int u\,dV, \\
&= \int_a^c \int_0^{2\pi} \int_0^L \left(\frac{1}{8\pi^2\epsilon_0}\frac{q^2}{L^2r^2}\right) r\,dr\,d\phi\,dz, \\
&= \frac{q^2}{8\pi^2\epsilon_0 L^2}\int_a^c \int_0^{2\pi} \int_0^L \frac{dr}{r}d\phi\,dz, \\
&= \frac{q^2}{4\pi\epsilon_0 L}\int_a^c \frac{dr}{r}, \\
&= \frac{q^2}{4\pi\epsilon_0 L}\ln\frac{c}{a}.
\end{aligned}$$

Now we equate this to the value for U that we found above, and we solve for c.

$$\begin{aligned}
\frac{1}{2}\frac{q^2}{2}\frac{\ln(b/a)}{2\pi\epsilon_0 L} &= \frac{q^2}{4\pi\epsilon_0 L}\ln\frac{c}{a}, \\
\ln(b/a) &= 2\ln(c/a), \\
(b/a) &= (c/a)^2, \\
\sqrt{ab} &= c.
\end{aligned}$$

E30-28 (a) $d = \epsilon_0 A/C$, or

$$d = (8.85\times10^{-12}\text{F/m})(0.350\,\text{m}^2)/(51.3\times10^{-12}\text{F}) = 6.04\times10^{-3}\text{m}.$$

(b) $C = (5.60)(51.3\times10^{-12}\,\text{F}) = 2.87\times10^{-10}\text{F}$.

E30-29 Originally, $C_1 = \epsilon_0 A/d_1$. After the changes, $C_2 = \kappa\epsilon_0 A/d_2$. Dividing C_2 by C_1 yields $C_2/C_1 = \kappa d_1/d_2$, so

$$\kappa = d_2C_2/d_1C_1 = (2)(2.57\times10^{-12}\text{F})/(1.32\times10^{-12}\text{F}) = 3.89.$$

E30-30 The required capacitance is found from $U = \frac{1}{2}C(\Delta V)^2$, or

$$C = 2(6.61\times10^{-6}\mathrm{J})/(630\,\mathrm{V})^2 = 3.33\times10^{-11}\mathrm{F}.$$

The dielectric constant required is $\kappa = (3.33\times10^{-11}\mathrm{F})/(7.40\times10^{-12}\mathrm{F}) = 4.50$. Try transformer oil.

$\boxed{\textbf{E30-31}}$ Capacitance with dielectric media is given by Eq. 30-31,

$$C = \frac{\kappa_\mathrm{e}\epsilon_0 A}{d}.$$

The various sheets have different dielectric constants and different thicknesses, and we want to maximize C, which means maximizing κ_e/d. For mica this ratio is 54 mm^{-1}, for glass this ratio is 35 mm^{-1}, and for paraffin this ratio is 0.20 mm^{-1}. Mica wins.

E30-32 The minimum plate separation is given by

$$d = (4.13\times10^{3}\mathrm{V})/(18.2\times10^{6}\mathrm{V/m}) = 2.27\times10^{-4}\mathrm{m}.$$

The minimum plate area is then

$$A = \frac{dC}{\kappa\epsilon_0} = \frac{(2.27\times10^{-4}\mathrm{m})(68.4\times10^{-9}\mathrm{F})}{(2.80)(8.85\times10^{-12}\mathrm{F/m})} = 0.627\,\mathrm{m}^2.$$

E30-33 The capacitance of a cylindrical capacitor is given by Eq. 30-11,

$$C = 2\pi(8.85\times10^{-12}\mathrm{F/m})(2.6)\frac{1.0\times10^{3}\mathrm{m}}{\ln(0.588/0.11)} = 8.63\times10^{-8}\mathrm{F}.$$

E30-34 (a) $U = C'(\Delta V)^2/2$, $C' = \kappa_\mathrm{e}\epsilon_0 A/d$, and $\Delta V/d$ is less than or equal to the dielectric strength (which we will call S). Then $\Delta V = Sd$ and

$$U = \frac{1}{2}\kappa_\mathrm{e}\epsilon_0 AdS^2,$$

so the volume is given by

$$V = 2U/\kappa_\mathrm{e}\epsilon_0 S^2.$$

This quantity is a minimum for mica, so

$$V = 2(250\times10^{3}\mathrm{J})/(5.4)(8.85\times10^{-12}\mathrm{F/m})(160\times10^{6}\mathrm{V/m})^2 = 0.41\,\mathrm{m}^3.$$

(b) $\kappa_\mathrm{e} = 2U/V\epsilon_0 S^2$, so

$$\kappa_\mathrm{e} = 2(250\times10^{3}\mathrm{J})/(0.087\mathrm{m}^3)(8.85\times10^{-12}\mathrm{F/m})(160\times10^{6}\mathrm{V/m})^2 = 25.$$

$\boxed{\textbf{E30-35}}$ (a) The capacitance of a cylindrical capacitor is given by Eq. 30-11,

$$C = 2\pi\epsilon_0\kappa_\mathrm{e}\frac{L}{\ln(b/a)}.$$

The factor of κ_e is introduced because there is now a dielectric (the Pyrex drinking glass) between the plates. We can look back to Table 29-2 to get the dielectric properties of Pyrex. The capacitance of our "glass" is then

$$C = 2\pi(8.85\times10^{-12}\mathrm{F/m})(4.7)\frac{(0.15\,\mathrm{m})}{\ln((3.8\text{ cm})/(3.6\text{ cm})} = 7.3\times10^{-10}\,\mathrm{F}.$$

(b) The breakdown potential is (14 kV/mm)(2 mm) = 28 kV.

E30-36 (a) $C' = \kappa_e C = (6.5)(13.5\times10^{-12}\text{F}) = 8.8\times10^{-11}\text{F}$.

(b) $Q = C'\Delta V = (8.8\times10^{-11}\text{F})(12.5\,\text{V}) = 1.1\times10^{-9}\text{C}$.

(c) $E = \Delta V/d$, but we don't know d.

(d) $E' = E/\kappa_e$, but we couldn't find E.

$\boxed{\textbf{E30-37}}$ (a) Insert the slab so that it is a distance a above the lower plate. Then the distance between the slab and the upper plate is $d - a - b$. Inserting the slab has the same effect as having two capacitors wired in series; the separation of the bottom capacitor is a, while that of the top capacitor is $d - a - b$.

The bottom capacitor has a capacitance of $C_1 = \epsilon_0 A/a$, while the top capacitor has a capacitance of $C_2 = \epsilon_0 A/(d - a - b)$. Adding these in series,

$$\begin{aligned}
\frac{1}{C_{\text{eq}}} &= \frac{1}{C_1} + \frac{1}{C_2}, \\
&= \frac{a}{\epsilon_0 A} + \frac{d-a-b}{\epsilon_0 A}, \\
&= \frac{d-b}{\epsilon_0 A}.
\end{aligned}$$

So the capacitance of the system after putting the copper slab in is $C = \epsilon_0 A/(d - b)$.

(b) The energy stored in the system before the slab is inserted is

$$U_{\text{i}} = \frac{q^2}{2C_{\text{i}}} = \frac{q^2}{2}\frac{d}{\epsilon_0 A}$$

while the energy stored after the slab is inserted is

$$U_{\text{f}} = \frac{q^2}{2C_{\text{f}}} = \frac{q^2}{2}\frac{d-b}{\epsilon_0 A}$$

The ratio is $U_{\text{i}}/U_{\text{f}} = d/(d - b)$.

(c) Since there was more energy *before* the slab was inserted, then the slab must have gone in willingly, *it was pulled in!*. To get the slab back out we will need to do work on the slab equal to the energy difference.

$$U_{\text{i}} - U_{\text{f}} = \frac{q^2}{2}\frac{d}{\epsilon_0 A} - \frac{q^2}{2}\frac{d-b}{\epsilon_0 A} = \frac{q^2}{2}\frac{b}{\epsilon_0 A}.$$

E30-38 (a) Insert the slab so that it is a distance a above the lower plate. Then the distance between the slab and the upper plate is $d - a - b$. Inserting the slab has the same effect as having two capacitors wired in series; the separation of the bottom capacitor is a, while that of the top capacitor is $d - a - b$.

The bottom capacitor has a capacitance of $C_1 = \epsilon_0 A/a$, while the top capacitor has a capacitance of $C_2 = \epsilon_0 A/(d - a - b)$. Adding these in series,

$$\begin{aligned}
\frac{1}{C_{\text{eq}}} &= \frac{1}{C_1} + \frac{1}{C_2}, \\
&= \frac{a}{\epsilon_0 A} + \frac{d-a-b}{\epsilon_0 A}, \\
&= \frac{d-b}{\epsilon_0 A}.
\end{aligned}$$

So the capacitance of the system after putting the copper slab in is $C = \epsilon_0 A/(d - b)$.

(b) The energy stored in the system before the slab is inserted is

$$U_{\rm i} = \frac{C_{\rm i}(\Delta V)^2}{2} = \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A}{d}$$

while the energy stored after the slab is inserted is

$$U_{\rm f} = \frac{C_{\rm f}(\Delta V)^2}{2} = \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A}{d-b}$$

The ratio is $U_{\rm i}/U_{\rm f} = (d-b)/d$.

(c) Since there was more energy *after* the slab was inserted, then the slab must not have gone in willingly, *it was being repelled!*. To get the slab in we will need to do work on the slab equal to the energy difference.

$$U_{\rm f} - U_{\rm i} = \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A}{d-b} - \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A}{d} = \frac{(\Delta V)^2}{2}\frac{\epsilon_0 A b}{d(d-b)}.$$

E30-39 $C = \kappa_{\rm e}\epsilon_0 A/d$, so $d = \kappa_{\rm e}\epsilon_0 A/C$.

(a) $E = \Delta V/d = C\Delta V/\kappa_{\rm e}\epsilon_0 A$, or

$$E = \frac{(112\times10^{-12}\mathrm{F})(55.0\,\mathrm{V})}{(5.4)(8.85\times10^{-12}\mathrm{F/m})(96.5\times10^{-4}\mathrm{m}^2)} = 13400\,\mathrm{V/m}.$$

(b) $Q = C\Delta V = (112\times10^{-12}\mathrm{F})(55.0\,\mathrm{V}) = 6.16\times10^{-9}\mathrm{C}$..

(c) $Q' = Q(1-1/\kappa_{\rm e}) = (6.16\times10^{-9}\mathrm{C})(1-1/(5.4)) = 5.02\times10^{-9}\mathrm{C}$.

E30-40 (a) $E = q/\kappa_{\rm e}\epsilon_0 A$, so

$$\kappa_{\rm e} = \frac{(890\times10^{-9}\mathrm{C})}{(1.40\times10^{6}\mathrm{V/m})(8.85\times10^{-12}\mathrm{F/m})(110\times10^{-4}\mathrm{m}^2)} = 6.53$$

(b) $q' = q(1-1/\kappa_{\rm e}) = (890\times10^{-9}\mathrm{C})(1-1/(6.53)) = 7.54\times10^{-7}\mathrm{C}$.

$\boxed{\textbf{P30-1}}$ The capacitance of the cylindrical capacitor is from Eq. 30-11,

$$C = \frac{2\pi\epsilon_0 L}{\ln(b/a)}.$$

If the cylinders are very close together we can write $b = a + d$, where d, the separation between the cylinders, is a small number, so

$$C = \frac{2\pi\epsilon_0 L}{\ln\left((a+d)/a\right)} = \frac{2\pi\epsilon_0 L}{\ln\left(1+d/a\right)}.$$

Expanding according to the hint,

$$C \approx \frac{2\pi\epsilon_0 L}{d/a} = \frac{2\pi a\epsilon_0 L}{d}$$

Now $2\pi a$ is the circumference of the cylinder, and L is the length, so $2\pi a L$ is the area of a cylindrical plate. Hence, for small separation between the cylinders we have

$$C \approx \frac{\epsilon_0 A}{d},$$

which is the expression for the parallel plates.

P30-2 (a) $C = \epsilon_0 A/x$; take the derivative and

$$\begin{aligned} \frac{dC}{dT} &= \frac{\epsilon_0}{x}\frac{dA}{dT} - \frac{\epsilon_0 A}{x^2}\frac{dx}{dT}, \\ &= C\left(\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right). \end{aligned}$$

(b) Since $(1/A)dA/dT = 2\alpha_\mathrm{a}$ and $(1/x)dx/dT = \alpha_\mathrm{s}$, we need

$$\alpha_\mathrm{s} = 2\alpha_\mathrm{a} = 2(23\times10^{-6}/\mathrm{C}^\circ) = 46\times10^{-6}/\mathrm{C}^\circ.$$

$\boxed{\textbf{P30-3}}$ Insert the slab so that it is a distance d above the lower plate. Then the distance between the slab and the upper plate is $a-b-d$. Inserting the slab has the same effect as having two capacitors wired in series; the separation of the bottom capacitor is d, while that of the top capacitor is $a-b-d$.

The bottom capacitor has a capacitance of $C_1 = \epsilon_0 A/d$, while the top capacitor has a capacitance of $C_2 = \epsilon_0 A/(a-b-d)$. Adding these in series,

$$\begin{aligned} \frac{1}{C_\mathrm{eq}} &= \frac{1}{C_1} + \frac{1}{C_2}, \\ &= \frac{d}{\epsilon_0 A} + \frac{a-b-d}{\epsilon_0 A}, \\ &= \frac{a-b}{\epsilon_0 A}. \end{aligned}$$

So the capacitance of the system after putting the slab in is $C = \epsilon_0 A/(a-b)$.

P30-4 The potential difference between any two adjacent plates is ΔV. Each interior plate has a charge q on *each* surface; the exterior plate (one pink, one gray) has a charge of q on the interior surface only.

The capacitance of one pink/gray plate pair is $C = \epsilon_0 A/d$. There are n plates, but only $n-1$ plate pairs, so the total charge is $(n-1)q$. This means the total capacitance is $C = \epsilon_0(n-1)A/d$.

$\boxed{\textbf{P30-5}}$ Let $\Delta V_0 = 96.6\,\mathrm{V}$.

As far as point e is concerned point a looks like it is originally positively charged, and point d is originally negatively charged. It is then convenient to define the charges on the capacitors in terms of the charges on the top sides, so the original charge on C_1 is $q_{1,\mathrm{i}} = C_1\Delta V_0$ while the original charge on C_2 is $q_{2,\mathrm{i}} = -C_2\Delta V_0$. Note the negative sign reflecting the opposite polarity of C_2.

(a) Conservation of charge requires

$$q_{1,\mathrm{i}} + q_{2,\mathrm{i}} = q_{1,\mathrm{f}} + q_{2,\mathrm{f}},$$

but since $q = C\Delta V$ and the two capacitors will be at the same potential after the switches are closed we can write

$$\begin{aligned} C_1\Delta V_0 - C_2\Delta V_0 &= C_1\Delta V + C_2\Delta V, \\ (C_1 - C_2)\,\Delta V_0 &= (C_1 + C_2)\,\Delta V, \\ \frac{C_1 - C_2}{C_1 + C_2}\Delta V_0 &= \Delta V. \end{aligned}$$

With numbers,

$$\Delta V = (96.6\,\mathrm{V})\frac{(1.16\,\mu\mathrm{F}) - (3.22\,\mu\mathrm{F})}{(1.16\,\mu\mathrm{F}) + (3.22\,\mu\mathrm{F})} = -45.4\,\mathrm{V}.$$

The negative sign means that the top sides of *both* capacitor will be negatively charged after the switches are closed.

(b) The charge on C_1 is $C_1\Delta V = (1.16\,\mu\mathrm{F})(45.4\,\mathrm{V}) = 52.7\mu\mathrm{C}$.

(c) The charge on C_2 is $C_2\Delta V = (3.22\,\mu\mathrm{F})(45.4\,\mathrm{V}) = 146\mu\mathrm{C}$.

P30-6 C_2 and C_3 form an effective capacitor with equivalent capacitance $C_a = C_2C_3/(C_2+C_3)$. The charge on C_1 is originally $q_0 = C_1\Delta V_0$. After throwing the switch the potential across C_1 is given by $q_1 = C_1\Delta V_1$. The same potential is across C_a; $q_2 = q_3$, so $q_2 = C_a\Delta V_1$. Charge is conserved, so $q_1 + q_2 = q_0$. Combining some of the above,

$$\Delta V_1 = \frac{q_0}{C_1+C_a} = \frac{C_1}{C_1+C_a}\Delta V_0,$$

and then

$$q_1 = \frac{C_1^2}{C_1+C_a}\Delta V_0 = \frac{C_1^2(C_2+C_3)}{C_1C_2+C_1C_3+C_2C_3}\Delta V_0.$$

Similarly,

$$q_2 = \frac{C_aC_1}{C_1+C_a}\Delta V_0 = \left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}\Delta V_0.$$

$q_3 = q_2$ because they are in series.

P30-7 (a) If terminal a is more positive than terminal b then current can flow that will charge the capacitor on the left, the current can flow through the diode on the top, and the current can charge the capacitor on the right. Current will not flow through the diode on the left. The capacitors are effectively in series.

Since the capacitors are identical and series capacitors have the same charge, we expect the capacitors to have the same potential difference across them. But the total potential difference across both capacitors is equal to 100 V, so the potential difference across either capacitor is 50 V.

The output pins are connected to the capacitor on the right, so the potential difference across the output is 50 V.

(b) If terminal b is more positive than terminal a the current can flow through the diode on the left. If we assume the diode is resistanceless in this configuration then the potential difference across it will be zero. The net result is that the potential difference across the output pins is 0 V.

In real life the potential difference across the diode would not be zero, even if forward biased. It will be somewhere around 0.5 Volts.

P30-8 Divide the strip of width a into N segments, each of width $\Delta x = a/N$. The capacitance of each strip is $\Delta C = \epsilon_0 a\Delta x/y$. If θ is small then

$$\frac{1}{y} = \frac{1}{d + x\sin\theta} \approx \frac{1}{d+x\theta} \approx \frac{d}{(}1 - x\theta/d).$$

Since parallel capacitances *add*,

$$C = \sum \Delta C = \frac{\epsilon_0 a}{d}\int_0^a (1 - x\theta/d)dx = \frac{\epsilon_0 a^2}{d}\left(1 - \frac{a\theta}{2d}\right).$$

P30-9 (a) When S_2 is open the circuit acts as two parallel capacitors. The branch on the left has an effective capacitance given by

$$\frac{1}{C_l} = \frac{1}{(1.0\times10^{-6}\text{F})} + \frac{1}{(3.0\times10^{-6}\text{F})} = \frac{1}{7.5\times10^{-7}\text{F}},$$

while the branch on the right has an effective capacitance given by

$$\frac{1}{C_l} = \frac{1}{(2.0\times10^{-6}\text{F})} + \frac{1}{(4.0\times10^{-6}\text{F})} = \frac{1}{1.33\times10^{-6}\text{F}}.$$

The charge on *either* capacitor in the branch on the left is

$$q = (7.5\times10^{-7}\text{F})(12\,\text{V}) = 9.0\times10^{-6}\text{C},$$

while the charge on *either* capacitor in the branch on the right is

$$q = (1.33\times10^{-6}\text{F})(12\,\text{V}) = 1.6\times10^{-5}\text{C}.$$

(b) After closing S_2 the circuit is effectively two capacitors in series. The top part has an effective capacitance of

$$C_\text{t} = (1.0\times10^{-6}\text{F}) + (2.0\times10^{-6}\text{F}) = (3.0\times10^{-6}\text{F}),$$

while the effective capacitance of the bottom part is

$$C_\text{b} = (3.0\times10^{-6}\text{F}) + (4.0\times10^{-6}\text{F}) = (7.0\times10^{-6}\text{F}).$$

The effective capacitance of the series combination is given by

$$\frac{1}{C_\text{eq}} = \frac{1}{(3.0\times10^{-6}\text{F})} + \frac{1}{(7.0\times10^{-6}\text{F})} = \frac{1}{2.1\times10^{-6}\text{F}}.$$

The charge on each part is $q = (2.1\times10^{-6}\text{F})(12\,\text{V}) = 2.52\times10^{-5}\text{C}$. The potential difference across the top part is

$$\Delta V_\text{t} = (2.52\times10^{-5}\text{C})/(3.0\times10^{-6}\text{F}) = 8.4\,\text{V},$$

and then the charge on the top two capacitors is $q_1 = (1.0\times10^{-6}\text{F})(8.4\,\text{V}) = 8.4\times10^{-6}\text{C}$ and $q_2 = (2.0\times10^{-6}\text{F})(8.4\,\text{V}) = 1.68\times10^{-5}\text{C}$. The potential difference across the bottom part is

$$\Delta V_\text{t} = (2.52\times10^{-5}\text{C})/(7.0\times10^{-6}\text{F}) = 3.6\,\text{V},$$

and then the charge on the top two capacitors is $q_1 = (3.0\times10^{-6}\text{F})(3.6\,\text{V}) = 1.08\times10^{-5}\text{C}$ and $q_2 = (4.0\times10^{-6}\text{F})(3.6\,\text{V}) = 1.44\times10^{-5}\text{C}$.

P30-10 Let $\Delta V = \Delta V_{xy}$. By symmetry $\Delta V_2 = 0$ and $\Delta V_1 = \Delta V_4 = \Delta V_5 = \Delta V_3 = \Delta V/2$. Suddenly the problem is *very* easy. The charges on each capacitor is q_1, except for $q_2 = 0$. Then the equivalent capacitance of the circuit is

$$C_\text{eq} = \frac{q}{\Delta V} = \frac{q_1 + q_4}{2\Delta V_1} = C_1 = 4.0\times10^{-6}\text{F}.$$

$\boxed{\textbf{P30-11}}$ (a) The charge on the capacitor with stored energy $U_0 = 4.0\,\text{J}$ is q_0, where

$$U_0 = \frac{q_0^2}{2C}.$$

When this capacitor is connected to an identical uncharged capacitor the charge is shared equally, so that the charge on either capacitor is now $q = q_0/2$. The stored energy in *one* capacitor is then

$$U = \frac{q^2}{2C} = \frac{q_0^2/4}{2C} = \frac{1}{4}U_0.$$

But there are two capacitors, so the total energy stored is $2U = U_0/2 = 2.0\,\text{J}$.

(b) Good question. Current had to flow through the connecting wires to get the charge from one capacitor to the other. Originally the second capacitor was uncharged, so the potential difference across that capacitor would have been zero, which means the potential difference across the connecting wires would have been equal to that of the first capacitor, and there would then have been energy dissipation in the wires according to

$$P = i^2 R.$$

That's where the missing energy went.

P30-12 $R = \rho L/A$ and $C = \epsilon_0 A/L$. Combining, $R = \rho\epsilon_0/C$, or

$$R = (9.40\,\Omega\cdot\text{m})(8.85\times10^{-12}\text{F/m})/(110\times10^{-12}\text{F}) = 0.756\,\Omega.$$

P30-13 (a) $u = \frac{1}{2}\epsilon_0 E^2 = e^2/32\pi^2\epsilon_0 r^4$.

(b) $U = \int u\,dV$ where $dV = 4\pi r^2 dr$. Then

$$U = 4pi\int_R^\infty \frac{e^2}{32\pi^2\epsilon_0 r^4}r^2\,dr = \frac{e^2}{8\pi\epsilon_0}\frac{1}{R}.$$

(c) $R = e^2/8\pi\epsilon_0 mc^2$, or

$$R = \frac{(1.60\times10^{-19}\text{C})^2}{8\pi(8.85\times10^{-12}\text{F/m})(9.11\times10^{-31}\text{kg})(3.00\times10^8\text{m/s})^2} = 1.40\times10^{-15}\text{m}.$$

P30-14 $U = \frac{1}{2}q^2/C = q^2x/2A\epsilon_0$. $F = dU/dx = q^2/2A\epsilon_0$.

$\boxed{\textbf{P30-15}}$ According to Problem 14, the force on a plate of a parallel plate capacitor is

$$F = \frac{q^2}{2\epsilon_0 A}.$$

The force per unit area is then

$$\frac{F}{A} = \frac{q^2}{2\epsilon_0 A^2} = \frac{\sigma^2}{2\epsilon_0},$$

where $\sigma = q/A$ is the surface charge density. But we know that the electric field near the surface of a conductor is given by $E = \sigma/\epsilon_0$, so

$$\frac{F}{A} = \frac{1}{2}\epsilon_0 E^2.$$

P30-16 A small surface area element dA carries a charge $dq = q\,dA/4\pi R^2$. There are three forces on the elements which balance, so

$$p(V_0/V)dA + q\,dq/4\pi\epsilon_0 R^2 = p\,dA,$$

or

$$pR_0^3 + q^2/16\pi^2\epsilon_0 R = pR^3.$$

This can be rearranged as

$$q^2 = 16\pi^2\epsilon_0 pR(R^3 - R_0^3).$$

P30-17 The magnitude of the electric field in the cylindrical region is given by $E = \lambda/2\pi\epsilon_0 r$, where λ is the linear charge density on the anode. The potential difference is given by $\Delta V = (\lambda/2\pi\epsilon_0)\ln(b/a)$, where a is the radius of the anode b the radius of the cathode. Combining, $E = \Delta V/r\ln(b/a)$, this will be a maximum when $r = a$, so

$$\Delta V = (0.180\times10^{-3}\mathrm{m})\ln[(11.0\times10^{-3}\mathrm{m})/(0.180\times10^{-3}\mathrm{m})](2.20\times10^{6}\mathrm{V/m}) = 1630\,\mathrm{V}.$$

P30-18 This is effectively two capacitors in parallel, each with an area of $A/2$. Then

$$C_{\mathrm{eq}} = \kappa_{\mathrm{e1}}\frac{\epsilon_0 A/2}{d} + \kappa_{\mathrm{e2}}\frac{\epsilon_0 A/2}{d} = \frac{\epsilon_0 A}{d}\left(\frac{\kappa_{\mathrm{e1}}+\kappa_{\mathrm{e2}}}{2}\right).$$

P30-19 We will treat the system as two capacitors in series by pretending there is an infinitesimally thin conductor between them. The slabs are (I assume) the same thickness. The capacitance of one of the slabs is then given by Eq. 30-31,

$$C_1 = \frac{\kappa_{\mathrm{e1}}\epsilon_0 A}{d/2},$$

where $d/2$ is the thickness of the slab. There would be a similar expression for the other slab. The equivalent series capacitance would be given by Eq. 30-21,

$$\begin{aligned}
\frac{1}{C_{\mathrm{eq}}} &= \frac{1}{C_1} + \frac{1}{C_2},\\
&= \frac{d/2}{\kappa_{\mathrm{e1}}\epsilon_0 A} + \frac{d/2}{\kappa_{\mathrm{e2}}\epsilon_0 A},\\
&= \frac{d}{2\epsilon_0 A}\frac{\kappa_{\mathrm{e2}}+\kappa_{\mathrm{e1}}}{\kappa_{\mathrm{e1}}\kappa_{\mathrm{e2}}},\\
C_{\mathrm{eq}} &= \frac{2\epsilon_0 A}{d}\frac{\kappa_{\mathrm{e1}}\kappa_{\mathrm{e2}}}{\kappa_{\mathrm{e2}}+\kappa_{\mathrm{e1}}}.
\end{aligned}$$

P30-20 Treat this as three capacitors. Find the equivalent capacitance of the series combination on the right, and then add on the parallel part on the left. The right hand side is

$$\frac{1}{C_{\mathrm{eq}}} = \frac{d}{\kappa_{\mathrm{e2}}\epsilon_0 A/2} + \frac{d}{\kappa_{\mathrm{e3}}\epsilon_0 A/2} = \frac{2d}{\epsilon_0 A}\left(\frac{\kappa_{\mathrm{e2}}+\kappa_{\mathrm{e3}}}{\kappa_{\mathrm{e2}}\kappa_{\mathrm{e3}}}\right).$$

Add this to the left hand side, and

$$\begin{aligned}
C_{\mathrm{eq}} &= \frac{\kappa_{\mathrm{e1}}\epsilon_0 A/2}{2d} + \frac{\epsilon_0 A}{2d}\left(\frac{\kappa_{\mathrm{e2}}\kappa_{\mathrm{e3}}}{\kappa_{\mathrm{e2}}+\kappa_{\mathrm{e3}}}\right),\\
&= \frac{\epsilon_0 A}{2d}\left(\frac{\kappa_{\mathrm{e1}}}{2} + \frac{\kappa_{\mathrm{e2}}\kappa_{\mathrm{e3}}}{\kappa_{\mathrm{e2}}+\kappa_{\mathrm{e3}}}\right).
\end{aligned}$$

P30-21 (a) q doesn't change, but $C' = C/2$. Then $\Delta V' = q/C = 2\Delta V$.
(b) $U = C(\Delta V)^2/2 = \epsilon_0 A(\Delta V)^2/2d$. $U' = C'(\Delta V')^2/2 = \epsilon_0 A(2\Delta V)^2/4d = 2U$.
(c) $W = U' - U = 2U - U = U = \epsilon_0 A(\Delta V)^2/2d$.

P30-22 The total energy is $U = q\delta V/2 = (7.02\times10^{-10}\text{C})(52.3\,\text{V})/2 = 1.84\times10^{-8}\text{J}$.
(a) In the air gap we have

$$U_\text{a} = \frac{\epsilon_0 E_0^2 V}{2} = \frac{(8.85\times10^{-12}\text{F/m})(6.9\times10^3\text{V/m})^2(1.15\times10^{-2}\text{m}^2)(4.6\times10^{-3}\text{m})}{2} = 1.11\times10^{-8}\text{J}.$$

That is $(1.11/1.85) = 60\%$ of the total.
(b) The remaining 40% is in the slab.

P30-23 (a) $C = \epsilon_0 A/d = (8.85\times10^{-12}\text{F/m})(0.118\,\text{m}^2)/(1.22\times10^{-2}\text{m}) = 8.56\times10^{-11}\text{F}$.
(b) Use the results of Problem 30-24.

$$C' = \frac{(4.8)(8.85\times10^{-12}\text{F/m})(0.118\,\text{m}^2)}{(4.8)(1.22\times10^{-2}\text{m}) - (4.3\times10^{-3}\text{m})(4.8-1)} = 1.19\times10^{-10}\text{F}$$

(c) $q = C\Delta V = (8.56\times10^{-11}\text{F})(120\,\text{V}) = 1.03\times10^{-8}\text{C}$; since the battery is disconnected $q' = q$.
(d) $E = q/\epsilon_0 A = (1.03\times10^{-8}\text{C})/(8.85\times10^{-12}\text{F/m})(0.118\,\text{m}^2) = 9860\,\text{V/m}$ in the space between the plates.
(e) $E' = E/\kappa_\text{e} = (9860\,\text{V/m})/(4.8) = 2050\,\text{V/m}$ in the dielectric.
(f) $\Delta V' = q/C' = (1.03\times10^{-8}\text{C})/(1.19\times10^{-10}\text{F}) = 86.6\,\text{V}$.
(g) $W = U' - U = q^2(1/C - 1/C')/2$, or

$$W = \frac{(1.03\times10^{-8}\text{C})^2}{2}[1/(8.56\times10^{-11}\text{F}) - 1/(1.19\times10^{-10}\text{F})] = 1.73\times10^{-7}\text{J}.$$

P30-24 The result is effectively three capacitors in series. Two are air filled with thicknesses of x and $d-b-x$, the third is dielectric filled with thickness b. All have an area A. The effective capacitance is given by

$$\begin{aligned}
\frac{1}{C} &= \frac{x}{\epsilon_0 A} + \frac{d-b-x}{\epsilon_0 A} + \frac{b}{\kappa_\text{e}\epsilon_0 A},\\
&= \frac{1}{\epsilon_0 A}\left((d-b) + \frac{b}{\kappa_\text{e}}\right),\\
C &= \frac{\epsilon_0 A}{d - b + b/\kappa_\text{e}},\\
&= \frac{\kappa_\text{e}\epsilon_0 A}{\kappa_\text{e} - b(\kappa_\text{e}-1)}.
\end{aligned}$$

E31-1 $(5.12\,\mathrm{A})(6.00\,\mathrm{V})(5.75\ \mathrm{min})(60\ \mathrm{s/min}) = 1.06\times10^{4}\mathrm{J}$.

E31-2 (a) $(12.0\,\mathrm{V})(1.60\times10^{-19}\mathrm{C}) = 1.92\times10^{-18}\mathrm{J}$.
(b) $(1.92\times10^{-18}\mathrm{J})(3.40\times10^{18}/\mathrm{s}) = 6.53\,\mathrm{W}$.

$\boxed{\textbf{E31-3}}$ If the energy is delivered at a rate of 110 W, then the current through the battery is

$$i = \frac{P}{\Delta V} = \frac{(110\,\mathrm{W})}{(12\,\mathrm{V})} = 9.17\,\mathrm{A}.$$

Current is the flow of charge in some period of time, so

$$\Delta t = \frac{\Delta q}{i} = \frac{(125\,\mathrm{A}\cdot\mathrm{h})}{(9.2\,\mathrm{A})} = 13.6\ \mathrm{h},$$

which is the same as 13 hours and 36 minutes.

E31-4 $(100\,\mathrm{W})(8\ \mathrm{h}) = 800\,\mathrm{W}\cdot\mathrm{h}$.
(a) $(800\,\mathrm{W}\cdot\mathrm{h})/(2.0\,\mathrm{W}\cdot\mathrm{h}) = 400$ batteries, at a cost of $(400)(\$0.80) = \$320$.
(b) $(800\,\mathrm{W}\cdot\mathrm{h})(\$0.12\times10^{-3}\,\mathrm{W}\cdot\mathrm{h}) = \$0.096$.

$\boxed{\textbf{E31-5}}$ Go all of the way around the circuit. It is a simple one loop circuit, and although it does not matter which way we go around, we will follow the direction of the larger emf. Then

$$(150\,\mathrm{V}) - i(2.0\,\Omega) - (50\,\mathrm{V}) - i(3.0\,\Omega) = 0,$$

where i is positive if it is counterclockwise. Rearranging,

$$100\,\mathrm{V} = i(5.0\,\Omega),$$

or $i = 20\,\mathrm{A}$.

Assuming the potential at P is $V_P = 100\,\mathrm{V}$, then the potential at Q will be given by

$$V_Q = V_P - (50\,\mathrm{V}) - i(3.0\,\Omega) = (100\,\mathrm{V}) - (50\,\mathrm{V}) - (20\,\mathrm{A})(3.0\,\Omega) = -10\,\mathrm{V}.$$

E31-6 (a) $R_{\mathrm{eq}} = (10\,\Omega) + (140\,\Omega) = 150\,\Omega$. $i = (12.0\,\mathrm{V})/(150\,\Omega) = 0.080\,\mathrm{A}$.
(b) $R_{\mathrm{eq}} = (10\,\Omega) + (80\,\Omega) = 90\,\Omega$. $i = (12.0\,\mathrm{V})/(90\,\Omega) = 0.133\,\mathrm{A}$.
(c) $R_{\mathrm{eq}} = (10\,\Omega) + (20\,\Omega) = 30\,\Omega$. $i = (12.0\,\mathrm{V})/(30\,\Omega) = 0.400\,\mathrm{A}$.

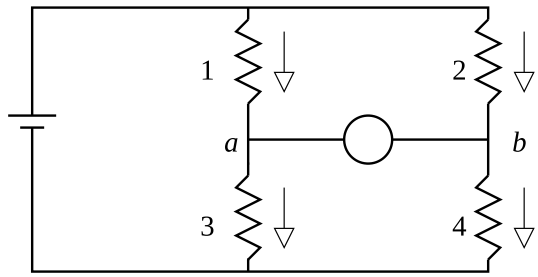
E31-7 (a) $R_{\mathrm{eq}} = (3.0\,\mathrm{V} - 2.0\,\mathrm{V})/(0.050\,\mathrm{A}) = 20\,\Omega$. Then $R = (20\,\Omega) - (3.0\,\Omega) - (3.0\,\Omega) = 14\,\Omega$.
(b) $P = i\Delta V = i^2R = (0.050\,\mathrm{A})^2(14\,\Omega) = 3.5\times10^{-2}\mathrm{W}$.

E31-8 $(5.0\,\mathrm{A})R_1 = \Delta V$. $(4.0\,\mathrm{A})(R_1 + 2.0\,\Omega) = \Delta V$. Combining, $5R_1 = 4R_1 + 8.0\,\Omega$, or $R_1 = 8.0\,\Omega$.

E31-9 (a) $(53.0\,\mathrm{W})/(1.20\,\mathrm{A}) = 44.2\,\mathrm{V}$.
(b) $(1.20\,\mathrm{A})(19.0\,\Omega) = 22.8\,\mathrm{V}$ is the potential difference across R. Then an additional potential difference of $(44.2\,\mathrm{V}) - (22.8\,\mathrm{V}) = 21.4\,\mathrm{V}$ must exist across C.
(c) The left side is positive; it is a reverse emf.

E31-10 (a) The current in the resistor is $\sqrt{(9.88\,\mathrm{W})/(0.108\,\Omega)} = 9.56\,\mathrm{A}$. The total resistance of the circuit is $(1.50\,\mathrm{V})/(9.56\,\mathrm{A}) = 0.157\,\Omega$. The internal resistance of the battery is then $(0.157\,\Omega) - (0.108\,\Omega) = 0.049\,\Omega$.
(b) $(9.88\,\mathrm{W})/(9.56\,\mathrm{A}) = 1.03\,\mathrm{V}$.

E31-11 We assign directions to the currents through the four resistors as shown in the figure.



Since the ammeter has no resistance the potential at a is the same as the potential at b. Consequently the potential difference ($\Delta V_{\rm b}$) across both of the bottom resistors is the same, and the potential difference ($\Delta V_{\rm t}$) across the two top resistors is also the same (but different from the bottom). We then have the following relationships:

$$\begin{aligned} \Delta V_{\rm t} + \Delta V_{\rm b} &= \mathcal{E}, \\ i_1 + i_2 &= i_3 + i_4, \\ \Delta V_j &= i_j R_j, \end{aligned}$$

where the j subscript in the last line refers to resistor 1, 2, 3, or 4.

For the top resistors,

$$\Delta V_1 = \Delta V_2 \text{ implies } 2i_1 = i_2;$$

while for the bottom resistors,

$$\Delta V_3 = \Delta V_4 \text{ implies } i_3 = i_4.$$

Then the junction rule requires $i_4 = 3i_1/2$, and the loop rule requires

$$(i_1)(2R) + (3i_1/2)(R) = \mathcal{E} \text{ or } i_1 = 2\mathcal{E}/(7R).$$

The current that flows through the ammeter is the difference between i_2 and i_4, or $4\mathcal{E}/(7R) - 3\mathcal{E}/(7R) = \mathcal{E}/(7R)$.

E31-12 (a) Define the current i_1 as moving to the left through r_1 and the current i_2 as moving to the left through r_2. $i_3 = i_1 + i_2$ is moving to the right through R. Then there are two loop equations:

$$\begin{aligned} \mathcal{E}_1 &= i_1 r_1 + i_3 R, \\ \mathcal{E}_2 &= (i_3 - i_1) r_2 + i_3 R. \end{aligned}$$

Multiply the top equation by r_2 and the bottom by r_1 and then add:

$$r_2\mathcal{E}_1 + r_1\mathcal{E}_2 = i_3 r_1 r_2 + i_3 R(r_1 + r_2),$$

which can be rearranged as

$$i_3 = \frac{r_2\mathcal{E}_1 + r_1\mathcal{E}_2}{r_1 r_2 + R r_1 + R r_2}.$$

(b) There is only one current, so

$$\mathcal{E}_1 + \mathcal{E}_2 = i(r_1 + r_2 + R),$$

or

$$i = \frac{\mathcal{E}_1 + \mathcal{E}_2}{r_1 + r_2 + R}.$$

$\boxed{\textbf{E31-13}}$ (a) Assume that the current flows through each source of emf in the same direction as the emf. The the loop rule will give us three equations

$$\begin{aligned}
\mathcal{E}_1 - i_1 R_1 + i_2 R_2 - \mathcal{E}_2 - i_1 R_1 &= 0, \\
\mathcal{E}_2 - i_2 R_2 + i_3 R_1 - \mathcal{E}_3 + i_3 R_1 &= 0, \\
\mathcal{E}_1 - i_1 R_1 + i_3 R_1 - \mathcal{E}_3 + i_3 R_1 - i_1 R_1 &= 0.
\end{aligned}$$

The junction rule (looks at point a) gives us $i_1 + i_2 + i_3 = 0$. Use this to eliminate i_2 from the second loop equation,

$$\mathcal{E}_2 + i_1 R_2 + i_3 R_2 + 2i_3 R_1 - \mathcal{E}_3 = 0,$$

and then combine this with the the third equation to eliminate i_3,

$$\mathcal{E}_1 R_2 - \mathcal{E}_3 R_2 + 2i_3 R_1 R_2 + 2\mathcal{E}_2 R_1 + 2i_3 R_1 R_2 + 4i_3 R_1^2 - 2\mathcal{E}_3 R_1 = 0,$$

or

$$i_3 = \frac{2\mathcal{E}_3 R_1 + \mathcal{E}_3 R_2 - \mathcal{E}_1 R_2 - 2\mathcal{E}_2 R_1}{4R_1 R_2 + 4R_1^2} = 0.582\,\mathrm{A}.$$

Then we can find i_1 from

$$i_1 = \frac{\mathcal{E}_3 - \mathcal{E}_2 - i_3 R_2 - 2i_3 R_1}{R_2} = -0.668\,\mathrm{A},$$

where the negative sign indicates the current is *down*.

Finally, we can find $i_2 = -(i_1 + i_3) = 0.0854\,\mathrm{A}$.

(b) Start at a and go to b (final minus initial!),

$$+i_2 R_2 - \mathcal{E}_2 = -3.60\,\mathrm{V}.$$

E31-14 (a) The current through the circuit is $i = \mathcal{E}/(r+R)$. The power delivered to R is then $P = i\Delta V = i^2 R = \mathcal{E}^2 R/(r+R)^2$. Evaluate dP/dR and set it equal to zero to find the maximum. Then

$$0 = \frac{dP}{dR} = \mathcal{E}^2 R \frac{r - R}{(r+R)^3},$$

which has the solution $r = R$.

(b) When $r = R$ the power is

$$P = \mathcal{E}^2 R \frac{1}{(R+R)^2} = \frac{\mathcal{E}^2}{4r}.$$

$\boxed{\textbf{E31-15}}$ (a) We first use $P = Fv$ to find the power output by the electric motor. Then $P = (2.0\,\mathrm{N})(0.50\,\mathrm{m/s}) = 1.0\,\mathrm{W}$.

The potential difference across the motor is $\Delta V_\mathrm{m} = \mathcal{E} - ir$. The power output from the motor is the rate of energy dissipation, so $P_\mathrm{m} = \Delta V_\mathrm{m} i$. Combining these two expressions,

$$\begin{aligned}
P_\mathrm{m} &= (\mathcal{E} - ir)\, i, \\
&= \mathcal{E} i - i^2 r, \\
0 &= -i^2 r + \mathcal{E} i - P_\mathrm{m}, \\
0 &= (0.50\,\Omega) i^2 - (2.0\,\mathrm{V}) i + (1.0\,\mathrm{W}).
\end{aligned}$$

Rearrange and solve for i,

$$i = \frac{(2.0\,\mathrm{V}) \pm \sqrt{(2.0\,\mathrm{V})^2 - 4(0.50\,\Omega)(1.0\,\mathrm{W})}}{2(0.50\,\Omega)},$$

which has solutions $i = 3.4\,\text{A}$ and $i = 0.59\,\text{A}$.

(b) The potential difference across the terminals of the motor is $\Delta V_{\text{m}} = \mathcal{E} - ir$ which if $i = 3.4\,\text{A}$ yields $\Delta V_{\text{m}} = 0.3\,\text{V}$, but if $i = 0.59\,\text{A}$ yields $\Delta V_{\text{m}} = 1.7\,\text{V}$. The battery provides an emf of 2.0 V; it isn't possible for the potential difference across the motor to be larger than this, but both solutions seem to satisfy this constraint, so we will move to the next part and see what happens.

(c) So what is the significance of the two possible solutions? It is a consequence of the fact that power is related to the current squared, and with any quadratics we expect two solutions. Both are possible, but it might be that only one is stable, or even that neither is stable, and a small perturbation to the friction involved in turning the motor will cause the system to break down. We will learn in a later chapter that the effective resistance of an electric motor depends on the speed at which it is spinning, and although that won't affect the problem here as worded, it will affect the physical problem that provided the numbers in this problem!

E31-16 $r_{\text{eq}} = 4r = 4(18\,\Omega) = 72\,\Omega$. The current is $i = (27\,\text{V})/(72\,\Omega) = 0.375\,\text{A}$.

E31-17 In parallel connections of two resistors the effective resistance is less than the smaller resistance but larger than half the smaller resistance. In series connections of two resistors the effective resistance is greater than the larger resistance but less than twice the larger resistance.

Since the effective resistance of the parallel combination is less than either single resistance and the effective resistance of the series combinations is larger than either single resistance we can conclude that $3.0\,\Omega$ must have been the parallel combination and $16\,\Omega$ must have been the series combination.

The resistors are then $4.0\,\Omega$ and $12\,\Omega$ resistors.

E31-18 Points B and C are effectively the same point!

(a) The three resistors are in parallel. Then $r_{\text{eq}} = R/3$.

(b) See (a).

(c) 0, since there is no resistance between B and C.

E31-19 Focus on the loop through the battery, the $3.0\,\Omega$, and the $5.0\,\Omega$ resistors. The loop rule yields

$$(12.0\,\text{V}) = i[(3.0\,\Omega) + (5.0\,\Omega)] = i(8.0\,\Omega).$$

The potential difference across the $5.0\,\Omega$ resistor is then

$$\Delta V = i(5.0\,\Omega) = (5.0\,\Omega)(12.0\,\text{V})/(8.0\,\Omega) = 7.5\,\text{V}.$$

E31-20 Each lamp draws a current of $(500\,\text{W})/(120\,\text{V}) = 4.17\,\text{A}$. Furthermore, the fuse can support $(15\,\text{A})/(4.17\,\text{A}) = 3.60$ lamps. That is a maximum of 3.

E31-21 The current in the series combination is $i_{\text{s}} = \mathcal{E}/(R_1 + R_2)$. The power dissipated is $P_{\text{s}} = i\mathcal{E} = \mathcal{E}^2/(R_1 + R_2)$.

In a parallel arrangement R_1 dissipates $P_1 = i_1\mathcal{E} = \mathcal{E}^2/R_1$. A similar expression exists for R_2, so the total power dissipated is $P_{\text{p}} = \mathcal{E}^2(1/R_1 + 1/R_2)$.

The ratio is 5, so $5 = P_{\text{p}}/P_{\text{s}} = (1/R_1 + 1/R_2)(R_1 + R_2)$, or $5R_1R_2 = (R_1 + R_2)^2$. Solving for R_2 yields $2.618R_1$ or $0.382R_1$. Then $R_2 = 262\,\Omega$ or $R_2 = 38.2\,\Omega$.

E31-22 Combining n identical resistors in series results in an equivalent resistance of $r_{\rm eq} = nR$. Combining n identical resistors in parallel results in an equivalent resistance of $r_{\rm eq} = R/n$. If the resistors are arranged in a square array consisting of n parallel branches of n series resistors, then the effective resistance is R. Each will dissipate a power P, together they will dissipate n^2P.

So we want nine resistors, since four would be too small.

$\boxed{\textbf{E31-23}}$ (a) Work through the circuit one step at a time. We first "add" R_2, R_3, and R_4 in parallel:

$$\frac{1}{R_{\rm eff}} = \frac{1}{42.0\,\Omega} + \frac{1}{61.6\,\Omega} + \frac{1}{75.0\,\Omega} = \frac{1}{18.7\,\Omega}$$

We then "add" this resistance in series with R_1,

$$R_{\rm eff} = (112\,\Omega) + (18.7\,\Omega) = 131\,\Omega.$$

(b) The current through the battery is $i = \mathcal{E}/R = (6.22\,\mathrm{V})/(131\,\Omega) = 47.5$ mA. This is also the current through R_1, since all the current through the battery must also go through R_1.

The potential difference across R_1 is $\Delta V_1 = (47.5\text{ mA})(112\,\Omega) = 5.32\,\mathrm{V}$. The potential difference across each of the three remaining resistors is $6.22\,\mathrm{V} - 5.32\,\mathrm{V} = 0.90\,\mathrm{V}$.

The current through each resistor is then

$$\begin{aligned} i_2 &= (0.90\,\mathrm{V})/(42.0\,\Omega) = 21.4\text{ mA}, \\ i_3 &= (0.90\,\mathrm{V})/(61.6\,\Omega) = 14.6\text{ mA}, \\ i_4 &= (0.90\,\mathrm{V})/(75.0\,\Omega) = 12.0\text{ mA}. \end{aligned}$$

E31-24 The equivalent resistance of the parallel part is $r' = R_2R/(R_2 + R)$. The equivalent resistance for the circuit is $r = R_1 + R_2R/(R_2 + R)$. The current through the circuit is $i' = \mathcal{E}/r$. The potential difference across R is $\Delta V = \mathcal{E} - i'R_1$, or

$$\begin{aligned} \Delta V &= \mathcal{E}(1 - R_1/r), \\ &= \mathcal{E}\left(1 - R_1\frac{R_2 + R}{R_1R_2 + R_1R + RR_2}\right), \\ &= \mathcal{E}\frac{RR_2}{R_1R_2 + R_1R + RR_2}. \end{aligned}$$

Since $P = i\Delta V = (\Delta V)^2/R$,

$$P = \mathcal{E}^2\frac{RR_2^2}{(R_1R_2 + R_1R + RR_2)^2}.$$

Set $dP/dR = 0$, the solution is $R = R_1R_2/(R_1 + R_2)$.

$\boxed{\textbf{E31-25}}$ (a) First "add" the left two resistors in series; the effective resistance of that branch is $2R$. Then "add" the right two resistors in series; the effective resistance of that branch is also $2R$.

Now we combine the three parallel branches and find the effective resistance to be

$$\frac{1}{R_{\rm eff}} = \frac{1}{2R} + \frac{1}{R} + \frac{1}{2R} = \frac{4}{2R},$$

or $R_{\rm eff} = R/2$.

(b) First we "add" the right two resistors in series; the effective resistance of that branch is $2R$. We then combine this branch with the resistor which connects points F and H. This is a parallel connection, so the effective resistance is

$$\frac{1}{R_{\rm eff}} = \frac{1}{2R} + \frac{1}{R} = \frac{3}{2R},$$

or $2R/3$.

This value is effectively in series with the resistor which connects G and H, so the "total" is $5R/3$.

Finally, we can combine this value in parallel with the resistor that directly connects F and G according to

$$\frac{1}{R_{\text{eff}}} = \frac{1}{R} + \frac{3}{5R} = \frac{8}{5R},$$

or $R_{\text{eff}} = 5R/8$.

E31-26 The resistance of the second resistor is $r_2 = (2.4\,\text{V})/(0.001\,\text{A}) = 2400\,\Omega$. The potential difference across the first resistor is $(12\,\text{V}) - (2.4\,\text{V}) = 9.6\,\text{V}$. The resistance of the first resistor is $(9.6\,\text{V})/(0.001\,\text{A}) = 9600\,\Omega$.

E31-27 See Exercise 31-26. The resistance ratio is

$$\frac{r_1}{r_1 + r_2} = \frac{(0.95 \pm 0.1\,\text{V})}{(1.50\,\text{V})},$$

or

$$\frac{r_2}{r_1} = \frac{(1.50\,\text{V})}{(0.95 \pm 0.1\,\text{V})} - 1.$$

The allowed range for the ratio r_2/r_1 is between 0.5625 and 0.5957.

We can choose any standard resistors we want, and we could use any tolerance, but then we will need to check our results. 22Ω and 39Ω would work; as would 27Ω and 47Ω. There are other choices.

$\boxed{\textbf{E31-28}}$ Consider any junction other than A or B. Call this junction point 0; label the four nearest junctions to this as points 1, 2, 3, and 4. The current through the resistor that links point 0 to point 1 is $i_1 = \Delta V_{01}/R$, where ΔV_{01} is the potential difference across the resistor, so $\Delta V_{01} = V_0 - V_1$, where V_0 is the potential at the junction 0, and V_1 is the potential at the junction 1. Similar expressions exist for the other three resistor.

For the junction 0 the net current must be zero; there is no way for charge to accumulate on the junction. Then $i_1 + i_2 + i_3 + i_4 = 0$, and this means

$$\Delta V_{01}/R + \Delta V_{02}/R + \Delta V_{03}/R + \Delta V_{04}/R = 0$$

or

$$\Delta V_{01} + \Delta V_{02} + \Delta V_{03} + \Delta V_{04} = 0.$$

Let $\Delta V_{0i} = V_0 - V_i$, and then rearrange,

$$4V_0 = V_1 + V_2 + V_3 + V_4,$$

or

$$V_0 = \frac{1}{4}\left(V_1 + V_2 + V_3 + V_4\right).$$

$\boxed{\textbf{E31-29}}$ The current through the radio is $i = P/\Delta V = (7.5\,\text{W})/(9.0\,\text{V}) = 0.83\,\text{A}$. The radio was left one for 6 hours, or 2.16×10^4 s. The total charge to flow through the radio in that time is $(0.83\,\text{A})(2.16\times10^4\,\text{s}) = 1.8\times10^4\,\text{C}$.

E31-30 The power dissipated by the headlights is $(9.7\,\text{A})(12.0\,\text{V}) = 116\,\text{W}$. The power required by the engine is $(116\,\text{W})/(0.82) = 142\,\text{W}$, which is equivalent to 0.190 hp.

E31-31 (a) $P = (120\,\text{V})(120\,\text{V})/(14.0\,\Omega) = 1030\,\text{W}$.

(b) $W = (1030\,\text{W})(6.42\text{ h}) = 6.61\text{ kW}\cdot\text{h}$. The cost is \$0.345.

E31-32

$\boxed{\textbf{E31-33}}$ We want to apply either Eq. 31-21,

$$P_R = i^2 R,$$

or Eq. 31-22,

$$P_R = (\Delta V_R)^2/R,$$

depending on whether we are in series (the current is the same through each bulb), or in parallel (the potential difference across each bulb is the same. The brightness of a bulb will be measured by P, even though P is not necessarily a measure of the rate radiant energy is emitted from the bulb.

(b) If the bulbs are in parallel then $P_R = (\Delta V_R)^2/R$ is how we want to compare the brightness. The potential difference across each bulb is the same, so the bulb with the smaller resistance is brighter.

(b) If the bulbs are in series then $P_R = i^2 R$ is how we want to compare the brightness. Both bulbs have the same current, so the larger value of R results in the brighter bulb.

One direct consequence of this can be tried at home. Wire up a 60 W, 120 V bulb and a 100 W, 120 V bulb in series. Which is brighter? You should observe that the 60 W bulb will be brighter.

E31-34 (a) $j = i/A = (25\,\text{A})/\pi(0.05\text{ in}) = 3180\text{ A/in}^2 = 4.93\times10^6\text{A/m}^2$.

(b) $E = \rho j = (1.69\times10^{-8}\Omega\cdot\text{m})(4.93\times10^6\text{A/m}^2) = 8.33\times10^{-2}\text{V/m}$.

(c) $\Delta V = Ed = (8.33\times10^{-2}\text{V/m})(305\,\text{m}) = 25\,\text{V}$.

(d) $P = i\Delta V = (25\,\text{A})(25\,\text{V}) = 625\,\text{W}$.

E31-35 (a) The bulb is on for 744 hours. The energy consumed is $(100\,\text{W})(744\text{ h}) = 74.4\text{ kW}\cdot\text{h}$, at a cost of $(74.4)(0.06) = \$4.46$.

(b) $r = V^2/P = (120\,\text{V})^2/(100\,\text{W}) = 144\,\Omega$.

(c) $i = P/V = (100\,\text{W})/(120\,\text{V}) = 0.83\,\text{A}$.

E31-36 $P = (\Delta V)^2/r$ and $r = r_0(1 + \alpha\Delta T)$. Then

$$P = \frac{P_0}{1+\alpha\Delta T} = \frac{(500\,\text{W})}{1 + (4.0\times10^{-4}/\text{C}^\circ)(-600\text{C}^\circ)} = 660\,\text{W}$$

E31-37 (a) $n = q/e = it/e$, so

$$n = (485\times10^{-3}\text{A})(95\times10^{-9}\text{s})/(1.6\times10^{-19}\text{C}) = 2.88\times10^{11}.$$

(b) $i_{\text{av}} = (520/\text{s})(485\times10^{-3}\text{A})(95\times10^{-9}\text{s}) = 2.4\times10^{-5}\text{A}$.

(c) $P_\text{p} = i_\text{p}\Delta V = (485\times10^{-3}\text{A})(47.7\times10^6\text{V}) = 2.3\times10^6\text{W}$; while $P_\text{a} = i_\text{a}\Delta V = (2.4\times10^{-5}\text{A})(47.7\times10^6\text{V}) = 1.14\times10^3\text{W}$.

E31-38 $r = \rho L/A = (3.5\times10^{-5}\Omega\cdot\text{m})(1.96\times10^{-2}\text{m})/\pi(5.12\times10^{-3}\text{m})^2 = 8.33\times10^{-3}\Omega$.

(a) $i = \sqrt{P/r} = \sqrt{(1.55\,\text{W})/(8.33\times10^{-3}\Omega)} = 13.6\,\text{A}$, so

$$j = i/A = (13.6\,\text{A})/\pi(5.12\times10^{-3}\text{m})^2 = 1.66\times10^5\text{A/m}^2.$$

(b) $\Delta V = \sqrt{Pr} = \sqrt{(1.55\,\text{W})(8.33\times10^{-3}\Omega)} = 0.114\,\text{V}$.

$\boxed{\textbf{E31-39}}$ (a) The current through the wire is

$$i = P/\Delta V = (4800\,\mathrm{W})/(75\,\mathrm{V}) = 64\,\mathrm{A},$$

The resistance of the wire is

$$R = \Delta V/i = (75\,\mathrm{V})/(64\,\mathrm{A}) = 1.17\,\Omega.$$

The length of the wire is then found from

$$L = \frac{RA}{\rho} = \frac{(1.17\,\Omega)(2.6\times10^{-6}\,\mathrm{m}^2)}{(5.0\times10^{-7}\,\Omega\mathrm{m})} = 6.1\,\mathrm{m}.$$

One could easily wind this much nichrome to make a toaster oven. Of course allowing 64 Amps to be drawn through household wiring will likely blow a fuse.

(b) We want to combine the above calculations into one formula, so

$$L = \frac{RA}{\rho} = \frac{A\Delta V/i}{\rho} = \frac{A(\Delta V)^2}{P\rho},$$

then

$$L = \frac{(2.6\times10^{-6}\,\mathrm{m}^2)(110\,\mathrm{V})^2}{(4800\,\mathrm{W})(5.0\times10^{-7}\,\Omega\mathrm{m})} = 13\,\mathrm{m}.$$

Hmm. We need more wire if the potential difference is increased? Does this make sense? Yes, it does. We need more wire because we need more resistance to *decrease* the current so that the same power output occurs.

E31-40 (a) The energy required to bring the water to boiling is $Q = mC\Delta T$. The time required is

$$t = \frac{Q}{0.77P} = \frac{(2.1\,\mathrm{kg})(4200\,\mathrm{J/kg})(100°\mathrm{C} - 18.5°\mathrm{C})}{0.77(420\,\mathrm{W})} = 2.22\times10^3\mathrm{s}$$

(b) The additional time required to boil half of the water away is

$$t = \frac{mL/2}{0.77P} = \frac{(2.1\,\mathrm{kg})(2.26\times10^6\mathrm{J/kg})/2}{0.77(420\,\mathrm{W})} = 7340\,\mathrm{s}.$$

$\boxed{\textbf{E31-41}}$ (a) Integrate both sides of Eq. 31-26;

$$\begin{aligned}
\int_0^q \frac{dq}{q-\mathcal{E}C} &= -\int_0^t \frac{dt}{RC},\\
\ln(q-\mathcal{E}C)|_0^q &= -\left.\frac{t}{RC}\right|_0^t,\\
\ln\left(\frac{q-\mathcal{E}C}{-\mathcal{E}C}\right) &= -\frac{t}{RC},\\
\frac{q-\mathcal{E}C}{-\mathcal{E}C} &= e^{-t/RC},\\
q &= \mathcal{E}C\left(1-e^{-t/RC}\right).
\end{aligned}$$

That wasn't so bad, was it?

(b) Rearrange Eq. 31-26 in order to get q terms on the left and t terms on the right, then integrate;

$$\begin{aligned}
\int_{q_0}^{q} \frac{dq}{q} &= -\int_0^t \frac{dt}{RC}, \\
\ln q|_{q_0}^{q} &= -\left.\frac{t}{RC}\right|_0^t, \\
\ln\left(\frac{q}{q_0}\right) &= -\frac{t}{RC}, \\
\frac{q}{q_0} &= e^{-t/RC}, \\
q &= q_0 e^{-t/RC}.
\end{aligned}$$

That wasn't so bad either, was it?

E31-42 (a) $\tau_C = RC = (1.42\times 10^6\Omega)(1.80\times 10^{-6}\text{F}) = 2.56\,\text{s}$.

(b) $q_0 = C\Delta V = (1.80\times 10^{-6}\text{F})(11.0\,\text{V}) = 1.98\times 10^{-5}\text{C}$.

(c) $t = -\tau_C \ln(1 - q/q_0)$, so

$$t = -(2.56\,\text{s}) \ln(1 - 15.5\times 10^{-6}\text{C}/1.98\times 10^{-5}\text{C}) = 3.91\,\text{s}.$$

E31-43 Solve $n = t/\tau_C = -\ln(1 - 0.99) = 4.61$.

E31-44 (a) $\Delta V = \mathcal{E}(1 - e^{-t/\tau_C})$, so

$$\tau_C = -(1.28\times 10^{-6}\text{s})/\ln(1 - 5.00\,\text{V}/13.0\,\text{V}) = 2.64\times 10^{-6}\text{s}$$

(b) $C = \tau_C/R = (2.64\times 10^{-6}\text{s})/(15.2\times 10^3\Omega) = 1.73\times 10^{-10}\text{F}$

E31-45 (a) $\Delta V = \mathcal{E}e^{-t/\tau_C}$, so

$$\tau_C = -(10.0\,\text{s})/\ln(1.06\,\text{V}/100\,\text{V}) = 2.20\,\text{s}$$

(b) $\Delta V = (100\,\text{V})e^{-17\,\text{s}/2.20\,\text{s}} = 4.4\times 10^{-2}\text{V}$.

E31-46 $\Delta V = \mathcal{E}e^{-t/\tau_C}$ and $\tau_C = RC$, so

$$R = -\frac{t}{C\ln(\Delta V/\Delta V_0)} = -\frac{t}{(220\times 10^{-9}\text{F})\ln(0.8\,\text{V}/5\,\text{V})} = \frac{t}{4.03\times 10^{-7}\text{F}}.$$

If t is between $10.0\,\mu\text{s}$ and $6.0\,\text{ms}$, then R is between

$$R = (10\times 10^{-6}\text{s})/(4.03\times 10^{-7}\text{F}) = 24.8\Omega,$$

and

$$R = (6\times 10^{-3}\text{s})/(4.03\times 10^{-7}\text{F}) = 14.9\times 10^3\Omega.$$

$\boxed{\textbf{E31-47}}$ The charge on the capacitor needs to build up to a point where the potential across the capacitor is $V_L = 72\,\text{V}$, and this needs to happen within 0.5 seconds. This means that we want to solve

$$C\Delta V_L = C\mathcal{E}\left(1 - e^{T/RC}\right)$$

for R knowing that $T = 0.5\,\text{s}$. This expression can be written as

$$R = -\frac{T}{C\ln(1 - V_L/\mathcal{E})} = -\frac{(0.5\,\text{s})}{(0.15\,\mu\text{C})\ln(1 - (72\,\text{V})/(95\,\text{V}))} = 2.35\times 10^6\Omega.$$

E31-48 (a) $q_0 = \sqrt{2UC} = \sqrt{2(0.50\,\mathrm{J})(1.0\times10^{-6}\mathrm{F})} = 1\times10^{-3}\mathrm{C}$.

(b) $i_0 = \Delta V_0/R = q_0/RC = (1\times10^{-3}\mathrm{C})/(1.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F}) = 1\times10^{-3}\mathrm{A}$.

(c) $\Delta V_C = \Delta V_0 e^{-t/\tau_C}$, so

$$\Delta V_C = \frac{(1\times10^{-3}\mathrm{C})}{(1.0\times10^{-6}\mathrm{F})} e^{-t/(1.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})} = (1000\,\mathrm{V})e^{-t/(1.0\,\mathrm{s})}$$

Note that $\Delta V_R = \Delta V_C$.

(d) $P_R = (\Delta V_R)^2/R$, so

$$P_R = (1000\,\mathrm{V})^2 e^{-2t/(1.0\,\mathrm{s})}/(1\times10^{6}\omega) = (1\,\mathrm{W})e^{-2t/(1.0\,\mathrm{s})}.$$

E31-49 (a) $i = dq/dt = \mathcal{E}e^{-t/\tau_C}/R$, so

$$i = \frac{(4.0\,\mathrm{V})}{(3.0\times10^{6}\Omega)} e^{-(1.0\,\mathrm{s})/(3.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})} = 9.55\times10^{-7}\mathrm{A}.$$

(b) $P_C = i\Delta V = (\mathcal{E}^2/R)e^{-t/\tau_C}(1-e^{-t/\tau_C})$, so

$$P_C = \frac{(4.0\,\mathrm{V})^2}{(3.0\times10^{6}\Omega)} e^{-(1.0\,\mathrm{s})/(3.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})}\left(1-e^{-(1.0\,\mathrm{s})/(3.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})}\right) = 1.08\times10^{-6}\mathrm{W}.$$

(c) $P_R = i^2R = (\mathcal{E}^2/R)e^{-2t/\tau_C}$, so

$$P_R = \frac{(4.0\,\mathrm{V})^2}{(3.0\times10^{6}\Omega)} e^{-2(1.0\,\mathrm{s})/(3.0\times10^{6}\Omega)(1.0\times10^{-6}\mathrm{F})} = 2.74\times10^{-6}\mathrm{W}.$$

(d) $P = P_R + P_C$, or

$$P = 2.74\times10^{-6}\mathrm{W} + 1.08\times10^{-6}\mathrm{W} = 3.82\times10^{-6}\mathrm{W}$$

E31-50 The rate of energy dissipation in the resistor is

$$P_R = i^2R = (\mathcal{E}^2/R)e^{-2t/\tau_C}.$$

Evaluating

$$\int_0^\infty P_R dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2t/RC}dt = \frac{\mathcal{E}^2}{2}C,$$

but that is the original energy stored in the capacitor.

$\boxed{\textbf{P31-1}}$ The terminal voltage of the battery is given by $V = \mathcal{E} - ir$, so the internal resistance is

$$r = \frac{\mathcal{E}-V}{i} = \frac{(12.0\,\mathrm{V})-(11.4\,\mathrm{V})}{(50\,\mathrm{A})} = 0.012\,\Omega,$$

so the battery appears within specs.

The resistance of the wire is given by

$$R = \frac{\Delta V}{i} = \frac{(3.0\,\mathrm{V})}{(50\,\mathrm{A})} = 0.06\,\Omega,$$

so the cable appears to be bad.

What about the motor? Trying it,

$$R = \frac{\Delta V}{i} = \frac{(11.4\,\mathrm{V})-(3.0\,\mathrm{V})}{(50\,\mathrm{A})} = 0.168\,\Omega,$$

so it appears to be within spec.

P31-2 Traversing the circuit we have

$$\mathcal{E} - ir_1 + \mathcal{E} - ir_2 - iR = 0,$$

so $i = 2\mathcal{E}/(r_1 + r_2 + R)$. The potential difference across the first battery is then

$$\Delta V_1 = \mathcal{E} - ir_1 = \mathcal{E}\left(1 - \frac{2r_1}{r_1 + r_2 + R}\right) = \mathcal{E}\frac{r_2 - r_1 + R}{r_1 + r_2 + R}$$

This quantity will only vanish if $r_2 - r_1 + R = 0$, or $r_1 = R + r_2$. Since $r_1 > r_2$ this is actually possible; $R = r_1 - r_2$.

P31-3 $\Delta V = \mathcal{E} - ir_\mathrm{i}$ and $i = \mathcal{E}/(r_\mathrm{i} + R)$, so

$$\Delta V = \mathcal{E}\frac{R}{r_\mathrm{i} + R},$$

There are then two simultaneous equations:

$$(0.10\,\mathrm{V})(500\,\Omega) + (0.10\,\mathrm{V})r_\mathrm{i} = \mathcal{E}(500\,\Omega)$$

and

$$(0.16\,\mathrm{V})(1000\,\Omega) + (0.16\,\mathrm{V})r_\mathrm{i} = \mathcal{E}(1000\,\Omega),$$

with solution

(a) $r_\mathrm{i} = 1.5\times10^3\Omega$ and

(b) $\mathcal{E} = 0.400\,\mathrm{V}$.

(c) The cell receives energy from the sun at a rate $(2.0\ \mathrm{mW/cm}^2)(5.0\ \mathrm{cm}^2) = 0.010\,\mathrm{W}$. The cell converts energy at a rate of $V^2/R = (0.16\,\mathrm{V})^2/(1000\,\Omega) = 0.26\,\%$

P31-4 (a) The emf of the battery can be found from

$$\mathcal{E} = ir_\mathrm{i} + \Delta V_\mathrm{l} = (10\,\mathrm{A})(0.05\,\Omega) + (12\,\mathrm{V}) = 12.5\,\mathrm{V}$$

(b) Assume that resistance is *not* a function of temperature. The resistance of the headlights is then

$$r_\mathrm{l} = (12.0\,\mathrm{V})/(10.0\,\mathrm{A}) = 1.2\,\Omega.$$

The potential difference across the lights when the starter motor is on is

$$\Delta V_\mathrm{l} = (8.0\,\mathrm{A})(1.2\,\Omega) = 9.6\,\mathrm{V},$$

and this is also the potential difference across the terminals of the battery. The current through the battery is then

$$i = \frac{\mathcal{E} - \Delta V}{r_\mathrm{i}} = \frac{(12.5\,\mathrm{V}) - (9.6\,\mathrm{V})}{(0.05\,\Omega)} = 58\,\mathrm{A},$$

so the current through the motor is 50 Amps.

P31-5 (a) The resistivities are

$$\rho_A = r_A A/L = (76.2\times10^{-6}\Omega)(91.0\times10^{-4}\mathrm{m}^2)/(42.6\,\mathrm{m}) = 1.63\times10^{-8}\Omega\cdot\mathrm{m},$$

and

$$\rho_B = r_B A/L = (35.0\times10^{-6}\Omega)(91.0\times10^{-4}\mathrm{m}^2)/(42.6\,\mathrm{m}) = 7.48\times10^{-9}\Omega\cdot\mathrm{m}.$$

(b) The current is $i = \Delta V/(r_A + r_B) = (630\,\mathrm{V})/(111.2\,\mu\Omega) = 5.67\times10^6\mathrm{A}$. The current density is then

$$j = (5.67\times10^6\mathrm{A})/(91.0\times10^{-4}\mathrm{m}^2) = 6.23\times10^8\mathrm{A/m}^2.$$

(c) $E_A = \rho_A j = (1.63\times10^{-8}\Omega\cdot\mathrm{m})(6.23\times10^8\mathrm{A/m}^2) = 10.2\,\mathrm{V/m}$ and $E_B = \rho_B j = (7.48\times10^{-9}\Omega\cdot\mathrm{m})(6.23\times10^8\mathrm{A/m}^2) = 4.66\,\mathrm{V/m}$.

(d) $\Delta V_A = E_A L = (10.2\,\mathrm{V/m})(42.6\,\mathrm{m}) = 435\,\mathrm{V}$ and $\Delta V_B = E_B L = (4.66\,\mathrm{V/m})(42.6\,\mathrm{m}) = 198\,\mathrm{V}$.

P31-6 Set up the problem with the traditional presentation of the Wheatstone bridge problem. Then the symmetry of the problem (flip it over on the line between x and y) implies that there is *no* current through r. As such, the problem is equivalent to two identical parallel branches each with two identical series resistances.

Each branch has resistance $R + R = 2R$, so the overall circuit has resistance

$$\frac{1}{R_{\text{eq}}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R},$$

so $R_{\text{eq}} = R$.

P31-7

P31-8 (a) The loop through R_1 is trivial: $i_1 = \mathcal{E}_2/R_1 = (5.0\,\text{V})/(100\,\Omega) = 0.05\,\text{A}$. The loop through R_2 is only slightly harder: $i_2 = (\mathcal{E}_2 + \mathcal{E}_3 - \mathcal{E}_1)/R_2 = 0.06\,\text{A}$.

(b) $\Delta V_{ab} = \mathcal{E}_3 + \mathcal{E}_2 = (5.0\,\text{V}) + (4.0\,\text{V}) = 9.0\,\text{V}$.

$\boxed{\textbf{P31-9}}$ (a) The three way light-bulb has two filaments (or so we are told in the question). There are four ways for these two filaments to be wired: either one alone, both in series, or both in parallel. Wiring the filaments in series will have the largest total resistance, and since $P = V^2/R$ this arrangement would result in the dimmest light. But we are told the light still operates at the lowest setting, and if a filament burned out in a series arrangement the light would go out.

We then conclude that the lowest setting is one filament, the middle setting is another filament, and the brightest setting is both filaments in parallel.

(b) The beauty of parallel settings is that then power is additive (it is also addictive, but that's a different field.) One filament dissipates 100 W at 120 V; the other filament (the one that burns out) dissipates 200 W at 120 V, and both together dissipate 300 W at 120 V.

The resistance of one filament is then

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\,\text{V})^2}{(100\,\text{W})} = 144\,\Omega.$$

The resistance of the other filament is

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\,\text{V})^2}{(200\,\text{W})} = 72\,\Omega.$$

P31-10 We can assume that R "contains" all of the resistance of the resistor, the battery and the ammeter, then

$$R = (1.50\,\text{V})/(1.0\text{ m/A}) = 1500\,\Omega.$$

For each of the following parts we apply $R + r = \Delta V/i$, so

(a) $r = (1.5\,\text{V})/(0.1\,\text{mA}) - (1500\,\Omega) = 1.35\times 10^4\Omega$,

(b) $r = (1.5\,\text{V})/(0.5\,\text{mA}) - (1500\,\Omega) = 1.5\times 10^3\Omega$,

(c) $r = (1.5\,\text{V})/(0.9\,\text{mA}) - (1500\,\Omega) = 167\Omega$.

(d) $R = (1500\,\Omega) - (18.5\,\Omega) = 1482\,\Omega$

P31-11 (a) The effective resistance of the parallel branches on the middle and the right is

$$\frac{R_2 R_3}{R_2 + R_3}.$$

The effective resistance of the circuit as seen by the battery is then

$$R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3},$$

The current through the battery is

$$i = \mathcal{E}\frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3},$$

The potential difference across R_1 is then

$$\Delta V_1 = \mathcal{E}\frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} R_1,$$

while $\Delta V_3 = \mathcal{E} - \Delta V_1$, or

$$\Delta V_3 = \mathcal{E}\frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3},$$

so the current through the ammeter is

$$i_3 = \frac{\Delta V_3}{R_3} = \mathcal{E}\frac{R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3},$$

or

$$i_3 = (5.0\,\mathrm{V})\frac{(4\,\Omega)}{(2\,\Omega)(4\,\Omega) + (2\,\Omega)(6\,\Omega) + (4\,\Omega)(6\,\Omega)} = 0.45\,\mathrm{A}.$$

(b) Changing the locations of the battery and the ammeter is equivalent to swapping R_1 and R_3. But since the expression for the current doesn't change, then the current is the same.

P31-12 $\Delta V_1 + \Delta V_2 = \Delta V_S + \Delta V_X$; if $V_a = V_b$, then $\Delta V_1 = \Delta V_S$. Using the first expression,

$$i_a(R_1 + R_2) = i_b(R_S + R_X),$$

using the second,

$$i_a R_1 = i_b R_2.$$

Dividing the first by the second,

$$1 + R_2/R_1 = 1 + R_X/R_S,$$

or $R_X = R_S(R_2/R_1)$.

P31-13

P31-14 $L_v = \Delta Q/\Delta m$ and $\Delta Q/\Delta t = P = i\Delta V$, so

$$L_v = \frac{i\Delta V}{\Delta m/\Delta t} = \frac{(5.2\,\mathrm{A})(12\,\mathrm{V})}{(21\times 10^{-6}\mathrm{kg/s})} = 2.97\times 10^{6}\mathrm{J/kg}.$$

P31-15 $P = i^2 R$. $W = p\Delta V$, where V is volume. $p = mg/A$ and $V = Ay$, where y is the height of the piston. Then $P = dW/dt = mgv$. Combining all of this,

$$v = \frac{i^2 R}{mg} = \frac{(0.240\,\mathrm{A})^2(550\,\Omega)}{(11.8\,\mathrm{kg})(9.8\,\mathrm{m/s^2})} = 0.274\,\mathrm{m/s}.$$

P31-16 (a) Since $q = CV$, then

$$q = (32\times10^{-6}\mathrm{F})\left[(6\,\mathrm{V}) + (4\,\mathrm{V/s})(0.5\,\mathrm{s}) - (2\,\mathrm{V/s^2})(0.5\,\mathrm{s})^2\right] = 2.4\times10^{-4}\mathrm{C}.$$

(b) Since $i = dq/dt = C\,dV/dt$, then

$$i = (32\times10^{-6}\mathrm{F})\left[(4\,\mathrm{V/s}) - 2(2\,\mathrm{V/s^2})(0.5\,\mathrm{s})\right] = 6.4\times10^{-5}\mathrm{A}.$$

(c) Since $P = iV$,

$$P = [\left[(4\,\mathrm{V/s}) - 2(2\,\mathrm{V/s^2})(0.5\,\mathrm{s})\right]\left[(6\,\mathrm{V}) + (4\,\mathrm{V/s})(0.5\,\mathrm{s}) - (2\,\mathrm{V/s^2})(0.5\,\mathrm{s})^2\right] = 4.8\times10^{-4}\mathrm{W}.$$

$\boxed{\textbf{P31-17}}$ (a) We have $P = 30P_0$ and $i = 4i_0$. Then

$$R = \frac{P}{i^2} = \frac{30P_0}{(4i_0)^2} = \frac{30}{16}R_0.$$

We don't really care what happened with the potential difference, since knowing the change in resistance of the wire should give all the information we need.

The volume of the wire is a constant, even upon drawing the wire out, so $LA = L_0A_0$; the product of the length and the cross sectional area must be a constant.

Resistance is given by $R = \rho L/A$, but $A = L_0A_0/L$, so the length of the wire is

$$L = \sqrt{\frac{A_0L_0R}{\rho}} = \sqrt{\frac{30}{16}\frac{A_0L_0R_0}{\rho}} = 1.37L_0.$$

(b) We know that $A = L_0A_0/L$, so

$$A = \frac{L}{L_0}A_0 = \frac{A_0}{1.37} = 0.73A_0.$$

P31-18 (a) The capacitor charge as a function of time is given by Eq. 31-27,

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right),$$

while the current through the circuit (and the resistor) is given by Eq. 31-28,

$$i = \frac{\mathcal{E}}{R}e^{-t/RC}.$$

The energy supplied by the emf is

$$U = \int \mathcal{E}i\,dt = \mathcal{E}\int dq = \mathcal{E}q;$$

but the energy in the capacitor is $U_C = q\Delta V/2 = \mathcal{E}q/2$.

(b) Integrating,

$$U_R = \int i^2 R dt = \frac{\mathcal{E}^2}{R}\int e^{-2t/RC}dt = \frac{\mathcal{E}^2}{2C} = \frac{\mathcal{E}q}{2}.$$

P31-19 The capacitor charge as a function of time is given by Eq. 31-27,

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right),$$

while the current through the circuit (and the resistor) is given by Eq. 31-28,

$$i = \frac{\mathcal{E}}{R}e^{-t/RC}.$$

The energy stored in the capacitor is given by

$$U = \frac{q^2}{2C},$$

so the rate that energy is being stored in the capacitor is

$$P_C = \frac{dU}{dt} = \frac{q}{C}\frac{dq}{dt} = \frac{q}{C}i.$$

The rate of energy dissipation in the resistor is

$$P_R = i^2R,$$

so the time at which the rate of energy dissipation in the resistor is equal to the rate of energy storage in the capacitor can be found by solving

$$\begin{aligned}
P_C &= P_R, \\
i^2R &= \frac{q}{C}i, \\
iRC &= q, \\
\mathcal{E}Ce^{-t/RC} &= C\mathcal{E}\left(1 - e^{-t/RC}\right), \\
e^{-t/RC} &= 1/2, \\
t &= RC\ln 2.
\end{aligned}$$

$\boxed{\textbf{E32-1}}$ Apply Eq. 32-3, $\vec{\mathbf{F}} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$.

All of the paths which involve left hand turns are positive particles (path 1); those paths which involve right hand turns are negative particle (path 2 and path 4); and those paths which don't turn involve neutral particles (path 3).

E32-2 (a) The greatest magnitude of force is $F = qvB = (1.6\times10^{-19}\text{C})(7.2\times10^6\text{m/s})(83\times10^{-3}\text{T}) = 9.6\times10^{-14}\text{N}$. The least magnitude of force is 0.

(b) The force on the electron is $F = ma$; the angle between the velocity and the magnetic field is θ, given by $ma = qvB\sin\theta$. Then

$$\theta = \arcsin\left(\frac{(9.1\times10^{-31}\text{kg})(4.9\times10^{16}\text{m/s}^2)}{(1.6\times10^{-19}\text{C})(7.2\times10^6\text{m/s})(83\times10^{-3}\text{T})}\right) = 28^\circ.$$

E32-3 (a) $v = E/B = (1.5\times10^3\text{V/m})/(0.44\,\text{T}) = 3.4\times10^3\text{m/s}$.

E32-4 (a) $v = F/qB\sin\theta = (6.48\times10^{-17}\text{N}/(1.60\times10^{-19}\text{C})(2.63\times10^{-3}\text{T})\sin(23.0^\circ) = 3.94\times10^5\text{m/s}$.

(b) $K = mv^2/2 = (938\text{ MeV}/c^2)(3.94\times10^5\text{m/s})^2/2 = 809\text{ eV}$.

$\boxed{\textbf{E32-5}}$ The magnetic force on the proton is

$$F_B = qvB = (1.6\times10^{-19}\text{ C})(2.8\times10^7\text{ m/s})(30eex{-}6\,\text{T}) = 1.3\times10^{-16}\text{N}.$$

The gravitational force on the proton is

$$mg = (1.7\times10^{-27}\text{kg})(9.8\,\text{m/s}^2) = 1.7\times10^{-26}\text{ N}.$$

The ratio is then 7.6×10^9. If, however, you carry the number of significant digits for the intermediate answers farther you will get the answer which is in the back of the book.

E32-6 The speed of the electron is given by $v = \sqrt{2q\Delta V/m}$, or

$$v = \sqrt{2(1000\text{ eV})/(5.1\times10^5\text{ eV}/c^2)} = 0.063c.$$

The electric field between the plates is $E = (100\,\text{V})/(0.020\,\text{m}) = 5000\,\text{V/m}$. The required magnetic field is then

$$B = E/v = (5000\,\text{V/m})/(0.063c) = 2.6\times10^{-4}\text{T}.$$

E32-7 Both have the same velocity. Then $K_\text{p}/K_\text{e} = m_\text{p}v^2/m_\text{e}v^2 = m_\text{p}/m_\text{e} =$.

E32-8 The speed of the ion is given by $v = \sqrt{2q\Delta V/m}$, or

$$v = \sqrt{2(10.8\text{ keV})/(6.01)(932\text{ MeV}/c^2)} = 1.96\times10^{-3}c.$$

The required electric field is $E = vB = (1.96\times10^{-3}c)(1.22\,\text{T}) = 7.17\times10^5\text{V/m}$.

$\boxed{\textbf{E32-9}}$ (a) For a charged particle moving in a circle in a magnetic field we apply Eq. 32-10;

$$r = \frac{mv}{|q|B} = \frac{(9.11\times10^{-31}\text{kg})(0.1)(3.00\times10^8\text{ m/s})}{(1.6\times10^{-19}\text{ C})(0.50\,\text{T})} = 3.4\times10^{-4}\text{ m}.$$

(b) The (non-relativistic) kinetic energy of the electron is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.511\text{ MeV})(0.10c)^2 = 2.6\times10^{-3}\text{ MeV}.$$

E32-10 (a) $v = \sqrt{2K/m} = \sqrt{2(1.22 \text{ keV})/(511 \text{ keV}/c^2)} = 0.0691c$.
(b) $B = mv/qr = (9.11\times10^{-31}\text{kg})(0.0691c)/(1.60\times10^{-19}\text{C})(0.247\,\text{m}) = 4.78\times10^{-4}\text{T}$.
(c) $f = qB/2\pi m = (1.60\times10^{-19}\text{C})(4.78\times10^{-4}\text{T})/2\pi(9.11\times10^{-31}\text{kg}) = 1.33\times10^{7}$ Hz.
(d) $T = 1/f = 1/(1.33\times10^{7}\text{ Hz}) = 7.48\times10^{-8}\text{s}$.

E32-11 (a) $v = \sqrt{2K/m} = \sqrt{2(350 \text{ eV})/(511 \text{ keV}/c^2)} = 0.037c$.
(b) $r = mv/qB = (9.11\times10^{-31}\text{kg})(0.037c)/(1.60\times10^{-19}\text{C})(0.20\text{T}) = 3.16\times10^{-4}\text{m}$.

E32-12 The frequency is $f = (7.00)/(1.29\times10^{-3}\text{s}) = 5.43\times10^{3}$ Hz. The mass is given by $m = qB/2\pi f$, or

$$m = \frac{(1.60\times10^{-19}\text{C})(45.0\times10^{-3}\text{T})}{2\pi(5.43\times10^{3}\text{ Hz})} = 2.11\times10^{-25}\text{kg} = 127 \text{ u}.$$

$\boxed{\textbf{E32-13}}$ (a) Apply Eq. 32-10, but rearrange it as

$$v = \frac{|q|rB}{m} = \frac{2(1.6\times10^{-19}\,\text{C})(0.045\,\text{m})(1.2\,\text{T})}{4.0(1.66\times10^{-27}\text{kg})} = 2.6\times10^{6}\,\text{m/s}.$$

(b) The speed is equal to the circumference divided by the period, so

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B} = \frac{2\pi 4.0(1.66\times10^{-27}\text{kg})}{2(1.6\times10^{-19}\,\text{C})(1.2\,\text{T})} = 1.1\times10^{-7}\,\text{s}.$$

(c) The (non-relativistic) kinetic energy is

$$K = \frac{|q|^2r^2B}{2m} = \frac{(2\times1.6\times10^{-19}\,\text{C})^2(0.045\,\text{m})^2(1.2\,\text{T})^2}{2(4.0\times1.66\times10^{-27}\text{kg}))} = 2.24\times10^{-14}\text{J}.$$

To change to electron volts we need merely divide this answer by the charge on one electron, so

$$K = \frac{(2.24\times10^{-14}\text{J})}{(1.6\times10^{-19}\,\text{C})} = 140 \text{ keV}.$$

(d) $\Delta V = \frac{K}{q} = (140 \text{ keV})/(2e) = 70\,\text{V}$.

E32-14 (a) $R = mv/qB = (938 \text{ MeV}/c^2)(0.100c)/e(1.40\,\text{T}) = 0.223\,\text{m}$.
(b) $f = qB/2\pi m = e(1.40\,\text{T})/2\pi(938 \text{ MeV}/c^2) = 2.13\times10^{7}$ Hz.

E32-15 (a) $K_\alpha/K_\text{p} = (q_\alpha^2/m_\alpha)/({q_\text{p}}^2/m_\text{p}) = 2^2/4 = 1$.
(b) $K_\text{d}/K_\text{p} = ({q_\text{d}}^2/m_\text{d})/({q_\text{p}}^2/m_\text{p}) = 1^2/2 = 1/2$.

E32-16 (a) $K = q\Delta V$. Then $K_\text{p} = e\Delta V$, $K_\text{d} = e\Delta V$, and $K_\alpha = 2e\Delta V$.
(b) $r = sqrt2mK/qB$. Then $r_\text{d}/r_\text{p} = \sqrt{(2/1)(1/1)}/(1/1) = \sqrt{2}$.
(c) $r = sqrt2mK/qB$. Then $r_\alpha/r_\text{p} = \sqrt{(4/1)(2/1)}/(2/1) = \sqrt{2}$.

$\boxed{\textbf{E32-17}}$ $r = \sqrt{2mK}/|q|B = (\sqrt{m}/|q|)(\sqrt{2K}/B)$. All three particles are traveling with the same kinetic energy in the same magnetic field. The relevant factors are in front; we just need to compare the mass and charge of each of the three particles.
(a) The radius of the deuteron path is $\frac{\sqrt{2}}{1}r_\text{p}$.
(b) The radius of the alpha particle path is $\frac{\sqrt{4}}{2}r_\text{p} = r_\text{p}$.

E32-18 The neutron, being neutral, is unaffected by the magnetic field and moves off in a line tangent to the original path. The proton moves at the same original speed as the deuteron and has the same charge, but since it has half the mass it moves in a circle with half the radius.

E32-19 (a) The proton momentum would be $pc = qcBR = e(3.0{\times}10^8\text{m/s})(41{\times}10^{-6}\text{T})(6.4{\times}10^6\text{m}) = 7.9{\times}10^4$ MeV. Since 79000 MeV is much, much greater than 938 MeV the proton is ultra-relativistic. Then $E \approx pc$, and since $\gamma = E/mc^2$ we have $\gamma = p/mc$. Inverting,

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{m^2c^2}{p^2}} \approx 1 - \frac{m^2c^2}{2p^2} \approx 0.99993.$$

E32-20 (a) Classically, $R = \sqrt{2mK}/qB$, or

$$R = \sqrt{2(0.511\text{ MeV}/c^2)(10.0\text{ MeV})}/e(2.20\,\text{T}) = 4.84\times10^{-3}\text{m}.$$

(b) This would be an ultra-relativistic electron, so $K \approx E \approx pc$, then $R = p/qB = K/qBc$, or

$$R = (10.0\text{ MeV})/e(2.2\,\text{T})(3.00\times10^8\text{m/s}) = 1.52\times10^{-2}\text{m}.$$

(c) The electron is effectively traveling at the speed of light, so $T = 2\pi R/c$, or

$$T = 2\pi(1.52\times10^{-2}\text{m})/(3.00\times10^8\text{ m/s}) = 3.18\times10^{-10}\text{s}.$$

This result *does* depend on the speed!

$\boxed{\textbf{E32-21}}$ Use Eq. 32-10, except we rearrange for the mass,

$$m = \frac{|q|rB}{v} = \frac{2(1.60\times10^{-19}\,\text{C})(4.72\,\text{m})(1.33\,\text{T})}{0.710(3.00\times10^8\text{ m/s})} = 9.43\times10^{-27}\text{ kg}$$

However, if it is moving at this velocity then the "mass" which we have here is not the true mass, but a relativistic correction. For a particle moving at $0.710c$ we have

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (0.710)^2}} = 1.42,$$

so the true mass of the particle is $(9.43\times10^{-27}\text{ kg})/(1.42) = 6.64\times10^{-27}$kg. The number of nucleons present in this particle is then $(6.64\times10^{-27}\text{kg})/(1.67\times10^{-27}\text{ kg}) = 3.97 \approx 4$. The charge was $+2$, which implies two protons, the other two nucleons would be neutrons, so this must be an alpha particle.

E32-22 (a) Since 950 GeV is much, much greater than 938 MeV the proton is ultra-relativistic. $\gamma = E/mc^2$, so

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{m^2c^4}{E^2}} \approx 1 - \frac{m^2c^4}{2E^2} \approx 0.9999995.$$

(b) Ultra-relativistic motion requires $pc \approx E$, so

$$B = pc/qRc = (950\text{ GeV})/e(750\,\text{m})(3.00\times10^8\text{m/s}) = 4.44\,\text{T}.$$

E32-23 First use $2\pi f = qB/m$. The use $K = q^2B^2R^2/2m = mR^2(2\pi f)^2/2$. The number of turns is $n = K/2q\Delta V$, on average the particle is located at a distance $R/\sqrt{2}$ from the center, so the distance traveled is $x = n2\pi R/\sqrt{2} = n\sqrt{2}\pi R$. Combining,

$$x = \frac{\sqrt{2}\pi^3 R^3 m f^2}{q\Delta V} = \frac{\sqrt{2}\pi^3(0.53\,\mathrm{m})^3(2 \times 932\times 10^3 \text{ keV}/c^2)(12\times 10^6/\mathrm{s})^2}{e(80 \text{ kV})} = 240\,\mathrm{m}.$$

E32-24 The particle moves in a circle. $x = R\sin\omega t$ and $y = R\cos\omega t$.

$\boxed{\textbf{E32-25}}$ We will use Eq. 32-20, $E_\mathrm{H} = v_\mathrm{d}B$, except we will not take the derivation through to Eq. 32-21. Instead, we will set the drift velocity equal to the speed of the strip. We will, however, set $E_\mathrm{H} = \Delta V_\mathrm{H}/w$. Then

$$v = \frac{E_\mathrm{H}}{B} = \frac{\Delta V_\mathrm{H}/w}{B} = \frac{(3.9\times 10^{-6}\mathrm{V})/(0.88\times 10^{-2}\mathrm{m})}{(1.2\times 10^{-3}\mathrm{T})} = 3.7\times 10^{-1}\mathrm{m/s}.$$

E32-26 (a) $v = E/B = (40\times 10^{-6}\mathrm{V})/(1.2\times 10^{-2}\mathrm{m})/(1.4\,\mathrm{T}) = 2.4\times 10^{-3}\mathrm{m/s}$.
(b) $n = (3.2\,\mathrm{A})(1.4\,\mathrm{T})/(1.6\times 10^{-19}\mathrm{C})(9.5\times 10^{-6}\mathrm{m})(40\times 10^{-6}\mathrm{V}) = 7.4\times 10^{28}/\mathrm{m}^3$.; Silver.

E32-27 $E_\mathrm{H} = v_\mathrm{d}B$ and $v_\mathrm{d} = j/ne$. Combine and rearrange.

E32-28 (a) Use the result of the previous exercise and $E_\mathrm{c} = \rho j$.
(b) $(0.65\,\mathrm{T})/(8.49\times 10^{28}/\mathrm{m}^3)(1.60\times 10^{-19}\mathrm{C})(1.69\times 10^{-8}\Omega\cdot\mathrm{m}) = 0.0028$.

$\boxed{\textbf{E32-29}}$ Since $\vec{\mathbf{L}}$ is perpendicular to $\vec{\mathbf{B}}$ can use

$$F_B = iLB.$$

Equating the two forces,

$$\begin{aligned} iLB &= mg, \\ i &= \frac{mg}{LB} = \frac{(0.0130\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(0.620\,\mathrm{m})(0.440\,\mathrm{T})} = 0.467\,\mathrm{A}. \end{aligned}$$

Use of an appropriate right hand rule will indicate that the current must be directed to the right in order to have a magnetic force directed upward.

E32-30 $F = iLB\sin\theta = (5.12\times 10^3\mathrm{A})(100\,\mathrm{m})(58\times 10^{-6}\mathrm{T})\sin(70°) = 27.9\,\mathrm{N}$. The direction is horizontally west.

$\boxed{\textbf{E32-31}}$ (a) We use Eq. 32-26 again, and since the (horizontal) axle is perpendicular to the vertical component of the magnetic field,

$$i = \frac{F}{BL} = \frac{(10,000\,\mathrm{N})}{(10\,\mu\mathrm{T})(3.0\,\mathrm{m})} = 3.3\times 10^8\mathrm{A}.$$

(b) The power lost per ohm of resistance in the rails is given by

$$P/r = i^2 = (3.3\times 10^8\mathrm{A})^2 = 1.1\times 10^{17}\,\mathrm{W}.$$

(c) If such a train were to be developed the rails would melt well before the train left the station.

E32-32 $F = idB$, so $a = F/m = idB/m$. Since a is constant, $v = at = idBt/m$. The direction is to the left.

E32-33 Only the $\hat{\mathbf{j}}$ component of $\vec{\mathbf{B}}$ is of interest. Then $F = \int dF = i \int B_y \, dx$, or

$$F = (5.0\,\mathrm{A})(8\times10^{-3}\mathrm{T/m^2}) \int_{1.2}^{3.2} x^2 \, dx = 0.414\,\mathrm{N}.$$

The direction is $-\hat{\mathbf{k}}$.

E32-34 The magnetic force will have two components: one will lift vertically ($F_y = F \sin\alpha$), the other push horizontally ($F_x = F \cos\alpha$). The rod will move when $F_x > \mu(W - F_y)$. We are interested in the minimum value for F as a function of α. This occurs when

$$\frac{dF}{d\alpha} = \frac{d}{d\alpha}\left(\frac{\mu W}{\cos\alpha + \mu \sin\alpha}\right) = 0.$$

This happens when $\mu = \tan\alpha$. Then $\alpha = \arctan(0.58) = 30°$, and

$$F = \frac{(0.58)(1.15\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{\cos(30°) + (0.58)\sin(30°)} = 5.66\,\mathrm{N}$$

is the minimum force. Then $B = (5.66\,\mathrm{N})/(53.2\,\mathrm{A})(0.95\,\mathrm{m}) = 0.112\,\mathrm{T}$.

E32-35 We choose that the field points from the shorter side to the longer side.

(a) The magnetic field is parallel to the 130 cm side so there is no magnetic force on that side. The magnetic force on the 50 cm side has magnitude

$$F_B = iLB \sin\theta,$$

where θ is the angle between the 50 cm side and the magnetic field. This angle is larger than 90°, but the sine can be found directly from the triangle,

$$\sin\theta = \frac{(120 \text{ cm})}{(130 \text{ cm})} = 0.923,$$

and then the force on the 50 cm side can be found by

$$F_B = (4.00\,\mathrm{A})(0.50\,\mathrm{m})(75.0\times10^{-3}\,\mathrm{T})\frac{(120 \text{ cm})}{(130 \text{ cm})} = 0.138\,\mathrm{N},$$

and is directed out of the plane of the triangle.

The magnetic force on the 120 cm side has magnitude

$$F_B = iLB \sin\theta,$$

where θ is the angle between the 1200 cm side and the magnetic field. This angle is larger than 180°, but the sine can be found directly from the triangle,

$$\sin\theta = \frac{(-50 \text{ cm})}{(130 \text{ cm})} = -0.385,$$

and then the force on the 50 cm side can be found by

$$F_B = (4.00\,\mathrm{A})(1.20\,\mathrm{m})(75.0\times10^{-3}\,\mathrm{T})\frac{(-50 \text{ cm})}{(130 \text{ cm})} = -0.138\,\mathrm{N},$$

and is directed into the plane of the triangle.

(b) Look at the three numbers above.

E32-36 $\tau = NiAB\sin\theta$, so

$$\tau = (20)(0.1\,\text{A})(0.12\,\text{m})(0.05\,\text{m})(0.5\,\text{T})\sin(90^\circ - 33^\circ) = 5.0\times10^{-3}\text{N}\cdot\text{m}.$$

E32-37 The external magnetic field must be in the plane of the clock/wire loop. The clockwise current produces a magnetic dipole moment directed into the plane of the clock.

(a) Since the magnetic field points along the 1 pm line and the torque is perpendicular to both the external field and the dipole, then the torque must point along either the 4 pm or the 10 pm line. Applying Eq. 32-35, the direction is along the 4 pm line. It will take the minute hand 20 minutes to get there.

(b) $\tau = (6)(2.0\,\text{A})\pi(0.15\,\text{m})^2(0.07\,\text{T}) = 0.059\,\text{N}\cdot\text{m}$.

P32-1 Since $\vec{\mathbf{F}}$ must be perpendicular to $\vec{\mathbf{B}}$ then $\vec{\mathbf{B}}$ must be along $\hat{\mathbf{k}}$. The magnitude of v is $\sqrt{(40)^2 + (35)^2}$ km/s = 53.1 km/s; the magnitude of F is $\sqrt{(-4.2)^2 + (4.8)^2}$fN = 6.38 fN. Then

$$B = F/qv = (6.38\times10^{-15}\text{N})/(1.6\times10^{-19}\text{C})(53.1\times10^3\text{m/s}) = 0.75\,\text{T}.$$

or $\vec{\mathbf{B}} = 0.75\,\text{T}\,\hat{\mathbf{k}}$.

P32-2 $\vec{\mathbf{a}} = (q/m)(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$. For the initial velocity given,

$$\vec{\mathbf{v}} \times \vec{\mathbf{B}} = (15.0\times10^3\text{m/s})(400\times10^{-6}\text{T})\hat{\mathbf{j}} - (12.0\times10^3\text{m/s})(400\times10^{-6}\text{T})\hat{\mathbf{k}}.$$

But since there is no acceleration in the $\hat{\mathbf{j}}$ or $\hat{\mathbf{k}}$ direction this must be offset by the electric field. Consequently, two of the electric field components are $E_y = -6.00\,\text{V/m}$ and $E_z = 4.80\,\text{V/m}$. The third component of the electric field is the source of the acceleration, so

$$E_x = ma_x/q = (9.11\times10^{-31}\text{kg})(2.00\times10^{12}\text{m/s}^2)/(-1.60\times10^{-19}\text{C}) = -11.4\,\text{V/m}.$$

P32-3 (a) Consider first the cross product, $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. The electron moves horizontally, there is a component of the $\vec{\mathbf{B}}$ which is down, so the cross product results in a vector which points to the left of the electron's path.

But the force on the electron is given by $\vec{\mathbf{F}} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$, and since the electron has a negative charge the force on the electron would be directed to the *right* of the electron's path.

(b) The kinetic energy of the electrons is much less than the rest mass energy, so this is non-relativistic motion. The speed of the electron is then $v = \sqrt{2K/m}$, and the magnetic force on the electron is $F_B = qvB$, where we are assuming $\sin\theta = 1$ because the electron moves horizontally through a magnetic field with a vertical component. We can ignore the effect of the magnetic field's horizontal component because the electron is moving parallel to this component.

The acceleration of the electron because of the magnetic force is then

$$\begin{aligned} a &= \frac{qvB}{m} = \frac{qB}{m}\sqrt{\frac{2K}{m}}, \\ &= \frac{(1.60\times10^{-19}\text{C})(55.0\times10^{-6}\text{T})}{(9.11\times10^{-31}\text{kg})}\sqrt{\frac{2(1.92\times10^{-15}\text{J})}{(9.11\times10^{-31}\text{kg})}} = 6.27\times10^{14}\,\text{m/s}^2. \end{aligned}$$

(c) The electron travels a horizontal distance of 20.0 cm in a time of

$$t = \frac{(20.0\text{ cm})}{\sqrt{2K/m}} = \frac{(20.0\text{ cm})}{\sqrt{2(1.92\times10^{-15}\text{J})/(9.11\times10^{-31}\text{kg})}} = 3.08\times10^{-9}\,\text{s}.$$

In this time the electron is accelerated to the side through a distance of

$$d = \frac{1}{2}at^2 = \frac{1}{2}(6.27\times10^{14}\,\text{m/s}^2)(3.08\times10^{-9}\,\text{s})^2 = 2.98\text{ mm}.$$

P32-4 (a) d needs to be larger than the turn radius, so $R \leq d$; but $2mK/q^2B^2 = R^2 \leq d^2$, or $B \geq \sqrt{2mK/q^2d^2}$.

(b) Out of the page.

P32-5 Only undeflected ions emerge from the velocity selector, so $v = E/B$. The ions are then deflected by B' with a radius of curvature of $r = mv/qB$; combining and rearranging, $q/m = E/rBB'$.

P32-6 The ions are given a kinetic energy $K = q\Delta V$; they are then deflected with a radius of curvature given by $R^2 = 2mK/q^2B^2$. But $x = 2R$. Combine all of the above, and $m = B^2qx^2/8\Delta V$.

$\boxed{\textbf{P32-7}}$ (a) Start with the equation in Problem 6, and take the square root of both sides to get

$$\sqrt{m} = \left(\frac{B^2q}{8\Delta V}\right)^{\frac{1}{2}} x,$$

and then take the derivative of x with respect to m,

$$\frac{1}{2}\frac{dm}{\sqrt{m}} = \left(\frac{B^2q}{8\Delta V}\right)^{\frac{1}{2}} dx,$$

and then consider finite differences instead of differential quantities,

$$\Delta m = \left(\frac{mB^2q}{2\Delta V}\right)^{\frac{1}{2}} \Delta x,$$

(b) Invert the above expression,

$$\Delta x = \left(\frac{2\Delta V}{mB^2q}\right)^{\frac{1}{2}} \Delta m,$$

and then put in the given values,

$$\begin{aligned}\Delta x &= \left(\frac{2(7.33\times10^3\text{V})}{(35.0)(1.66\times10^{-27}\text{kg})(0.520\,\text{T})^2(1.60\times10^{-19}\text{C})}\right)^{\frac{1}{2}} (2.0)(1.66\times10^{-27}\text{kg}),\\ &= 8.02 \text{ mm}.\end{aligned}$$

Note that we used 35.0 u for the mass; if we had used 37.0 u the result would have been closer to the answer in the back of the book.

P32-8 (a) $B = \sqrt{2\Delta V m/qr^2} = \sqrt{2(0.105\text{ MV})(238)(932\text{ MeV}/c^2)/2e(0.973\,\text{m})^2} = 5.23\times10^{-7}\text{T}$.

(b) The number of atoms in a gram is $6.02\times10^{23}/238 = 2.53\times10^{21}$. The current is then

$$(0.090)(2.53\times10^{21})(2)(1.6\times10^{-19}\text{C})/(3600\,\text{s}) = 20.2\text{ mA}.$$

P32-9 (a) $-q$.

(b) Regardless of speed, the orbital period is $T = 2\pi m/qB$. But they collide halfway around a complete orbit, so $t = \pi m/qB$.

P32-10

$\boxed{\textbf{P32-11}}$ (a) The period of motion can be found from the reciprocal of Eq. 32-12,

$$T = \frac{2\pi m}{|q|B} = \frac{2\pi(9.11\times10^{-31}\text{kg})}{(1.60\times10^{-19}\text{C})(455\times10^{-6}\text{T})} = 7.86\times10^{-8}\text{s}.$$

(b) We need to find the velocity of the electron from the kinetic energy,

$$v = \sqrt{2K/m} = \sqrt{2(22.5\text{ eV})(1.60\times10^{-19}\text{ J/eV})/(9.11\times10^{-31}\text{kg})} = 2.81\times10^{6}\text{ m/s}.$$

The velocity can written in terms of components which are parallel and perpendicular to the magnetic field. Then

$$v_{||} = v\cos\theta \text{ and } v_\perp = v\sin\theta.$$

The pitch is the parallel distance traveled by the electron in one revolution, so

$$p = v_{||}T = (2.81\times10^{6}\text{m/s})\cos(65.5^\circ)(7.86\times10^{-8}\text{s}) = 9.16\text{ cm}.$$

(c) The radius of the helical path is given by Eq. 32-10, except that we use the perpendicular velocity component, so

$$R = \frac{mv_\perp}{|q|B} = \frac{(9.11\times10^{-31}\text{kg})(2.81\times10^{6}\text{m/s})\sin(65.5^\circ)}{(1.60\times10^{-19}\text{C})(455\times10^{-6}\text{T})} = 3.20\text{ cm}$$

P32-12 $\vec{\mathbf{F}} = i\int_a^b d\vec{\mathbf{l}} \times \vec{\mathbf{B}}$. $d\vec{\mathbf{l}}$ has two components, those parallel to the path, say $d\vec{\mathbf{x}}$ and those perpendicular, say $d\vec{\mathbf{y}}$. Then the integral can be written as

$$\vec{\mathbf{F}} = \int_a^b d\vec{\mathbf{x}} \times \vec{\mathbf{B}} + \int_a^b d\vec{\mathbf{y}} \times \vec{\mathbf{B}}.$$

But $\vec{\mathbf{B}}$ is constant, and can be removed from the integral. $\int_a^b d\vec{\mathbf{x}} = \vec{\mathbf{l}}$, a vector that points from a to b. $\int_a^b d\vec{\mathbf{y}} = 0$, because there is no net motion perpendicular to $\vec{\mathbf{l}}$.

P32-13 $qv_yB = F_x = m\,dv_x/dt$; $-qv_xB = F_y = m\,dv_y/dt$. Taking the time derivative of the second expression and inserting into the first we get

$$qv_yB = m\left(-\frac{m}{qB}\right)\frac{d^2v_y}{dt^2},$$

which has solution $v_y = -v\sin(mt/qB)$, where v is a constant. Using the second equation we find that there is a similar solution for v_x, except that it is out of phase, and so $v_x = v\cos(mt/qB)$.

Integrating,

$$x = \int v_x dt = v\int \cos(mt/qB) = \frac{qBv}{m}\sin(mt/qB).$$

Similarly,

$$y = \int v_y dt = -v\int \sin(mt/qB) = \frac{qBv}{m}\cos(mt/qB).$$

This is the equation of a circle.

P32-14 $d\vec{\mathbf{L}} = \hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy + \hat{\mathbf{k}}dz$. $\vec{\mathbf{B}}$ is uniform, so that the integral can be written as

$$\vec{\mathbf{F}} = i\oint(\hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy + \hat{\mathbf{k}}dz) \times \vec{\mathbf{B}} = i\hat{\mathbf{i}} \times \vec{\mathbf{B}}\oint dx + i\hat{\mathbf{j}} \times \vec{\mathbf{B}}\oint dy + i\hat{\mathbf{k}} \times \vec{\mathbf{B}}\oint dz,$$

but since $\oint dx = \oint dy = \oint dz = 0$, the entire expression vanishes.

P32-15 The current pulse provides an impulse which is equal to

$$\int F\,dt = \int BiL\,dt = BL\int i\,dt = BLq.$$

This gives an initial velocity of $v_0 = BLq/m$, which will cause the rod to hop to a height of

$$h = v_0^2/2g = B^2L^2q^2/2m^2g.$$

Solving for q,

$$q = \frac{m}{BL}\sqrt{2gh} = \frac{(0.013\,\mathrm{kg})}{(0.12\,\mathrm{T})(0.20\,\mathrm{m})}\sqrt{2(9.8\,\mathrm{m/s^2})(3.1\,\mathrm{m})} = 4.2\,\mathrm{C}.$$

P32-16

$\boxed{\textbf{P32-17}}$ The torque on a current carrying loop depends on the orientation of the loop; the maximum torque occurs when the plane of the loop is parallel to the magnetic field. In this case the magnitude of the torque is from Eq. 32-34 with $\sin\theta = 1$—

$$\tau = NiAB.$$

The area of a circular loop is $A = \pi r^2$ where r is the radius, but since the circumference is $C = 2\pi r$, we can write

$$A = \frac{C^2}{4\pi}.$$

The circumference is *not* the length of the wire, because there may be more than one turn. Instead, $C = L/N$, where N is the number of turns.

Finally, we can write the torque as

$$\tau = Ni\frac{L^2}{4\pi N^2}B = \frac{iL^2B}{4\pi N},$$

which is a maximum when N is a minimum, or $N = 1$.

P32-18 $d\vec{\mathbf{F}} = i\,d\vec{\mathbf{L}} \times \vec{\mathbf{B}}$; the direction of $d\vec{\mathbf{F}}$ will be upward and somewhat toward the center. $\vec{\mathbf{L}}$ and $\vec{\mathbf{B}}$ are a right angles, but only the upward component of $d\vec{\mathbf{F}}$ will survive the integration as the central components will cancel out by symmetry. Hence

$$F = iB\sin\theta\int dL = 2\pi riB\sin\theta.$$

P32-19 The torque on the cylinder from gravity is

$$\tau_{\mathrm{g}} = mgr\sin\theta,$$

where r is the radius of the cylinder. The torque from magnetism needs to balance this, so

$$mgr\sin\theta = NiAB\sin\theta = Ni2rLB\sin\theta,$$

or

$$i = \frac{mg}{2NLB} = \frac{(0.262\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{2(13)(0.127\,\mathrm{m})(0.477\,\mathrm{T})} = 1.63\,\mathrm{A}.$$

$\boxed{\textbf{E33-1}}$ (a) The magnetic field from a moving charge is given by Eq. 33-5. If the protons are moving side by side then the angle is $\phi = \pi/2$, so

$$B = \frac{\mu_0}{4\pi}\frac{qv}{r^2}$$

and we are interested is a distance $r = d$. The electric field at that distance is

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2},$$

where in both of the above expressions q is the charge of the source proton.

On the receiving end is the other proton, and the force on that proton is given by

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}).$$

The velocity is the same as that of the first proton (otherwise they wouldn't be moving side by side.) This velocity is then perpendicular to the magnetic field, and the resulting direction for the cross product will be opposite to the direction of $\vec{\mathbf{E}}$. Then for balance,

$$\begin{aligned} E &= vB, \\ \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} &= v\frac{\mu_0}{4\pi}\frac{qv}{r^2}, \\ \frac{1}{\epsilon_0\mu_0} &= v^2. \end{aligned}$$

We can solve this easily enough, and we find $v \approx 3 \times 10^8\,\text{m/s}$.

(b) This is clearly a relativistic speed!

E33-2 $B = \mu_0 i/2\pi d = (4\pi\times10^{-7}\text{T}\cdot\text{m/A})(120\,\text{A})/2\pi(6.3\,\text{m}) = 3.8\times10^{-6}\text{T}$. This will deflect the compass needle by as much as one degree. However, there is unlikely to be a place on the Earth's surface where the magnetic field is 210 μT. This was likely a typo, and should probably have been 21.0 μT. The deflection would then be some ten degrees, and that *is* significant.

E33-3 $B = \mu_0 i/2\pi d = (4\pi\times10^{-7}\text{T}\cdot\text{m/A})(50\,\text{A})/2\pi(1.3\times10^{-3}\,\text{m}) = 37.7\times10^{-3}\text{T}$.

E33-4 (a) $i = 2\pi dB/\mu_0 = 2\pi(8.13\times10^{-2}\text{m})(39.0\times10^{-6}\text{T})/(4\pi\times10^{-7}\text{T}\cdot\text{m/A}) = 15.9\,\text{A}$.

(b) Due East.

$\boxed{\textbf{E33-5}}$ Use

$$B = \frac{\mu_0 i}{2\pi d} = \frac{(4\pi\times10^{-7}\text{N/A}^2)(1.6\times10^{-19}\,\text{C})(5.6\times10^{14}\,\text{s}^{-1})}{2\pi(0.0015\,\text{m})} = 1.2\times10^{-8}\text{T}.$$

E33-6 Zero, by symmetry. Any contributions from the top wire are exactly canceled by contributions from the bottom wire.

E33-7 $B = \mu_0 i/2\pi d = (4\pi\times10^{-7}\text{T}\cdot\text{m/A})(48.8\,\text{A})/2\pi(5.2\times10^{-2}\,\text{m}) = 1.88\times10^{-4}\text{T}$.

$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. All cases are either parallel or perpendicular, so either $F = 0$ or $F = qvB$.

(a) $F = qvB = (1.60\times10^{-19}\text{C})(1.08\times10^{7}\text{m/s})(1.88\times10^{-4}\text{T}) = 3.24\times10^{-16}\text{N}$. The direction of $\vec{\mathbf{F}}$ is parallel to the current.

(b) $F = qvB = (1.60\times10^{-19}\text{C})(1.08\times10^{7}\text{m/s})(1.88\times10^{-4}\text{T}) = 3.24\times10^{-16}\text{N}$. The direction of $\vec{\mathbf{F}}$ is radially outward from the current.

(c) $F = 0$.

E33-8 We want $B_1 = B_2$, but with opposite directions. Then $i_1/d_1 = i_2/d_2$, since all constants cancel out. Then $i_2 = (6.6\,\text{A})(1.5\text{ cm})/(2.25\text{ cm}) = 4.4\,\text{A}$, directed out of the page.

$\boxed{\textbf{E33-9}}$ For a single long straight wire, $B = \mu_0 i/2\pi d$ but we need a factor of "2" since there are two wires, then $i = \pi d B/\mu_0$. Finally

$$i = \frac{\pi d B}{\mu_0} = \frac{\pi(0.0405\,\text{m})(296, \mu\text{T})}{(4\pi\times10^{-7}\text{N/A}^2)} = 30\,\text{A}$$

E33-10 (a) The semi-circle part contributes half of Eq. 33-21, or $\mu_0 i/4R$. Each long straight wire contributes half of Eq. 33-13, or $\mu_0 i/4\pi R$. Add the three contributions and get

$$B_a = \frac{\mu_0 i}{4R}\left(\frac{2}{\pi}+1\right) = \frac{(4\pi\times10^{-7}\text{N/A}^2)(11.5\,\text{A})}{4(5.20\times10^{-3}\text{m})}\left(\frac{2}{\pi}+1\right) = 1.14\times10^{-3}\text{T}.$$

The direction is out of the page.

(b) Each long straight wire contributes Eq. 33-13, or $\mu_0 i/2\pi R$. Add the two contributions and get

$$B_a = \frac{\mu_0 i}{\pi R} = \frac{(4\pi\times10^{-7}\text{N/A}^2)(11.5\,\text{A})}{\pi(5.20\times10^{-3}\text{m})} = 8.85\times10^{-4}\text{T}.$$

The direction is out of the page.

E33-11 $z^3 = \mu_0 i R^2/2B = (4\pi\times10^{-7}\text{N/A}^2)(320)(4.20\,\text{A})(2.40\times10^{-2}\text{m})^2/2(5.0\times10^{-6}\text{T}) = 9.73\times 10^{-2}\text{m}^3$. Then $z = 0.46\,\text{m}$.

E33-12 The circular part contributes a fraction of Eq. 33-21, or $\mu_0 i\theta/4\pi R$. Each long straight wire contributes half of Eq. 33-13, or $\mu_0 i/4\pi R$. Add the three contributions and get

$$B = \frac{\mu_0 i}{4\pi R}(\theta - 2).$$

The goal is to get $B = 0$ that will happen if $\theta = 2$ radians.

$\boxed{\textbf{E33-13}}$ There are four current segments that could contribute to the magnetic field. The straight segments, however, contribute nothing because the straight segments carry currents either directly toward or directly away from the point P.

That leaves the two rounded segments. Each contribution to $\vec{\mathbf{B}}$ can be found by starting with Eq. 33-21, or $\mu_0 i\theta/4\pi b$. The direction is out of the page.

There is also a contribution from the top arc; the calculations are almost identical except that this is pointing into the page and $r = a$, so $\mu_0 i\theta/4\pi a$. The net magnetic field at P is then

$$B = B_1 + B_2 = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right).$$

E33-14 For each straight wire segment use Eq. 33-12. When the length of wire is L, the distance to the center is $W/2$; when the length of wire is W the distance to the center is $L/2$. There are four terms, but they are equal in pairs, so

$$\begin{aligned} B &= \frac{\mu_0 i}{4\pi}\left(\frac{4L}{W\sqrt{L^2/4+W^2/4}} + \frac{4W}{L\sqrt{L^2/4+W^2/4}}\right), \\ &= \frac{2\mu_0 i}{\pi\sqrt{L^2+W^2}}\left(\frac{L^2}{WL}+\frac{W^2}{WL}\right) = \frac{2\mu_0 i}{\pi}\frac{\sqrt{L^2+W^2}}{WL}. \end{aligned}$$

$\boxed{\textbf{E33-15}}$ We imagine the ribbon conductor to be a collection of thin wires, each of thickness dx and carrying a current di. di and dx are related by $di/dx = i/w$. The contribution of one of these thin wires to the magnetic field at P is $dB = \mu_0\, di/2\pi x$, where x is the distance from this thin wire to the point P. We want to change variables to x and integrate, so

$$B = \int dB = \int \frac{\mu_0 i\, dx}{2\pi w x} = \frac{\mu_0 i}{2\pi w}\int \frac{dx}{x}.$$

The limits of integration are from d to $d + w$, so

$$B = \frac{\mu_0 i}{2\pi w} \ln\left(\frac{d+w}{d}\right).$$

E33-16 The fields from each wire are perpendicular at P. Each contributes an amount $B' = \mu_0 i/2\pi d$, but since they are perpendicular there is a net field of magnitude $B = \sqrt{2B'^2} = \sqrt{2}\mu_0 i/2\pi d$. Note that $a = \sqrt{2}d$, so $B = \mu_0 i/\pi a$.

(a) $B = (4\pi\times10^{-7}\mathrm{T\cdot m/A})(115\,\mathrm{A})/\pi(0.122\,\mathrm{m}) = 3.77\times10^{-4}\mathrm{T}$. The direction is to the left.

(b) Same numerical result, except the direction is up.

E33-17 Follow along with Sample Problem 33-4.

Reversing the direction of the second wire (so that now both currents are directed out of the page) will also reverse the direction of B_2. Then

$$\begin{aligned} B &= B_1 - B_2 = \frac{\mu_0 i}{2\pi}\left(\frac{1}{b+x} - \frac{1}{b-x}\right), \\ &= \frac{\mu_0 i}{2\pi}\left(\frac{(b-x)-(b+x)}{b^2-x^2}\right), \\ &= \frac{\mu_0 i}{\pi}\left(\frac{x}{x^2-b^2}\right). \end{aligned}$$

E33-18 (b) By symmetry, only the horizontal component of $\vec{\mathbf{B}}$ survives, and must point to the right.

(a) The horizontal component of the field contributed by the top wire is given by

$$B = \frac{\mu_0 i}{2\pi h}\sin\theta = \frac{\mu_0 i}{2\pi h}\frac{b/2}{h} = \frac{\mu_0 i b}{\pi(4R^2+b^2)},$$

since h is the hypotenuse, or $h = \sqrt{R^2 + b^2/4}$. But there are two such components, one from the top wire, and an identical component from the bottom wire.

$\boxed{\textbf{E33-19}}$ (a) We can use Eq. 33-21 to find the magnetic field strength at the center of the large loop,

$$B = \frac{\mu_0 i}{2R} = \frac{(4\pi\times10^{-7}\mathrm{T\cdot m/A})(13\,\mathrm{A})}{2(0.12\,\mathrm{m})} = 6.8\times10^{-5}\mathrm{T}.$$

(b) The torque on the smaller loop in the center is given by Eq. 32-34,

$$\vec{\tau} = Ni\vec{\mathbf{A}}\times\vec{\mathbf{B}},$$

but since the magnetic field from the large loop is perpendicular to the plane of the large loop, and the plane of the small loop is also perpendicular to the plane of the large loop, the magnetic field is in the plane of the small loop. This means that $|\vec{\mathbf{A}}\times\vec{\mathbf{B}}| = AB$. Consequently, the magnitude of the torque on the small loop is

$$\tau = NiAB = (50)(1.3\,\mathrm{A})(\pi)(8.2\times10^{-3}\mathrm{m})^2(6.8\times10^{-5}\mathrm{T}) = 9.3\times10^{-7}\mathrm{N\cdot m}.$$

E33-20 (a) There are two contributions to the field. One is from the circular loop, and is given by $\mu_0 i/2R$. The other is from the long straight wire, and is given by $\mu_0 i/2\pi R$. The two fields are out of the page and parallel, so

$$B = \frac{\mu_0 i}{2R}(1 + 1/\pi).$$

(b) The two components are now at right angles, so

$$B = \frac{\mu_0 i}{2R}\sqrt{1 + 1/\pi^2}.$$

The direction is given by $\tan\theta = 1/\pi$ or $\theta = 18°$.

E33-21 The force per meter for any pair of parallel currents is given by Eq. 33-25, $F/L = \mu_0 i^2/2\pi d$, where d is the separation. The direction of the force is along the line connecting the intersection of the currents with the perpendicular plane. Each current experiences three forces; two are at right angles and equal in magnitude, so $|\vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{14}|/L = \sqrt{F_{12}^2 + F_{14}^2}/L = \sqrt{2}\mu_0 i^2/2\pi a$. The third force points parallel to this sum, but $d = \sqrt{a}$, so the resultant force is

$$\frac{F}{L} = \frac{\sqrt{2}\mu_0 i^2}{2\pi a} + \frac{\mu_0 i^2}{2\pi\sqrt{2}a} = \frac{4\pi \times \times 10^{-7}\mathrm{N/A^2}(18.7\,\mathrm{A})^2}{2\pi(0.245\,\mathrm{m})}(\sqrt{2} + 1/\sqrt{2}) = 6.06\times 10^{-4}\mathrm{N/m}.$$

It is directed toward the center of the square.

E33-22 By symmetry we expect the middle wire to have a net force of zero; the two on the outside will each be attracted toward the center, but the answers will be symmetrically distributed.

For the wire which is the farthest left,

$$\frac{F}{L} = \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a} + \frac{1}{4a}\right) = \frac{4\pi \times \times 10^{-7}\mathrm{N/A^2}(3.22\,\mathrm{A})^2}{2\pi(0.083\,\mathrm{m})}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = 5.21\times 10^{-5}\mathrm{N/m}.$$

For the second wire over, the contributions from the two adjacent wires should cancel. This leaves

$$\frac{F}{L} = \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{2a} + \frac{1}{3a} +\right) = \frac{4\pi \times \times 10^{-7}\mathrm{N/A^2}(3.22\,\mathrm{A})^2}{2\pi(0.083\,\mathrm{m})}\left(\frac{1}{2} + \frac{1}{3}\right) = 2.08\times 10^{-5}\mathrm{N/m}.$$

$\boxed{\textbf{E33-23}}$ (a) The force on the projectile is given by the integral of

$$d\vec{\mathbf{F}} = i\,d\vec{\mathbf{l}} \times \vec{\mathbf{B}}$$

over the length of the projectile (which is w). The magnetic field strength can be found from adding together the contributions from each rail. If the rails are circular and the distance between them is small compared to the length of the wire we can use Eq. 33-13,

$$B = \frac{\mu_0 i}{2\pi x},$$

where x is the distance from the center of the rail. There is one problem, however, because these are not wires of infinite length. Since the current *stops* traveling along the rail when it reaches the projectile we have a rod that is only half of an infinite rod, so we need to multiply by a factor of 1/2. But there are two rails, and each will contribute to the field, so the net magnetic field strength between the rails is

$$B = \frac{\mu_0 i}{4\pi x} + \frac{\mu_0 i}{4\pi(2r + w - x)}.$$

In that last term we have an expression that is a measure of the distance from the center of the lower rail in terms of the distance x from the center of the upper rail.

The magnitude of the force on the projectile is then

$$\begin{aligned} F &= i\int_r^{r+w} B\,dx, \\ &= \frac{\mu_0 i^2}{4\pi}\int_r^{r+w}\left(\frac{1}{x}+\frac{1}{2r+w-x}\right)dx, \\ &= \frac{\mu_0 i^2}{4\pi}2\ln\left(\frac{r+w}{r}\right) \end{aligned}$$

The current through the projectile is down the page; the magnetic field through the projectile is into the page; so the force on the projectile, according to $\vec{\mathbf{F}} = i\vec{\mathbf{l}}\times\vec{\mathbf{B}}$, is to the right.

(b) Numerically the magnitude of the force on the rail is

$$F = \frac{(450\times 10^3\,\text{A})^2(4\pi\times 10^{-7}\text{N/A}^2)}{2\pi}\ln\left(\frac{(0.067\,\text{m})+(0.012\,\text{m})}{(0.067\,\text{m})}\right) = 6.65\times 10^3\,\text{N}$$

The speed of the rail can be found from either energy conservation so we first find the work done on the projectile,

$$W = Fd = (6.65\times 10^3\,\text{N})(4.0\,\text{m}) = 2.66\times 10^4\,\text{J}.$$

This work results in a change in the kinetic energy, so the final speed is

$$v = \sqrt{2K/m} = \sqrt{2(2.66\times 10^4\,\text{J})/(0.010\,\text{kg})} = 2.31\times 10^3\,\text{m/s}.$$

E33-24 The contributions from the left end and the right end of the square cancel out. This leaves the top and the bottom. The net force is the difference, or

$$\begin{aligned} F &= \frac{(4\pi\times 10^{-7}\text{N/A}^2)(28.6\,\text{A})(21.8\,\text{A})(0.323\,\text{m})}{2\pi}\left(\frac{1}{(1.10\times 10^{-2}\text{m})}-\frac{1}{(10.30\times 10^{-2}\text{m})}\right), \\ &= 3.27\times 10^{-3}\text{N}. \end{aligned}$$

E33-25 The magnetic force on the upper wire near the point d is

$$F_B = \frac{\mu_0 i_a i_b L}{2\pi(d+x)} \approx \frac{\mu_0 i_a i_b L}{2\pi d} - \frac{\mu_0 i_a i_b L}{2\pi d^2}x,$$

where x is the distance from the equilibrium point d. The equilibrium magnetic force is equal to the force of gravity mg, so near the equilibrium point we can write

$$F_B = mg - mg\frac{x}{d}.$$

There is then a restoring force against small perturbations of magnitude mgx/d which corresponds to a spring constant of $k = mg/d$. This would give a frequency of oscillation of

$$f = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\sqrt{g/d},$$

which is identical to the pendulum.

E33-26 $B = (4\pi\times 10^{-7}\text{N/A}^2)(3.58\,\text{A})(1230)/(0.956\text{m}) = 5.79\times 10^{-3}\text{T}.$

$\boxed{\textbf{E33-27}}$ The magnetic field inside an ideal solenoid is given by Eq. 33-28 $B = \mu_0 in$, where n is the turns per unit length. Solving for n,

$$n = \frac{B}{\mu_0 i} = \frac{(0.0224\,\text{T})}{(4\pi\times 10^{-7}\text{N/A}^2)(17.8\,\text{A})} = 1.00\times 10^3/\text{m}^{-1}.$$

The solenoid has a length of 1.33 m, so the total number of turns is

$$N = nL = (1.00\times 10^3/\text{m}^{-1})(1.33\,\text{m}) = 1330,$$

and since each turn has a length of one circumference, then the total length of the wire which makes up the solenoid is $(1330)\pi(0.026\,\text{m}) = 109\,\text{m}$.

E33-28 From the solenoid we have

$$B_\text{s} = \mu_0 n i_\text{s} = \mu_0(11500/\text{m})(1.94\,\text{mA}) = \mu_0(22.3\text{A/m}).$$

From the wire we have

$$B_\text{w} = \frac{\mu_0 i_\text{w}}{2\pi r} = \frac{\mu_0(6.3\,\text{A})}{2\pi r} = (1.002\,\text{A})\frac{\mu_0}{r}$$

These fields are at right angles, so we are interested in when $\tan(40°) = B_\text{w}/B_\text{s}$, or

$$r = \frac{(1.002\,\text{A})}{\tan(40°)(22.3\,\text{A/m})} = 5.35\times 10^{-2}\text{m}.$$

E33-29 Let $u = z - d$. Then

$$\begin{aligned} B &= \frac{\mu_0 n i R^2}{2}\int_{d-L/2}^{d+L/2}\frac{du}{[R^2+u^2]^{3/2}}, \\ &= \frac{\mu_0 n i R^2}{2}\left.\frac{u}{R^2\sqrt{R^2+u^2}}\right|_{d-L/2}^{d+L/2}, \\ &= \frac{\mu_0 n i}{2}\left(\frac{d+L/2}{\sqrt{R^2+(d+L/2)^2}} - \frac{d-L/2}{\sqrt{R^2+(d-L/2)^2}}\right). \end{aligned}$$

If L is much, much greater than R and d then $|L/2 \pm d| >> R$, and R can be ignored in the denominator of the above expressions, which then simplify to

$$\begin{aligned} B &= \frac{\mu_0 n i}{2}\left(\frac{d+L/2}{\sqrt{R^2+(d+L/2)^2}} - \frac{d-L/2}{\sqrt{R^2+(d-L/2)^2}}\right). \\ &= \frac{\mu_0 n i}{2}\left(\frac{d+L/2}{\sqrt{(d+L/2)^2}} - \frac{d-L/2}{\sqrt{(d-L/2)^2}}\right). \\ &= \mu_0 i n. \end{aligned}$$

It *is* important that we consider the relative size of $L/2$ and d!

E33-30 The net current in the loop is $1i_0 + 3i_0 + 7i_0 - 6i_0 = 5i_0$. Then $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = 5\mu_0 i_0$.

E33-31 (a) The path is clockwise, so a positive current is *into* page. The net current is 2.0 A out, so $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = -\mu_0 i_0 = -2.5\times 10^{-6}\text{T}\cdot\text{m}$.

(b) The net current is zero, so $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = 0$.

E33-32 Let R_0 be the radius of the wire. On the surface of the wire $B_0 = \mu_0 i/2\pi R_0$.

Outside the wire we have $B = \mu_0 i/2\pi R$, this is half B_0 when $R = 2R_0$.

Inside the wire we have $B = \mu_0 i R/2\pi R_0^2$, this is half B_0 when $R = R_0/2$.

$\boxed{\textbf{E33-33}}$ (a) We don't want to reinvent the wheel. The answer is found from Eq. 33-34, except it looks like

$$B = \frac{\mu_0 i r}{2\pi c^2}.$$

(b) In the region between the wires the magnetic field looks like Eq. 33-13,

$$B = \frac{\mu_0 i}{2\pi r}.$$

This is derived on the right hand side of page 761.

(c) Ampere's law (Eq. 33-29) is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 i$, where i is the current enclosed. Our Amperian loop will still be a circle centered on the axis of the problem, so the left hand side of the above equation will reduce to $2\pi r B$, just like in Eq. 33-32. The right hand side, however, depends on the *net* current enclosed which is the current i in the center wire minus the fraction of the current enclosed in the outer conductor. The cross sectional area of the outer conductor is $\pi(a^2 - b^2)$, so the fraction of the outer current enclosed in the Amperian loop is

$$i\frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)} = i\frac{r^2 - b^2}{a^2 - b^2}.$$

The net current in the loop is then

$$i - i\frac{r^2 - b^2}{a^2 - b^2} = i\frac{a^2 - r^2}{a^2 - b^2},$$

so the magnetic field in this region is

$$B = \frac{\mu_0 i}{2\pi r}\frac{a^2 - r^2}{a^2 - b^2}.$$

(d) This part is easy since the net current is zero; consequently $B = 0$.

E33-34 (a) Ampere's law (Eq. 33-29) is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 i$, where i is the current enclosed. Our Amperian loop will still be a circle centered on the axis of the problem, so the left hand side of the above equation will reduce to $2\pi r B$, just like in Eq. 33-32. The right hand side, however, depends on the *net* current enclosed which is the fraction of the current enclosed in the conductor. The cross sectional area of the conductor is $\pi(a^2 - b^2)$, so the fraction of the current enclosed in the Amperian loop is

$$i\frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)} = i\frac{r^2 - b^2}{a^2 - b^2}.$$

The magnetic field in this region is

$$B = \frac{\mu_0 i}{2\pi r}\frac{r^2 - b^2}{a^2 - b^2}.$$

(b) If $r = a$, then

$$B = \frac{\mu_0 i}{2\pi a}\frac{a^2 - b^2}{a^2 - b^2} = \frac{\mu_0 i}{2\pi a},$$

which is what we expect.

If $r = b$, then

$$B = \frac{\mu_0 i}{2\pi b}\frac{b^2 - b^2}{a^2 - b^2} = 0,$$

which is what we expect.

If $b = 0$, then

$$B = \frac{\mu_0 i}{2\pi r}\frac{r^2 - 0^2}{a^2 - 0^2} = \frac{\mu_0 i r}{2\pi a^2}$$

which is what I expected.

E33-35 The magnitude of the magnetic field due to the cylinder will be *zero* at the center of the cylinder and $\mu_0 i_0/2\pi(2R)$ at point P. The magnitude of the magnetic field field due to the wire will be $\mu_0 i/2\pi(3R)$ at the center of the cylinder but $\mu_0 i/2\pi R$ at P. In order for the net field to have different directions in the two locations the currents in the wire and pipe must be in different direction. The net field at the center of the pipe is $\mu_0 i/2\pi(3R)$, while that at P is then $\mu_0 i_0/2\pi(2R) - \mu_0 i/2\pi R$. Set these equal and solve for i;

$$i/3 = i_0/2 - i,$$

or $i = 3i_0/8$.

E33-36 (a) $B = (4\pi\times10^{-7}\mathrm{N/A^2})(0.813\,\mathrm{A})(535)/2\pi(0.162\,\mathrm{m}) = 5.37\times10^{-4}\mathrm{T}$.

(b) $B = (4\pi\times10^{-7}\mathrm{N/A^2})(0.813\,\mathrm{A})(535)/2\pi(0.162\,\mathrm{m} + 0.052\,\mathrm{m}) = 4.07\times10^{-4}\mathrm{T}$.

E33-37 (a) A positive particle would experience a magnetic force directed to the right for a magnetic field out of the page. This particle is going the other way, so it must be negative.

(b) The magnetic field of a toroid is given by Eq. 33-36,

$$B = \frac{\mu_0 i N}{2\pi r},$$

while the radius of curvature of a charged particle in a magnetic field is given by Eq. 32-10

$$R = \frac{mv}{|q|B}.$$

We use the R to distinguish it from r. Combining,

$$R = \frac{2\pi m v}{\mu_0 i N|q|}r,$$

so the two radii are directly proportional. This means

$$R/(11\text{ cm}) = (110\text{ cm})/(125\text{ cm}),$$

so $R = 9.7$ cm.

P33-1 The field from one coil is given by Eq. 33-19

$$B = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.$$

There are N turns in the coil, so we need a factor of N. There are two coils and we are interested in the magnetic field at P, a distance $R/2$ from each coil. The magnetic field strength will be twice the above expression but with $z = R/2$, so

$$B = \frac{2\mu_0 N i R^2}{2(R^2 + (R/2)^2)^{3/2}} = \frac{8\mu_0 N i}{(5)^{3/2}R}.$$

P33-2 (a) Change the limits of integration that lead to Eq. 33-12:

$$\begin{aligned} B &= \frac{\mu_0 i d}{4\pi}\int_0^L \frac{dz}{(z^2+d^2)^{3/2}}, \\ &= \frac{\mu_0 i d}{4\pi}\frac{z}{(z^2+d^2)^{1/2}}\bigg|_0^L, \\ &= \frac{\mu_0 i d}{4\pi}\frac{L}{(L^2+d^2)^{1/2}}. \end{aligned}$$

(b) The angle ϕ in Eq. 33-11 would always be 0, so $\sin\phi = 0$, and therefore $B = 0$.

$\boxed{\textbf{P33-3}}$ This problem is the all important derivation of the Helmholtz coil properties.

(a) The magnetic field from one coil is

$$B_1 = \frac{\mu_0 N i R^2}{2(R^2+z^2)^{3/2}}.$$

The magnetic field from the other coil, located a distance s away, but for points measured from the first coil, is

$$B_2 = \frac{\mu_0 N i R^2}{2(R^2+(z-s)^2)^{3/2}}.$$

The magnetic field on the axis between the coils is the sum,

$$B = \frac{\mu_0 N i R^2}{2(R^2+z^2)^{3/2}} + \frac{\mu_0 N i R^2}{2(R^2+(z-s)^2)^{3/2}}.$$

Take the derivative with respect to z and get

$$\frac{dB}{dz} = -\frac{3\mu_0 N i R^2}{2(R^2+z^2)^{5/2}}z - \frac{3\mu_0 N i R^2}{2(R^2+(z-s)^2)^{5/2}}(z-s).$$

At $z = s/2$ this expression vanishes! We expect this by symmetry, because the magnetic field will be strongest in the plane of either coil, so the mid-point should be a local minimum.

(b) Take the derivative again and

$$\begin{aligned} \frac{d^2B}{dz^2} = & -\frac{3\mu_0 N i R^2}{2(R^2+z^2)^{5/2}} + \frac{15\mu_0 N i R^2}{2(R^2+z^2)^{5/2}}z^2 \\ & -\frac{3\mu_0 N i R^2}{2(R^2+(z-s)^2)^{5/2}} + \frac{15\mu_0 N i R^2}{2(R^2+(z-s)^2)^{5/2}}(z-s)^2. \end{aligned}$$

We could try and simplify this, but we don't really want to; we instead want to set it equal to zero, then let $z = s/2$, and then solve for s. The second derivative will equal zero when

$$-3(R^2+z^2) + 15z^2 - 3(R^2+(z-s)^2) + 15(z-s)^2 = 0,$$

and is $z = s/2$ this expression will simplify to

$$\begin{aligned} 30(s/2)^2 &= 6(R^2+(s/2)^2), \\ 4(s/2)^2 &= R^2, \\ s &= R. \end{aligned}$$

P33-4 (a) Each of the side of the square is a straight wire segment of length a which contributes a field strength of

$$B = \frac{\mu_0 i}{4\pi r}\frac{a}{\sqrt{a^2/4 + r^2}},$$

where r is the distance to the point on the axis of the loop, so

$$r = \sqrt{a^2/4 + z^2}.$$

This field is *not* parallel to the z axis; the z component is $B_z = B(a/2)/r$. There are four of these contributions. The off axis components cancel. Consequently, the field for the square is

$$\begin{aligned} B &= 4\frac{\mu_0 i}{4\pi r}\frac{a}{\sqrt{a^2/4 + r^2}}\frac{a/2}{r}, \\ &= \frac{\mu_0 i}{2\pi r^2}\frac{a^2}{\sqrt{a^2/4 + r^2}}, \\ &= \frac{\mu_0 i}{2\pi (a^2/4 + z^2)}\frac{a^2}{\sqrt{a^2/2 + z^2}}, \\ &= \frac{4\mu_0 i}{\pi (a^2 + 4z^2)}\frac{a^2}{\sqrt{2a^2 + 4z^2}}. \end{aligned}$$

(b) When $z = 0$ this reduces to

$$B = \frac{4\mu_0 i}{\pi (a^2)}\frac{a^2}{\sqrt{2a^2}} = \frac{4\mu_0 i}{\sqrt{2}\,\pi a}.$$

P33-5 (a) The polygon has n sides. A perpendicular bisector of each side can be drawn to the center and has length x where $x/a = \cos(\pi/n)$. Each side has a length $L = 2a\sin(\pi/n)$. Each of the side of the polygon is a straight wire segment which contributes a field strength of

$$B = \frac{\mu_0 i}{4\pi x}\frac{L}{\sqrt{L^2/4 + x^2}},$$

This field *is* parallel to the z axis. There are n of these contributions. The off axis components cancel. Consequently, the field for the polygon

$$\begin{aligned} B &= n\frac{\mu_0 i}{4\pi x}\frac{L}{\sqrt{L^2/4 + x^2}}, \\ &= n\frac{\mu_0 i}{4\pi}\frac{2}{\sqrt{L^2/4 + x^2}}\tan(\pi/n), \\ &= n\frac{\mu_0 i}{2\pi}\frac{1}{a}\tan(\pi/n), \end{aligned}$$

since $(L/2)^2 + x^2 = a^2$.

(b) Evaluate:

$$\lim_{n\to\infty} n\tan(\pi/n) = \lim_{n\to\infty} n\sin(\pi/n) \approx n\pi/n = \pi.$$

Then the answer to part (a) simplifies to

$$B = \frac{\mu_0 i}{2a}.$$

P33-6 For a square loop of wire we have four finite length segments each contributing a term which looks like Eq. 33-12, except that L is replaced by $L/4$ and d is replaced by $L/8$. Then at the center,

$$B = 4\frac{\mu_0 i}{4\pi L/8}\frac{L/4}{\sqrt{L^2/64 + L^2/64}} = \frac{16\mu_0 i}{\sqrt{2}\pi L}.$$

For a circular loop $R = L/2\pi$ so

$$B = \frac{\mu_0 i}{2R} = \frac{\pi\mu_0}{L}.$$

Since $16/\sqrt{2}\,\pi > \pi$, the square wins. But only by some 7%!

$\boxed{\textbf{P33-7}}$ We want to use the differential expression in Eq. 33-11, except that the limits of integration are going to be different. We have four wire segments. From the top segment,

$$\begin{aligned} B_1 &= \frac{\mu_0 i}{4\pi}\frac{d}{\sqrt{z^2+d^2}}\bigg|_{-L/4}^{3L/4}, \\ &= \frac{\mu_0 i}{4\pi d}\left(\frac{3L/4}{\sqrt{(3L/4)^2+d^2}} - \frac{-L/4}{\sqrt{(-L/4)^2+d^2}}\right). \end{aligned}$$

For the top segment $d = L/4$, so this simplifies even further to

$$B_1 = \frac{\mu_0 i}{10\pi L}\left(\sqrt{2}(3\sqrt{5}+5)\right).$$

The bottom segment has the same integral, but $d = 3L/4$, so

$$B_3 = \frac{\mu_0 i}{30\pi L}\left(\sqrt{2}(\sqrt{5}+5)\right).$$

By symmetry, the contribution from the right hand side is the same as the bottom, so $B_2 = B_3$, and the contribution from the left hand side is the same as that from the top, so $B_4 = B_1$. Adding all four terms,

$$\begin{aligned} B &= \frac{2\mu_0 i}{30\pi L}\left(3\sqrt{2}(3\sqrt{5}+5) + \sqrt{2}(\sqrt{5}+5)\right), \\ &= \frac{2\mu_0 i}{3\pi L}(2\sqrt{2}+\sqrt{10}). \end{aligned}$$

P33-8 Assume a current ring has a radius r and a width dr, the charge on the ring is $dq = 2\pi\sigma r\,dr$, where $\sigma = q/\pi R^2$. The current in the ring is $di = \omega\,dq/2\pi = \omega\sigma r\,dr$. The ring contributes a field $dB = \mu_0\,di/2r$. Integrate over all the rings:

$$B = \int_0^R \mu_0\omega\sigma r\,dr/2r = \mu_0\omega R/2 = \mu\omega q/2\pi R.$$

P33-9 $B = \mu_0 in$ and $mv = qBr$. Combine, and

$$i = \frac{mv}{\mu_0 qrn} = \frac{(5.11\times10^5\mathrm{eV}/c^2)(0.046c)}{(4\pi\times10^{-7}\mathrm{N/A^2})e(0.023\,\mathrm{m})(10000/\mathrm{m})} = 0.271\,\mathrm{A}.$$

P33-10 This shape is a triangle with area $A = (4d)(3d)/2 = 6d^2$. The enclosed current is then

$$i = jA = (15\,\mathrm{A/m^2})6(0.23\,\mathrm{m})^2 = 4.76\,\mathrm{A}$$

The line integral is then

$$\mu_0 i = 6.0\times10^{-6}\mathrm{T}\cdot\mathrm{m}.$$

P33-11 Assume that B does vary as the picture implies. Then the line integral along the path shown *must* be nonzero, since $\vec{\mathbf{B}} \cdot \vec{\mathbf{l}}$ on the right is not zero, while it is along the three other sides. Hence $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ is non zero, implying some current passes through the dotted path. But it doesn't, so $\vec{\mathbf{B}}$ cannot have an abrupt change.

P33-12 (a) Sketch an Amperian loop which is a rectangle which enclosed N wires, has a vertical sides with height h, and horizontal sides with length L. Then $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 N i$. Evaluate the integral along the four sides. The vertical side contribute nothing, since $\vec{\mathbf{B}}$ is perpendicular to $\vec{\mathbf{h}}$, and then $\vec{\mathbf{B}} \cdot \vec{\mathbf{h}} = 0$. If the integral is performed in a counterclockwise direction (it must, since the sense of integration was determined by assuming the current is positive), we get BL for each horizontal section. Then

$$B = \frac{\mu_0 i N}{2L} = \frac{1}{2}\mu_0 i n.$$

(b) As $a \to \infty$ then $\tan^{-1}(a/2R) \to \pi/2$. Then $B \to \mu_0 i/2a$. If we assume that i is made up of several wires, each with current i_0, then $i/a = i_0 n$.

$\boxed{\textbf{P33-13}}$ Apply Ampere's law with an Amperian loop that is a circle centered on the center of the wire. Then

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \oint B\,ds = B \oint ds = 2\pi r B,$$

because $\vec{\mathbf{B}}$ is tangent to the path and B is uniform along the path by symmetry. The current enclosed is

$$i_{\text{enc}} = \int j\,dA.$$

This integral is best done in polar coordinates, so $dA = (dr)(r\,d\theta)$, and then

$$\begin{aligned} i_{\text{enc}} &= \int_0^r \int_0^{2\pi} (j_0 r/a)\, r dr\, d\theta, \\ &= 2\pi j_0/a \int_0^r r^2 dr, \\ &= \frac{2\pi j_0}{3a} r^3. \end{aligned}$$

When $r = a$ the current enclosed is i, so

$$i = \frac{2\pi j_0 a^2}{3} \text{ or } j_0 = \frac{3i}{2\pi a^2}.$$

The magnetic field strength inside the wire is found by gluing together the two parts of Ampere's law,

$$\begin{aligned} 2\pi r B &= \mu_0 \frac{2\pi j_0}{3a} r^3, \\ B &= \frac{\mu_0 j_0 r^2}{3a}, \\ &= \frac{\mu_0 i r^2}{2\pi a^3}. \end{aligned}$$

P33-14 (a) According to Eq. 33-34, the magnetic field inside the wire *without a hole* has magnitude $B = \mu_0 ir/2\pi R^2 = \mu_0 jr/2$ and is directed radially. If we superimpose a second current to create the hole, the additional field at the center of the hole is zero, so $B = \mu_0 jb/2$. But the current in the remaining wire is

$$i = jA = j\pi(R^2 - a^2),$$

so

$$B = \frac{\mu_0 ib}{2\pi(R^2 - a^2)}.$$

E34-1 $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = (42\times10^{-6}\mathrm{T})(2.5\,\mathrm{m}^2)\cos(57^\circ) = 5.7\times10^{-5}\mathrm{Wb}.$

E34-2 $|\mathcal{E}| = |d\Phi_B/dt| = A\,dB/dt = (\pi/4)(0.112\,\mathrm{m})^2(0.157\,\mathrm{T/s}) = 1.55\,\mathrm{mV}.$

$\boxed{\textbf{E34-3}}$ (a) The magnitude of the emf induced in a loop is given by Eq. 34-4,

$$\begin{aligned} |\mathcal{E}| &= N\left|\frac{d\Phi_B}{dt}\right|, \\ &= N\left|(12\,\mathrm{mWb/s^2})t + (7\,\mathrm{mWb/s})\right| \end{aligned}$$

There is only one loop, and we want to evaluate this expression for $t = 2.0\,\mathrm{s}$, so

$$|\mathcal{E}| = (1)\left|(12\,\mathrm{mWb/s^2})(2.0\,\mathrm{s}) + (7\,\mathrm{mWb/s})\right| = 31\,\mathrm{mV}.$$

(b) This part isn't harder. The magnetic flux through the loop is increasing when $t = 2.0\,\mathrm{s}$. The induced current needs to flow in such a direction to create a *second* magnetic field to oppose this increase. The original magnetic field is out of the page and we oppose the increase by pointing the other way, so the *second* field will point into the page (inside the loop).

By the right hand rule this means the induced current is clockwise through the loop, or to the left through the resistor.

E34-4 $\mathcal{E} = -d\Phi_B/dt = -A\,dB/dt.$

(a) $\mathcal{E} = -\pi(0.16\,\mathrm{m})^2(0.5\,\mathrm{T})/(2\,\mathrm{s}) = -2.0\times10^{-2}\mathrm{V}.$

(b) $\mathcal{E} = -\pi(0.16\,\mathrm{m})^2(0.0\,\mathrm{T})/(2\,\mathrm{s}) = 0.0\times10^{-2}\mathrm{V}.$

(c) $\mathcal{E} = -\pi(0.16\,\mathrm{m})^2(-0.5\,\mathrm{T})/(4\,\mathrm{s}) = 1.0\times10^{-2}\mathrm{V}.$

E34-5 (a) $R = \rho L/A = (1.69\times10^{-8}\Omega\cdot\mathrm{m})[(\pi)(0.104\,\mathrm{m})]/[(\pi/4)(2.50\times10^{-3}\mathrm{m})^2] = 1.12\times10^{-3}\Omega.$

(b) $\mathcal{E} = iR = (9.66\,\mathrm{A})(1.12\times10^{-3}\Omega) = 1.08\times10^{-2}\mathrm{V}$. The required dB/dt is then given by

$$\frac{dB}{dt} = \frac{\mathcal{E}}{A} = (1.08\times10^{-2}\,\mathrm{V})/(\pi/4)(0.104\,\mathrm{m})^2 = 1.27\,\mathrm{T/s}.$$

E34-6 $\mathcal{E} = -A\,\Delta B/\Delta t = AB/\Delta t$. The power is $P = i\mathcal{E} = \mathcal{E}^2/R$. The energy dissipated is

$$E = P\Delta t = \frac{\mathcal{E}^2\Delta t}{R} = \frac{A^2B^2}{R\Delta t}.$$

$\boxed{\textbf{E34-7}}$ (a) We could re-derive the steps in the sample problem, or we could start with the end result. We'll start with the result,

$$\mathcal{E} = NA\mu_0 n\left|\frac{di}{dt}\right|,$$

except that we have gone ahead and used the derivative instead of the Δ.

The rate of change in the current is

$$\frac{di}{dt} = (3.0\,\mathrm{A/s}) + (1.0\,\mathrm{A/s^2})t,$$

so the induced emf is

$$\begin{aligned} \mathcal{E} &= (130)(3.46\times10^{-4}\mathrm{m}^2)(4\pi\times10^{-7}\mathrm{Tm/A})(2.2\times10^4/\mathrm{m})\left((3.0\mathrm{A/s}) + (2.0\mathrm{A/s^2})t\right), \\ &= (3.73\times10^{-3}\,\mathrm{V}) + (2.48\times10^{-3}\,\mathrm{V/s})t. \end{aligned}$$

(b) When $t = 2.0\,\mathrm{s}$ the induced emf is $8.69\times10^{-3}\,\mathrm{V}$, so the induced current is

$$i = (8.69\times10^{-3}\,\mathrm{V})/(0.15\,\Omega) = 5.8\times10^{-2}\,\mathrm{A}.$$

E34-8 (a) $i = \mathcal{E}/R = NA\,dB/dt$. Note that A refers to the area enclosed by the outer solenoid where B is non-zero. This A is then the cross sectional area of the inner solenoid! Then

$$i = \frac{1}{R}NA\mu_0 n\frac{di}{dt} = \frac{(120)(\pi/4)(0.032\,\text{m})^2(4\pi\times 10^{-7}\text{N/A}^2)(220\times 10^2/\text{m})}{(5.3\,\Omega)}\frac{(1.5\,\text{A})}{(0.16\,\text{s})} = 4.7\times 10^{-3}\text{A}.$$

E34-9 $P = \mathcal{E}i = \mathcal{E}^2/R = (A\,dB/dt)^2/(\rho L/a)$, where A is the area of the loop and a is the cross sectional area of the wire. But $a = \pi d^2/4$ and $A = L^2/4\pi$, so

$$P = \frac{L^3 d^2}{64\pi\rho}\left(\frac{dB}{dt}\right)^2 = \frac{(0.525\,\text{m})^3(1.1\times 10^{-3}\text{m})^2}{64\pi(1.69\times 10^{-8}\Omega\cdot\text{m})}(9.82\times 10^{-3}\text{T/s})^2 = 4.97\times 10^{-6}\text{W}.$$

E34-10 $\Phi_B = BA = B(2.3\,\text{m})^2/2$. $\mathcal{E}_B = -d\Phi_B/dt = -AdB/dt$, or

$$\mathcal{E}_B = -\frac{(2.3\,\text{m})^2}{2}[-(0.87\,\text{T/s})] = 2.30\,\text{V},$$

so $\mathcal{E} = (2.0\,\text{V}) + (2.3\,\text{V}) = 4.3\,\text{V}$.

$\boxed{\textbf{E34-11}}$ (a) The induced emf, as a function of time, is given by Eq. 34-5, $\mathcal{E}(t) = -d\Phi_B(t)/dt$ This emf drives a current through the loop which obeys $\mathcal{E}(t) = i(t)R$ Combining,

$$i(t) = -\frac{1}{R}\frac{d\Phi_B(t)}{dt}.$$

Since the current is defined by $i = dq/dt$ we can write

$$\frac{dq(t)}{dt} = -\frac{1}{R}\frac{d\Phi_B(t)}{dt}.$$

Factor out the dt from both sides, and then integrate:

$$\begin{aligned} dq(t) &= -\frac{1}{R}\,d\Phi_B(t), \\ \int dq(t) &= -\int \frac{1}{R}\,d\Phi_B(t), \\ q(t) - q(0) &= \frac{1}{R}\left(\Phi_B(0) - \Phi_B(t)\right) \end{aligned}$$

(b) No. The induced current could have increased from zero to some positive value, then decreased to zero and became negative, so that the net charge to flow through the resistor was zero. This would be like sloshing the charge back and forth through the loop.

E34-12 $\Delta Phi_B = 2\Phi_B = 2NBA$. Then the charge to flow through is

$$q = 2(125)(1.57\,\text{T})(12.2\times 10^{-4}\text{m}^2)/(13.3\,\Omega) = 3.60\times 10^{-2}\text{C}.$$

E34-13 The part above the long straight wire (a distance $b-a$ above it) cancels out contributions below the wire (a distance $b-a$ beneath it). The flux through the loop is then

$$\Phi_B = \int_{2a-b}^{a}\frac{\mu_0 i}{2\pi r}b\,dr = \frac{\mu_0 ib}{2\pi}\ln\left(\frac{a}{2a-b}\right).$$

The emf in the loop is then

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{2a-b}\right) [2(4.5\,\mathrm{A/s^2})t - (10\,\mathrm{A/s})].$$

Evaluating,

$$\mathcal{E} = \frac{4\pi\times 10^{-7}\mathrm{N/A^2}(0.16\,\mathrm{m})}{2\pi} \ln\left(\frac{(0.12\,\mathrm{m})}{2(0.12\,\mathrm{m}) - (0.16\,\mathrm{m})}\right) [2(4.5\,\mathrm{A/s^2})(3.0\,\mathrm{s}) - (10\,\mathrm{A/s})] = 2.20\times 10^{-7}\mathrm{V}.$$

E34-14 Use Eq. 34-6: $\mathcal{E} = BDv = (55\times 10^{-6}\mathrm{T})(1.10\,\mathrm{m})(25\,\mathrm{m/s}) = 1.5\times 10^{-3}\mathrm{V}$.

E34-15 If the angle doesn't vary then the flux, given by

$$\Phi = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$$

is constant, so there is no emf.

E34-16 (a) Use Eq. 34-6: $\mathcal{E} = BDv = (1.18\mathrm{T})(0.108\,\mathrm{m})(4.86\,\mathrm{m/s}) = 0.619\,\mathrm{V}$.
(b) $i = (0.619\,\mathrm{V})/(0.415\,\Omega) = 1.49\,\mathrm{A}$.
(c) $P = (0.619\,\mathrm{V})(1.49\,\mathrm{A}) = 0.922\,\mathrm{W}$.
(d) $F = iLB = (1.49\,\mathrm{A})(0.108\,\mathrm{m})(1.18\mathrm{T}) = 0.190\,\mathrm{N}$.
(e) $P = Fv = (0.190\,\mathrm{V})(4.86\,\mathrm{m/s}) = 0.923\,\mathrm{W}$.

E34-17 The magnetic field is out of the page, and the current through the rod is down. Then Eq. 32-26 $\vec{\mathbf{F}} = i\vec{\mathbf{L}} \times \vec{\mathbf{B}}$ shows that the direction of the magnetic force is to the right; furthermore, since everything is perpendicular to everything else, we can get rid of the vector nature of the problem and write $F = iLB$. Newton's second law gives $F = ma$, and the acceleration of an object from rest results in a velocity given by $v = at$. Combining,

$$v(t) = \frac{iLB}{m}t.$$

E34-18 (b) The rod will accelerate as long as there is a net force on it. This net force comes from $F = iLB$. The current is given by $iR = \mathcal{E} - BLv$, so as v increases i decreases. When $i = 0$ the rod stops accelerating and assumes a terminal velocity.
(a) $\mathcal{E} = BLv$ will give the terminal velocity. In this case, $v = \mathcal{E}/BL$.

E34-19

E34-20 The acceleration is $a = R\omega^2$; since $\mathcal{E} = B\omega R^2/2$, we can find

$$a = 4\mathcal{E}^2/B^2R^3 = 4(1.4\,\mathrm{V})^2/(1.2\,\mathrm{T})^2(5.3\times 10^{-2}\mathrm{m})^3 = 3.7\times 10^4\mathrm{m/s^2}.$$

E34-21 We will use the results of Exercise 11 that were worked out above. All we need to do is find the initial flux; flipping the coil up-side-down will simply change the sign of the flux.
So

$$\Phi_B(0) = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = (59\,\mu\mathrm{T})(\pi)(0.13\,\mathrm{m})^2 \sin(20^\circ) = 1.1\times 10^{-6}\,\mathrm{Wb}.$$

Then using the results of Exercise 11 we have

$$\begin{aligned} q &= \frac{N}{R}(\Phi_B(0) - \Phi_B(t)), \\ &= \frac{950}{85\,\Omega}((1.1\times 10^{-6}\,\mathrm{Wb}) - (-1.1\times 10^{-6}\,\mathrm{Wb})), \\ &= 2.5\times 10^{-5}\,\mathrm{C}. \end{aligned}$$

E34-22 (a) The flux through the loop is

$$\Phi_B = \int_0^{vt} dx \int_a^{a+L} dr \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i v t}{2\pi} \ln \frac{a+L}{a}.$$

The emf is then

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 i v}{2\pi} \ln \frac{a+L}{a}.$$

Putting in the numbers,

$$\mathcal{E} = \frac{(4\pi\times 10^{-7}\mathrm{N/A^2})(110\,\mathrm{A})(4.86\,\mathrm{m/s})}{2\pi} \ln \frac{(0.0102\,\mathrm{m}) + (0.0983\,\mathrm{m})}{(0.0102\,\mathrm{m})} = 2.53\times 10^{-4}\mathrm{V}.$$

(b) $i = \mathcal{E}/R = (2.53\times 10^{-4}\mathrm{V})/(0.415\,\Omega) = 6.10\times 10^{-4}\mathrm{A}$.
(c) $P = i^2 R = (6.10\times 10^{-4}\mathrm{A})^2(0.415\,\Omega) = 1.54\times 10^{-7}\mathrm{W}$.
(d) $F = \int B i_l\, dl$, or

$$F = i_l \int_a^{a+L} dr \frac{\mu_0 i}{2\pi r} = i_l \frac{\mu_0 i}{2\pi} \ln \frac{a+L}{a}.$$

Putting in the numbers,

$$F = (6.10\times 10^{-4}\mathrm{A})\frac{(4\pi\times 10^{-7}\mathrm{N/A^2})(110\,\mathrm{A})}{2\pi} \ln \frac{(0.0102\,\mathrm{m}) + (0.0983\,\mathrm{m})}{(0.0102\,\mathrm{m})} = 3.17\times 10^{-8}\mathrm{N}.$$

(e) $P = Fv = (3.17\times 10^{-8}\mathrm{N})(4.86\,\mathrm{m/s}) = 1.54\times 10^{-7}\mathrm{W}$.

E34-23 (a) Starting from the beginning, Eq. 33-13 gives

$$B = \frac{\mu_0 i}{2\pi y}.$$

The flux through the loop is given by

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}},$$

but since the magnetic field from the long straight wire goes through the loop perpendicular to the plane of the loop this expression simplifies to a scalar integral. The loop is a rectangular, so use $dA = dx\, dy$, and let x be parallel to the long straight wire.

Combining the above,

$$\begin{aligned} \Phi_B &= \int_D^{D+b} \int_0^a \left(\frac{\mu_0 i}{2\pi y}\right) dx\, dy, \\ &= \frac{\mu_0 i}{2\pi} a \int_D^{D+b} \frac{dy}{y}, \\ &= \frac{\mu_0 i}{2\pi} a \ln\left(\frac{D+b}{D}\right) \end{aligned}$$

(b) The flux through the loop is a function of the distance D from the wire. If the loop moves away from the wire at a constant speed v, then the distance D varies as vt. The induced emf is then

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt}, \\ &= \frac{\mu_0 i}{2\pi} a \frac{b}{t(vt+b)}. \end{aligned}$$

The current will be this emf divided by the resistance R. The "back-of-the-book" answer is somewhat different; the answer is expressed in terms of D instead if t. The two answers are otherwise identical.

E34-24 (a) The area of the triangle is $A = x^2 \tan\theta/2$. In this case $x = vt$, so

$$\Phi_B = B(vt)^2 \tan\theta/2,$$

and then

$$\mathcal{E} = 2Bv^2 t \tan\theta/2,$$

(b) $t = \mathcal{E}/2Bv^2 \tan\theta/2$, so

$$t = \frac{(56.8\,\mathrm{V})}{2(0.352\,\mathrm{T})(5.21\,\mathrm{m/s})^2 \tan(55^\circ)} = 2.08\,\mathrm{s}.$$

E34-25 $\mathcal{E} = NBA\omega$, so

$$\omega = \frac{(24\,\mathrm{V})}{(97)(0.33\,\mathrm{T})(0.0190\,\mathrm{m}^2)} = 39.4\,\mathrm{rad/s}.$$

That's 6.3 rev/second.

E34-26 (a) The frequency of the emf is the same as the frequency of rotation, f.

(b) The flux changes by $BA = B\pi a^2$ during a half a revolution. This is a sinusoidal change, so the amplitude of the sinusoidal variation in the emf is $\mathcal{E} = \Phi_B\omega/2$. Then $\mathcal{E} = B\pi^2 a^2 f$.

$\boxed{\textbf{E34-27}}$ We can use Eq. 34-10; the emf is $\mathcal{E} = BA\omega \sin\omega t$, This will be a maximum when $\sin\omega t = 1$. The angular frequency, ω is equal to $\omega = (1000)(2\pi)/(60)$ rad/s $= 105$ rad/s The maximum emf is then

$$\mathcal{E} = (3.5\,\mathrm{T})\left[(100)(0.5\,\mathrm{m})(0.3\,\mathrm{m})\right](105\text{ rad/s}) = 5.5\text{ kV}.$$

E34-28 (a) The amplitude of the emf is $\mathcal{E} = BA\omega$, so

$$A = \mathcal{E}/2\pi f B = (150\,\mathrm{V})/2\pi(60/\mathrm{s})(0.50\,\mathrm{T}) = 0.798\mathrm{m}^2.$$

(b) Divide the previous result by 100. $A = 79.8$ cm^2.

E34-29 $d\Phi_B/dt = A\,dB/dt = A(-8.50\,\mathrm{mT/s})$.

(a) For this path

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -d\Phi_B/dt = --\,\pi(0.212\,\mathrm{m})^2(-8.50\,\mathrm{mT/s}) = -1.20\,\mathrm{mV}.$$

(b) For this path

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -d\Phi_B/dt = --\,\pi(0.323\,\mathrm{m})^2(-8.50\,\mathrm{mT/s}) = -2.79\,\mathrm{mV}.$$

(c) For this path

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -d\Phi_B/dt = - - \pi(0.323\,\mathrm{m})^2(-8.50\,\mathrm{mT/s}) - \pi(0.323\,\mathrm{m})^2(-8.50\,\mathrm{mT/s}) = 1.59\,\mathrm{mV}.$$

E34-30 $d\Phi_B/dt = A\,dB/dt = A(-6.51\,\mathrm{mT/s})$, while $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 2\pi r E$.

(a) The path of integration is *inside* the solenoid, so

$$E = \frac{-\pi r^2(-6.51\,\mathrm{mT/s})}{2\pi r} = \frac{(0.022\,\mathrm{m})(-6.51\,\mathrm{mT/s})}{2} = 7.16\times 10^{-5}\mathrm{V/m}.$$

(b) The path of integration is *outside* the solenoid, so

$$E = \frac{-\pi r^2(-6.51\,\mathrm{mT/s})}{2\pi R} = \frac{(0.063\,\mathrm{m})^2(-6.51\,\mathrm{mT/s})}{2(0.082\,\mathrm{m})} = 1.58\times 10^{-4}\mathrm{V/m}$$

E34-31 The induced electric field can be found from applying Eq. 34-13,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}.$$

We start with the left hand side of this expression. The problem has cylindrical symmetry, so the induced electric field lines should be circles centered on the axis of the cylindrical volume. If we choose the path of integration to lie along an electric field line, then the electric field $\vec{\mathbf{E}}$ will be parallel to $d\vec{\mathbf{s}}$, and E will be uniform along this path, so

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = \oint E\,ds = E \oint ds = 2\pi r E,$$

where r is the radius of the circular path.

Now for the right hand side. The flux is contained in the path of integration, so $\Phi_B = B\pi r^2$. All of the time dependence of the flux is contained in B, so we can immediately write

$$2\pi r E = -\pi r^2 \frac{dB}{dt} \text{ or } E = -\frac{r}{2}\frac{dB}{dt}.$$

What does the negative sign mean? The path of integration is chosen so that if our right hand fingers curl around the path our thumb gives the direction of the magnetic field which cuts through the path. Since the field points into the page a positive electric field would have a clockwise orientation. Since B is decreasing the derivative is negative, but we get another negative from the equation above, so the electric field has a positive direction.

Now for the magnitude.

$$E = (4.82\times 10^{-2}\,\mathrm{m})(10.7\times 10^{-3}\,T/\mathrm{s})/2 = 2.58\times 10^{-4}\,\mathrm{N/C}.$$

The acceleration of the electron at either a or c then has magnitude

$$a = Eq/m = (2.58\times 10^{-4}\,\mathrm{N/C})(1.60\times 10^{-19}\,\mathrm{C})/(9.11\times 10^{-31}\,\mathrm{kg}) = 4.53\times 10^{7}\mathrm{m/s^2}.$$

P34-1 The induced current is given by $i = \mathcal{E}/R$. The resistance of the loop is given by $R = \rho L/A$, where A is the cross sectional area. Combining, and writing in terms of the radius of the wire, we have

$$i = \frac{\pi r^2 \mathcal{E}}{\rho L}.$$

The length of the wire is related to the radius of the wire because we have a fixed mass. The total volume of the wire is $\pi r^2 L$, and this is related to the mass and density by $m = \delta \pi r^2 L$. Eliminating r we have

$$i = \frac{m\mathcal{E}}{\rho \delta L^2}.$$

The length of the wire loop is the same as the circumference, which is related to the radius R of the loop by $L = 2\pi R$. The emf is related to the changing flux by $\mathcal{E} = -d\Phi_B/dt$, but if the shape of the loop is fixed this becomes $\mathcal{E} = -A\,dB/dt$. Combining all of this,

$$i = \frac{mA}{\rho\delta(2\pi R)^2}\frac{dB}{dt}.$$

We dropped the negative sign because we are only interested in absolute values here.

Now $A = \pi R^2$, so this expression can also be written as

$$i = \frac{m\pi R^2}{\rho\delta(2\pi R)^2}\frac{dB}{dt} = \frac{m}{4\pi\rho\delta}\frac{dB}{dt}.$$

P34-2 For the lower surface $\vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = (76\times10^{-3}\mathrm{T})(\pi/2)(0.037\,\mathrm{m})^2\cos(62^\circ) = 7.67\times10^{-5}\mathrm{Wb}$. For the upper surface $\vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = (76\times10^{-3}\mathrm{T})(\pi/2)(0.037\,\mathrm{m})^2\cos(28^\circ) = 1.44\times10^{-4}\mathrm{Wb}$.. The induced emf is then

$$\mathcal{E} = (7.67\times10^{-5}\mathrm{Wb} + 1.44\times10^{-4}\mathrm{Wb})/(4.5\times10^{-3}\mathrm{s}) = 4.9\times10^{-2}\mathrm{V}.$$

P34-3 (a) We are only interested in the portion of the ring in the yz plane. Then $\mathcal{E} = (3.32\times 10^{-3}\mathrm{T/s})(\pi/4)(0.104\,\mathrm{m})^2 = 2.82\times10^{-5}\mathrm{V}$.

(b) From c to b. Point your right thumb along $-x$ to oppose the increasing $\vec{\mathbf{B}}$ field. Your right fingers will curl from c to b.

P34-4 $\mathcal{E} \propto NA$, but $A = \pi r^2$ and $N2\pi r = L$, so $\mathcal{E} \propto 1/N$. This means use only one loop to maximize the emf.

$\boxed{\textbf{P34-5}}$ This is a integral best performed in rectangular coordinates, then $dA = (dx)(dy)$. The magnetic field is perpendicular to the surface area, so $\vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = B\,dA$. The flux is then

$$\begin{aligned}
\Phi_B &= \int \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = \int B\,dA,\\
&= \int_0^a\int_0^a (4\,\mathrm{T/m\cdot s^2})t^2 y\,dy\,dx,\\
&= (4\,\mathrm{T/m\cdot s^2})t^2\left(\frac{1}{2}a^2\right)a,\\
&= (2\,\mathrm{T/m\cdot s^2})a^3t^2.
\end{aligned}$$

But $a = 2.0$ cm, so this becomes

$$\Phi_B = (2\,\mathrm{T/m\cdot s^2})(0.02\,\mathrm{m})^3t^2 = (1.6\times10^{-5}\mathrm{Wb/s^2})t^2.$$

The emf around the square is given by

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -(3.2\times10^{-5}\mathrm{Wb/s^2})t,$$

and at $t = 2.5\,\mathrm{s}$ this is $-8.0\times10^{-5}\mathrm{V}$. Since the magnetic field is directed out of the page, a positive emf would be counterclockwise (hold your right thumb in the direction of the magnetic field and your fingers will give a counter clockwise sense around the loop). But the answer was negative, so the emf must be clockwise.

P34-6 (a) Far from the plane of the large loop we can approximate the large loop as a dipole, and then

$$B = \frac{\mu_0 i \pi R^2}{2x^3}.$$

The flux through the small loop is then

$$\Phi_B = \pi r^2 B = \frac{\mu_0 i \pi^2 r^2 R^2}{2x^3}.$$

(b) $\mathcal{E} = -d\Phi_B/dt$, so

$$\mathcal{E} = \frac{3\mu_0 i \pi^2 r^2 R^2}{2x^4} v.$$

(c) Anti-clockwise when viewed from above.

$\boxed{\textbf{P34-7}}$ The magnetic field is perpendicular to the surface area, so $\vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = B\,dA$. The flux is then

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = BA,$$

since the magnetic field is uniform. The area is $A = \pi r^2$, where r is the radius of the loop. The induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -2\pi r B \frac{dr}{dt}.$$

It is given that $B = 0.785\,\mathrm{T}$, $r = 1.23\,\mathrm{m}$, and $dr/dt = -7.50\times10^{-2}\mathrm{m/s}$. The negative sign indicate a decreasing radius. Then

$$\mathcal{E} = -2\pi(1.23\,\mathrm{m})(0.785\,\mathrm{T})(-7.50\times10^{-2}\mathrm{m/s}) = 0.455\,\mathrm{V}.$$

P34-8 (a) $d\Phi_B/dt = B\,dA/dt$, but dA/dt is $\Delta A/\Delta t$, where ΔA is the area swept out during one rotation and $\Delta t = 1/f$. But the area swept out is πR^2, so

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \pi f B R^2.$$

(b) If the output current is i then the power is $P = \mathcal{E}i$. But $P = \tau\omega = \tau 2\pi f$, so

$$\tau = \frac{P}{2\pi f} = iBR^2/2.$$

P34-9 (a) $\mathcal{E} = -d\Phi_B/dt$, and $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$,so

$$\mathcal{E} = BLv\cos\theta.$$

The component of the force of gravity on the rod which pulls it down the incline is $F_G = mg\sin\theta$. The component of the magnetic force on the rod which pulls it up the incline is $F_B = BiL\cos\theta$. Equating,

$$BiL\cos\theta = mg\sin\theta,$$

and since $\mathcal{E} = iR$,

$$v = \frac{\mathcal{E}}{BL\cos\theta} = \frac{mgR\sin\theta}{B^2L^2\cos^2\theta}.$$

(b) $P = i\mathcal{E} = \mathcal{E}^2/R = B^2L^2v^2\cos^2\theta/R = mgv\sin\theta$. This is identical to the rate of change of gravitational potential energy.

P34-10 Let the cross section of the wire be a.

(a) $R = \rho L/a = \rho(r\theta + 2r)/a$; with numbers,

$$R = (3.4\times 10^{-3}\Omega)(2 + \theta).$$

(b) $\Phi_B = B\theta r^2/2$; with numbers,

$$\Phi_B = (4.32\times 10^{-3}\text{Wb})\theta.$$

(c) $i = \mathcal{E}/R = B\omega r^2/2R = Ba\omega r/2\rho(\theta + 2)$, or

$$i = \frac{Ba\alpha tr}{\rho(\alpha t^2 + 4)}.$$

Take the derivative and set it equal to zero,

$$0 = \frac{4 - at^2}{(\alpha t^2 + 4)^2},$$

so $at^2 = 4$, or $\theta = \frac{1}{2}at^2 = 2$ rad.

(d) $\omega = \sqrt{2\alpha\theta}$, so

$$i = \frac{(0.15\,\text{T})(1.2\times 10^{-6}\text{m}^2)\sqrt{2(12\text{ rad/s}^2)(2\text{ rad})}(0.24\,\text{m})}{(1.7\times 10^{-8}\Omega\cdot\text{m})(6\text{ rad})} = 2.2\,\text{A}.$$

$\boxed{\textbf{P34-11}}$ It does say approximate, so we will be making some rather bold assumptions here. First we will find an expression for the emf. Since B is constant, the emf must be caused by a change in the area; in this case a shift in position. The small square where $B \neq 0$ has a width a and sweeps around the disk with a speed $r\omega$. An approximation for the emf is then $\mathcal{E} = Bar\omega$. This emf causes a current. We don't know exactly where the current flows, but we can reasonably assume that it occurs near the location of the magnetic field. Let us assume that it is constrained to that region of the disk. The resistance of this portion of the disk is the approximately

$$R = \frac{1}{\sigma}\frac{L}{A} = \frac{1}{\sigma}\frac{a}{at} = \frac{1}{\sigma t},$$

where we have assumed that the current is flowing radially when defining the cross sectional area of the "resistor". The induced current is then (on the order of)

$$\frac{\mathcal{E}}{R} = \frac{Bar\omega}{1/(\sigma t)} = Bar\omega\sigma t.$$

This current experiences a breaking force according to $F = BIl$, so

$$F = B^2a^2r\omega\sigma t,$$

where l is the length through which the current flows, which is a.

Finally we can find the torque from $\tau = rF$, and

$$\tau = B^2a^2r^2\omega\sigma t.$$

P34-12 The induced electric field in the ring is given by Eq. 34-11: $2\pi RE = |d\Phi_B/dt|$. This electric field will result in a force on the free charge carries (electrons?), given by $F = Ee$. The acceleration of the electrons is then $a = Ee/m_\mathrm{e}$. Then

$$a = \frac{e}{2\pi R m_\mathrm{e}}\frac{d\Phi_B}{dt}.$$

Integrate both sides with respect to time to find the speed of the electrons.

$$\begin{aligned}\int a\,dt &= \int \frac{e}{2\pi R m_\mathrm{e}}\frac{d\Phi_B}{dt}dt,\\ v &= \frac{e}{2\pi R m_\mathrm{e}}\int \frac{d\Phi_B}{,}\\ &= \frac{e}{2\pi R m_\mathrm{e}}\Delta\Phi_B.\end{aligned}$$

The current density is given by $j = nev$, and the current by $iA = i\pi a^2$. Combining,

$$i = \frac{ne^2a^2}{2Rm_\mathrm{e}}\Delta Phi_B.$$

Actually, it should be pointed out that ΔPhi_B refers to the change in flux from *external* sources. The current induced in the wire will produce a flux which will exactly offset ΔPhi_B so that the *net* flux through the superconducting ring is fixed at the value present when the ring became superconducting.

P34-13 Assume that E does vary as the picture implies. Then the line integral along the path shown *must* be nonzero, since $\vec{\mathbf{E}}\cdot\vec{\mathbf{l}}$ on the right is not zero, while it is along the three other sides. Hence $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}$ is non zero, implying a change in the magnetic flux through the dotted path. But it doesn't, so $\vec{\mathbf{E}}$ cannot have an abrupt change.

P34-14 The electric field a distance r from the center is given by

$$E = \frac{\pi r^2 dB/dT}{2\pi r} = \frac{r}{2}\frac{dB}{dt}.$$

This field is directed perpendicular to the radial lines.

Define h to be the distance from the center of the circle to the center of the rod, and evaluate $\mathcal{E} = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$,

$$\begin{aligned}\mathcal{E} &= \frac{dB}{dt}\int \frac{r}{2}\frac{h}{r}dx,\\ &= \frac{dB}{dt}\frac{L}{2}h.\end{aligned}$$

But $h^2 = R^2 - (L/2)^2$, so

$$\mathcal{E} = \frac{dB}{dt}\frac{L}{2}\sqrt{R^2-(L/2)^2}.$$

P34-15 (a) $\Phi_B = \pi r^2 B_\mathrm{av}$, so

$$E = \frac{\mathcal{E}}{2\pi r} = \frac{(0.32\,\mathrm{m})}{2}2(0.28\,\mathrm{T})(120\,\pi) = 34\,\mathrm{V/m}.$$

(b) $a = F/m = Eq/m = (33.8\,\mathrm{V/m})(1.6\times10^{-19}\mathrm{C})/(9.1\times10^{-31}\mathrm{kg}) = 6.0\times10^{12}\mathrm{m/s^2}$.

$\boxed{\textbf{E35-1}}$ If the Earth's magnetic dipole moment were produced by a single current around the core, then that current would be

$$i = \frac{\mu}{A} = \frac{(8.0 \times 10^{22}\,\mathrm{J/T})}{\pi(3.5 \times 10^6\mathrm{m})^2} = 2.1\times 10^9\,\mathrm{A}$$

E35-2 (a) $i = \mu/A = (2.33\,\mathrm{A}\cdot\mathrm{m}^2)/(160)\pi(0.0193\,\mathrm{m})^2 = 12.4\,\mathrm{A}$.
(b) $\tau = \mu B = (2.33\,\mathrm{A}\cdot\mathrm{m}^2)(0.0346\,\mathrm{T}) = 8.06\times 10^{-2}\mathrm{N}\cdot\mathrm{m}$.

E35-3 (a) Using the right hand rule a clockwise current would generate a magnetic moment which would be into the page. Both currents are clockwise, so add the moments:

$$\mu = (7.00\,\mathrm{A})\pi(0.20\,\mathrm{m})^2 + (7.00\,\mathrm{A})\pi(0.30\,\mathrm{m})^2 = 2.86\,\mathrm{A}\cdot\mathrm{m}^2.$$

(b) Reversing the current reverses the moment, so

$$\mu = (7.00\,\mathrm{A})\pi(0.20\,\mathrm{m})^2 - (7.00\,\mathrm{A})\pi(0.30\,\mathrm{m})^2 = -1.10\,\mathrm{A}\cdot\mathrm{m}^2.$$

E35-4 (a) $\mu = iA = (2.58\,\mathrm{A})\pi(0.16\,\mathrm{m})^2 = 0.207\,\mathrm{A}\cdot\mathrm{m}^2$.
(b) $\tau = \mu B\sin\theta = (0.207\,\mathrm{A}\cdot\mathrm{m}^2)(1.20\,\mathrm{T})\sin(41^\circ) = 0.163\,\mathrm{N}\cdot\mathrm{m}$.

$\boxed{\textbf{E35-5}}$ (a) The result from Problem 33-4 for a square loop of wire was

$$B(z) = \frac{4\mu_0 i a^2}{\pi(4z^2 + a^2)(4z^2 + 2a^2)^{1/2}}.$$

For z much, much larger than a we can ignore any a terms which are added to or subtracted from z terms. This means that

$$4z^2 + a^2 \rightarrow 4z^2 \text{ and } (4z^2 + 2a^2)^{1/2} \rightarrow 2z,$$

but we can't ignore the a^2 in the numerator.

The expression for B then simplifies to

$$B(z) = \frac{\mu_0 i a^2}{2\pi z^3},$$

which certainly looks like Eq. 35-4.

(b) We can rearrange this expression and get

$$B(z) = \frac{\mu_0}{2\pi z^3} i a^2,$$

where it is rather evident that ia^2 must correspond to $\vec{\mu}$, the dipole moment, in Eq. 35-4. So that must be the answer.

E35-6 $\mu = iA = (0.2\,\mathrm{A})\pi(0.08\,\mathrm{m})^2 = 4.02\times 10^{-3}\mathrm{A}\cdot\mathrm{m}^2$; $\vec{\mu} = \mu\hat{\mathbf{n}}$.
(a) For the torque,

$$\vec{\tau} = \vec{\mu}\times\vec{\mathbf{B}} = (-9.65\times 10^{-4}\mathrm{N}\cdot\mathrm{m})\hat{\mathbf{i}} + (-7.24\times 10^{-4}\mathrm{N}\cdot\mathrm{m})\hat{\mathbf{j}} + (8.08\times 10^{-4}\mathrm{N}\cdot\mathrm{m})\hat{\mathbf{k}}.$$

(b) For the magnetic potential energy,

$$U = \vec{\mu}\cdot\vec{\mathbf{B}} = \mu[(0.60)(0.25\,\mathrm{T})] = 0.603\times 10^{-3}\mathrm{J}.$$

E35-7 $\mu = iA = i\pi(a^2 + b^2/2) = i\pi(a^2 + b^2)/2.$

E35-8 If the distance to P is very large compared to a or b we can write the Law of Biot and Savart as

$$\vec{\mathbf{B}} = \frac{\mu_0 i}{4\pi}\frac{\vec{\mathbf{s}} \times \vec{\mathbf{r}}}{r^3}.$$

$\vec{\mathbf{s}}$ is perpendicular to $\vec{\mathbf{r}}$ for the left and right sides, so the left side contributes

$$B_1 = \frac{\mu_0 i}{4\pi}\frac{b}{(x + a/2)^2},$$

and the right side contributes

$$B_3 = -\frac{\mu_0 i}{4\pi}\frac{b}{(x - a/2)^2}.$$

The top and bottom sides each contribute an equal amount

$$B_2 = B_4 = \frac{\mu_0 i}{4\pi}\frac{a \sin\theta}{x^2 + b^2/4} \approx \frac{\mu_0 i}{4\pi}\frac{a(b/2)}{x^3}.$$

Add the four terms, expand the denominators, and keep only terms in x^3,

$$B = -\frac{\mu_0 i}{4\pi}\frac{ab}{x^3} = -\frac{\mu_0}{4\pi}\frac{\mu}{x^3}.$$

The negative sign indicates that it is into the page.

$\boxed{\textbf{E35-9}}$ (a) The electric field at this distance from the proton is

$$E = \frac{1}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)}\frac{(1.60\times10^{-19}\mathrm{C})}{(5.29\times10^{-11}\mathrm{m})^2} = 5.14\times10^{11}\mathrm{N/C}.$$

(b) The magnetic field at this from the proton is given by the dipole approximation,

$$\begin{aligned} B(z) &= \frac{\mu_0 \mu}{2\pi z^3}, \\ &= \frac{(4\pi\times10^{-7}\mathrm{N/A}^2)(1.41\times10^{-26}\mathrm{A/m}^2)}{2\pi(5.29\times10^{-11}\mathrm{m})^3}, \\ &= 1.90\times10^{-2}\,\mathrm{T} \end{aligned}$$

E35-10 1.50 g of water has $(2)(6.02\times10^{23})(1.5)/(18) = 1.00\times10^{23}$ hydrogen nuclei. If all are aligned the net magnetic moment would be $\mu = (1.00\times10^{23})(1.41\times10^{-26}\mathrm{J/T}) = 1.41\times10^{-3}\mathrm{J/T}$. The field strength is then

$$B = \frac{\mu_0}{4\pi}\frac{\mu}{x^3} = (1.00\times10^{-7}\mathrm{N/A}^2)\frac{(1.41\times10^{-3}\mathrm{J/T})}{(5.33\,\mathrm{m})^3} = 9.3\times10^{-13}\mathrm{T}.$$

E35-11 (a) There is effectively a current of $i = fq = q\omega/2\pi$. The dipole moment is then $\mu = iA = (q\omega/2\pi)(\pi r^2) = \frac{1}{2}q\omega r^2$.

(b) The rotational inertia of the ring is mr^2 so $L = I\omega = mr^2\omega$. Then

$$\frac{\mu}{L} = \frac{(1/2)q\omega r^2}{mr^2\omega} = \frac{q}{2m}.$$

E35-12 The mass of the bar is

$$m = \rho V = (7.87 \text{ g/cm}^3)(4.86 \text{ cm})(1.31 \text{ cm}^2) = 50.1 \text{ g}.$$

The number of atoms in the bar is

$$N = (6.02\times10^{23})(50.1 \text{ g})/(55.8 \text{ g}) = 5.41\times10^{23}.$$

The dipole moment of the bar is then

$$\mu = (5.41\times10^{23})(2.22)(9.27\times10^{-24}\,\text{J/T}) = 11.6\,\text{J/T}.$$

(b) The torque on the magnet is $\tau = (11.6\,\text{J/T})(1.53\,\text{T}) = 17.7\,\text{N}\cdot\text{m}$.

$\boxed{\textbf{E35-13}}$ The magnetic dipole moment is given by $\mu = MV$, Eq. 35-13. Then

$$\mu = (5,300\,\text{A/m})(0.048\,\text{m})\pi(0.0055\,\text{m})^2 = 0.024\,\text{A}\cdot\text{m}^2.$$

E35-14 (a) The original field is $B_0 = \mu_0 in$. The field will increase to $B = \kappa_\text{m} B_0$, so the increase is

$$\Delta B = (\kappa_1 - 1)\mu_0 in = (3.3\times10^{-4})(4\pi\times10^{-7}\text{N/A}^2)(1.3\,\text{A})(1600/\text{m}) = 8.6\times10^{-7}\text{T}.$$

(b) $M = (\kappa_1 - 1)B_0/\mu_0 = (\kappa_1 - 1)in = (3.3\times10^{-4})(1.3\,\text{A})(1600/\text{m}) = 0.69\,\text{A/m}$.

E35-15 The energy to flip the dipoles is given by $U = 2\mu B$. The temperature is then

$$T = \frac{2\mu B}{3k/2} = \frac{4(1.2\times10^{-23}\text{J/T})(0.5\,\text{T})}{3(1.38\times10^{-23}\text{J/K})} = 0.58\,\text{K}.$$

E35-16 The Curie temperature of iron is 770°C, which is 750°C higher than the surface temperature. This occurs at a depth of (750°C)/(30 C°/km) = 25 km.

$\boxed{\textbf{E35-17}}$ (a) Look at the figure. At 50% (which is 0.5 on the vertical axis), the curve is at $B_0/T \approx 0.55\,\text{T/K}$. Since $T = 300\,\text{K}$, we have $B_0 \approx 165\,\text{T}$.

(b) Same figure, but now look at the 90% mark. $B_0/T \approx 1.60\,\text{T/K}$, so $B_0 \approx 480\,\text{T}$.

(c) Good question. I think both fields are far beyond our current abilities.

E35-18 (a) Look at the figure. At 50% (which is 0.5 on the vertical axis), the curve is at $B_0/T \approx 0.55\,\text{T/K}$. Since $B_0 = 1.8\,\text{T}$, we have $T \approx (1.8\,\text{T})/(0.55\,\text{T/K}) = 3.3\,\text{K}$.

(b) Same figure, but now look at the 90% mark. $B_0/T \approx 1.60\,\text{T/K}$, so $T \approx (1.8\,\text{T})/(1.60\,\text{T/K}) = 1.1\,\text{K}$.

E35-19 Since $(0.5\,\text{T})/(10\,\text{K}) = 0.05\,\text{T/K}$, and all higher temperatures have lower values of the ratio, and this puts all points in the region near where Curie's Law (the straight line) is valid, then the answer is yes.

E35-20 Using Eq. 35-19,

$$\mu_n = \frac{VM}{N} = \frac{M_r M}{A\rho} = \frac{(108\text{g/mol})(511\times10^3\text{A/m})}{(10490\,\text{kg/m}^3)(6.02\times10^{23}/\text{mol})} = 8.74\times10^{-21}\text{A/m}^2$$

E35-21 (a) $B = \mu_0\mu/2z^3$, so

$$B = \frac{(4\pi\times10^{-7}\text{N/A}^2)(1.5\times10^{-23}\text{J/T})}{2(10\times10^{-9}\text{m})^3} = 9.4\times10^{-6}\text{T}.$$

(b) $U = 2\mu B = 2(1.5\times10^{-23}\text{J/T})(9.4\times10^{-6}\text{T}) = 2.82\times10^{-28}\text{J}.$

E35-22 $\Phi_B = (43\times10^{-6}\text{T})(295,000\times10^6\text{m}^2) = 1.3\times10^7\text{Wb}.$

$\boxed{\textbf{E35-23}}$ (a) We'll assume, however, that all of the iron atoms are perfectly aligned. Then the dipole moment of the earth will be related to the dipole moment of one atom by

$$\mu_{\text{Earth}} = N\mu_{\text{Fe}},$$

where N is the number of iron atoms in the magnetized sphere. If m_A is the relative atomic mass of iron, then the total mass is

$$m = \frac{Nm_A}{A} = \frac{m_A}{A}\frac{\mu_{\text{Earth}}}{\mu_{\text{Fe}}},$$

where A is Avogadro's number. Next, the volume of a sphere of mass m is

$$V = \frac{m}{\rho} = \frac{m_A}{\rho A}\frac{\mu_{\text{Earth}}}{\mu_{\text{Fe}}},$$

where ρ is the density.

And finally, the radius of a sphere with this volume would be

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3\mu_{\text{Earth}}m_A}{4\pi\rho\mu_{\text{Fe}}A}\right)^{1/3}.$$

Now we find the radius by substituting in the known values,

$$r = \left(\frac{3(8.0\times10^{22}\text{J/T})(56\text{ g/mol})}{4\pi(14\times10^6\text{g/m}^3)(2.1\times10^{-23}\text{J/T})(6.0\times10^{23}\text{/mol})}\right)^{1/3} = 1.8\times10^5\text{m}.$$

(b) The fractional volume is the cube of the fractional radius, so the answer is

$$(1.8\times10^5\text{ m}/6.4\times10^6)^3 = 2.2\times10^{-5}.$$

E35-24 (a) At magnetic equator $L_\text{m} = 0$, so

$$B = \frac{\mu_0\mu}{4\pi r^3} = \frac{(1.00\times10^{-7}\text{N/A}^2)(8.0\times10^{22}\text{J/T})}{(6.37\times10^6\text{m})^3} = 31\mu\text{T}.$$

There is no vertical component, so the inclination is zero.

(b) Here $L_\text{m} = 60°$, so

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2 L_\text{m}} = \frac{(1.00\times10^{-7}\text{N/A}^2)(8.0\times10^{22}\text{J/T})}{(6.37\times10^6\text{m})^3}\sqrt{1+3\sin^2(60°)} = 56\mu\text{T}.$$

The inclination is given by

$$\arctan(B_\text{v}/B_\text{h}) = \arctan(2\tan L_\text{m}) = 74°.$$

(c) At magnetic north pole $L_\text{m} = 90°$, so

$$B = \frac{\mu_0\mu}{2\pi r^3} = \frac{2(1.00\times10^{-7}\text{N/A}^2)(8.0\times10^{22}\text{J/T})}{(6.37\times10^6\text{m})^3} = 62\mu\text{T}.$$

There is no horizontal component, so the inclination is 90°.

E35-25 This problem is effectively solving $1/r^3 = 1/2$ for r measured in Earth radii. Then $r = 1.26r_\mathrm{E}$, and the altitude above the Earth is $(0.26)(6.37\times10^6\mathrm{m}) = 1.66\times10^6\mathrm{m}$.

E35-26 The radial distance from the center is $r = (6.37\times10^6\mathrm{m}) - (2900\times10^3\mathrm{m}) = 3.47\times10^6\mathrm{m}$. The field strength is

$$B = \frac{\mu_0\mu}{2\pi r^3} = \frac{2(1.00\times10^{-7}\mathrm{N/A^2})(8.0\times10^{22}\mathrm{J/T})}{(3.47\times10^6\mathrm{m})^3} = 380\mu\mathrm{T}.$$

E35-27 Here $L_\mathrm{m} = 90° - 11.5° = 78.5°$, so

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2 L_\mathrm{m}} = \frac{(1.00\times10^{-7}\mathrm{N/A^2})(8.0\times10^{22}\mathrm{J/T})}{(6.37\times10^6\mathrm{m})^3}\sqrt{1+3\sin^2(78.5°)} = 61\mu\mathrm{T}.$$

The inclination is given by

$$\arctan(B_\mathrm{v}/B_\mathrm{h}) = \arctan(2\tan L_\mathrm{m}) = 84°.$$

E35-28 The flux out the "other" end is $(1.6\times10^{-3}\mathrm{T})\pi(0.13\,\mathrm{m})^2 = 85\mu\mathrm{Wb}$. The net flux through the surface is zero, so the flux through the curved surface is $0-(85\mu\mathrm{Wb})-(-25\mu\mathrm{Wb}) = -60\mu\mathrm{Wb}$.. The negative indicates inward.

$\boxed{\textbf{E35-29}}$ The total magnetic flux through a closed surface is zero. There is inward flux on faces one, three, and five for a total of -9 Wb. There is outward flux on faces two and four for a total of +6 Wb. The difference is +3 Wb; consequently the outward flux on the sixth face must be +3 Wb.

E35-30 The stable arrangements are (a) and (c). The torque in each case is zero.

E35-31 The field on the x axis between the wires is

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{2r+x} + \frac{1}{2r-x}\right).$$

Since $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = 0$, we can assume the flux through the curved surface is equal to the flux through the xz plane within the cylinder. This flux is

$$\begin{aligned}\Phi_B &= L\int_{-r}^{r}\left[\frac{\mu_0 i}{2\pi}\left(\frac{1}{2r+x}+\frac{1}{2r-x}\right)\right]dx,\\ &= L\frac{\mu_0 i}{2\pi}\left(\ln\frac{3r}{r} - \ln\frac{r}{3r}\right),\\ &= L\frac{\mu_0 i}{\pi}\ln 3.\end{aligned}$$

$\boxed{\textbf{P35-1}}$ We can imagine the rotating disk as being composed of a number of rotating rings of radius r, width dr, and circumference $2\pi r$. The surface charge density on the disk is $\sigma = q/\pi R^2$, and consequently the (differential) charge on any ring is

$$dq = \sigma(2\pi r)(dr) = \frac{2qr}{R^2}dr$$

The rings "rotate" with angular frequency ω, or period $T = 2\pi/\omega$. The effective (differential) current for each ring is then

$$di = \frac{dq}{T} = \frac{qr\omega}{\pi R^2}dr.$$

Each ring contributes to the magnetic moment, and we can glue all of this together as

$$\begin{aligned}\mu &= \int d\mu,\\ &= \int \pi r^2\, di,\\ &= \int_0^R \frac{qr^3\omega}{R^2}dr,\\ &= \frac{qR^2\omega}{4}.\end{aligned}$$

P35-2 (a) The sphere can be sliced into disks. The disks can be sliced into rings. Each ring has some charge q_i, radius r_i, and mass m_i; the period of rotation for a ring is $T = 2\pi/\omega$, so the current in the ring is $q_i/T = \omega q_i/2\pi$. The magnetic moment is

$$\mu_i = (\omega q_i/2\pi)\pi r_i^2 = \omega q_i r_i^2/2.$$

Note that this is *closely* related to the expression for angular momentum of a ring: $l_i = \omega m_i r_i^2$. Equating,

$$\mu_i = q_i l_i/2m_i.$$

If both mass density and charge density are uniform then we can write $q_i/m_i = q/m$,

$$\mu = \int d\mu = (q/2m)\int dl = qL/2m$$

For a solid sphere $L = \omega I = 2\omega m R^2/5$, so

$$\mu = q\omega R^2/5.$$

(b) See part (a)

P35-3 (a) The orbital speed is given by $K = mv^2/2$. The orbital radius is given by $mv = qBr$, or $r = mv/qB$. The frequency of revolution is $f = v/2\pi r$. The effective current is $i = qf$. Combining all of the above to find the dipole moment,

$$\mu = iA = q\frac{v}{2\pi r}\pi r^2 = q\frac{vr}{2} = q\frac{mv^2}{2qB} = \frac{K}{B}.$$

(b) Since q and m cancel out of the above expression the answer is the same!
(c) Work it out:

$$M = \frac{\mu}{V} = \frac{(5.28\times10^{21}/\mathrm{m}^3)(6.21\times10^{-20}\,\mathrm{J})}{(1.18\,\mathrm{T})} + \frac{(5.28\times10^{21}/\mathrm{m}^3)(7.58\times10^{-21}\,\mathrm{J})}{(1.18\,\mathrm{T})} = 312\,\mathrm{A/m}.$$

P35-4 (b) Point the thumb or your right hand to the right. Your fingers curl in the direction of the current in the wire loop.

(c) In the vicinity of the wire of the loop $\vec{\mathbf{B}}$ has a component which is directed radially outward. Then $\vec{\mathbf{B}} \times d\vec{\mathbf{s}}$ has a component directed to the left. Hence, the net force is directed to the left.

P35-5 (b) Point the thumb or your right hand to the left. Your fingers curl in the direction of the current in the wire loop.

(c) In the vicinity of the wire of the loop $\vec{\mathbf{B}}$ has a component which is directed radially outward. Then $\vec{\mathbf{B}} \times d\vec{\mathbf{s}}$ has a component directed to the right. Hence, the net force is directed to the right.

P35-6 (a) Let $x = \mu B/kT$. Adopt the convention that N_+ refers to the atoms which have parallel alignment and N_- those which are anti-parallel. Then $N_+ + N_- = N$, so

$$N_+ = Ne^x/(e^x + e^{-x}),$$

and

$$N_- = Ne^{-x}/(e^x + e^{-x}),$$

Note that the denominators are necessary so that $N_+ + N_- = N$. Finally,

$$M = \mu(N_+ - N_-) = \mu N \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

(b) If $\mu B \ll kT$ then x is very small and $e^{\pm x} \approx 1 \pm x$. The above expression reduces to

$$M = \mu N \frac{(1+x) - (1-x)}{(1+x) + (1-x)} = \mu N x = \frac{\mu^2 B}{kT}.$$

(c) If $\mu B \gg kT$ then x is very large and $e^{\pm x} \to \infty$ while $e^{-x} \to 0$. The above expression reduces to

$$N = \mu N.$$

$\boxed{\textbf{P35-7}}$ (a) Centripetal acceleration is given by $a = r\omega^2$. Then

$$\begin{aligned} a - a_0 &= r(\omega_0 + \Delta\omega)^2 - r\omega_0^2, \\ &= 2r\omega_0\Delta\omega + r(\Delta\omega_0)^2, \\ &\approx 2r\omega_0\Delta\omega. \end{aligned}$$

(b) The change in centripetal acceleration is caused by the additional magnetic force, which has magnitude $F_B = qvB = er\omega B$. Then

$$\Delta\omega = \frac{a - a_0}{2r\omega_0} = \frac{eB}{2m}.$$

Note that we boldly canceled ω against ω_0 in this last expression; we are assuming that $\Delta\omega$ is small, and for these problems it is.

P35-8 (a) $i = \mu/A = (8.0\times10^{22}\mathrm{J/T})/\pi(6.37\times10^6\mathrm{m})^2 = 6.3\times10^8\mathrm{A}$.

(b) Far enough away both fields act like perfect dipoles, and can then cancel.

(c) Close enough neither field acts like a perfect dipole and the fields will not cancel.

P35-9 (a) $B = \sqrt{{B_\mathrm{h}}^2 + {B_\mathrm{v}}^2}$, so

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2 L_\mathrm{m} + 4\sin^2 L_\mathrm{m}} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1 + 3\sin^2 L_\mathrm{m}}.$$

(b) $\tan\phi_\mathrm{i} = B_\mathrm{v}/B_\mathrm{h} = 2\sin L_\mathrm{m}/\cos L_\mathrm{m} = 2\tan L_\mathrm{m}$.

$\boxed{\textbf{E36-1}}$ The important relationship is Eq. 36-4, written as

$$\Phi_B = \frac{iL}{N} = \frac{(5.0\,\mathrm{mA})(8.0\,\mathrm{mH})}{(400)} = 1.0\times10^{-7}\mathrm{Wb}$$

E36-2 (a) $\Phi = (34)(2.62\times10^{-3}\mathrm{T})\pi(0.103\,\mathrm{m})^2 = 2.97\times10^{-3}\mathrm{Wb}$.
(b) $L = \Phi/i = (2.97\times10^{-3}\mathrm{Wb})/(3.77\,\mathrm{A}) = 7.88\times10^{-4}\mathrm{H}$.

E36-3 $n = 1/d$, where d is the diameter of the wire. Then

$$\frac{L}{l} = \mu_0 n^2 A = \frac{\mu_0 A}{d^2} = \frac{(4\pi\times10^{-7}\mathrm{H/m})(\pi/4)(4.10\times10^{-2}\mathrm{m})^2}{(2.52\times10^{-3}\mathrm{m})^2} = 2.61\times10^{-4}\mathrm{H/m}.$$

E36-4 (a) The emf supports the current, so the current must be decreasing.
(b) $L = \mathcal{E}/(di/dt) = (17\,\mathrm{V})/(25\times10^3\mathrm{A/s}) = 6.8\times10^{-4}\mathrm{H}$.

$\boxed{\textbf{E36-5}}$ (a) Eq. 36-1 can be used to find the inductance of the coil.

$$L = \frac{\mathcal{E}_L}{di/dt} = \frac{(3.0\,\mathrm{mV})}{(5.0\,\mathrm{A/s})} = 6.0\times10^{-4}\mathrm{H}.$$

(b) Eq. 36-4 can then be used to find the number of turns in the coil.

$$N = \frac{iL}{\Phi_B} = \frac{(8.0\,\mathrm{A})(6.0\times10^{-4}\mathrm{H})}{(40\mu\mathrm{Wb})} = 120$$

E36-6 Use the equation between Eqs. 36-9 and 36-10.

$$\begin{aligned}\Phi_B &= \frac{(4\pi\times10^{-7}\mathrm{H/m})(0.81\,\mathrm{A})(536)(5.2\times10^{-2}\mathrm{m})}{2\pi}\ln\frac{(5.2\times10^{-2}\mathrm{m})+(15.3\times10^{-2}\mathrm{m})}{(15.3\times10^{-2}\mathrm{m})},\\ &= 1.32\times10^{-6}\mathrm{Wb}.\end{aligned}$$

E36-7 $L = \kappa_\mathrm{m}\mu_0 n^2 Al = \kappa_\mathrm{m}\mu_0 N^2 A/l$, or

$$L = (968)(4\pi\times10^{-7}\mathrm{H/m})(1870)^2(\pi/4)(5.45\times10^{-2}\mathrm{m})^2/(1.26\,\mathrm{m}) = 7.88\,\mathrm{H}.$$

E36-8 In each case apply $\mathcal{E} = L\Delta i/\Delta t$.
(a) $\mathcal{E} = (4.6\,\mathrm{H})(7\,\mathrm{A})/(2\times10^{-3}\mathrm{s}) = 1.6\times10^4\mathrm{V}$.
(b) $\mathcal{E} = (4.6\,\mathrm{H})(2\,\mathrm{A})/(3\times10^{-3}\mathrm{s}) = 3.1\times10^3\mathrm{V}$.
(c) $\mathcal{E} = (4.6\,\mathrm{H})(5\,\mathrm{A})/(1\times10^{-3}\mathrm{s}) = 2.3\times10^4\mathrm{V}$.

$\boxed{\textbf{E36-9}}$ (a) If two inductors are connected in parallel then the current through each inductor will add to the total current through the circuit, $i = i_1 + i_2$, Take the derivative of the current with respect to time and then $di/dt = di_1/dt + di_2/dt$,

The potential difference across each inductor is the same, so if we divide by $\mathcal{E}$ and apply we get

$$\frac{di/dt}{\mathcal{E}} = \frac{di_1/dt}{\mathcal{E}} + \frac{di_2/dt}{\mathcal{E}},$$

But

$$\frac{di/dt}{\mathcal{E}} = \frac{1}{L},$$

so the previous expression can also be written as

$$\frac{1}{L_{\rm eq}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

(b) If the inductors are close enough together then the magnetic field from one coil will induce currents in the other coil. Then we will need to consider mutual induction effects, but that is a topic not covered in this text.

E36-10 (a) If two inductors are connected in series then the emf across each inductor will add to the total emf across both, $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$,

Then the current through each inductor is the same, so if we divide by di/dt and apply we get

$$\frac{\mathcal{E}}{di/dt} = \frac{\mathcal{E}_1}{di/dt} + \frac{\mathcal{E}_2}{di/dt},$$

But

$$\frac{\mathcal{E}}{di/dt} = L,$$

so the previous expression can also be written as

$$L_{\rm eq} = L_1 + L_2.$$

(b) If the inductors are close enough together then the magnetic field from one coil will induce currents in the other coil. Then we will need to consider mutual induction effects, but that is a topic not covered in this text.

E36-11 Use Eq. 36-17, but rearrange:

$$\tau_L = \frac{t}{\ln[i_0/i]} = \frac{(1.50\,{\rm s})}{\ln[(1.16\,{\rm A})/(10.2\times 10^{-3}{\rm A})]} = 0.317\,{\rm s}.$$

Then $R = L/\tau_L = (9.44\,{\rm H})/(0.317\,{\rm s}) = 29.8\,\Omega$.

E36-12 (a) There is no current through the resistor, so $\mathcal{E}_R = 0$ and then $\mathcal{E}_L = \mathcal{E}$.

(b) $\mathcal{E}_L = \mathcal{E}e^{-2} = (0.135)\mathcal{E}$.

(c) $n = -\ln(\mathcal{E}_L/\mathcal{E}) = -ln(1/2) = 0.693$.

$\boxed{\textbf{E36-13}}$ (a) From Eq. 36-4 we find the inductance to be

$$L = \frac{N\Phi_B}{i} = \frac{(26.2\times 10^{-3}{\rm Wb})}{(5.48\,{\rm A})} = 4.78\times 10^{-3}{\rm H}.$$

Note that Φ_B is the *flux*, while the quantity $N\Phi_B$ is the *number of flux linkages.*

(b) We can find the time constant from Eq. 36-14,

$$\tau_L = L/R = (4.78\times 10^{-3}{\rm H})/(0.745\,\Omega) = 6.42\times 10^{-3}\,{\rm s}.$$

The we can invert Eq. 36-13 to get

$$\begin{aligned} t &= -\tau_L \ln\left(1 - \frac{Ri(t)}{\mathcal{E}}\right), \\ &= -(6.42\times 10^{-3}\,{\rm s}) \ln\left(1 - \frac{(0.745\,{\rm A})(2.53\,{\rm A})}{(6.00\,{\rm V})}\right) = 2.42\times 10^{-3}\,{\rm s}. \end{aligned}$$

E36-14 (a) Rearrange:

$$\begin{aligned}
\mathcal{E} &= iR + L\frac{di}{dt}, \\
\frac{\mathcal{E}}{R} - i &= \frac{L}{R}\frac{di}{dt}, \\
\frac{R}{L}dt &= \frac{di}{\mathcal{E}/R - i}.
\end{aligned}$$

(b) Integrate:

$$\begin{aligned}
-\int_0^t \frac{R}{L}dt &= \int_0^i \frac{di}{i - \mathcal{E}/R}, \\
-\frac{R}{L}t &= \ln\frac{i + \mathcal{E}/R}{\mathcal{E}/R}, \\
\frac{\mathcal{E}}{R}e^{-t/\tau_L} &= i + \mathcal{E}/R, \\
\frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) &= i.
\end{aligned}$$

E36-15 $di/dt = (5.0\,\mathrm{A/s})$. Then

$$\mathcal{E} = iR + L\frac{di}{dt} = (3.0\,\mathrm{A})(4.0\,\Omega) + (5.0\,\mathrm{A/s})t(4.0\,\Omega) + (6.0\,\mathrm{H})(5.0\,\mathrm{A/s}) = 42\,\mathrm{V} + (20\,\mathrm{V/s})t.$$

E36-16 $(1/3) = (1 - e^{-t/\tau_L})$, so

$$\tau_L = -\frac{t}{\ln(2/3)} = -\frac{(5.22\,\mathrm{s})}{\ln(2/3)} = 12.9\,\mathrm{s}.$$

$\boxed{\textbf{E36-17}}$ We want to take the derivative of the current in Eq. 36-13 with respect to time,

$$\frac{di}{dt} = \frac{\mathcal{E}}{R}\frac{1}{\tau_L}e^{-t/\tau_L} = \frac{\mathcal{E}}{L}e^{-t/\tau_L}.$$

Then $\tau_L = (5.0\times10^{-2}\mathrm{H})/(180\,\Omega) = 2.78\times10^{-4}\mathrm{s}$. Using this we find the rate of change in the current when $t = 1.2$ ms to be

$$\frac{di}{dt} = \frac{(45\,\mathrm{V})}{((5.0\times10^{-2}\mathrm{H})}e^{-(1.2\times10^{-3}\mathrm{s})/(2.78\times10^{-4}\mathrm{s})} = 12\,\mathrm{A/s}.$$

E36-18 (b) Consider some time t_i:

$$\mathcal{E}_L(t_i) = \mathcal{E}e^{-t_i/\tau_L}.$$

Taking a ratio for two different times,

$$\frac{\mathcal{E}_L(t_1)}{\mathcal{E}_L(t_2)} = e^{(t_2-t_1)/\tau_L},$$

or

$$\tau_L = \frac{t_2 - t_1}{\ln[\mathcal{E}_L(t_1)/\mathcal{E}_L(t_2)]} = \frac{(2\,\mathrm{ms}) - (1\,\mathrm{ms})}{\ln[(18.24\,\mathrm{V})/(13.8\,\mathrm{V})]} = 3.58\,\mathrm{ms}$$

(a) Choose any time, and

$$\mathcal{E} = \mathcal{E}_L e^{t/\tau_L} = (18.24\,\mathrm{V})e^{(1\,\mathrm{ms})/(3.58\,\mathrm{ms})} = 24\,\mathrm{V}.$$

E36-19 (a) When the switch is just closed there is *no* current through the inductor. So $i_1 = i_2$ is given by

$$i_1 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{(100\,\mathrm{V})}{(10\,\Omega) + (20\,\Omega)} = 3.33\,\mathrm{A}.$$

(b) A long time later there is current through the inductor, but it is as if the inductor has no effect on the circuit. Then the effective resistance of the circuit is found by first finding the equivalent resistance of the parallel part

$$1/(30\,\Omega) + 1/(20\,\Omega) = 1/(12\,\Omega),$$

and then finding the equivalent resistance of the circuit

$$(10\,\Omega) + (12\,\Omega) = 22\,\Omega.$$

Finally, $i_1 = (100\,\mathrm{V})/(22\,\Omega) = 4.55\,\mathrm{A}$ and

$$\Delta V_2 = (100\,\mathrm{V}) - (4.55\,\mathrm{A})(10\,\Omega) = 54.5\,\mathrm{V};$$

consequently, $i_2 = (54.5\,\mathrm{V})/(20\,\Omega) = 2.73\,\mathrm{A}$. It didn't ask, but $i_2 = (4.55\,\mathrm{A}) - (2.73\,\mathrm{A}) = 1.82\,\mathrm{A}$.

(c) After the switch is just opened the current through the battery stops, while that through the inductor continues on. Then $i_2 = i_3 = 1.82\,\mathrm{A}$.

(d) All go to zero.

E36-20 (a) For toroids $L = \mu_0 N^2 h \ln(b/a)/2\pi$. The number of turns is limited by the inner radius: $Nd = 2\pi a$. In this case,

$$N = 2\pi(0.10\,\mathrm{m})/(0.00096\,\mathrm{m}) = 654.$$

The inductance is then

$$L = \frac{(4\pi\times10^{-7}\mathrm{H/m})(654)^2(0.02\,\mathrm{m})}{2\pi}\ln\frac{(0.12\,\mathrm{m})}{(0.10\,\mathrm{m})} = 3.1\times10^{-4}\mathrm{H}.$$

(b) Each turn has a length of $4(0.02\,\mathrm{m}) = 0.08\,\mathrm{m}$. The resistance is then

$$R = N(0.08\,\mathrm{m})(0.021\,\Omega/\mathrm{m}) = 1.10\,\Omega$$

The time constant is

$$\tau_L = L/R = (3.1\times10^{-4}\mathrm{H})/(1.10\,\Omega) = 2.8\times10^{-4}\mathrm{s}.$$

$\boxed{\textbf{E36-21}}$ (I) When the switch is just closed there is *no* current through the inductor or R_2, so the potential difference across the inductor must be 10 V. The potential difference across R_1 is always 10 V when the switch is closed, regardless of the amount of time elapsed since closing.

(a) $i_1 = (10\,\mathrm{V})/(5.0\,\Omega) = 2.0\,\mathrm{A}$.

(b) Zero; read the above paragraph.

(c) The current through the switch is the sum of the above two currents, or 2.0 A.

(d) Zero, since the current through R_2 is zero.

(e) 10 V, since the potential across R_2 is zero.

(f) Look at the results of Exercise 36-17. When $t = 0$ the rate of change of the current is $di/dt = \mathcal{E}/L$. Then

$$di/dt = (10\,\mathrm{V})/(5.0\,\mathrm{H}) = 2.0\,\mathrm{A/s}.$$

(II) After the switch has been closed for a long period of time the currents are stable and the inductor no longer has an effect on the circuit. Then the circuit is a simple two resistor parallel network, each resistor has a potential difference of 10 V across it.

(a) Still 2.0 A; nothing has changed.
(b) $i_2 = (10\,\text{V})/(10\,\Omega) = 1.0\,\text{A}$.
(c) Add the two currents and the current through the switch will be 3.0 A.
(d) 10 V; see the above discussion.
(e) Zero, since the current is no longer changing.
(f) Zero, since the current is no longer changing.

E36-22 $U = (71\,\text{J/m}^3)(0.022\,\text{m}^3) = 1.56\,\text{J}$. Then using $U = i^2L/2$ we get

$$i = \sqrt{2U/L} = \sqrt{2(1.56\,\text{J})/(0.092\,\text{H})} = 5.8\,\text{A}.$$

E36-23 (a) $L = 2U/i^2 = 2(0.0253\,\text{J})/(0.062\,\text{A})^2 = 13.2\,\text{H}$.

(b) Since the current is squared in the energy expression, doubling the current would quadruple the energy. Then $i' = 2i_0 = 2(0.062\,\text{A}) = 0.124\,\text{A}$.

E36-24 (a) $B = \mu_0 in$ and $u = B^2/2\mu_0$, or

$$u = \mu_0 i^2 n^2/2 = (4\pi\times10^{-7}\text{N/A}^2)(6.57\,\text{A})^2(950/0.853\,\text{m})^2/2 = 33.6\,\text{J/m}^3.$$

(b) $U = uAL = (33.6\,\text{J/m}^3)(17.2\times10^{-4}\text{m}^2)(0.853\,\text{m}) = 4.93\times10^{-2}\text{J}$.

E36-25 $u_B = B^2/2\mu_0$, and from Sample Problem 33-2 we know B, hence

$$u_B = \frac{(12.6\,\text{T})^2}{2(4\pi\times10^{-7}\text{N/A}^2)} = 6.32\times10^7\text{J/m}^3.$$

E36-26 (a) $u_B = B^2/2\mu_0$, so

$$u_B = \frac{(100\times10^{-12}\text{T})^2}{2(4\pi\times10^{-7}\text{N/A}^2)}\frac{1}{(1.6\times10^{-19}\text{J/eV})} = 2.5\times10^{-2}\ \text{eV/cm}^3.$$

(b) $x = (10)(9.46\times10^{15}\text{m}) = 9.46\times10^{16}\text{m}$. Using the results from part (a) expressed in J/m^3 we find the energy contained is

$$U = (3.98\times10^{-15}\text{J/m}^3)(9.46\times10^{16}\text{m})^3 = 3.4\times10^{36}\text{J}$$

$\boxed{\textbf{E36-27}}$ The energy density of an electric field is given by Eq. 36-23; that of a magnetic field is given by Eq. 36-22. Equating,

$$\begin{aligned}\frac{\epsilon_0}{2}E^2 &= \frac{1}{2\mu_0}B^2,\\ E &= \frac{B}{\sqrt{\epsilon_0\mu_0}}.\end{aligned}$$

The answer is then

$$E = (0.50\,\text{T})/\sqrt{(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\text{N/A}^2)} = 1.5\times10^8\,\text{V/m}.$$

E36-28 The rate of internal energy increase in the resistor is given by $P = i\Delta V_R$. The rate of energy storage in the inductor is $dU/dt = Li\,di/dt = i\Delta V_L$. Since the current is the same through both we want to find the time when $\Delta V_R = \Delta V_L$. Using Eq. 36-15 we find

$$\begin{aligned}1 - e^{-t/\tau_L} &= e^{-t/\tau_L},\\ \ln 2 &= t/\tau_L,\end{aligned}$$

so $t = (37.5\,\text{ms})\ln 2 = 26.0\,\text{ms}$.

E36-29 (a) Start with Eq. 36-13:

$$\begin{aligned}
i &= \mathcal{E}(1 - e^{-t/\tau_L})/R, \\
1 - \frac{iR}{\mathcal{E}} &= e^{-t/\tau_L}, \\
\tau_L &= \frac{-t}{\ln(1 - iR/\mathcal{E})}, \\
&= \frac{-(5.20\times10^{-3}\mathrm{s})}{\ln[1 - (1.96\times10^{-3}\mathrm{A})(10.4\times10^{3}\Omega)/(55.0\,\mathrm{V})]}, \\
&= 1.12\times10^{-2}\mathrm{s}.
\end{aligned}$$

Then $L = \tau_L R = (1.12\times10^{-2}\mathrm{s})(10.4\times10^{3}\Omega) = 116\,\mathrm{H}$.

(b) $U = (1/2)(116\,\mathrm{H})(1.96\times10^{-3}\mathrm{A})^2 = 2.23\times10^{-4}\mathrm{J}$.

E36-30 (a) $U = \mathcal{E}\Delta q$; $q = \int i dt$.

$$\begin{aligned}
U &= \mathcal{E}\int \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) dt, \\
&= \frac{\mathcal{E}^2}{R}\left(t + \tau_L e^{-t/\tau_L}\right)_0^2, \\
&= \frac{\mathcal{E}^2}{R}t + \frac{\mathcal{E}^2 L}{R^2}(e^{-t/\tau_L} - 1).
\end{aligned}$$

Using the numbers provided,

$$\tau_L = (5.48\,\mathrm{H})/(7.34\,\Omega) = 0.7466\,\mathrm{s}.$$

Then

$$U = \frac{(12.2\,\mathrm{V})^2}{(7.34\,\Omega)}\left[(2\,\mathrm{s}) + (0.7466\,\mathrm{s})(e^{-(2\,\mathrm{s})/0.7466\,\mathrm{s})} - 1)\right] = 26.4\,\mathrm{J}$$

(b) The energy stored in the inductor is $U_L = Li^2/2$, or

$$\begin{aligned}
U_L &= \frac{L\mathcal{E}^2}{2R^2}\int\left(1 - e^{-t/\tau_L}\right)^2 dt, \\
&= 6.57\,\mathrm{J}.
\end{aligned}$$

(c) $U_R = U - U_L = 19.8\,\mathrm{J}$.

$\boxed{\textbf{E36-31}}$ This shell has a volume of

$$V = \frac{4\pi}{3}\left((R_\mathrm{E} + a)^3 - {R_\mathrm{E}}^3\right).$$

Since $a << R_\mathrm{E}$ we can expand the polynomials but keep only the terms which are linear in a. Then

$$V \approx 4\pi {R_\mathrm{E}}^2 a = 4\pi(6.37\times10^{6}\mathrm{m})^2(1.6\times10^{4}\mathrm{m}) = 8.2\times10^{18}\mathrm{m}^3.$$

The magnetic energy density is found from Eq. 36-22,

$$u_B = \frac{1}{2\mu_0}B^2 = \frac{(60\times10^{-6}\,\mathrm{T})^2}{2(4\pi\times10^{-7}\mathrm{N/A}^2)} = 1.43\times10^{-3}\mathrm{J/m}^3.$$

The total energy is then $(1.43\times10^{-3}\mathrm{J/m}^3)(8.2eex18\mathrm{m}^3) = 1.2\times10^{16}\mathrm{J}$.

E36-32 (a) $B = \mu_0 i/2\pi r$ and $u_B = B^2/2\mu_0 = \mu_0 i^2/8\pi^2 r^2$, or

$$u_B = (4\pi\times10^{-7}\text{H/m})(10\,\text{A})^2/8\pi^2(1.25\times10^{-3}\text{m})^2 = 1.0\,\text{J/m}^3.$$

(b) $E = \Delta V/l = iR/l$ and $u_E = \epsilon_0 E^2/2 = \epsilon_0 i^2(R/l)^2/2$. Then

$$u_E = (8.85\times10^{-12}\text{F/m})(10\,\text{A})^2(3.3\times10^{-3}\Omega/\text{m})^2/2 = 4.8\times10^{-15}\text{J/m}^3.$$

E36-33 $i = \sqrt{2U/L} = \sqrt{2(11.2\times10^{-6}\text{J})/(1.48\times10^{-3}\text{H})} = 0.123\,\text{A}.$

E36-34 $C = q^2/2U = (1.63\times10^{-6}\text{C})^2/2(142\times10^{-6}\text{J}) = 9.36\times10^{-9}\text{F}.$

E36-35 $1/2\pi f = \sqrt{LC}$ so $L = 1/4\pi^2 f^2 C$, or

$$L = 1/4\pi^2(10\times10^3\text{Hz})^2(6.7\times10^{-6}\text{F}) = 3.8\times10^{-5}\text{H}.$$

E36-36 ${q_\text{max}}^2/2C = L{i_\text{max}}^2/2$, or

$$i_\text{max} = q_\text{max}/\sqrt{LC} = (2.94\times10^{-6}\text{C})/\sqrt{(1.13\times10^{-3}\text{H})(3.88\times10^{-6}\text{F})} = 4.44\times10^{-2}\text{A}.$$

$\boxed{\textbf{E36-37}}$ Closing a switch has the effect of "shorting" out the relevant circuit element, which effectively removes it from the circuit. If S_1 is closed we have $\tau_C = RC$ or $C = \tau_C/R$, if instead S_2 is closed we have $\tau_L = L/R$ or $L = R\tau_L$, but if instead S_3 is closed we have a LC circuit which will oscillate with period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}.$$

Substituting from the expressions above,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\tau_L\tau_C}.$$

E36-38 The capacitors can be used individually, or in series, or in parallel. The four possible capacitances are then $2.00\mu\text{F}$, $5.00\mu\text{F}$, $2.00\mu_F + 5.00\mu\text{F} = 7.00\mu\text{F}$, and $(2.00\mu_F)(5.00\mu\text{F})(2.00\mu_F + 5.00\mu\text{F}) = 1.43\mu\text{F}$. The possible resonant frequencies are then

$$\begin{aligned}
\frac{1}{2\pi}\sqrt{\frac{1}{LC}} &= f,\\
\frac{1}{2\pi}\sqrt{\frac{1}{(10.0\,\text{mH})(1.43\mu_F)}} &= 1330\text{ Hz},\\
\frac{1}{2\pi}\sqrt{\frac{1}{(10.0\,\text{mH})(2.00\mu_F)}} &= 1130\text{ Hz},\\
\frac{1}{2\pi}\sqrt{\frac{1}{(10.0\,\text{mH})(5.00\mu_F)}} &= 712\text{ Hz},\\
\frac{1}{2\pi}\sqrt{\frac{1}{(10.0\,\text{mH})(7.00\mu_F)}} &= 602\text{ Hz}.
\end{aligned}$$

E36-39 (a) $k = (8.13\,\text{N})/(0.0021\,\text{m}) = 3.87\times10^3\text{N/m}$. $\omega = \sqrt{k/m} = \sqrt{(3870\,\text{N/m})/(0.485\,\text{kg})} =$ 89.3 rad/s.

(b) $T = 2\pi/\omega = 2\pi/(89.3\text{ rad/s}) = 7.03\times10^{-2}\text{s}$.

(c) $LC = 1/\omega^2$, so

$$C = 1/(89.3\text{ rad/s})^2(5.20\,\text{H}) = 2.41\times10^{-5}\text{F}.$$

E36-40 The period of oscillation is $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(52.2\text{mH})(4.21\mu\text{F})} = 2.95\,\text{ms}$. It requires one-quarter period for the capacitor to charge, or 0.736 ms.

$\boxed{\textbf{E36-41}}$ (a) An LC circuit oscillates so that the energy is converted from all magnetic to all electrical *twice* each cycle. It occurs twice because once the energy is magnetic with the current flowing in one direction through the inductor, and later the energy is magnetic with the current flowing the other direction through the inductor.

The period is then *four* times $1.52\mu\text{s}$, or $6.08\mu\text{s}$.

(b) The frequency is the reciprocal of the period, or 164000 Hz.

(c) Since it occurs twice during each oscillation it is equal to half a period, or $3.04\mu\text{s}$.

E36-42 (a) $q = C\Delta V = (1.13\times10^{-9}\text{F})(2.87\,\text{V}) = 3.24\times10^{-9}\text{C}$.

(c) $U = q^2/2C = (3.24\times10^{-9}\text{C})^2/2(1.13\times10^{-9}\text{F}) = 4.64\times10^{-9}\text{J}$.

(b) $i = \sqrt{2U/L} = \sqrt{2(4.64\times10^{-9}\text{J})/(3.17\times10^{-3}\text{H})} = 1.71\times10^{-3}\text{A}$.

E36-43 (a) $i_\text{m} = q_\text{m}\omega$ and $q_\text{m} = CV_\text{m}$. Multiplying the second expression by L we get $Lq_\text{m} = V_\text{m}/\omega^2$. Combining, $Li_\text{m}\omega = V_\text{m}$. Then

$$f = \frac{\omega}{2\pi} = \frac{(50\,\text{V})}{2\pi(0.042\,\text{H})(0.031\,\text{A})} = 6.1\times10^3/\text{s}.$$

(b) See (a) above.

(c) $C = 1/\omega^2 L = 1/(2\pi 6.1\times10^3/\text{s})^2(0.042\,\text{H}) = 1.6\times10^{-8}\text{F}$.

E36-44 (a) $f = 1/2\pi\sqrt{LC} = 1/2\pi\sqrt{(6.2\times10^{-6}\text{F})(54\times10^{-3}\text{H})} = 275$ Hz.

(b) Note that from Eq. 36-32 we can deduce $i_\text{max} = \omega q_\text{max}$. The capacitor starts with a charge $q = C\Delta V = (6.2\times10^{-6}\text{F})(34\,\text{V}) = 2.11\times10^{-4}\text{C}$. Then the current amplitude is

$$i_\text{max} = q_\text{max}/\sqrt{LC} = (2.11\times10^{-4}\text{C})/\sqrt{(6.2\times10^{-6}\text{F})(54\times10^{-3}\text{H})} = 0.365\,\text{A}.$$

E36-45 (a) $\omega = 1/\sqrt{LC} = 1/\sqrt{(10\times10^{-6}\text{F})(3.0\times10^{-3}\text{H})} = 5800$ rad/s.

(b) $T = 2\pi/\omega = 2\pi/(5800\text{ rad/s}) = 1.1\times10^{-3}\text{s}$.

E36-46 $f = (2\times10^5\text{Hz})(1+\theta/180°)$. $C = 4\pi^2/f^2L$, so

$$C = \frac{4\pi^2}{(2\times10^5\text{Hz})^2(1+\theta/180°)^2(1\,\text{mH})} = \frac{(9.9\times10^{-7}\text{F})}{(1+\theta/180°)^2}.$$

E36-47 (a) $U_E = U_B/2$ and $U_E + U_B = U$, so $3U_E = U$, or $3(q^2/2C) = q_\text{max}^2/2C$, so $q = q_\text{max}/\sqrt{3}$.

(b) Solve $q = q_\text{max}\cos\omega t$, or

$$t = \frac{T}{2\pi}\arccos 1/\sqrt{3} = 0.152T.$$

E36-48 (a) Add the contribution from the inductor and the capacitor,

$$U = \frac{(24.8\times10^{-3}\text{H})(9.16\times10^{-3}\text{A})^2}{2} + \frac{(3.83\times10^{-6}\text{C})^2}{2(7.73\times10^{-6}\text{F})} = 1.99\times10^{-6}\text{J}.$$

(b) $q_\text{m} = \sqrt{2(7.73\times10^{-6}\text{F})(1.99\times10^{-6}\text{J})} = 5.55\times10^{-6}\text{C}$.
(c) $i_\text{m} = \sqrt{2(1.99\times10^{-6}\text{J})/(24.8\times10^{-3}\text{H})} = 1.27\times10^{-2}\text{A}$.

E36-49 (a) The frequency of such a system is given by Eq. 36-26, $f = 1/2\pi\sqrt{LC}$. Note that maximum frequency occurs with minimum capacitance. Then

$$\frac{f_1}{f_2} = \sqrt{\frac{C_2}{C_1}} = \sqrt{\frac{(365\text{ pF})}{(10\text{ pF})}} = 6.04.$$

(b) The desired ratio is $1.60/0.54 = 2.96$ Adding a capacitor in parallel will result in an effective capacitance given by

$$C_{1,\text{eff}} = C_1 + C_\text{add},$$

with a similar expression for C_2. We want to choose C_add so that

$$\frac{f_1}{f_2} = \sqrt{\frac{C_{2,\text{eff}}}{C_{1,\text{eff}}}} = 2.96.$$

Solving,

$$\begin{aligned} C_{2,\text{eff}} &= C_{1,\text{eff}}(2.96)^2, \\ C_2 + C_\text{add} &= (C_1 + C_\text{add})8.76, \\ C_\text{add} &= \frac{C_2 - 8.76C_1}{7.76}, \\ &= \frac{(365\text{ pF}) - 8.76(10\text{ pF})}{7.76} = 36\text{ pF}. \end{aligned}$$

The necessary inductance is then

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(0.54\times10^6\text{Hz})^2(401\times10^{-12}\text{F})} = 2.2\times10^{-4}\text{H}.$$

E36-50 The key here is that $U_E = C(\Delta V)^2/2$. We want to charge a capacitor with one-ninth the capacitance to have three times the potential difference. Since $3^2 = 9$, it is reasonable to assume that we want to take *all* of the energy from the first capacitor and give it to the second. Closing S_1 and S_2 will not work, because the energy will be shared. Instead, close S_2 until the capacitor has completely discharged into the inductor, then simultaneously open S_2 while closing S_1. The inductor will then discharge into the second capacitor. Open S_1 when it is "full".

E36-51 (a) $\omega = 1/\sqrt{LC}$.

$$q_\text{m} = \frac{i_\text{m}}{\omega} = (2.0\,\text{A})\sqrt{(3.0\times10^{-3}\text{H})(2.7\times10^{-6}\text{F})} = 1.80\times10^{-4}\text{C}$$

(b) $dU_C/dt = qi/C$. Since $q = q_\text{m}\sin\omega t$ and $i = i_\text{m}\cos\omega t$ then dU_C/dt is a maximum when $\sin\omega t\cos\omega t$ is a maximum, or when $\sin 2\omega t$ is a maximum. This happens when $2\omega t = \pi/2$, or $t = T/8$.

(c) $dU_C/dt = q_\text{m}i_\text{m}/2C$, or

$$dU_C/dt = (1.80\times10^{-4}\text{C})(2.0\,\text{A})/2(2.7\times10^{-6}\text{F}) = 67\,\text{W}.$$

E36-52 After only complete cycles $q = q_{\max}e^{-Rt/2L}$. Not only that, but $t = N\tau$, where $\tau = 2\pi/\omega'$. Finally, $\omega' = \sqrt{(1/LC) - (R/2L)^2}$. Since the first term under the square root is so much larger than the second, we can ignore the effect of damping on the frequency, and simply use $\omega' \approx \omega = 1/\sqrt{LC}$. Then

$$q = q_{\max}e^{-NR\tau/2L} = q_{\max}e^{-N\pi R\sqrt{LC}/L} = q_{\max}e^{-N\pi R\sqrt{C/L}}.$$

Finally, $\pi R\sqrt{C/L} = \pi(7.22\,\Omega)\sqrt{(3.18\,\mu\text{F})/(12.3\,\text{H})} = 1.15\times10^{-2}$. Then

$$\begin{aligned}
N = 5 &\quad:\quad q = (6.31\mu\text{C})e^{-5(0.0115)} = 5.96\mu\text{C},\\
N = 5 &\quad:\quad q = (6.31\mu\text{C})e^{-10(0.0115)} = 5.62\mu\text{C},\\
N = 5 &\quad:\quad q = (6.31\mu\text{C})e^{-100(0.0115)} = 1.99\mu\text{C}.
\end{aligned}$$

E36-53 Use Eq. 36-40, and since $U \propto q^2$, we want to solve $e^{-Rt/L} = 1/2$, then

$$t = \frac{L}{R}\ln 2.$$

E36-54 Start by assuming that the presence of the resistance does not significantly change the frequency. Then $\omega = 1/\sqrt{LC}$, $q = q_{\max}e^{-Rt/2L}$, $t = N\tau$, and $\tau = 2\pi/\omega$. Combining,

$$q = q_{\max}e^{-NR\tau/2L} = q_{\max}e^{-N\pi R\sqrt{LC}/L} = q_{\max}e^{-N\pi R\sqrt{C/L}}.$$

Then

$$R = -\frac{\sqrt{L/C}}{N\pi}\ln(q/q_{\max}) = -\frac{\sqrt{(220\text{mH})/(12\mu\text{F})}}{(50)\pi}\ln(0.99) = 8700\,\Omega.$$

It remains to be verified that $1/LC \gg (R/2L)^2$.

$\boxed{\textbf{E36-55}}$ The damped (angular) frequency is given by Eq. 36-41; the fractional change would then be

$$\frac{\omega - \omega'}{\omega} = 1 - \sqrt{1 - (R/2L\omega)^2} = 1 - \sqrt{1 - (R^2C/4L)}.$$

Setting this equal to 0.01% and then solving for R,

$$R = \sqrt{\frac{4L}{C}\left(1 - (1 - 0.0001)^2\right)} = \sqrt{\frac{4(12.6\times10^{-3}\text{H})}{(1.15\times10^{-6}\text{F})}(1.9999\times10^{-4})} = 2.96\,\Omega.$$

$\boxed{\textbf{P36-1}}$ The inductance of a toroid is

$$L = \frac{\mu_0 N^2 h}{2\pi}\ln\frac{b}{a}.$$

If the toroid is very large and thin then we can write $b = a + \delta$, where $\delta << a$. The natural log then can be approximated as

$$\ln\frac{b}{a} = \ln\left(1 + \frac{\delta}{a}\right) \approx \frac{\delta}{a}.$$

The product of δ and h is the cross sectional area of the toroid, while $2\pi a$ is the circumference, which we will call l. The inductance of the toroid then reduces to

$$L \approx \frac{\mu_0 N^2}{2\pi}\frac{\delta}{a} = \frac{\mu_0 N^2 A}{l}.$$

But N is the number of turns, which can also be written as $N = nl$, where n is the turns per unit length. Substitute this in and we arrive at Eq. 36-7.

P36-2 (a) Since ni is the net current per unit length and is this case i/W, we can simply write $B = \mu_0 i/W$.

(b) There is only one loop of wire, so

$$L = \phi_B/i = BA/i = \mu_0 i\pi R^2/Wi = \mu_0 \pi R^2/W.$$

P36-3 Choose the y axis so that it is parallel to the wires and directly between them. The field in the plane between the wires is

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{d/2+x} + \frac{1}{d/2-x}\right).$$

The flux per length l of the wires is

$$\begin{aligned}
\Phi_B &= l\int_{-d/2+a}^{d/2-a} B\,dx = l\frac{\mu_0 i}{2\pi}\int_{-d/2+a}^{d/2-a}\left(\frac{1}{d/2+x}+\frac{1}{d/2-x}\right)dx,\\
&= 2l\frac{\mu_0 i}{2\pi}\int_{-d/2+a}^{d/2-a}\left(\frac{1}{d/2+x}\right)dx,\\
&= 2l\frac{\mu_0 i}{2\pi}\ln\frac{d-a}{a}.
\end{aligned}$$

The inductance is then

$$L = \frac{\phi_B}{i} = \frac{\mu_0 l}{\pi}\ln\frac{d-a}{a}.$$

P36-4 (a) Choose the y axis so that it is parallel to the wires and directly between them. The field in the plane between the wires is

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{d/2+x} + \frac{1}{d/2-x}\right).$$

The flux per length l between the wires is

$$\begin{aligned}
\Phi_1 &= \int_{-d/2+a}^{d/2-a} B\,dx = \frac{\mu_0 i}{2\pi}\int_{-d/2+a}^{d/2-a}\left(\frac{1}{d/2+x}+\frac{1}{d/2-x}\right)dx,\\
&= 2\frac{\mu_0 i}{2\pi}\int_{-d/2+a}^{d/2-a}\left(\frac{1}{d/2+x}\right)dx,\\
&= 2\frac{\mu_0 i}{2\pi}\ln\frac{d-a}{a}.
\end{aligned}$$

The field in the plane *inside* one of the wires, but still between the centers is

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{d/2+x} + \frac{d/2-x}{a^2}\right).$$

The additional flux is then

$$\begin{aligned}
\Phi_2 &= 2\int_{d/2-a}^{d/2} B\,dx = 2\frac{\mu_0 i}{2\pi}\int_{d/2-a}^{d/2}\left(\frac{1}{d/2+x}+\frac{d/2-x}{a^2}\right)dx,\\
&= 2\frac{\mu_0 i}{2\pi}\left(\ln\frac{d}{d-a}+\frac{1}{2}\right).
\end{aligned}$$

The flux per meter between the axes of the wire is the sum, or

$$\begin{aligned}
\Phi_B &= \frac{\mu_0 i}{\pi}\left(\ln\frac{d}{a}+\frac{1}{2}\right),\\
&= \frac{(4\pi\times10^{-7}\mathrm{H/m})(11.3\,\mathrm{A})}{\pi}\left(\ln\frac{(21.8,\mathrm{mm})}{(1.3\,\mathrm{mm})}+\frac{1}{2}\right),\\
&= 1.5\times10^{-5}\mathrm{Wb/m}.
\end{aligned}$$

(b) The fraction f inside the wires is

$$\begin{aligned}
f &= \left(\ln\frac{d}{d-a}+\frac{1}{2}\right)/\left(\ln\frac{d}{a}+\frac{1}{2}\right),\\
&= \left(\frac{(21.8,\mathrm{mm})}{(21.8,\mathrm{mm})-(1.3\,\mathrm{mm})}+\frac{1}{2}\right)/\left(\frac{(21.8,\mathrm{mm})}{(1.3\,\mathrm{mm})}+\frac{1}{2}\right),\\
&= 0.09.
\end{aligned}$$

(c) The net flux is zero for parallel currents.

P36-5 The magnetic field in the region between the conductors of a coaxial cable is given by

$$B=\frac{\mu_0 i}{2\pi r},$$

so the flux through an area of length l, width $b-a$, and perpendicular to $\vec{\mathbf{B}}$ is

$$\begin{aligned}
\Phi_B &= \int \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = \int B\,dA,\\
&= \int_a^b\int_0^l \frac{\mu_0 i}{2\pi r}dz\,dr,\\
&= \frac{\mu_0 il}{2\pi}\ln\frac{b}{a}.
\end{aligned}$$

We evaluated this integral is cylindrical coordinates: $dA=(dr)(dz)$. As you have been warned so many times before, learn these differentials!

The inductance is then

$$L=\frac{\Phi_B}{i}=\frac{\mu_0 l}{2\pi}\ln\frac{b}{a}.$$

P36-6 (a) So long as the fuse is not blown there is effectively no resistance in the circuit. Then the equation for the current is $\mathcal{E}=L\,di/dt$, but since $\mathcal{E}$ is constant, this has a solution $i=\mathcal{E}t/L$. The fuse blows when $t=i_{\mathrm{max}}L/\mathcal{E}=(3.0\,\mathrm{A})(5.0\,\mathrm{H})/(10\,\mathrm{V})=1.5\,\mathrm{s}$.

(b) Note that once the fuse blows the maximum steady state current is $2/3\,\mathrm{A}$, so there must be an exponential approach to that current.

P36-7 The initial rate of increase is $di/dt=\mathcal{E}/L$. Since the steady state current is $\mathcal{E}/R$, the current will reach the steady state value in a time given by $\mathcal{E}/R=i=\mathcal{E}t/L$, or $t=L/R$. But that's τ_L.

P36-8 (a) $U=\frac{1}{2}Li^2=(152\,\mathrm{H})(32\,\mathrm{A})^2/2=7.8\times10^4\mathrm{J}$.

(b) If the coil develops at finite resistance then all of the energy in the field will be dissipated as heat. The mass of Helium that will boil off is

$$m=Q/L_v=(7.8\times10^4\mathrm{J})/(85\,\mathrm{J/mol})/(4.00\mathrm{g/mol})=3.7\,\mathrm{kg}.$$

P36-9 (a) $B = (\mu_0 N i)/(2\pi r)$, so

$$u = \frac{B^2}{2\mu_0} = \frac{\mu_0 N^2 i^2}{8\pi^2 r^2}.$$

(b) $U = \int u\, dV = \int u r dr\, d\theta\, dz$. The field inside the toroid is uniform in z and θ, so

$$\begin{aligned} U &= 2\pi h \int_a^b \frac{\mu_0 N^2 i^2}{8\pi^2 r^2} r\, dr, \\ &= \frac{h\mu_0 N^2 i^2}{4\pi} \ln \frac{b}{a}. \end{aligned}$$

(c) The answers are the same!

P36-10 The energy in the inductor is originally $U = Li_0^2/2$. The internal energy in the resistor increases at a rate $P = i^2 R$. Then

$$\int_0^\infty P\, dt = R \int_0^\infty i_0^2 e^{-2t/\tau_L}\, dt = \frac{R i_0^2 \tau_L}{2} = \frac{L i_0^2}{2}.$$

$\boxed{\textbf{P36-11}}$ (a) In Chapter 33 we found the magnetic field inside a wire carrying a uniform current density is

$$B = \frac{\mu_0 i r}{2\pi R^2}.$$

The magnetic energy density in this wire is

$$u_B = \frac{1}{2\mu_0} B^2 = \frac{\mu_0 i^2 r^2}{8\pi^2 R^4}.$$

We want to integrate in cylindrical coordinates over the volume of the wire. Then the volume element is $dV = (dr)(r\, d\theta)(dz)$, so

$$\begin{aligned} U_B &= \int u_B dV, \\ &= \int_0^R \int_0^l \int_0^{2\pi} \frac{\mu_0 i^2 r^2}{8\pi^2 R^4} d\theta\, dz\, r dr, \\ &= \frac{\mu_0 i^2 l}{4\pi R^4} \int_0^R r^3\, dr, \\ &= \frac{\mu_0 i^2 l}{16\pi}. \end{aligned}$$

(b) Solve

$$U_B = \frac{L}{2} i^2$$

for L, and

$$L = \frac{2U_B}{i^2} = \frac{\mu_0 l}{8\pi}.$$

P36-12 $1/C = 1/C_1 + 1/C_2$ and $L = L_1 + L_2$. Then

$$\frac{1}{\omega} = \sqrt{LC} = \sqrt{(L_1 + L_2)\frac{C_1 C_2}{C_1 + C_2}} = \sqrt{\frac{C_2/\omega_0^2 + C_1/\omega_0^2}{C_1 + C_2}} = \frac{1}{\omega_0}.$$

P36-13 (a) There is no current in the middle inductor; the loop equation becomes

$$L\frac{d^2q}{dt^2} + \frac{q}{C} + L\frac{d^2q}{dt^2} + \frac{q}{C} = 0.$$

Try $q = q_{\rm m} \cos \omega t$ as a solution:

$$-L\omega^2 + \frac{1}{C} - L\omega^2 + \frac{1}{C} = 0;$$

which requires $\omega = 1/\sqrt{LC}$.

(b) Look at only the left hand loop; the loop equation becomes

$$L\frac{d^2q}{dt^2} + \frac{q}{C} + 2L\frac{d^2q}{dt^2} = 0.$$

Try $q = q_{\rm m} \cos \omega t$ as a solution:

$$-L\omega^2 + \frac{1}{C} - 2L\omega^2 = 0;$$

which requires $\omega = 1/\sqrt{3LC}$.

P36-14 (b) $(\omega' - \omega)/\omega$ is the fractional shift; this can also be written as

$$\begin{aligned}
\frac{\omega'}{\omega - 1} &= \sqrt{1 - (LC)(R/2L)^2} - 1, \\
&= \sqrt{1 - R^2C/4L} - 1, \\
&= \sqrt{1 - \frac{(100\,\Omega)^2(7.3\times10^{-6}\mathrm{F})}{4(4.4\,\mathrm{H})}} - 1 = -2.1\times10^{-3}.
\end{aligned}$$

$\boxed{\textbf{P36-15}}$ We start by focusing on the charge on the capacitor, given by Eq. 36-40 as

$$q = q_{\rm m}e^{-Rt/2L}\cos(\omega' t + \phi).$$

After one oscillation the cosine term has returned to the original value but the exponential term has attenuated the charge on the capacitor according to

$$q = q_{\rm m}e^{-RT/2L},$$

where T is the period. The fractional energy loss on the capacitor is then

$$\frac{U_0 - U}{U_0} = 1 - \frac{q^2}{q_m^2} = 1 - e^{-RT/L}.$$

For small enough damping we can expand the exponent. Not only that, but $T = 2\pi/\omega$, so

$$\frac{\Delta U}{U} \approx 2\pi R/\omega L.$$

P36-16 We are given $1/2 = e^{-t/2\tau_L}$ when $t = 2\pi n/\omega'$. Then

$$\omega' = \frac{2\pi n}{t} = \frac{2\pi n}{2(L/R)\ln 2} = \frac{\pi n R}{L \ln 2}.$$

From Eq. 36-41,

$$\begin{aligned}
\omega^2 - \omega'^2 &= (R/2L)^2, \\
(\omega - \omega')(\omega + \omega') &= (R/2L)^2, \\
(\omega - \omega')2\omega' &\approx (R/2L)^2, \\
\frac{\omega - \omega'}{\omega} &= \approx \frac{(R/2L)^2}{2\omega'^2}, \\
&= \frac{(\ln 2)^2}{8\pi^2 n^2}, \\
&= \frac{0.0061}{n^2}.
\end{aligned}$$

$\boxed{\textbf{E37-1}}$ The frequency, f, is related to the angular frequency ω by

$$\omega = 2\pi f = 2\pi(60 \text{ Hz}) = 377 \text{ rad/s}$$

The current is alternating because that is what the generator is designed to produce. It does this through the configuration of the magnets and coils of wire. One complete turn of the generator will (could?) produce one "cycle"; hence, the generator is turning 60 times per second. Not only does this set the frequency, it also sets the emf, since the emf is proportional to the speed at which the coils move through the magnetic field.

E37-2 (a) $X_L = \omega L$, so

$$f = X_L/2\pi L = (1.28\times10^3\Omega)/2\pi(0.0452\,\text{H}) = 4.51\times10^3/\text{s}.$$

(b) $X_C = 1/\omega C$, so

$$C = 1/2\pi f X_C = 1/2\pi(4.51\times10^3/\text{s})(1.28\times10^3\Omega) = 2.76\times10^{-8}\text{F}.$$

(c) The inductive reactance doubles while the capacitive reactance is cut in half.

E37-3 (a) $X_L = X_C$ implies $\omega L = 1/\omega C$ or $\omega = 1/\sqrt{LC}$, so

$$\omega = 1/\sqrt{(6.23\times10^{-3}\text{H})(11.4\times10^{-6}\text{F})} = 3750 \text{ rad/s}.$$

(b) $X_L = \omega L = (3750 \text{ rad/s})(6.23\times10^{-3}\text{H}) = 23.4\,\Omega$
(c) See (a) above.

E37-4 (a) $i_\text{m} = \mathcal{E}/X_L = \mathcal{E}/\omega L$, so

$$i_\text{m} = (25.0\,\text{V})/(377 \text{ rad/s})(12.7\,\text{H}) = 5.22\times10^{-3}\text{A}.$$

(b) The current and emf are 90° out of phase. When the current is a maximum, $\mathcal{E} = 0$.
(c) $\omega t = \arcsin[\mathcal{E}(t)/\mathcal{E}_\text{m}]$, so

$$\omega t = \arcsin\frac{(-13.8\,\text{V})}{(25.0\,\text{V})} = 0.585 \text{ rad}.$$

and

$$i = (5.22\times10^{-3}\text{A})\cos(0.585) = 4.35\times10^{-3}\text{A}.$$

(d) Taking energy.

$\boxed{\textbf{E37-5}}$ (a) The reactance of the capacitor is from Eq. 37-11, $X_C = 1/\omega C$. The AC generator from Exercise 4 has $\mathcal{E} = (25.0\,\text{V})\sin(377\,\text{rad/s})t$. So the reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\,\text{rad/s})(4.1\mu\text{F})} = 647\,\Omega.$$

The maximum value of the current is found from Eq. 37-13,

$$i_\text{m} = \frac{(\Delta V_C)_\text{max}}{X_C} = \frac{(25.0\,\text{V})}{(647\,\Omega)} = 3.86\times10^{-2}\text{A}.$$

(b) The generator emf is 90° out of phase with the current, so when the current is a maximum the emf is zero.

(c) The emf is -13.8 V when

$$\omega t = \arcsin\frac{(-13.8\,\mathrm{V})}{(25.0\,\mathrm{V})} = 0.585 \text{ rad}.$$

The current leads the voltage by $90^\circ = \pi/2$, so

$$i = i_\mathrm{m}\sin(\omega t - \phi) = (3.86\times10^{-2}\mathrm{A})\sin(0.585 - \pi/2) = -3.22\times10^{-2}\mathrm{A}.$$

(d) Since both the current and the emf are negative the product is positive and the generator is supplying energy to the circuit.

E37-6 $R = (\omega L - 1/omegaC)/\tan\phi$ and $\omega = 2\pi f = 2\pi(941/\mathrm{s}) = 5910$ rad/s , so

$$R = \frac{(5910\text{ rad/s})(88.3\times10^{-3}\mathrm{H}) - 1/(5910\text{ rad/s})(937\times10^{-9}\mathrm{F})}{\tan(75^\circ)} = 91.5\,\Omega.$$

E37-7

E37-8 (a) X_L doesn't change, so $X_L = 87\,\Omega$.
(b) $X_C = 1/\omega C = 1/2\pi(60/\mathrm{s})(70\times10^{-6}\mathrm{F}) = 37.9\Omega$.
(c) $Z = \sqrt{(160\,\Omega)^2 + (87\,\Omega - 37.9\,\Omega)^2} = 167\,\Omega$.
(d) $i_\mathrm{m} = (36\,\mathrm{V})/(167\,\Omega) = 0.216\,\mathrm{A}$.
(e) $\tan\phi = (87\,\Omega - 37.9\,\Omega)/(160\,\Omega) = 0.3069$, so

$$\phi = \arctan(0.3069) = 17^\circ.$$

$\boxed{\textbf{E37-9}}$ A circuit is considered inductive if $X_L > X_C$, this happens when i_m lags $\mathcal{E}_\mathrm{m}$. If, on the other hand, $X_L < X_C$, and i_m leads $\mathcal{E}_\mathrm{m}$, we refer to the circuit as capacitive. This is discussed on page 850, although it is slightly hidden in the text of column one.

(a) At resonance, $X_L = X_C$. Since $X_L = \omega L$ and $X_C = 1/\omega C$ we expect that X_L grows with increasing frequency, while X_C decreases with increasing frequency.

Consequently, at frequencies above the resonant frequency $X_L > X_C$ and the circuit is predominantly inductive. But what does this really mean? It means that the inductor plays a major role in the current through the circuit while the capacitor plays a minor role. The more inductive a circuit is, the less significant any capacitance is on the behavior of the circuit.

For frequencies below the resonant frequency the reverse is true.

(b) Right at the resonant frequency the inductive effects are exactly canceled by the capacitive effects. The impedance is equal to the resistance, and it is (almost) as if neither the capacitor or inductor are even in the circuit.

E37-10 The net y component is $X_C - X_L$. The net x component is R. The magnitude of the resultant is

$$Z = \sqrt{R^2 + (X_C - X_L)^2},$$

while the phase angle is

$$\tan\phi = \frac{-(X_C - X_L)}{R}.$$

E37-11 Yes.

At resonance $\omega = 1/\sqrt{(1.2\,\mathrm{H})(1.3\times10^{-6}\mathrm{F})} = 800$ rad/s and $Z = R$. Then $i_\mathrm{m} = \mathcal{E}/Z = (10\,\mathrm{V})/(9.6\,\Omega) = 1.04\,\mathrm{A}$, so

$$[\Delta V_L]_\mathrm{m} = i_\mathrm{m} X_L = (1.08\,\mathrm{A})(800\text{ rad/s})(1.2\,\mathrm{H}) = 1000\,\mathrm{V}.$$

E37-12 (a) Let $O = X_L - X_C$ and $A = R$, then $H^2 = A^2 + O^2 = Z^2$, so

$$\sin\phi = (X_L - X_C)/Z$$

and

$$\cos\phi = R/Z.$$

$\boxed{\textbf{E37-13}}$ (a) The voltage across the generator is the generator emf, so when it is a maximum from Sample Problem 37-3, it is 36 V. This corresponds to $\omega t = \pi/2$.

(b) The current through the circuit is given by $i = i_{\rm m}\sin(\omega t - \phi)$. We found in Sample Problem 37-3 that $i_{\rm m} = 0.196\,{\rm A}$ and $\phi = -29.4^\circ = 0.513$ rad.

For a resistive load we apply Eq. 37-3,

$$\Delta V_R = i_{\rm m}R\sin(\omega t - \phi) = (0.196\,{\rm A})(160\Omega)\sin((\pi/2) - (-0.513)) = 27.3\,{\rm V}.$$

(c) For a capacitive load we apply Eq. 37-12,

$$\Delta V_C = i_{\rm m}X_C\sin(\omega t - \phi - \pi/2) = (0.196\,{\rm A})(177\Omega)\sin(-(-0.513)) = 17.0\,{\rm V}.$$

(d) For an inductive load we apply Eq. 37-7,

$$\Delta V_L = i_{\rm m}X_L\sin(\omega t - \phi + \pi/2) = (0.196\,{\rm A})(87\Omega)\sin(\pi - (-0.513)) = -8.4\,{\rm V}.$$

(e) $(27.3\,{\rm V}) + (17.0\,{\rm V}) + (-8.4\,{\rm V}) = 35.9\,{\rm V}$.

E37-14 If circuit 1 and 2 have the same resonant frequency then $L_1C_1 = L_2C_2$. The series combination for the inductors is

$$L = L_1 + L_2,$$

The series combination for the capacitors is

$$1/C = 1/C_1 + 1/C_2,$$

so

$$LC = (L_1 + L_2)\frac{C_1C_2}{C_1 + C_2} = \frac{L_1C_1C_2 + L_2C_2C_1}{C_1 + C_2} = L_1C_1,$$

which is the same as both circuit 1 and 2.

E37-15 (a) $Z = (125\,{\rm V})/(3.20\,{\rm A}) = 39.1\,\Omega$.

(b) Let $O = X_L - X_C$ and $A = R$, then $H^2 = A^2 + O^2 = Z^2$, so

$$\cos\phi = R/Z.$$

Using this relation,

$$R = (39.1\,\Omega)\cos(56.3^\circ) = 21.7\,\Omega.$$

(c) If the current leads the emf then the circuit is capacitive.

E37-16 (a) Integrating over a single cycle,

$$\begin{aligned}\frac{1}{T}\int_0^T \sin^2\omega t\,dt &= \frac{1}{T}\int_0^T \frac{1}{2}(1 - \cos 2\omega t)\,dt,\\ &= \frac{1}{2T}T = \frac{1}{2}.\end{aligned}$$

(b) Integrating over a single cycle,

$$\begin{aligned}\frac{1}{T}\int_0^T \sin\omega t\cos\omega t\,dt &= \frac{1}{T}\int_0^T \frac{1}{2}\sin 2\omega t dt,\\ &= 0.\end{aligned}$$

$\boxed{\textbf{E37-17}}$ The resistance would be given by Eq. 37-32,

$$R = \frac{P_{\rm av}}{{i_{\rm rms}}^2} = \frac{(0.10)(746\,{\rm W})}{(0.650\,{\rm A})^2} = 177\,\Omega.$$

This would not be the same as the direct current resistance of the coils of a stopped motor, because there would be no inductive effects.

E37-18 Since $i_{\rm rms} = \mathcal{E}_{\rm rms}/Z$, then

$$P_{\rm av} = i^2{}_{\rm rms}R = \frac{\mathcal{E}^2{}_{\rm rms}R}{Z^2}.$$

E37-19 (a) $Z = \sqrt{(160\,\Omega)^2 + (177\,\Omega)^2} = 239\,\Omega$; then

$$P_{\rm av} = \frac{1}{2}\frac{(36\,{\rm V})^2(160\,\Omega)}{(239\,\Omega)^2} = 1.82\,{\rm W}.$$

(b) $Z = \sqrt{(160\,\Omega)^2 + (87\,\Omega)^2} = 182\,\Omega$; then

$$P_{\rm av} = \frac{1}{2}\frac{(36\,{\rm V})^2(160\,\Omega)}{(182\,\Omega)^2} = 3.13\,{\rm W}.$$

E37-20 (a) $Z = \sqrt{(12.2\,\Omega)^2 + (2.30\,\Omega)^2} = 12.4\,\Omega$
(b) $P_{\rm av} = (120\,{\rm V})^2(12.2\,\Omega)/(12.4\,\Omega)^2 = 1140\,{\rm W}$.
(c) $i_{\rm rms} = (120\,{\rm V})/(12.4\,\Omega) = 9.67\,{\rm A}$.

$\boxed{\textbf{E37-21}}$ The rms value of any sinusoidal quantity is related to the maximum value by $\sqrt{2}\,v_{\rm rms} = v_{\rm max}$. Since this factor of $\sqrt{2}$ appears in all of the expressions, we can conclude that if the rms values are equal then so are the maximum values. This means that

$$(\Delta V_R)_{\rm max} = (\Delta V_C)_{\rm max} = (\Delta V_L)_{\rm max}$$

or $i_mR = i_mX_C = i_mX_L$ or, with one last simplification, $R = X_L = X_C$. Focus on the right hand side of the last equality. If $X_C = X_L$ then we have a resonance condition, and the impedance (see Eq. 37-20) is a minimum, and is equal to R. Then, according to Eq. 37-21,

$$i_m = \frac{\mathcal{E}_{\rm m}}{R},$$

which has the immediate consequence that the rms voltage across the resistor is the same as the rms voltage across the generator. So everything is 100 V.

E37-22 (a) The antenna is "in-tune" when the impedance is a minimum, or $\omega = 1/\sqrt{LC}$. So

$$f = \omega/2\pi = 1/2\pi\sqrt{(8.22\times10^{-6}{\rm H})(0.270\times10^{-12}{\rm F})} = 1.07\times10^{8}{\rm Hz}.$$

(b) $i_{\rm rms} = (9.13\,\mu{\rm V})/(74.7\,\Omega) = 1.22\times10^{-7}{\rm A}$.
(c) $X_C = 1/2\pi fC$, so

$$V_C = iX_C = (1.22\times10^{-7}{\rm A})/2\pi(1.07\times10^{8}{\rm Hz})(0.270\times10^{-12}{\rm F}) = 6.72\times10^{-4}{\rm V}.$$

E37-23 Assuming no inductors or capacitors in the circuit, then the circuit effectively behaves as a DC circuit. The current through the circuit is $i = \mathcal{E}/(r+R)$. The power delivered to R is then $P = i\Delta V = i^2R = \mathcal{E}^2R/(r+R)^2$. Evaluate dP/dR and set it equal to zero to find the maximum. Then

$$0 = \frac{dP}{dR} = \mathcal{E}^2R\frac{r-R}{(r+R)^3},$$

which has the solution $r = R$.

E37-24 (a) Since $P_{\text{av}} = i_{\text{m}}{}^2R/2 = \mathcal{E}_{\text{m}}{}^2R/2Z^2$, then P_{av} is a maximum when Z is a minimum, and vise-versa. Z is a minimum at resonance, when $Z = R$ and $f = 1/2\pi\sqrt{LC}$. When Z is a minimum

$$C = 1/4\pi^2 f^2 L = 1/4\pi^2(60\text{ Hz})^2(60\,\text{mH}) = 1.2\times10^{-7}\text{F}.$$

(b) Z is a maximum when X_C is a maximum, which occurs when C is very small, like zero.

(c) When X_C is a maximum $P = 0$. When P is a maximum $Z = R$ so

$$P = (30\,\text{V})^2/2(5.0\,\Omega) = 90\,\text{W}.$$

(d) The phase angle is zero for resonance; it is 90° for infinite X_C or X_L.

(e) The power factor is zero for a system which has no power. The power factor is one for a system in resonance.

$\boxed{\textbf{E37-25}}$ (a) The resistance is $R = 15.0\,\Omega$. The inductive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(550\,\text{s}^{-1})(4.72\mu\text{F})} = 61.3\,\Omega.$$

The inductive reactance is given by

$$X_L = \omega L = 2\pi(550\,\text{s}^{-1})(25.3\,\text{mH}) = 87.4\,\Omega.$$

The impedance is then

$$Z = \sqrt{(15.0\,\Omega)^2 + ((87.4\,\Omega) - (61.3\,\Omega))^2} = 30.1\,\Omega.$$

Finally, the rms current is

$$i_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{(75.0\,\text{V})}{(30.1\,\Omega)} = 2.49\,\text{A}.$$

(b) The rms voltages between any two points is given by

$$(\Delta V)_{\text{rms}} = i_{\text{rms}}Z,$$

where Z is *not* the impedance of the circuit but instead the impedance between the two points in question. When only one device is between the two points the impedance is equal to the reactance (or resistance) of that device.

We're not going to show *all* of the work here, but we will put together a nice table for you

Points	Impedance Expression	Impedance Value	$(\Delta V)_{\text{rms}}$,
ab	$Z = R$	$Z = 15.0\,\Omega$	37.4 V,
bc	$Z = X_C$	$Z = 61.3\,\Omega$	153 V,
cd	$Z = X_L$	$Z = 87.4\,\Omega$	218 V,
bd	$Z = \lvert X_L - X_C\rvert$	$Z = 26.1\,\Omega$	65 V,
ac	$Z = \sqrt{R^2 + X_C^2}$	$Z = 63.1\,\Omega$	157 V,

Note that this last one was ΔV_{ac}, and not ΔV_{ad}, because it is more entertaining. You probably should use ΔV_{ad} for your homework.

(c) The average power dissipated from a capacitor or inductor is zero; that of the resistor is

$$P_R = [(\Delta V_R)_{\text{rms}}]^2/R = (37.4\,\text{V})^2/(15.0\Omega) = 93.3\,\text{W}.$$

E37-26 (a) The energy stored in the capacitor is a function of the charge on the capacitor; although the charge does vary with time it varies periodically and at the end of the cycle has returned to the original values. As such, the energy stored in the capacitor doesn't change from one period to the next.

(b) The energy stored in the inductor is a function of the current in the inductor; although the current does vary with time it varies periodically and at the end of the cycle has returned to the original values. As such, the energy stored in the inductor doesn't change from one period to the next.

(c) $P = \mathcal{E}i = \mathcal{E}_\mathrm{m} i_\mathrm{m} \sin(\omega t)\sin(\omega t - \phi)$, so the energy generated in one cycle is

$$\begin{aligned} U &= \int_0^T P\,dt = \mathcal{E}_\mathrm{m} i_\mathrm{m} \int_0^T \sin(\omega t)\sin(\omega t - \phi)dt, \\ &= \mathcal{E}_\mathrm{m} i_\mathrm{m} \int_0^T \sin(\omega t)\sin(\omega t - \phi)dt, \\ &= \frac{T}{2}\mathcal{E}_\mathrm{m} i_\mathrm{m} \cos\phi. \end{aligned}$$

(d) $P = {i_\mathrm{m}}^2 R \sin^2(\omega t - \phi)$, so the energy dissipated in one cycle is

$$\begin{aligned} U &= \int_0^T P\,dt = {i_\mathrm{m}}^2 R \int_0^T \sin^2(\omega t - \phi)dt, \\ &= {i_\mathrm{m}}^2 R \int_0^T \sin^2(\omega t - \phi)dt, \\ &= \frac{T}{2}{i_\mathrm{m}}^2 R. \end{aligned}$$

(e) Since $\cos\phi = R/Z$ and $\mathcal{E}_\mathrm{m}/Z = i_\mathrm{m}$ we can equate the answers for (c) and (d).

$\boxed{\textbf{E37-27}}$ Apply Eq. 37-41,

$$\Delta V_\mathrm{s} = \Delta V_\mathrm{p}\frac{N_\mathrm{s}}{N_\mathrm{p}} = (150\,\mathrm{V})\frac{(780)}{(65)} = 1.8\times 10^3\,\mathrm{V}.$$

E37-28 (a) Apply Eq. 37-41,

$$\Delta V_\mathrm{s} = \Delta V_\mathrm{p}\frac{N_\mathrm{s}}{N_\mathrm{p}} = (120\,\mathrm{V})\frac{(10)}{(500)} = 2.4\,\mathrm{V}.$$

(b) $i_\mathrm{s} = (2.4\,\mathrm{V})/(15\,\Omega) = 0.16\,\mathrm{A}$;

$$i_\mathrm{p} = i_\mathrm{s}\frac{N_\mathrm{s}}{N_\mathrm{p}} = (0.16\,\mathrm{A})\frac{(10)}{(500)} = 3.2\times 10^{-3}\mathrm{A}.$$

$\boxed{\textbf{E37-29}}$ The autotransformer could have a primary connected between taps T_1 and T_2 (200 turns), T_1 and T_3 (1000 turns), and T_2 and T_3 (800 turns).

The same possibilities are true for the secondary connections. Ignoring the one-to-one connections there are 6 choices— three are step up, and three are step down. The step up ratios are $1000/200 = 5$, $800/200 = 4$, and $1000/800 = 1.25$. The step down ratios are the reciprocals of these three values.

E37-30 $\rho = (1.69\times10^{-8}\Omega\cdot\text{m})[1 - (4.3\times10^{-3}/\text{C}^\circ)(14.6^\circ\text{C})] = 1.58\times10^{-8}\Omega\cdot\text{m}$. The resistance of the two wires is

$$R = \frac{\rho L}{A} = \frac{(1.58\times10^{-8}\Omega\cdot\text{m})2(1.2\times10^{3}\text{m})}{\pi(0.9\times10^{-3}\text{m})^2} = 14.9\,\Omega.$$

$P = i^2R = (3.8\,\text{A})^2(14.9\,\Omega) = 220\,\text{W}$.

E37-31 The supply current is

$$i_\text{p} = (0.270\,\text{A})(74\times10^{3}\text{V}/\sqrt{2})/(220\,\text{V}) = 64.2\,\text{A}.$$

The potential drop across the supply lines is

$$\Delta V = (64.2\,\text{A})(0.62\,\Omega) = 40\,\text{V}.$$

This is the amount by which the supply voltage must be increased.

E37-32 Use Eq. 37-46:

$$N_\text{p}/N_\text{s} = \sqrt{(1000\,\Omega)/(10\,\Omega)} = 10.$$

P37-1 (a) The emf is a maximum when $\omega t - \pi/4 = \pi/2$, so $t = 3\pi/4\omega = 3\pi/4(350\text{ rad/s}) = 6.73\times10^{-3}\text{s}$.

(b) The current is a maximum when $\omega t - 3\pi/4 = \pi/2$, so $t = 5\pi/4\omega = 5\pi/4(350\text{ rad/s}) = 1.12\times10^{-2}\text{s}$.

(c) The current lags the emf, so the circuit contains an inductor.

(d) $X_L = \mathcal{E}_\text{m}/i_\text{m}$ and $X_L = \omega L$, so

$$L = \frac{\mathcal{E}_\text{m}}{i_\text{m}\omega} = \frac{(31.4\,\text{V})}{(0.622\,\text{A})(350\text{ rad/s})} = 0.144\,\text{H}.$$

P37-2 (a) The emf is a maximum when $\omega t - \pi/4 = \pi/2$, so $t = 3\pi/4\omega = 3\pi/4(350\text{ rad/s}) = 6.73\times10^{-3}\text{s}$.

(b) The current is a maximum when $\omega t + \pi/4 = \pi/2$, so $t = \pi/4\omega = \pi/4(350\text{ rad/s}) = 2.24\times10^{-3}\text{s}$.

(c) The current leads the emf, so the circuit contains an capacitor.

(d) $X_C = \mathcal{E}_\text{m}/i_\text{m}$ and $X_C = 1/\omega C$, so

$$C = \frac{i_\text{m}}{\mathcal{E}_\text{m}\omega} = \frac{(0.622\,\text{A})}{(31.4\,\text{V})(350\text{ rad/s})} = 5.66\times10^{-5}\text{F}.$$

$\boxed{\textbf{P37-3}}$ (a) Since the maximum values for the voltages across the individual devices are proportional to the reactances (or resistances) for devices in series (the constant of proportionality is the maximum current), we have $X_L = 2R$ and $X_C = R$.

From Eq. 37-18,

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{2R - R}{R} = 1,$$

or $\phi = 45^\circ$.

(b) The impedance of the circuit, in terms of the resistive element, is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2R - R)^2} = \sqrt{2}\,R.$$

But $\mathcal{E}_\text{m} = i_\text{m}Z$, so $Z = (34.4\,\text{V})/(0.320\,\text{A}) = 108\Omega$. Then we can use our previous work to finds that $R = 76\Omega$.

P37-4 When the switch is open the circuit is an LRC circuit. In position 1 the circuit is an RLC circuit, but the capacitance is equal to the two capacitors of C in parallel, or $2C$. In position 2 the circuit is a simple LC circuit with no resistance.

The impedance when the switch is in position 2 is $Z_2 = |\omega L - 1/\omega C|$. But

$$Z_2 = (170\,\mathrm{V})/(2.82\,\mathrm{A}) = 60.3\,\Omega.$$

The phase angle when the switch is open is $\phi_0 = 20°$. But

$$\tan\phi_0 = \frac{\omega L - 1/\omega C}{R} = \frac{Z_2}{R},$$

so $R = (60.3\,\Omega)/\tan(20°) = 166\,\Omega$.

The phase angle when the switch is in position 1 is

$$\tan\phi_1 = \frac{\omega L - 1/\omega 2C}{R},$$

so $\omega L - 1/\omega 2C = (166\,\Omega)\tan(10°) = 29.2\,\Omega$. Equating the ωL part,

$$\begin{aligned}
(29.2\,\Omega) + 1/\omega 2C &= (-60.3\,\Omega) + 1/\omega C, \\
C &= 1/2(377\text{ rad/s})[(60.3\,\Omega) + (29.2\,\Omega)] = 1.48\times 10^{-5}\mathrm{F}.
\end{aligned}$$

Finally,

$$L = \frac{(-60.3\Omega) + 1/(377\text{ rad/s})(1.48\times 10^{-5}\mathrm{F})}{(377\text{ rad/s})} = 0.315\,\mathrm{H}.$$

$\boxed{\textbf{P37-5}}$ All three wires have emfs which vary sinusoidally in time; if we choose *any* two wires the phase difference will have an absolute value of 120°. We can then choose any two wires and expect (by symmetry) to get the same result. We choose 1 and 2. The potential difference is then

$$V_1 - V_2 = V_\mathrm{m}\left(\sin\omega t - \sin(\omega t - 120°)\right).$$

We need to add these two sine functions to get just one. We use

$$\sin\alpha - \sin\beta = 2\sin\frac{1}{2}(\alpha-\beta)\cos\frac{1}{2}(\alpha+\beta).$$

Then

$$\begin{aligned}
V_1 - V_2 &= 2V_\mathrm{m}\sin\frac{1}{2}(120°)\cos\frac{1}{2}(2\omega t - 120°), \\
&= 2V_\mathrm{m}(\frac{\sqrt{3}}{2})\cos(\omega t - 60°), \\
&= \sqrt{3}V_\mathrm{m}\sin(\omega t + 30°).
\end{aligned}$$

P37-6 (a) $\cos\phi = \cos(-42°) = 0.74$.

(b) The current leads.

(c) The circuit is capacitive.

(d) No. Resonance circuits have a power factor of one.

(e) There must be at least a capacitor and a resistor.

(f) $P = (75\,\mathrm{V})(1.2\,\mathrm{A})(0.74)/2 = 33\,\mathrm{W}$.

P37-7 (a) $\omega = 1/\sqrt{LC} = 1/\sqrt{(0.988\,\text{H})(19.3\times10^{-6}\text{F})} = 229$ rad/s.

(b) $i_\text{m} = (31.3\,\text{V})/(5.12\,\Omega) = 6.11\,\text{A}$.

(c) The current amplitude will be halved when the impedance is doubled, or when $Z = 2R$. This occurs when $3R^2 = (\omega L - 1/\omega C)^2$, or

$$3R^2\omega^2 = \omega^4 L^2 - 2\omega^2 L/C + 1/C^2.$$

The solution to this quadratic is

$$\omega^2 = \frac{2L + 3CR^2 \pm \sqrt{9C^2R^4 + 12CR^2L}}{2L^2C},$$

so $\omega_1 = 224.6$ rad/s and $\omega_2 = 233.5$ rad/s.

(d) $\Delta\omega/\omega = (8.9 \text{ rad/s})/(229 \text{ rad/s}) = 0.039$.

P37-8 (a) The current amplitude will be halved when the impedance is doubled, or when $Z = 2R$. This occurs when $3R^2 = (\omega L - 1/\omega C)^2$, or

$$3R^2\omega^2 = \omega^4 L^2 - 2\omega^2 L/C + 1/C^2.$$

The solution to this quadratic is

$$\omega^2 = \frac{2L + 3CR^2 \pm \sqrt{9C^2R^4 + 12CR^2L}}{2L^2C},$$

Note that $\Delta\omega = \omega_+ - \omega_-$; with a wee bit of algebra,

$$\Delta\omega(\omega_+ + \omega_-) = \omega_+^2 - \omega_-^2.$$

Also, $\omega_+ + \omega_- \approx 2\omega$. Hence,

$$\begin{aligned}
\omega\Delta\omega &\approx \frac{\sqrt{9C^2R^4 + 12CR^2L}}{2L^2C}, \\
\omega\Delta\omega &\approx \frac{\omega^2 R\sqrt{9C^2R^2 + 12LC}}{2L}, \\
\omega\Delta\omega &\approx \frac{\omega R\sqrt{9\omega^2C^2R^2 + 12}}{2L}, \\
\frac{\Delta\omega}{\omega} &\approx \frac{R\sqrt{9CR^2/L + 12}}{2L\omega}, \\
&\approx \frac{\sqrt{3}R}{\omega L},
\end{aligned}$$

assuming that $CR^2 \ll 4L/3$.

P37-9

P37-10 Use Eq. 37-46.

$\boxed{\textbf{P37-11}}$ (a) The resistance of this bulb is

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\,\text{V})^2}{(1000\,\text{W})} = 14.4\,\Omega.$$

The power is directly related to the brightness; if the bulb is to be varied in brightness by a factor of 5 then it would have a minimum power of 200 W. The rms current through the bulb at this power would be

$$i_{\rm rms} = \sqrt{P/R} = \sqrt{(200\,{\rm W})/(14.4\,\Omega)} = 3.73\,{\rm A}.$$

The impedance of the circuit must have been

$$Z = \frac{\mathcal{E}_{\rm rms}}{i_{\rm rms}} = \frac{(120\,{\rm V})}{(3.73\,{\rm A})} = 32.2\,\Omega.$$

The inductive reactance would then be

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(32.2\,\Omega)^2 - (14.4\,\Omega)^2} = 28.8\,\Omega.$$

Finally, the inductance would be

$$L = X_L/\omega = (28.8\,\Omega)/(2\pi(60.0\,{\rm s}^{-1})) = 7.64\,{\rm H}.$$

(b) One could use a variable resistor, and since it would be in series with the lamp a value of

$$32.2\,\Omega - 14.4\Omega = 17.8\,\Omega$$

would work. But the resistor would get hot, while on average there is no power radiated from a pure inductor.

E38-1 The maximum value occurs where $r = R$; there $B_{\max} = \frac{1}{2}\mu_0\epsilon_0 R\, dE/dt$. For $r < R$ B is half of $B_{\max}$ when $r = R/2$. For $r > R$ B is half of $B_{\max}$ when $r = 2R$. Then the two values of r are 2.5 cm and 10.0 cm.

E38-2 For a parallel plate capacitor $E = \sigma/\epsilon_0$ and the flux is then $\Phi_E = \sigma A/\epsilon_0 = q/\epsilon_0$. Then

$$i_{\rm d} = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dq}{dt} = \frac{d}{dt}CV = C\frac{dV}{dt}.$$

$\boxed{\textbf{E38-3}}$ Use the results of Exercise 2, and change the potential difference across the plates of the capacitor at a rate

$$\frac{dV}{dt} = \frac{i_{\rm d}}{C} = \frac{(1.0\,{\rm mA})}{(1.0\mu{\rm F})} = 1.0 \text{ kV/s}.$$

Provide a constant current to the capacitor

$$i = \frac{dQ}{dt} = \frac{d}{dt}CV = C\frac{dV}{dt} = i_{\rm d}.$$

E38-4 Since E is uniform between the plates $\Phi_E = EA$, regardless of the size of the region of interest. Since $j_{\rm d} = i_{\rm d}/A$,

$$j_{\rm d} = \frac{i_{\rm d}}{A} = \frac{1}{A}\epsilon_0\frac{d\Phi_E}{dt} = \epsilon_0\frac{dE}{dt}.$$

E38-5 (a) In this case $i_{\rm d} = i = 1.84\,{\rm A}$.

(b) Since $E = q/\epsilon_0 A$, $dE/dt = i/\epsilon_0 A$, or

$$dE/dt = (1.84\,{\rm A})/(8.85\times10^{-12}{\rm F/m})(1.22\,{\rm m})^2 = 1.40\times10^{11}{\rm V/m}.$$

(c) $i_{\rm d} = \epsilon_0 d\Phi_E/dt = \epsilon_0 a dE/dt$. a here refers to the area of the smaller square. Combine this with the results of part (b) and

$$i_{\rm d} = ia/A = (1.84\,{\rm A})(0.61\,{\rm m}/1.22\,{\rm m})^2 = 0.46\,{\rm A}.$$

(d) $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 i_{\rm d} = (4\pi\times10^{-7}{\rm H/m})(0.46\,{\rm A}) = 5.78\times10^{-7}{\rm T\cdot m}$.

E38-6 Substitute Eq. 38-8 into the results obtained in Sample Problem 38-1. Outside the capacitor $\Phi_E = \pi R^2 E$, so

$$B = \frac{\mu_0}{2\pi r}\frac{\epsilon_0\pi R^2 dE}{dt} = \frac{\mu_0}{2\pi r}i_{\rm d}.$$

Inside the capacitor the enclosed flux is $\Phi_E = \pi r^2 E$; but we want instead to define $i_{\rm d}$ in terms of the total Φ_E inside the capacitor as was done above. Consequently, inside the conductor

$$B = \frac{\mu_0 r}{2\pi R^2}\frac{\epsilon_0\pi R^2 dE}{dt} = \frac{\mu_0 r}{2\pi R^2}i_{\rm d}.$$

$\boxed{\textbf{E38-7}}$ Since the electric field is uniform in the area and perpendicular to the surface area we have

$$\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int E\,dA = E\int dA = EA.$$

The displacement current is then

$$i_{\rm d} = \epsilon_0 A\frac{dE}{dt} = (8.85\times10^{-12}{\rm F/m})(1.9\,{\rm m}^2)\frac{dE}{dt}.$$

(a) In the first region the electric field decreases by 0.2 MV/m in 4μs, so

$$i_{\rm d} = (8.85\times10^{-12}{\rm F/m})(1.9\,{\rm m}^2)\frac{(-0.2\times10^6{\rm V/m})}{(4\times10^{-6}{\rm s})} = -0.84\,{\rm A}.$$

(b) The electric field is constant so there is no change in the electric flux, and hence there is no displacement current.

(c) In the last region the electric field decreases by 0.4 MV/m in 5μs, so

$$i_{\rm d} = (8.85\times10^{-12}{\rm F/m})(1.9\,{\rm m}^2)\frac{(-0.4\times10^6{\rm V/m})}{(5\times10^{-6}{\rm s})} = -1.3\,{\rm A}.$$

E38-8 (a) Because of the circular symmetry $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = 2\pi r B$, where r is the distance from the center of the circular plates. Not only that, but $i_{\rm d} = j_{\rm d}A = \pi r^2 j_{\rm d}$. Equate these two expressions, and

$$B = \mu_0 r j_{\rm d}/2 = (4\pi\times10^{-7}{\rm H/m})(0.053\,{\rm m})(1.87\times10^1{\rm A/m})/2 = 6.23\times10^{-7}{\rm T}.$$

(b) $dE/dt = i_{\rm d}/\epsilon_0 A = j_{\rm d}/\epsilon_0 = (1.87\times10^1{\rm A/m})/(8.85\times10^{-12}{\rm F/m}) = 2.11\times10^{-12}{\rm V/m}.$

E38-9 The magnitude of E is given by

$$E = \frac{(162\,{\rm V})}{(4.8\times10^{-3}{\rm m})}\sin 2\pi(60/{\rm s})t;$$

Using the results from Sample Problem 38-1,

$$\begin{aligned} B_{\rm m} &= \frac{\mu_0\epsilon_0 R}{2}\left.\frac{dE}{dt}\right|_{t=0}, \\ &= \frac{(4\pi\times10^{-7}{\rm H/m})(8.85\times10^{-12}{\rm F/m})(0.0321\,{\rm m})}{2}2\pi(60/{\rm s})\frac{(162\,{\rm V})}{(4.8\times10^{-3}{\rm m})}, \\ &= 2.27\times10^{-12}{\rm T}. \end{aligned}$$

E38-10 (a) Eq. 33-13 from page 764 and Eq. 33-34 from page 762.

(b) Eq. 27-11 from page 618 and the equation from Ex. 27-25 on page 630.

(c) The equations from Ex. 38-6 on page 876.

(d) Eqs. 34-16 and 34-17 from page 785.

$\boxed{\textbf{E38-11}}$ (a) Consider the path $abefa$. The closed line integral consists of *two* parts: $b \to e$ and $e \to f \to a \to b$. Then

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d\Phi}{dt}$$

can be written as

$$\int_{b\to e}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} + \int_{e\to f\to a\to b}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\Phi_{abef}.$$

Now consider the path $bcdeb$. The closed line integral consists of *two* parts: $b \to c \to d \to e$ and $e \to b$. Then

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d\Phi}{dt}$$

can be written as

$$\int_{b\to c\to d\to e}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} + \int_{e\to b}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\Phi_{bcde}.$$

These two expressions can be added together, and since

$$\int_{e\to b} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_{b\to e} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

we get

$$\int_{e\to f\to a\to b} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} + \int_{b\to c\to d\to e} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\left(\Phi_{abef} + \Phi_{bcde}\right).$$

The left hand side of this is just the line integral over the closed path $efadcde$; the right hand side is the net change in flux through the two surfaces. Then we can simplify this expression as

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi}{dt}.$$

(b) Do everything above again, except substitute B for E.

(c) If the equations were not self consistent we would arrive at different values of E and B depending on how we defined our surfaces. This multi-valued result would be quite unphysical.

E38-12 (a) Consider the part on the left. It has a shared surface s, and the other surfaces l. Applying Eq. I,

$$q_l/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_s \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_l \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

Note that $d\vec{\mathbf{A}}$ is directed to the right on the shared surface.

Consider the part on the right. It has a shared surface s, and the other surfaces r. Applying Eq. I,

$$q_r/\epsilon_0 = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_s \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_r \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

Note that $d\vec{\mathbf{A}}$ is directed to the left on the shared surface.

Adding these two expressions will result in a canceling out of the part

$$\int_s \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

since one is oriented opposite the other. We are left with

$$\frac{q_r + q_l}{\epsilon_0} = \int_r \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_l \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

E38-13

E38-14 (a) Electric dipole is because the charges are separating like an electric dipole. Magnetic dipole because the current loop acts like a magnetic dipole.

$\boxed{\textbf{E38-15}}$ A series LC circuit will oscillate naturally at a frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

We will need to combine this with $v = f\lambda$, where $v = c$ is the speed of EM waves.

We want to know the inductance required to produce an EM wave of wavelength $\lambda = 550\times10^{-9}$m, so

$$L = \frac{\lambda^2}{4\pi^2 c^2 C} = \frac{(550\times10^{-9}\text{m})^2}{4\pi^2(3.00\times10^8\text{m/s})^2(17\times10^{-12}\,\text{F})} = 5.01\times10^{-21}\,\text{H}.$$

This is a small inductance!

E38-16 (a) $B = E/c$, and B must be pointing in the negative y direction in order that the wave be propagating in the positive x direction. Then $B_x = B_z = 0$, and

$$B_y = -E_z/c = -(2.34\times 10^{-4}\text{V/m})/(3.00\times 10^8\text{m/s}) = (-7.80\times 10^{-13}\text{T})\sin k(x - ct).$$

(b) $\lambda = 2\pi/k = 2\pi/(9.72\times 10^6/\text{m}) = 6.46\times 10^{-7}\text{m}$.

$\boxed{\textbf{E38-17}}$ The electric and magnetic field of an electromagnetic wave are related by Eqs. 38-15 and 38-16,

$$B = \frac{E}{c} = \frac{(321\mu\text{V/m})}{(3.00\times 10^8\text{m/s})} = 1.07\text{ pT}.$$

E38-18 Take the partial of Eq. 38-14 with respect to x,

$$\begin{aligned} \frac{\partial}{\partial x}\frac{\partial E}{\partial x} &= -\frac{\partial}{\partial x}\frac{\partial B}{\partial t}, \\ \frac{\partial^2 E}{\partial x^2} &= -\frac{\partial^2 B}{\partial x \partial t}. \end{aligned}$$

Take the partial of Eq. 38-17 with respect to t,

$$\begin{aligned} -\frac{\partial}{\partial t}\frac{\partial B}{\partial x} &= \mu_0\epsilon_0\frac{\partial}{\partial t}\frac{\partial E}{\partial t}, \\ -\frac{\partial^2 B}{\partial t \partial x} &= \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}. \end{aligned}$$

Equate, and let $\mu_0\epsilon_0 = 1/c^2$, then

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}.$$

Repeat, except now take the partial of Eq. 38-14 with respect to t, and then take the partial of Eq. 38-17 with respect to x.

E38-19 (a) Since $\sin(kx - \omega t)$ is of the form $f(kx \pm \omega t)$, then we only need do part (b).

(b) The constant E_m drops out of the wave equation, so we need only concern ourselves with $f(kx \pm \omega t)$. Letting $g = kx \pm \omega t$,

$$\begin{aligned} \frac{\partial^2 f}{\partial t^2} &= c^2\frac{\partial^2 f}{\partial x^2}, \\ \frac{\partial^2 f}{\partial g^2}\left(\frac{\partial g}{\partial t}\right)^2 &= c^2\frac{\partial^2 f}{\partial g^2}\left(\frac{\partial g}{\partial x}\right)^2, \\ \frac{\partial g}{\partial t} &= c\frac{\partial g}{\partial x}, \\ \omega &= ck. \end{aligned}$$

E38-20 Use the right hand rule.

E38-21 $U = Pt = (100\times 10^{12}\text{W})(1.0\times 10^{-9}\text{s}) = 1.0\times 10^5\text{J}$.

E38-22 $E = Bc = (28\times 10^{-9}\text{T})(3.0\times 10^8\text{m/s}) = 8.4\,\text{V/m}$. It is in the positive x direction.

$\boxed{\textbf{E38-23}}$ Intensity is given by Eq. 38-28, which is simply an expression of power divided by surface area. To find the intensity of the TV signal at α-Centauri we need to find the distance in meters;

$$r = (4.30 \text{ light-years})(3.00\times10^8\,\text{m/s})(3.15\times10^7 \text{ s/year}) = 4.06 \times 10^{16}\,\text{m}.$$

The intensity of the signal when it has arrived at out nearest neighbor is then

$$I = \frac{P}{4\pi r^2} = \frac{(960 \text{ kW})}{4\pi(4.06 \times 10^{16}\,\text{m})^2} = 4.63 \times 10^{-29} \text{ W/m}^2$$

E38-24 (a) From Eq. 38-22, $S = cB^2/\mu_0$. $B = B_\text{m}\sin\omega t$. The time average is defined as

$$\frac{1}{T}\int_0^T S\,dt = \frac{c{B_\text{m}}^2}{\mu_0 T}\int_0^T \cos^2\omega t\,dt = \frac{c{B_\text{m}}^2}{2\mu_0}.$$

(b) $S_\text{av} = (3.0\times10^8\text{m/s})(1.0\times10^{-4}\text{T})^2/2(4\pi\times10^{-7}\text{H/m}) = 1.2\times10^6\text{W/m}^2$.

E38-25 $I = P/4\pi r^2$, so

$$r = \sqrt{P/4\pi I} = \sqrt{(1.0\times10^3\text{W})/4\pi(130\,\text{W/m}^2)} = 0.78\,\text{m}.$$

E38-26 $u_E = \epsilon_0 E^2/2 = \epsilon_0(cB)^2/2 = B^2/2\mu_0 = u_B$.

$\boxed{\textbf{E38-27}}$ (a) Intensity is related to distance by Eq. 38-28. If r_1 is the original distance from the street lamp and I_1 the intensity at that distance, then

$$I_1 = \frac{P}{4\pi r_1^2}.$$

There is a similar expression for the closer distance $r_2 = r_1 - 162\,\text{m}$ and the intensity at that distance $I_2 = 1.50 I_1$. We can combine the two expression for intensity,

$$\begin{aligned} I_2 &= 1.50 I_1, \\ \frac{P}{4\pi r_2^2} &= 1.50\frac{P}{4\pi r_1^2}, \\ r_1^2 &= 1.50 r_2^2, \\ r_1 &= \sqrt{1.50}\,(r_1 - 162\,\text{m}). \end{aligned}$$

The last line is easy enough to solve and we find $r_1 = 883\,\text{m}$.

(b) No, we can't find the power output from the lamp, because we were never provided with an absolute intensity reference.

E38-28 (a) $E_\text{m} = \sqrt{2\mu_0 c I}$, so

$$E_\text{m} = \sqrt{2(4\pi\times10^{-7}\text{H/m})(3.00\times10^8\text{m/s})(1.38\times10^3\text{W/m}^2)} = 1.02\times10^3\text{V/m}.$$

(b) $B_\text{m} = E_\text{m}/c = (1.02\times10^3\text{V/m})/(3.00\times10^8\text{m/s}) = 3.40\times10^{-6}\text{T}$.

E38-29 (a) $B_\text{m} = E_\text{m}/c = (1.96\,\text{V/m})/(3.00\times10^8\text{m/s}) = 6.53\times10^{-9}\text{T}$.

(b) $I = {E_\text{m}}^2/2\mu_0 c = (1.96\,\text{V})^2/2(4\pi\times10^{-7}\text{H/m})(3.00\times10^8\text{m/s}) = 5.10\times10^{-3}\text{W/m}^2$.

(c) $P = 4\pi r^2 I = 4\pi(11.2\,\text{m})^2(5.10\times10^{-3}\text{W/m}^2) = 8.04\,\text{W}$.

E38-30 (a) The intensity is

$$I = \frac{P}{A} = \frac{(1\times10^{-12}\mathrm{W})}{4\pi(6.37\times10^{6}\mathrm{m})^2} = 1.96\times10^{-27}\mathrm{W/m^2}.$$

The power received by the Arecibo antenna is

$$P = IA = (1.96\times10^{-27}\mathrm{W/m^2})\pi(305\,\mathrm{m})^2/4 = 1.4\times10^{-22}\mathrm{W}.$$

(b) The power of the transmitter at the center of the galaxy would be

$$P = IA = (1.96\times10^{-27}\mathrm{W})\pi(2.3\times10^{4}\mathrm{ly})^2(9.46\times10^{15}\mathrm{m/ly})^2 = 2.9\times10^{14}\mathrm{W}.$$

$\boxed{\textbf{E38-31}}$ (a) The electric field amplitude is related to the intensity by Eq. 38-26,

$$I = \frac{E_\mathrm{m}^2}{2\mu_0 c},$$

or

$$E_\mathrm{m} = \sqrt{2\mu_0 cI} = \sqrt{2(4\pi\times10^{-7}\mathrm{H/m})(3.00\times10^{8}\mathrm{m/s})(7.83\mu\mathrm{W/m^2})} = 7.68\times10^{-2}\,\mathrm{V/m}.$$

(b) The magnetic field amplitude is given by

$$B_\mathrm{m} = \frac{E_\mathrm{m}}{c} = \frac{(7.68\times10^{-2}\,\mathrm{V/m})}{(3.00\times10^{8}\mathrm{m/s})} = 2.56\times10^{-10}\,\mathrm{T}$$

(c) The power radiated by the transmitter can be found from Eq. 38-28,

$$P = 4\pi r^2 I = 4\pi(11.3\text{ km})^2(7.83\mu\mathrm{W/m^2}) = 12.6\text{ kW}.$$

E38-32 (a) The power incident on (and then reflected by) the target craft is $P_1 = I_1 A = P_0 A/2\pi r^2$. The intensity of the reflected beam is $I_2 = P_1/2\pi r^2 = P_0 A/4\pi^2 r^4$. Then

$$I_2 = (183\times10^{3}\mathrm{W})(0.222\,\mathrm{m^2})/4\pi^2(88.2\times10^{3}\mathrm{m})^4 = 1.70\times10^{-17}\mathrm{W/m^2}.$$

(b) Use Eq. 38-26:

$$E_\mathrm{m} = \sqrt{2\mu_0 cI} = \sqrt{2(4\pi\times10^{-7}\mathrm{H/m})(3.00\times10^{8}\mathrm{m/s})(1.70\times10^{-17}\mathrm{W/m^2})} = 1.13\times10^{-7}\,\mathrm{V/m}.$$

(c) $B_\mathrm{rms} = E_\mathrm{m}/\sqrt{2}c = (1.13\times10^{-7}\,\mathrm{V/m})/\sqrt{2}(3.00\times10^{8}\mathrm{m/s}) = 2.66\times10^{-16}\mathrm{T}$.

$\boxed{\textbf{E38-33}}$ Radiation pressure for absorption is given by Eq. 38-34, but we need to find the energy absorbed before we can apply that. We are given an intensity, a surface area, and a time, so

$$\Delta U = (1.1\times10^{3}\mathrm{W/m^2})(1.3\,\mathrm{m^2})(9.0\times10^{3}\mathrm{s}) = 1.3\times10^{7}\mathrm{J}.$$

The momentum delivered is

$$p = (\Delta U)/c = (1.3\times10^{7}\mathrm{J})/(3.00\times10^{8}\mathrm{m/s}) = 4.3\times10^{-2}\mathrm{kg\cdot m/s}.$$

E38-34 (a) $F/A = I/c = (1.38\times10^{3}\mathrm{W/m^2})/(3.00\times10^{8}\mathrm{m/s}) = 4.60\times10^{-6}\mathrm{Pa}$.
(b) $(4.60\times10^{-6}\mathrm{Pa})/(101\times10^{5}\mathrm{Pa}) = 4.55\times10^{-11}$.

E38-35 $F/A = 2P/Ac = 2(1.5\times10^{9}\mathrm{W})/(1.3\times10^{-6}\mathrm{m^2})(3.0\times10^{8}\mathrm{m/s}) = 7.7\times10^{6}\mathrm{Pa}$.

E38-36 $F/A = P/4\pi r^2 c$, so

$$F/A = (500\,\mathrm{W})/4\pi(1.50\,\mathrm{m})^2(3.00\times 10^8\mathrm{m/s}) = 5.89\times 10^{-8}\mathrm{Pa}.$$

E38-37 (a) $F = IA/c$, so

$$F = \frac{(1.38\times 10^3\mathrm{W/m^2})\pi(6.37\times 10^6\mathrm{m})^2}{(3.00\times 10^8\mathrm{m/s})} = 5.86\times 10^8\mathrm{N}.$$

E38-38 (a) Assuming MKSA, the units are

$$\frac{\mathrm{m}}{\mathrm{s}}\frac{\mathrm{F}}{\mathrm{m}}\frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{N}}{\mathrm{Am}} = \frac{\mathrm{m}}{\mathrm{s}}\frac{\mathrm{C}}{\mathrm{Vm}}\frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{sN}}{\mathrm{Cm}} = \frac{\mathrm{Ns}}{\mathrm{m^2 s}}.$$

(b) Assuming MKSA, the units are

$$\frac{\mathrm{A^2}}{\mathrm{N}}\frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{N}}{\mathrm{Am}} = \frac{\mathrm{A^2}}{\mathrm{N}}\frac{\mathrm{J}}{\mathrm{Cm}}\frac{\mathrm{N}}{\mathrm{Am}} = \frac{1}{\mathrm{sm}}\frac{\mathrm{J}}{\mathrm{m}} = \frac{\mathrm{J}}{\mathrm{m^2 s}}.$$

$\boxed{\textbf{E38-39}}$ We can treat the object as having two surfaces, one completely reflecting and the other completely absorbing. If the entire surface has an area A then the absorbing part has an area fA while the reflecting part has area $(1-f)A$. The average force is then the sum of the force on each part,

$$F_{\mathrm{av}} = \frac{I}{c}fA + \frac{2I}{c}(1-f)A,$$

which can be written in terms of pressure as

$$\frac{F_{\mathrm{av}}}{A} = \frac{I}{c}(2-f).$$

E38-40 We can treat the object as having two surfaces, one completely reflecting and the other completely absorbing. If the entire surface has an area A then the absorbing part has an area fA while the reflecting part has area $(1-f)A$. The average force is then the sum of the force on each part,

$$F_{\mathrm{av}} = \frac{I}{c}fA + \frac{2I}{c}(1-f)A,$$

which can be written in terms of pressure as

$$\frac{F_{\mathrm{av}}}{A} = \frac{I}{c}(2-f).$$

The intensity I is that of the incident beam; the reflected beam will have an intensity $(1-f)I$. Each beam will contribute to the energy density— I/c and $(1-f)I/c$, respectively. Add these two energy densities to get the net energy density outside the surface. The result is $(2-f)I/c$, which is the left hand side of the pressure relation above.

E38-41 The bullet density is $\rho = Nm/V$. Let $V = Ah$; the kinetic energy density is $K/V = \frac{1}{2}Nmv^2/Ah$. h/v, however, is the time taken for N balls to strike the surface, so that

$$P = \frac{F}{A} = \frac{Nmv}{At} = \frac{Nmv^2}{Ah} = \frac{2K}{V}.$$

E38-42 $F = IA/c$; $P = IA$; $a = F/m$; and $v = at$. Combine:

$$v = Pt/mc = (10\times10^3\mathrm{W})(86400\,\mathrm{s})/(1500\,\mathrm{kg})(3\times10^8\mathrm{m/s}) = 1.9\times10^{-3}\mathrm{m/s}.$$

E38-43 The force of radiation on the bottom of the cylinder is $F = 2IA/c$. The force of gravity on the cylinder is

$$W = mg = \rho HAg.$$

Equating, $2I/c = \rho Hg$. The intensity of the beam is given by $I = 4P/\pi d^2$. Solving for H,

$$H = \frac{8P}{\pi c\rho g d^2} = \frac{8(4.6\,\mathrm{W})}{\pi(3.0\times10^8\mathrm{m/s})(1200\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(2.6\times10^{-3}\mathrm{m})^2} = 4.9\times10^{-7}\mathrm{m}.$$

E38-44 $F = 2IA/c$. The value for I is in Ex. 38-37, among other places. Then

$$F = (1.38\times10^3\mathrm{W/m^2})(3.1\times10^6\mathrm{m^2})/(3.00\times10^8\mathrm{m/s}) = 29\,\mathrm{N}.$$

P38-1 For the two outer circles use Eq. 33-13. For the inner circle use $E = V/d$, $Q = CV$, $C = \epsilon_0 A/d$, and $i = dQ/dt$. Then

$$i = \frac{dQ}{dt} = \frac{\epsilon_0 A}{d}\frac{dV}{dt} = \epsilon_0 A\frac{dE}{dt}.$$

The change in flux is $d\Phi_E/dt = A\,dE/dt$. Then

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} = \mu_0\epsilon_0\frac{d\Phi_E}{dt} = \mu_0 i,$$

so $B = \mu_0 i/2\pi r$.

P38-2 (a) $i_\mathrm{d} = i$. Assuming $\Delta V = (174\times10^3\mathrm{V})\sin\omega t$, then $q = C\Delta V$ and $i = dq/dt = Cd(\Delta V)/dt$. Combine, and use $\omega = 2\pi(50.0/\mathrm{s})$,

$$i_\mathrm{d} = (100\times10^{-12}\mathrm{F})(174\times10^3\mathrm{V})2\pi(50.0/\mathrm{s}) = 5.47\times10^{-3}\mathrm{A}.$$

P38-3 (a) $i = i_\mathrm{d} = 7.63\mu\mathrm{A}$.

(b) $d\Phi_E/dt = i_\mathrm{d}/\epsilon_0 = (7.63\mu\mathrm{A})/(8.85\times10^{-12}\mathrm{F/m}) = 8.62\times10^5\mathrm{V/m}$.

(c) $i = dq/dt = Cd(\Delta V)/dt$; $C = \epsilon_0 A/d$; $[d(\Delta V)/dt]_\mathrm{m} = \mathcal{E}_\mathrm{m}\omega$. Combine, and

$$d = \frac{\epsilon_0 A}{C} = \frac{\epsilon_0 A\mathcal{E}_\mathrm{m}\omega}{i} = \frac{(8.85\times10^{-12}\mathrm{F/m})\pi(0.182\,\mathrm{m})^2(225\,\mathrm{V})(128\text{ rad/s})}{(7.63\mu\mathrm{A})} = 3.48\times10^{-3}\mathrm{m}.$$

P38-4 (a) $q = \int i\,dt = \alpha\int t\,dt = \alpha t^2/2$.

(b) $E = \sigma/\epsilon_0 = q/\epsilon_0 A = \alpha t^2/2\pi R^2\epsilon_0$.

(d) $2\pi rB = \mu_0\epsilon_0\pi r^2 dE/dt$, so

$$B = \mu_0 r(dE/dt)/2 = \mu_0\alpha rt/2\pi R^2.$$

(e) Check Exercise 38-10!

P38-5 (a) $\vec{\mathbf{E}} = E\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = B\hat{\mathbf{k}}$. Then $\vec{\mathbf{S}} = \vec{\mathbf{E}}\times\vec{\mathbf{B}}/\mu_0$, or

$$\vec{\mathbf{S}} = -EB/mu_0\,\hat{\mathbf{i}}.$$

Energy only passes through the yz faces; it goes in one face and out the other. The rate is $P = SA = EBa^2/mu_0$.

(b) The net change is zero.

P38-6 (a) For a sinusoidal time dependence $|dE/dt|_{\mathrm{m}} = \omega E_{\mathrm{m}} = 2\pi f E_{\mathrm{m}}$. Then

$$|dE/dt|_{\mathrm{m}} = 2\pi(2.4\times10^9/\mathrm{s})(13\times10^3\mathrm{V/m}) = 1.96\times10^{14}\mathrm{V/m\cdot s}.$$

(b) Using the result of part (b) of Sample Problem 38-1,

$$B = \frac{1}{2}(4\pi\times10^{-7}\mathrm{H/m})(8.9\times10^{-12}\mathrm{F/m})(2.4\times10^{-2}\mathrm{m})\frac{1}{2}(1.96\times10^{14}\mathrm{V/m\cdot s}) = 1.3\times10^{-5}\mathrm{T}.$$

$\boxed{\textbf{P38-7}}$ Look back to Chapter 14 for a discussion on the elliptic orbit. On page 312 it is pointed out that the closest distance to the sun is $R_{\mathrm{p}} = a(1-e)$ while the farthest distance is $R_{\mathrm{a}} = a(1+e)$, where a is the semi-major axis and e the eccentricity.

The fractional variation in intensity is

$$\begin{aligned}
\frac{\Delta I}{I} &\approx \frac{I_{\mathrm{p}} - I_{\mathrm{a}}}{I_{\mathrm{a}}}, \\
&= \frac{I_{\mathrm{p}}}{I_{\mathrm{a}}} - 1, \\
&= \frac{{R_{\mathrm{a}}}^2}{{R_{\mathrm{p}}}^2} - 1, \\
&= \frac{(1+e)^2}{(1-e)^2} - 1.
\end{aligned}$$

We need to expand this expression for small e using $(1+e)^2 \approx 1+2e$, and $(1-e)^{-2} \approx 1+2e$, and finally $(1+2e)^2 \approx 1+4e$. Combining,

$$\frac{\Delta I}{I} \approx (1+2e)^2 - 1 \approx 4e.$$

P38-8 The beam radius grows as $r = (0.440\,\mu\mathrm{rad})R$, where R is the distance from the origin. The beam intensity is

$$I = \frac{P}{\pi r^2} = \frac{(3850\,\mathrm{W})}{\pi(0.440\,\mu\mathrm{rad})^2(3.82\times10^8\mathrm{m})^2} = 4.3\times10^{-2}\mathrm{W}.$$

$\boxed{\textbf{P38-9}}$ Eq. 38-14 requires

$$\begin{aligned}
\frac{\partial E}{\partial x} &= -\frac{\partial B}{\partial t}, \\
E_{\mathrm{m}}k\cos kx\sin\omega t &= B_{\mathrm{m}}\omega\cos kx\sin\omega t, \\
E_{\mathrm{m}}k &= B_{\mathrm{m}}\omega.
\end{aligned}$$

Eq. 38-17 requires

$$\begin{aligned}
\mu_0\epsilon_0\frac{\partial E}{\partial t} &= -\frac{\partial B}{\partial x}, \\
\mu_0\epsilon_0 E_{\mathrm{m}}\omega\sin kx\cos\omega t &= B_{\mathrm{m}}k\sin kx\cos\omega t, \\
\mu_0\epsilon_0 E_{\mathrm{m}}\omega &= B_{\mathrm{m}}k.
\end{aligned}$$

Dividing one expression by the other,

$$\mu_0\epsilon_0 k^2 = \omega^2,$$

or

$$\frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

.

Not only that, but $E_\mathrm{m} = cB_\mathrm{m}$. You've seen an expression similar to this before, and you'll see expressions similar to it again.

(b) We'll assume that Eq. 38-21 is applicable here. Then

$$\begin{aligned} S &= \frac{1}{\mu_0} = \frac{E_\mathrm{m}B_\mathrm{m}}{\mu_0} \sin kx \sin \omega t \cos kx \cos \omega t, \\ &= \frac{E_\mathrm{m}^2}{4\mu_0 c} \sin 2kx \sin 2\omega t \end{aligned}$$

is the magnitude of the instantaneous Poynting vector.

(c) The time averaged power flow across any surface is the value of

$$\frac{1}{T}\int_0^T \int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}}\, dt,$$

where T is the period of the oscillation. We'll just gloss over any concerns about direction, and assume that the $\vec{\mathbf{S}}$ will be constant in direction so that we will, at most, need to concern ourselves about a constant factor $\cos\theta$. We can then deal with a scalar, instead of vector, integral, and we can integrate it in any order we want. We want to do the t integration first, because an integral over $\sin \omega t$ for a period $T = 2\pi/\omega$ is zero. Then we are done!

(d) There is no energy flow; the energy remains inside the container.

P38-10 (a) The electric field is parallel to the wire and given by

$$E = V/d = iR/d = (25.0\,\mathrm{A})(1.00\,\Omega/300\,\mathrm{m}) = 8.33\times 10^{-2}\mathrm{V/m}$$

(b) The magnetic field is in rings around the wire. Using Eq. 33-13,

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi\times 10^{-7}\mathrm{H/m})(25\,\mathrm{A})}{2\pi(1.24\times 10^{-3}\mathrm{m})} = 4.03\times 10^{-3}\mathrm{T}.$$

(c) $S = EB/\mu_0$, so

$$S = (8.33\times 10^{-2}\mathrm{V/m})(4.03\times 10^{-3}\mathrm{T})/(4\pi\times 10^{-7}\mathrm{H/m}) = 267\,\mathrm{W/m^2}.$$

P38-11 (a) We've already calculated B previously. It is

$$B = \frac{\mu_0 i}{2\pi r} \text{ where } i = \frac{\mathcal{E}}{R}.$$

The electric field of a long straight wire has the form $E = k/r$, where k is some constant. But

$$\Delta V = -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_a^b E\, dr = -k \ln(b/a).$$

In this problem the inner conductor is at the higher potential, so

$$k = \frac{-\Delta V}{\ln(b/a)} = \frac{\mathcal{E}}{\ln(b/a)},$$

and then the electric field is

$$E = \frac{\mathcal{E}}{r\ln(b/a)}.$$

This is also a vector field, and if $\mathcal{E}$ is positive the electric field points radially out from the central conductor.

(b) The Poynting vector is

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}};$$

$\vec{\mathbf{E}}$ is radial while $\vec{\mathbf{B}}$ is circular, so they are perpendicular. Assuming that $\mathcal{E}$ is positive the direction of $\vec{\mathbf{S}}$ is away from the battery. Switching the sign of $\mathcal{E}$ (connecting the battery in reverse) will flip the direction of both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$, so $\vec{\mathbf{S}}$ will pick up *two* negative signs and therefore *still* point away from the battery.

The magnitude is

$$S = \frac{EB}{\mu_0} = \frac{\mathcal{E}^2}{2\pi R\ln(b/a)r^2}$$

(c) We want to evaluate a surface integral in polar coordinates and so $dA = (dr)(rd\theta)$. We have already established that $\vec{\mathbf{S}}$ is pointing away from the battery parallel to the central axis. Then we can integrate

$$\begin{aligned} P &= \int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}} = \int S\,dA, \\ &= \int_a^b \int_0^{2\pi} \frac{\mathcal{E}^2}{2\pi R\ln(b/a)r^2} d\theta\, r\, dr, \\ &= \int_a^b \frac{\mathcal{E}^2}{R\ln(b/a)r} dr, \\ &= \frac{\mathcal{E}^2}{R}. \end{aligned}$$

(d) Read part (b) above.

P38-12 (a) $\vec{\mathbf{B}}$ is oriented as rings around the cylinder. If the thumb is in the direction of current then the fingers of the right hand grip ion the direction of the magnetic field lines. $\vec{\mathbf{E}}$ is directed parallel to the wire in the direction of the current. $\vec{\mathbf{S}}$ is found from the cross product of these two, and must be pointing radially inward.

(b) The magnetic field on the surface is given by Eq. 33-13:

$$B = \mu_0 i/2\pi a.$$

The electric field on the surface is given by

$$E = V/l = iR/l$$

Then S has magnitude

$$S = EB/\mu_0 = \frac{i}{2\pi a}\frac{iR}{l} = \frac{i^2R}{2\pi al}.$$

$\int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}}$ is only evaluated on the surface of the cylinder, not the end caps. $\vec{\mathbf{S}}$ is everywhere parallel to $d\vec{\mathbf{A}}$, so the dot product reduces to $S\,dA$; S is uniform, so it can be brought out of the integral; $\int dA = 2\pi al$ on the surface.

Hence,

$$\int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}} = i^2R,$$

as it should.

P38-13 (a) $f = vlambda = (3.00\times10^8\text{m/s})/(3.18\,\text{m}) = 9.43\times10^7$ Hz.

(b) $\vec{\mathbf{B}}$must be directed along the z axis. The magnitude is

$$B = E/c = (288\,\text{V/m})/(3.00\times10^8\text{m/s}) = 9.6\times10^{-7}\text{T}.$$

(c) $k = 2\pi/\lambda = 2\pi/(3.18\,\text{m}) = 1.98/\text{m}$ while $\omega = 2\pi f$, so

$$\omega = 2\pi(9.43\times10^7\text{Hz}) = 5.93\times10^8 \text{ rad/s}.$$

(d) $I = E_\text{m}B_\text{m}/2\mu_0$, so

$$I = \frac{(288\,\text{V})(9.6\times10^{-7}\text{T})}{2(4\pi\times10^{-7}\text{H/m})} = 110\,\text{W}.$$

(e) $P = I/c = (110\,\text{W})/(3.00\times10^8\text{m/s}) = 3.67\times10^{-7}\text{Pa}.$

P38-14 (a) $\vec{\mathbf{B}}$ is oriented as rings around the cylinder. If the thumb is in the direction of current then the fingers of the right hand grip ion the direction of the magnetic field lines. $\vec{\mathbf{E}}$ is directed parallel to the wire in the direction of the current. $\vec{\mathbf{S}}$ is found from the cross product of these two, and must be pointing radially inward.

(b) The magnitude of the electric field is

$$E = \frac{V}{d} = \frac{Q}{Cd} = \frac{Q}{\epsilon_0 A} = \frac{it}{\epsilon_0 A}.$$

The magnitude of the magnetic field on the outside of the plates is given by Sample Problem 38-1,

$$B = \frac{\mu_0\epsilon_0 R}{2}\frac{dE}{dt} = \frac{\mu_0\epsilon_0 iR}{2\epsilon_0 A} = \frac{\mu_0\epsilon_0 R}{2t}E.$$

$\vec{\mathbf{S}}$ has magnitude

$$S = \frac{EB}{\mu_0} = \frac{\epsilon_0 R}{2t}E^2.$$

Integrating,

$$\int \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = \frac{\epsilon_0 R}{2t}E^2 2\pi R d = Ad\frac{\epsilon_0 E^2}{t}.$$

But E is linear in t, so $d(E^2)/dt = 2E^2/t$; and then

$$\int \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = Ad\frac{d}{dt}\left(\frac{1}{2}\epsilon_0 E^2\right).$$

P38-15 (a) $I = P/A = (5.00\times10^{-3}\text{W})/\pi(1.05)^2(633\times10^{-9}\text{m})^2 = 3.6\times10^9\text{W/m}^2.$

(b) $p = I/c = (3.6\times10^9\text{W/m}^2)/(3.00\times10^8\text{m/s}) = 12\,\text{Pa}$

(c) $F = pA = P/c = (5.00\times10^{-3}\text{W})/(3.00\times10^8\text{m/s}) = 1.67\times10^{-11}\text{N}.$

(d) $a = F/m = F/\rho V$, so

$$a = \frac{(1.67\times10^{-11}\text{N})}{4(4880\,\text{kg/m}^3)(1.05)^3(633\times10^{-9})^3/3} = 2.9\times10^3\text{m/s}^2.$$

P38-16 The force from the sun is $F = GMm/r^2$. The force from radiation pressure is

$$F = \frac{2IA}{c} = \frac{2PA}{4\pi r^2 c}.$$

Equating,

$$A = \frac{4\pi GMm}{2P/c},$$

so

$$A = \frac{4\pi(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(1.99\times10^{30}\mathrm{kg})(1650\,\mathrm{kg})}{2(3.9\times10^{26}\mathrm{W})/(3.0\times10^{8}\mathrm{m/s})} = 1.06\times10^{6}\mathrm{m}^2.$$

That's about one square kilometer.

E39-1 Both scales are logarithmic; choose any data point from the right hand side such as

$$c = f\lambda \approx (1\text{ Hz})(3\times10^8\text{m}) = 3\times10^8\text{m/s},$$

and another from the left hand side such as

$$c = f\lambda \approx (1\times10^{21}\text{ Hz})(3\times10^{-13}\text{m}) = 3\times10^8\text{m/s}.$$

E39-2 (a) $f = v/\lambda = (3.0\times10^8\text{m/s})/(1.0\times10^4)(6.37\times10^6\text{m}) = 4.7\times10^{-3}$ Hz. If we assume that this is the data transmission rate in bits per second (a generous assumption), then it would take 140 days to download a web-page which would take only 1 second on a 56K modem!

(b) $T = 1/f = 212\,\text{s} = 3.5$ min.

$\boxed{\textbf{E39-3}}$ (a) Apply $v = f\lambda$. Then

$$f = (3.0\times10^8\text{m/s})/(0.067\times10^{-15}\text{m}) = 4.5\times10^{24}\text{ Hz}.$$

(b) $\lambda = (3.0\times10^8\text{m/s})/(30\text{ Hz}) = 1.0\times10^7\text{m}$.

E39-4 Don't simply take reciprocal of linewidth! $f = c/\lambda$, so $\delta f = (-c/\lambda^2)\delta\lambda$. Ignore the negative, and

$$\delta f = (3.00\times10^8\text{m/s})(0.010\times10^{-9}\text{m})/(632.8\times10^{-9}\text{m})^2 = 7.5\times10^9\text{ Hz}.$$

$\boxed{\textbf{E39-5}}$ (a) We refer to Fig. 39-6 to answer this question. The limits are approximately 520 nm and 620 nm.

(b) The wavelength for which the eye is most sensitive is 550 nm. This corresponds to to a frequency of

$$f = c/\lambda = (3.00\times10^8\text{ m/s})/(550\times10^{-9}\text{m}) = 5.45\times10^{14}\text{ Hz}.$$

This frequency corresponds to a period of $T = 1/f = 1.83\times10^{-15}$s.

E39-6 $f = c/\lambda$. The number of complete pulses is ft, or

$$ft = ct/\lambda = (3.00\times10^8\text{m/s})(430\times10^{-12}\text{s})/(520\times10^{-9}\text{m}) = 2.48\times10^5.$$

E39-7 (a) $2(4.34\text{ y}) = 8.68$ y.

(b) $2(2.2\times10^6\text{ y}) = 4.4\times10^6$ y.

E39-8 (a) $t = (150\times10^3\text{m})/(3\times10^8\text{m/s}) = 5\times10^{-4}$s.

(b) The distance traveled by the light is $(1.5\times10^{11}\text{m}) + 2(3.8\times10^8\text{m})$, so

$$t = (1.51\times10^{11}\text{m})/(3\times10^8\text{m/s}) = 503\,\text{s}.$$

(c) $t = 2(1.3\times10^{12}\text{m})/(3\times10^8\text{m/s}) = 8670\,\text{s}$.

(d) $1054 - 6500 \approx 5400$ BC.

$\boxed{\textbf{E39-9}}$ This is a question of how much time it takes light to travel 4 cm, because the light traveled from the Earth to the moon, bounced off of the reflector, and then traveled back. The time to travel 4 cm is $\Delta t = (0.04\,\text{m})/(3\times10^8\text{ m/s}) = 0.13$ ns. Note that I interpreted the question differently than the answer in the back of the book.

E39-10 Consider any incoming ray. The path of the ray can be projected onto the xy plane, the xz plane, or the yz plane. If the projected rays is exactly reflected in all three cases then the three dimensional incoming ray will be reflected exactly reversed. But the problem is symmetric, so it is sufficient to show that any plane works.

Now the problem has been reduced to Sample Problem 39-2, so we are done.

$\boxed{\textbf{E39-11}}$ We will choose the mirror to lie in the xy plane at $z = 0$. There is no loss of generality in doing so; we had to define our coordinate system somehow. The choice is convenient in that any normal is then parallel to the z axis. Furthermore, we can arbitrarily define the incident ray to originate at $(0, 0, z_1)$. Lastly, we can rotate the coordinate system about the z axis so that the reflected ray passes through the point $(0, y_3, z_3)$.

The point of reflection for this ray is somewhere on the surface of the mirror, say $(x_2, y_2, 0)$. This distance traveled from the point 1 to the reflection point 2 is

$$d_{12} = \sqrt{(0 - x_2)^2 + (0 - y_2)^2 + (z_1 - 0)^2} = \sqrt{x_2^2 + y_2^2 + z_1^2}$$

and the distance traveled from the reflection point 2 to the final point 3 is

$$d_{23} = \sqrt{(x_2 - 0)^2 + (y_2 - y_3)^2 + (0 - z_3)^2} = \sqrt{x_2^2 + (y_2 - y_3)^2 + z_3^2}.$$

The only point which is free to move is the reflection point, $(x_2, y_2, 0)$, and that point can only move in the xy plane. Fermat's principle states that the reflection point will be such to minimize the total distance,

$$d_{12} + d_{23} = \sqrt{x_2^2 + y_2^2 + z_1^2} + \sqrt{x_2^2 + (y_2 - y_3)^2 + z_3^2}.$$

We do this minimization by taking the partial derivative with respect to both x_2 and y_2. But we can do part by inspection alone. Any non-zero value of x_2 can only *add* to the total distance, regardless of the value of any of the other quantities. Consequently, $x_2 = 0$ is one of the conditions for minimization.

We are done! Although you are invited to finish the minimization process, once we know that $x_2 = 0$ we have that point 1, point 2, and point 3 all lie in the yz plane. The normal is parallel to the z axis, so it also lies in the yz plane. Everything is then in the yz plane.

E39-12 Refer to Page 442 of Volume 1.

E39-13 (a) $\theta_1 = 38°$.

(b) $(1.58)\sin(38°) = (1.22)\sin\theta_2$. Then $\theta_2 = \arcsin(0.797) = 52.9°$.

E39-14 $n_{\rm g} = n_{\rm v}\sin\theta_1/\sin\theta_2 = (1.00)\sin(32.5°)/\sin(21.0°) = 1.50$.

E39-15 $n = c/v = (3.00\times10^8\text{m/s})/(1.92\times10^8\text{m/s}) = 1.56$.

E39-16 $v = c/n = (3.00\times10^8\text{m/s})/(1.46) = 2.05\times10^8\text{m/s}$.

$\boxed{\textbf{E39-17}}$ The speed of light in a substance with index of refraction n is given by $v = c/n$. An electron will then emit Cerenkov radiation in this particular liquid if the speed exceeds

$$v = c/n = (3.00 \times 10^8\text{ m/s})/(1.54) = 1.95\times10^8\text{ m/s}.$$

E39-18 Since $t = d/v = nd/c$, $\Delta t = \Delta n\, d/c$. Then

$$\Delta t = (1.00029 - 1.00000)(1.61\times10^3\text{m})/(3.00\times10^8\text{m/s}) = 1.56\times10^{-9}\text{s}.$$

$\boxed{\textbf{E39-19}}$ The angle of the refracted ray is $\theta_2 = 90°$, the angle of the incident ray can be found by trigonometry,

$$\tan\theta_1 = \frac{(1.14\,\text{m})}{(0.85\,\text{m})} = 1.34,$$

or $\theta_1 = 53.3°$.

We can use these two angles, along with the index of refraction of air, to find that the index of refraction of the liquid from Eq. 39-4,

$$n_1 = n_2\frac{\sin\theta_2}{\sin\theta_1} = (1.00)\frac{(\sin 90°)}{(\sin 53.3°)} = 1.25.$$

There are no units attached to this quantity.

E39-20 For an equilateral prism $\phi = 60°$. Then

$$n = \frac{\sin[\psi+\phi]/2}{\sin[\phi/2]} = \frac{\sin[(37°)+(60°)]/2}{\sin[(60°)/2]} = 1.5.$$

E39-21

E39-22 $t = d/v$; but $L/d = \cos\theta_2 = \sqrt{1-\sin^2\theta_2}$ and $v = c/n$. Combining,

$$t = \frac{nL}{c\sqrt{1-\sin^2\theta_2}} = \frac{n^2L}{c\sqrt{n^2-\sin^2\theta_1}} = \frac{(1.63)^2(0.547\,\text{m})}{(3\times10^8\text{m/s})\sqrt{(1.63^2)-\sin^2(24°)}} = 3.07\times10^{-9}\text{s}.$$

E39-23 The ray of light from the top of the smokestack to the life ring is θ_1, where $\tan\theta_1 = L/h$ with h the height and L the distance of the smokestack.

Snell's law gives $n_1\sin\theta_1 = n_2\sin\theta_2$, so

$$\theta_1 = \arcsin[(1.33)\sin(27°)/(1.00)] = 37.1°.$$

Then $L = h\tan\theta_1 = (98\,\text{m})\tan(37.1°) = 74\,\text{m}$.

E39-24 The length of the shadow on the surface of the water is

$$x_1 = (0.64\,\text{m})/\tan(55°) = 0.448\,\text{m}.$$

The ray of light which forms the "end" of the shadow has an angle of incidence of 35°, so the ray travels into the water at an angle of

$$\theta_2 = \arcsin\left(\frac{(1.00)}{(1.33)}\sin(35°)\right) = 25.5°.$$

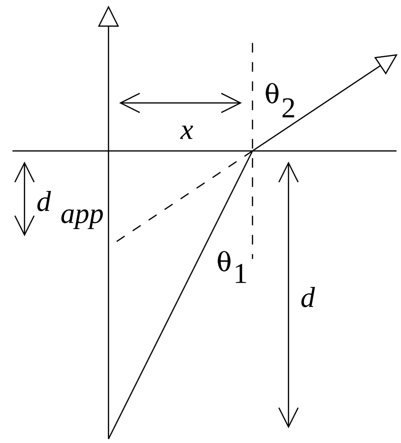
The ray travels an additional distance

$$x_2 = (2.00\,\text{m} - 0.64\,\text{m})/\tan(90° - 25.5°) = 0.649\,\text{m}$$

The total length of the shadow is

$$(0.448\,\text{m}) + (0.649\,\text{m}) = 1.10\,\text{m}.$$

$\boxed{\textbf{E39-25}}$ We'll rely heavily on the figure for our arguments. Let x be the distance between the points on the surface where the vertical ray crosses and the bent ray crosses.



In this exercise we will take advantage of the fact that, for small angles θ, $\sin\theta \approx \tan\theta \approx \theta$ In this approximation Snell's law takes on the particularly simple form $n_1\theta_1 = n_2\theta_2$ The two angles here are conveniently found from the figure,

$$\theta_1 \approx \tan\theta_1 = \frac{x}{d},$$

and

$$\theta_2 \approx \tan\theta_2 = \frac{x}{d_{\text{app}}}.$$

Inserting these two angles into the simplified Snell's law, as well as substituting $n_1 = n$ and $n_2 = 1.0$,

$$\begin{aligned} n_1\theta_1 &= n_2\theta_2, \\ n\frac{x}{d} &= \frac{x}{d_{\text{app}}}, \\ d_{\text{app}} &= \frac{d}{n}. \end{aligned}$$

E39-26 (a) You need to address the issue of total internal reflection to answer this question.

(b) Rearrange

$$n = \frac{\sin[\psi+\phi]/2}{/}\sin[\phi/2]$$

and $\theta = (\psi+\phi)/2$ to get

$$\theta = \arcsin\left(n\sin[\phi/2]\right) = \arcsin\left((1.60)\sin[(60^\circ)/2]\right) = 53.1^\circ.$$

E39-27 Use the results of Ex. 39-35. The apparent thickness of the carbon tetrachloride layer, as viewed by an observer in the water, is

$$d_{\text{c,w}} = n_{\text{w}}d_{\text{c}}/n_{\text{c}} = (1.33)(41\,\text{mm})/(1.46) = 37.5\,\text{mm}.$$

The total "thickness" from the water perspective is then $(37.5\,\text{mm}) + (20\,\text{mm}) = 57.5\,\text{mm}$. The apparent thickness of the entire system as view from the air is then

$$d_{\text{app}} = (57.5\,\text{mm})/(1.33) = 43.2\,\text{mm}.$$

E39-28 (a) Use the results of Ex. 39-35. $d_{\text{app}} = (2.16\,\text{m})/(1.33) = 1.62\,\text{m}$.

(b) Need a diagram here!

E39-29 (a) $\lambda_n = \lambda/n = (612\text{ nm})/(1.51) = 405\text{ nm}$.

(b) $L = nL_n = (1.51)(1.57\text{ pm}) = 2.37\text{ pm}$. There is actually a typo: the "p" in "pm" was supposed to be a μ. This makes a huge difference for part (c)!

E39-30 (a) $f = c/\lambda = (3.00\times10^8\text{m/s})/(589\text{ nm}) = 5.09\times10^{14}\text{Hz}$.

(b) $\lambda_n = \lambda/n = (589\text{ nm})/(1.53) = 385\text{ nm}$.

(c) $v = f\lambda = (5.09\times10^{14}\text{Hz})(385\text{ nm}) = 1.96\times10^8\text{m/s}$.

E39-31 (a) The second derivative of

$$L = \sqrt{a^2 + x^2} + \sqrt{b^2 + (d-x)^2}$$

is

$$\frac{a^2(b^2 + (d-2)^2)^{3/2} + b^2(a^2 + x^2)^{3/2}}{(b^2 + (d-2)^2)^{3/2}(a^2 + x^2)^{3/2}}.$$

This is *always* a positive number, so $dL/dx = 0$ is a minimum.

(a) The second derivative of

$$L = n_1\sqrt{a^2 + x^2} + n_2\sqrt{b^2 + (d-x)^2}$$

is

$$\frac{n_1a^2(b^2 + (d-2)^2)^{3/2} + n_2b^2(a^2 + x^2)^{3/2}}{(b^2 + (d-2)^2)^{3/2}(a^2 + x^2)^{3/2}}.$$

This is *always* a positive number, so $dL/dx = 0$ is a minimum.

E39-32 (a) The angle of incidence on the face ac will be $90^\circ - \phi$. Total internal reflection occurs when $\sin(90^\circ - \phi) > 1/n$, or

$$\phi < 90^\circ - \arcsin[1/(1.52)] = 48.9^\circ.$$

(b) Total internal reflection occurs when $\sin(90^\circ - \phi) > n_{\text{w}}/n$, or

$$\phi < 90^\circ - \arcsin[(1.33)/(1.52)] = 29.0^\circ.$$

$\boxed{\textbf{E39-33}}$ (a) The critical angle is given by Eq. 39-17,

$$\theta_{\text{c}} = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{(1.586)}{(1.667)} = 72.07^\circ.$$

(b) Critical angles only exist when "attempting" to travel from a medium of higher index of refraction to a medium of lower index of refraction; in this case from A to B.

E39-34 If the fire is at the water's edge then the light travels along the surface, entering the water near the fish with an angle of incidence of effectively 90°. Then the angle of refraction in the water is numerically equivalent to a critical angle, so the fish needs to look up at an angle of $\theta = \arcsin(1/1.33) = 49^\circ$ with the vertical. That's the same as 41° with the horizontal.

E39-35 Light can only emerge from the water if it has an angle of incidence less than the critical angle, or

$$\theta < \theta_c = \arcsin 1/n = \arcsin 1/(1.33) = 48.8^\circ.$$

The radius of the circle of light is given by $r/d = \tan\theta_c$, where d is the depth. The diameter is twice this radius, or

$$2(0.82\,\text{m})\tan(48.8^\circ) = 1.87\,\text{m}.$$

E39-36 The refracted angle is given by $n \sin\theta_1 = \sin(39^\circ)$. This ray strikes the left surface with an angle of incidence of $90^\circ - \theta_1$. Total internal reflection occurs when

$$\sin(90^\circ - \theta_1) = 1/n;$$

but $\sin(90^\circ - \theta_1) = \cos\theta_1$, so we can combine and get $\tan\theta = \sin(39^\circ)$ with solution $\theta_1 = 32.2^\circ$. The index of refraction of the glass is then

$$n = \sin(39^\circ)/\sin(32.2) = 1.18.$$

$\boxed{\textbf{E39-37}}$ The light strikes the quartz-air interface from the inside; it is originally "white", so if the reflected ray is to appear "bluish" (reddish) then the refracted ray should have been "reddish" (bluish). Since part of the light undergoes total internal reflection while the other part does not, then the angle of incidence must be approximately equal to the critical angle.

(a) Look at Fig. 39-11, the index of refraction of fused quartz is given as a function of the wavelength. As the wavelength increases the index of refraction decreases. The critical angle is a function of the index of refraction; for a substance in air the critical angle is given by $\sin\theta_c = 1/n$. As n decreases $1/n$ increases so θ_c increases. For fused quartz, then, as wavelength increases θ_c also increases.

In short, red light has a larger critical angle than blue light. If the angle of incidence is midway between the critical angle of red and the critical angle of blue, then the blue component of the light will experience total internal reflection while the red component will pass through as a refracted ray.

So yes, the light can be made to appear bluish.

(b) No, the light can't be made to appear reddish. See above.

(c) Choose an angle of incidence between the two critical angles as described in part (a). Using a value of $n = 1.46$ from Fig. 39-11,

$$\theta_c = \sin^{-1}(1/1.46) = 43.2^\circ.$$

Getting the effect to work will require considerable sensitivity.

E39-38 (a) There needs to be an opaque spot in the center of each face so that no refracted ray emerges. The radius of the spot will be large enough to cover rays which meet the surface at less than the critical angle. This means $\tan\theta_c = r/d$, where d is the distance from the surface to the spot, or 6.3 mm. Since

$$\theta_c = \arcsin 1/(1.52) = 41.1^\circ,$$

then $r = (6.3\,\text{mm})\tan(41.1^\circ) = 5.50\,\text{mm}$.

(b) The circles have an area of $a = \pi(5.50\,\text{mm})^2 = 95.0\,\text{mm}^2$. Each side has an area of $(12.6\,\text{mm})^2$; the fraction covered is then $(95.0\,\text{mm}^2)/(12.6\,\text{mm})^2 = 0.598$.

E39-39 For $u \ll c$ the relativistic Doppler shift simplifies to

$$\Delta f = -f_0 u/c = -u/\lambda_0,$$

so

$$u = \lambda_0 \Delta f = (0.211\,\text{m})\Delta f.$$

E39-40 $c = f\lambda$, so $0 = f\Delta\lambda + \lambda\Delta f$. Then $\Delta\lambda/\lambda = -\Delta f/f$. Furthermore, $f_0 - f$, from Eq. 39-21, is $f_0 u/c$ for small enough u. Then

$$\frac{\Delta\lambda}{\lambda} = -\frac{f - f_0}{f_0} = \frac{u}{c}.$$

$\boxed{\textbf{E39-41}}$ The Doppler theory for light gives

$$f = f_0 \frac{1 - u/c}{\sqrt{1 - u^2/c^2}} = f_0 \frac{1 - (0.2)}{\sqrt{1 - (0.2)^2}} = 0.82\, f_0.$$

The frequency is shifted down to about 80%, which means the wavelength is shifted up by an additional 25%. Blue light (480 nm) would appear yellow/orange (585 nm).

E39-42 Use Eq. 39-20:

$$f = f_0 \frac{1 - u/c}{\sqrt{1 - u^2/c^2}} = (100 \text{ Mhz}) \frac{1 - (0.892)}{\sqrt{1 - (0.892)^2}} = 23.9 \text{ MHz}.$$

E39-43 (a) If the wavelength is three times longer then the frequency is one-third, so for the classical Doppler shift

$$f_0/3 = f_0(1 - u/c),$$

or $u = 2c$.

(b) For the relativistic shift,

$$\begin{aligned} f_0/3 &= f_0 \frac{1 - u/c}{\sqrt{1 - u^2/c^2}}, \\ \sqrt{1 - u^2/c^2} &= 3(1 - u/c), \\ c^2 - u^2 &= 9(c - u)^2, \\ 0 &= 10u^2 - 18uc + 8c^2. \end{aligned}$$

The solution is $u = 4c/5$.

E39-44 (a) $f_0/f = \lambda/\lambda_0$. This shift is *small*, so we apply the approximation:

$$u = c\left(\frac{\lambda_0}{\lambda} - 1\right) = (3\times10^8 \text{m/s})\left(\frac{(462 \text{ nm})}{(434 \text{ nm})} - 1\right) = 1.9\times10^7 \text{m/s}.$$

(b) A red shift corresponds to objects moving away from us.

$\boxed{\textbf{E39-45}}$ The sun rotates once every 26 days at the equator, while the radius is 7.0×10^8m. The speed of a point on the equator is then

$$v = \frac{2\pi R}{T} = \frac{2\pi(7.0\times10^8 \text{m})}{(2.2\times10^6 \text{s})} = 2.0\times10^3 \text{ m/s}.$$

This corresponds to a velocity parameter of

$$\beta = u/c = (2.0\times10^3 \text{ m/s})/(3.0\times10^8 \text{ m/s}) = 6.7\times10^{-6}.$$

This is a case of small numbers, so we'll use the formula that you derived in Exercise 39-40:

$$\Delta\lambda = \lambda\beta = (553 \text{ nm})(6.7\times10^{-6}) = 3.7\times10^{-3} \text{ nm}.$$

E39-46 Use Eq. 39-23 written as

$$(1 - u/c)\lambda^2 = \lambda_0^2(1 + u/c),$$

which can be rearranged as

$$u/c = \frac{\lambda^2 - \lambda_0^2}{\lambda^2 + \lambda_0^2} = \frac{(540\text{ nm})^2 - (620\text{ nm})^2}{(540\text{ nm})^2 + (620\text{ nm})^2} = -0.137.$$

The negative sign means that you should be going toward the red light.

E39-47 (a) $f_1 = cf/(c + v)$ and $f_2 = cf/(c - v)$.

$$\Delta f = (f_2 - f) - (f - f_1) = f_2 + f_1 - 2f,$$

so

$$\begin{aligned}
\frac{\Delta f}{f} &= \frac{c}{c + v} + \frac{c}{c - v} - 2, \\
&= \frac{2v^2}{c^2 - v^2}, \\
&= \frac{2(8.65\times 10^5\text{m/s})^2}{(3.00\times 10^8\text{m/s})^2 - (8.65\times 10^5\text{m/s})^2}, \\
&= 1.66\times 10^{-5}.
\end{aligned}$$

(b) $f_1 = f(c - u)/sqrtc^2 - u^2$ and $f_2 = f(c + u)/\sqrt{c^2 - u^2}$.

$$\Delta f = (f_2 - f) - (f - f_1) = f_2 + f_1 - 2f,$$

so

$$\begin{aligned}
\frac{\Delta f}{f} &= \frac{2c}{\sqrt{c^2 - u^2}} - 2, \\
&= \frac{2(3.00\times 10^8\text{m/s})}{\sqrt{(3.00\times 10^8\text{m/s})^2 - (8.65\times 10^5\text{m/s})^2}} - 2, \\
&= 8.3\times 10^{-6}.
\end{aligned}$$

E39-48 (a) No relative motion, so every 6 minutes.

(b) The Doppler effect at this speed is

$$\frac{1 - u/c}{\sqrt{1 - u^2/c^2}} = \frac{1 - (0.6)}{\sqrt{1 - (0.6)^2}} = 0.5;$$

this means the frequency is one half, so the period is doubled to 12 minutes.

(c) If C send the signal at the instant the signal from A passes, then the two signals travel together to C, so C would get B's signals at the same rate that it gets A's signals: every six minutes.

E39-49

E39-50 The transverse Doppler effect is $\lambda = \lambda_0/\sqrt{1 - u^2/c^2}$. Then

$$\lambda = (589.00\text{ nm})/\sqrt{1 - (0.122)^2} = 593.43\text{ nm}.$$

The shift is $(593.43\text{ nm}) - (589.00\text{ nm}) = 4.43\text{ nm}$.

$\boxed{\textbf{E39-51}}$ The frequency observed by the detector from the first source is (Eq. 39-31)

$$f = f_1\sqrt{1-(0.717)^2} = 0.697 f_1.$$

The frequency observed by the detector from the second source is (Eq. 39-30)

$$f = f_2\frac{\sqrt{1-(0.717)^2}}{1+(0.717)\cos\theta} = \frac{0.697 f_2}{1+(0.717)\cos\theta}.$$

We need to equate these and solve for θ. Then

$$\begin{aligned} 0.697 f_1 &= \frac{0.697 f_2}{1+0.717\cos\theta}, \\ 1+0.717\cos\theta &= f_2/f_1, \\ \cos\theta &= (f_2/f_1 - 1)/0.717, \\ \theta &= 101.1^\circ. \end{aligned}$$

Subtract from 180° to find the angle with the line of sight.

E39-52

P39-1 Consider the triangle in Fig. 39-45. The true position corresponds to the speed of light, the opposite side corresponds to the velocity of earth in the orbit. Then

$$\theta = \arctan(29.8\times10^3\text{m/s})/(3.00\times10^8\text{m/s}) = 20.5''.$$

P39-2 The distance to Jupiter from point x is $d_x = r_j - r_e$. The distance to Jupiter from point y is

$$d_2 = \sqrt{r_e^2 + r_j^2}.$$

The difference in distance is related to the time according to

$$(d_2 - d_1)/t = c,$$

so

$$\frac{\sqrt{(778\times10^9\text{m})^2 + (150\times10^9\text{m})^2} - (778\times10^9\text{m}) + (150\times10^9\text{m})}{(600\,\text{s})} = 2.7\times10^8\text{m/s}.$$

P39-3 $\sin(30^\circ)/(4.0\,\text{m}/s) = \sin\theta/(3.0\,\text{m/s})$. Then $\theta = 22^\circ$. Water waves travel more slowly in shallower water, which means they always bend toward the normal as they approach land.

P39-4 (a) If the ray is normal to the water's surface then it passes into the water undeflected. Once in the water the problem is identical to Sample Problem 39-2. The reflected ray in the water is parallel to the incident ray in the water, so it also strikes the water normal, and is transmitted normal.

(b) Assume the ray strikes the water at an angle θ_1. It then passes into the water at an angle θ_2, where

$$n_w \sin\theta_2 = n_a \sin\theta_1.$$

Once the ray is in the water then the problem is identical to Sample Problem 39-2. The reflected ray in the water is parallel to the incident ray in the water, so it also strikes the water at an angle θ_2. When the ray travels back into the air it travels with an angle θ_3, where

$$n_w \sin\theta_2 = n_a \sin\theta_3.$$

Comparing the two equations yields $\theta_1 = \theta_3$, so the outgoing ray in the air is parallel to the incoming ray.

$\boxed{\textbf{P39-5}}$ (a) As was done in Ex. 39-25 above we use the small angle approximation of

$$\sin\theta \approx \theta \approx \tan\theta$$

The incident angle is θ; if the light were to go in a straight line we would expect it to strike a distance y_1 beneath the normal on the right hand side. The various distances are related to the angle by

$$\theta \approx \tan\theta \approx y_1/t.$$

The light, however, does *not* go in a straight line, it is refracted according to (the small angle approximation to) Snell's law, $n_1\theta_1 = n_2\theta_2$, which we will simplify further by letting $\theta_1 = \theta$, $n_2 = n$, and $n_1 = 1$, $\theta = n\theta_2$. The point where the refracted ray *does* strike is related to the angle by $\theta_2 \approx \tan\theta_2 = y_2/t$. Combining the three expressions,

$$y_1 = ny_2.$$

The difference, $y_1 - y_2$ is the vertical distance between the displaced ray and the original ray as measured on the plate glass. A little algebra yields

$$\begin{aligned} y_1 - y_2 &= y_1 - y_1/n, \\ &= y_1\left(1 - 1/n\right), \\ &= t\theta\frac{n-1}{n}. \end{aligned}$$

The perpendicular distance x is related to this difference by

$$\cos\theta = x/(y_1 - y_2).$$

In the small angle approximation $\cos\theta \approx 1 - \theta^2/2$. If θ is sufficiently small we can ignore the square term, and $x \approx y_2 - y_1$.

(b) Remember to use *radians* and not degrees whenever the small angle approximation is applied. Then

$$x = (1.0 \text{ cm})(0.175 \text{ rad})\frac{(1.52) - 1}{(1.52)} = 0.060 \text{ cm}.$$

P39-6 (a) At the top layer,

$$n_1 \sin\theta_1 = \sin\theta;$$

at the next layer,

$$n_2 \sin\theta_2 = n_1 \sin\theta_1;$$

at the next layer,

$$n_3 \sin\theta_3 = n_2 \sin\theta_2.$$

Combining all three expressions,

$$n_3 \sin\theta_3 = \sin\theta.$$

(b) $\theta_3 = \arcsin[sin(50°)/(1.00029)] = 49.98°$. Then shift is $(50°) - (49.98°) = 0.02°$.

$\boxed{\textbf{P39-7}}$ The "big idea" of Problem 6 is that when light travels through layers the angle that it makes in any layer depends only on the incident angle, the index of refraction where that incident angle occurs, and the index of refraction at the current point.

That means that light which leaves the surface of the runway at 90° to the normal will make an angle

$$n_0 \sin 90° = n_0(1 + ay)\sin\theta$$

at some height y above the runway. It is mildly entertaining to note that the value of n_0 is unimportant, only the value of a!

The expression

$$\sin\theta = \frac{1}{1+ay} \approx 1 - ay$$

can be used to find the angle made by the curved path against the normal as a function of y. The slope of the curve at any point is given by

$$\frac{dy}{dx} = \tan(90^\circ - \theta) = \cot\theta = \frac{\cos\theta}{\sin\theta}.$$

Now we need to know $\cos\theta$. It is

$$\cos\theta = \sqrt{1-\sin^2\theta} \approx \sqrt{2ay}.$$

Combining

$$\frac{dy}{dx} \approx \frac{\sqrt{2ay}}{1-ay},$$

and now we integrate. We will ignore the ay term in the denominator because it will always be small compared to 1. Then

$$\begin{aligned} \int_0^d dx &= \int_0^h \frac{dy}{\sqrt{2ay}}, \\ d &= \sqrt{\frac{2h}{a}} = \sqrt{\frac{2(1.7\,\mathrm{m})}{(1.5\times 10^{-6}\mathrm{m}^{-1})}} = 1500\,\mathrm{m}. \end{aligned}$$

P39-8 The energy of a particle is given by $E^2 = p^2c^2 + m^2c^4$. This energy is related to the mass by $E = \gamma mc^2$. γ is related to the speed by $\gamma = 1/\sqrt{1-u^2/c^2}$. Rearranging,

$$\begin{aligned} \frac{u}{c} &= \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{m^2c^2}{p^2+m^2c^2}}, \\ &= \sqrt{\frac{p^2}{p^2+m^2c^2}}. \end{aligned}$$

Since $n = c/u$ we can write this as

$$n = \sqrt{1+\frac{m^2c^2}{p^2}} = \sqrt{1+\left(\frac{mc^2}{pc}\right)^2}.$$

For the pion,

$$n = \sqrt{1+\left(\frac{(135\text{ MeV})}{(145\text{ MeV})}\right)^2} = 1.37.$$

For the muon,

$$n = \sqrt{1+\left(\frac{(106\text{ MeV})}{(145\text{ MeV})}\right)^2} = 1.24.$$

P39-9 (a) Before adding the drop of liquid project the light ray along the angle θ so that $\theta = 0$. Increase θ slowly until total internal reflection occurs at angle θ_1. Then

$$n_\mathrm{g} \sin\theta_1 = 1$$

is the equation which can be solved to find n_g.

Now put the liquid on the glass and repeat the above process until total internal reflection occurs at angle θ_2. Then

$$n_\mathrm{g} \sin\theta_2 = n_\mathrm{l}.$$

Note that $n_\mathrm{g} < n_\mathrm{g}$ for this method to work.

(b) This is not terribly practical.

P39-10 Let the internal angle at Q be θ_Q. Then $n \sin\theta_Q = 1$, because it is a critical angle. Let the internal angle at P be θ_P. Then $\theta_P + \theta_Q = 90°$. Combine this with the other formula and

$$1 = n\sin(90 - \theta_P) = n\cos\theta_Q = n\sqrt{1 - \sin^2\theta_P}.$$

Not only that, but $\sin\theta_1 = n\sin\theta_P$, or

$$1 = n\sqrt{1 - (\sin\theta_1)^2/n^2},$$

which can be solved for n to yield

$$n = \sqrt{1 + \sin^2\theta_1}.$$

(b) The largest value of the sine function is one, so $n_\mathrm{max} = \sqrt{2}$.

$\boxed{\textbf{P39-11}}$ (a) The fraction of light energy which escapes from the water is dependent on the critical angle. Light radiates in all directions from the source, but only that which strikes the surface at an angle less than the critical angle will escape. This critical angle is

$$\sin\theta_\mathrm{c} = 1/n.$$

We want to find the solid angle of the light which escapes; this is found by integrating

$$\Omega = \int_0^{2\pi}\int_0^{\theta_\mathrm{c}} \sin\theta\, d\theta\, d\phi.$$

This is *not* a hard integral to do. The result is

$$\Omega = 2\pi(1 - \cos\theta_\mathrm{c}).$$

There are 4π steradians in a spherical surface, so the fraction which escapes is

$$f = \frac{1}{2}(1 - \cos\theta_\mathrm{c}) = \frac{1}{2}(1 - \sqrt{1 - \sin^2\theta_\mathrm{c}}).$$

The last substitution is easy enough. We never needed to know the depth h.

(b) $f = \frac{1}{2}(1 - \sqrt{1 - (1/(1.3))^2}) = 0.18$.

P39-12 (a) The beam of light strikes the face of the fiber at an angle θ and is refracted according to

$$n_1 \sin\theta_1 = \sin\theta.$$

The beam then travels inside the fiber until it hits the cladding interface; it does so at an angle of $90^\circ - \theta_1$ to the normal. It will be reflected if it exceeds the critical angle of

$$n_1 \sin\theta_\mathrm{c} = n_2,$$

or if

$$\sin(90^\circ - \theta_1) \geq n_2/n_1,$$

which can be written as

$$\cos\theta_1 \geq n_2/n_1.$$

but if this is the cosine, then we can use $\sin^2 + \cos^2 = 1$ to find the sine, and

$$\sin\theta_1 \leq \sqrt{1 - n_2^2/n_1^2}.$$

Combine this with the first equation and

$$\theta \leq \arcsin\sqrt{n_1^2 - n_2^2}.$$

(b) $\theta = \arcsin\sqrt{(1.58)^2 - (1.53)^2} = 23.2^\circ$.

$\boxed{\textbf{P39-13}}$ Consider the two possible extremes: a ray of light can propagate in a straight line directly down the axis of the fiber, or it can reflect off of the sides with the minimum possible angle of incidence. Start with the harder option.

The minimum angle of incidence that will still involve reflection is the critical angle, so

$$\sin\theta_\mathrm{c} = \frac{n_2}{n_1}.$$

This light ray has farther to travel than the ray down the fiber axis because it is traveling at an angle. The distance traveled by this ray is

$$L' = L/\sin\theta_\mathrm{c} = L\frac{n_1}{n_2},$$

The time taken for this bouncing ray to travel a length L down the fiber is then

$$t' = \frac{L'}{v} = \frac{L'n_1}{c} = \frac{L}{c}\frac{n_1^2}{n_2}.$$

Now for the easier ray. It travels straight down the fiber in a time

$$t = \frac{L}{c}n_1.$$

The difference is

$$t' - t = \Delta t = \frac{L}{c}\left(\frac{n_1^2}{n_2} - n_1\right) = \frac{Ln_1}{cn_2}(n_1 - n_2).$$

(b) For the numbers in Problem 12 we have

$$\Delta t = \frac{(350\times 10^3\,\mathrm{m})(1.58)}{(3.00\times 10^8\mathrm{m/s})(1.53)}((1.58) - (1.53)) = 6.02\times 10^{-5}\mathrm{s}.$$

P39-14

P39-15 We can assume the airplane speed is small compared to the speed of light, and use Eq. 39-21. $\Delta f = 990$ Hz; so

$$|\Delta f| = f_0 u/c = u/\lambda_0,$$

hence $u = (990/\mathrm{s})(0.12\,\mathrm{m}) = 119\,\mathrm{m/s}$. The actual answer for the speed of the airplane is *half* this because there were two Doppler shifts: once when the microwaves struck the plane, and one when the reflected beam was received by the station. Hence, the plane approaches with a speed of $59.4\,\mathrm{m/s}$.

E40-1 (b) Since $i = -o$, $v_i = di/dt = -do/dt = -v_o$.

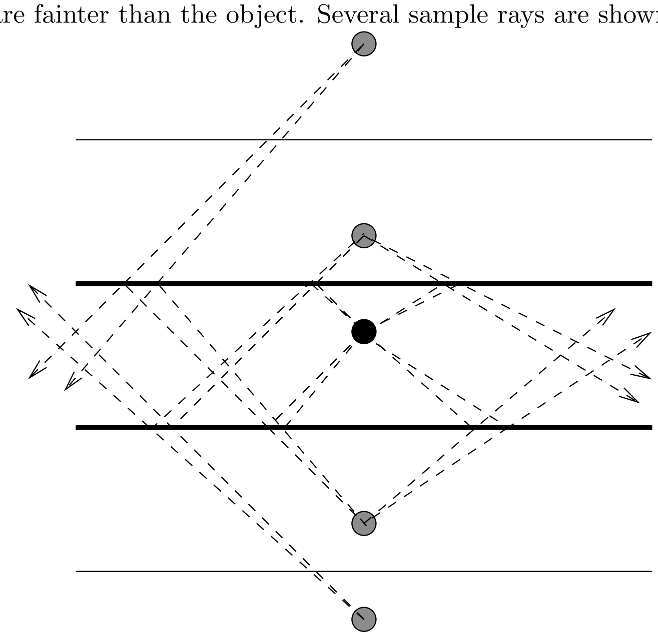
(a) In order to change from the frame of reference of the mirror to your own frame of reference you need to subtract v_o from all velocities. Then your velocity is $v_o - v_0 = 0$, the mirror is moving with velocity $0 - v_o = -v_o$ and your image is moving with velocity $-v_o - v_o = -2v_o$.

E40-2 You are 30 cm from the mirror, the image is 10 cm behind the mirror. You need to focus 40 cm away.

$\boxed{\textbf{E40-3}}$ If the mirror rotates through an angle α then the angle of incidence will increase by an angle α, and so will the angle of reflection. But that means that the angle between the incident angle and the reflected angle has increased by α twice.

E40-4 Sketch a line from Sarah through the right edge of the mirror and then beyond. Sarah can see any image which is located between that line and the mirror. By similar triangles, the image of Bernie will be $d/2 = (3.0\,\text{m})/2 = 1/5\,\text{m}$ from the mirror when it becomes visible. Since $i = -o$, Bernie will also be 1.5 m from the mirror.

E40-5 The images are fainter than the object. Several sample rays are shown.



E40-6 The image is displaced. The eye would need to look up to see it.

$\boxed{\textbf{E40-7}}$ The apparent depth of the swimming pool is given by the work done for Exercise 39-25, $d_{\text{app}} = d/n$ The water then "appears" to be only 186 cm/1.33 = 140 cm deep. The apparent distance between the light and the mirror is then 250 cm + 140 cm = 390 cm; consequently the image of the light is 390 cm beneath the surface of the mirror.

E40-8 Three. There is a single direct image in each mirror and one more image of an image in one of the mirrors.

$\boxed{\textbf{E40-9}}$ We want to know over what surface area of the mirror are rays of light reflected from the object into the eye. By similar triangles the diameter of the pupil and the diameter of the part of the mirror (d) which reflects light into the eye are related by

$$\frac{d}{(10\text{ cm})} = \frac{(5.0\text{ mm})}{(24\text{ cm}) + (10\text{ cm})},$$

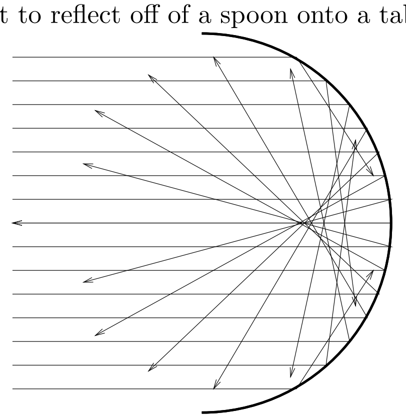
which has solution $d = 1.47$ mm The area of the circle on the mirror is

$$A = \pi(1.47\text{ mm})^2/4 = 1.7\text{ mm}^2.$$

E40-10 (a) Seven; (b) Five; and (c) Two. This is a problem of symmetry.

E40-11 Seven. Three images are the ones from Exercise 8. But each image has an image in the ceiling mirror. That would make a total of six, except that you also have an image in the ceiling mirror (look up, eh?). So the total is seven!

E40-12 A point focus is not formed. The envelope of rays is called the caustic. You can see a similar effect when you allow light to reflect off of a spoon onto a table.



$\boxed{\textbf{E40-13}}$ The image is magnified by a factor of 2.7, so the image distance is 2.7 times farther from the mirror than the object. An important question to ask is whether or not the image is real or virtual. If it is a virtual image it is behind the mirror and someone looking at the mirror could see it. If it were a real image it would be in front of the mirror, and the man, who serves as the object and is therefore closer to the mirror than the image, would not be able to see it.

So we shall assume that the image is virtual. The image distance is then a negative number. The focal length is half of the radius of curvature, so we want to solve Eq. 40-6, with $f = 17.5$ cm and $i = -2.7o$

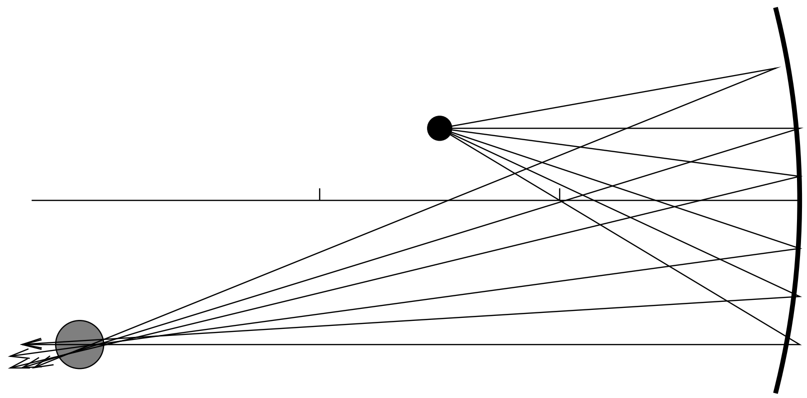
$$\frac{1}{(17.5\text{ cm})} = \frac{1}{o} + \frac{1}{-2.7o} = \frac{0.63}{o},$$

which has solution $o = 11$ cm.

E40-14 The image will be located at a point given by

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{(10\text{ cm})} - \frac{1}{(15\text{ cm})} = \frac{1}{(30\text{ cm})}.$$

The vertical scale is three times the horizontal scale in the figure below.



E40-15 This problem requires repeated application of $1/f = 1/o + 1/i$, $r = 2f$, $m = -i/o$, or the properties of plane, convex, or concave mirrors. All dimensioned variables below (f, r, i, o) are measured in centimeters.

(a) Concave mirrors have positive focal lengths, so $f = +20$; $r = 2f = +40$;

$$1/i = 1/f - 1/o = 1/(20) - 1/(10) = 1/(-20);$$

$m = -i/o = -(-20)/(10) = 2$; the image is virtual and upright.

(b) $m = +1$ for plane mirrors only; $r = \infty$ for flat surface; $f = \infty/2 = \infty$; $i = -o = -10$; the image is virtual and upright.

(c) If f is positive the mirror is concave; $r = 2f = +40$;

$$1/i = 1/f - 1/o = 1/(20) - 1/(30) = 1/(60);$$

$m = -i/o = -(60)/(30) = -2$; the image is real and inverted.

(d) If m is negative then the image is real and inverted; only Concave mirrors produce real images (from real objects); $i = -mo = -(-0.5)(60) = 30$;

$$1/f = 1/o + 1/i = 1/(30) + 1/(60) = 1/(20);$$

$r = 2f = +40$.

(e) If r is negative the mirror is convex; $f = r/2 = (-40)/2 = -20$;

$$1/o = 1/f - 1/i = 1/(-20) - 1/(-10) = 1/(20);$$

$m = -(-10)/(20) = 0.5$; the image is virtual and upright.

(f) If m is positive the image is virtual and upright; if m is less than one the image is reduced, but only convex mirrors produce reduced virtual images (from real objects); $f = -20$ for convex mirrors; $r = 2f = -40$; let $i = -mo = -o/10$, then

$$1/f = 1/o + 1/i = 1/o - 10/o = -9/o,$$

so $o = -9f = -9(-20) = 180$; $i = -o/10 = -(180)/10 = -18$.

(g) r is negative for convex mirrors, so $r = -40$; $f = r/2 = -20$; convex mirrors produce only virtual upright images (from real objects); so i is negative; and

$$1/o = 1/f - 1/i = 1/(-20) - 1/(-4) = 1/(5);$$

$m = -i/o = -(-4)/(5) = 0.8$.

(h) Inverted images are real; only concave mirrors produce real images (from real objects); inverted images have negative m; $i = -mo = -(-0.5)(24) = 12$;

$$1/f = 1/o + 1/i = 1/(24) + 1/(12) = 1/(8);$$

$r = 2f = 16$.

E40-16 Use the angle definitions provided by Eq. 40-8. From triangle OaI we have

$$\alpha + \gamma = 2\theta,$$

while from triangle IaC we have

$$\beta + \theta = \gamma.$$

Combining to eliminate θ we get

$$\alpha - \gamma = -2\beta.$$

Substitute Eq. 40-8 and eliminate s,

$$\frac{1}{o} - \frac{1}{i} = -\frac{2}{r},$$

or

$$\frac{1}{o} + \frac{1}{-i} = \frac{2}{-r},$$

which is the same as Eq. 40-4 if $i \to -i$ and $r \to -r$.

$\boxed{\textbf{E40-17}}$ (a) Consider the point A. Light from this point travels along the line ABC and will be parallel to the horizontal center line from the center of the cylinder. Since the tangent to a circle defines the outer limit of the intersection with a line, this line must describe the apparent size.

(b) The angle of incidence of ray AB is given by

$$\sin\theta_1 = r/R.$$

The angle of refraction of ray BC is given by

$$\sin\theta_2 = r^*/R.$$

Snell's law, and a little algebra, yields

$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2, \\ n_1 \frac{r}{R} &= n_2 \frac{r^*}{R}, \\ nr &= r^*. \end{aligned}$$

In the last line we used the fact that $n_2 = 1$, because it is in the air, and $n_1 = n$, the index of refraction of the glass.

E40-18 This problem requires repeated application of $(n_2 - n_1)/r = n_1/o + n_2/i$. All dimensioned variables below (r, i, o) are measured in centimeters.

(a)

$$\frac{(1.5) - (1.0)}{(30)} - \frac{(1.0)}{(10)} = -0.08333,$$

so $i = (1.5)/(-0.08333) = -18$, and the image is virtual.

(b)

$$\frac{(1.0)}{(10)} + \frac{(1.5)}{(-13)} = -0.015385,$$

so $r = (1.5 - 1.0)/(-0.015385) = -32.5$, and the image is virtual.

(c)

$$\frac{(1.5) - (1.0)}{(30)} - \frac{(1.5)}{(600)} = 0.014167,$$

so $o = (1.0)/(0.014167) = 71$. The image was real since $i > 0$.

(d) Rearrange the formula to solve for n_2, then

$$n_2 \left(\frac{1}{r} - 1i \right) = n_1 \left(\frac{1}{r} + \frac{1}{o} \right).$$

Substituting the numbers,

$$n_2 \left(\frac{1}{(-20)} - \frac{1}{(-20)} \right) = (1.0) \left(\frac{1}{(-20)} + \frac{1}{(20)} \right),$$

which has *any* solution for n_2! Since $i < 0$ the image is virtual.

(e)

$$\frac{(1.5)}{(10)} + \frac{(1.0)}{(-6)} = -0.016667,$$

so $r = (1.0 - 1.5)/(-0.016667) = 30$, and the image is virtual.

(f)

$$\frac{(1.0) - (1.5)}{(-30)} - \frac{(1.0)}{(-7.5)} = 0.15,$$

so $o = (1.5)/(0.15) = 10$. The image was virtual since $i < 0$.

(g)

$$\frac{(1.0) - (1.5)}{(30)} - \frac{(1.5)}{(70)} = -3.81 \times 10^{-2},$$

so $i = (1.0)/(-3.81 \times 10^{-2}) = -26$, and the image is virtual.

(h) Solving Eq. 40-10 for n_2 yields

$$n_2 = n_1 \frac{1/o + 1/r}{1/r - 1/i},$$

so

$$n_2 = (1.5) \frac{1/(100) + 1/(-30)}{1/(-30) - 1/(600)} = 1.0$$

and the image is real.

E40-19 (b) If the beam is small we can use Eq. 40-10. Parallel incoming rays correspond to an object at infinity. Solving for n_2 yields

$$n_2 = n_1 \frac{1/o + 1/r}{1/r - 1/i},$$

so if $o \to \infty$ and $i = 2r$, then

$$n_2 = (1.0) \frac{1/\infty + 1/r}{1/r - 1/2r} = 2.0$$

(c) There is no solution if $i = r$!

E40-20 The image will be located at a point given by

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{(10 \text{ cm})} - \frac{1}{(6 \text{ cm})} = \frac{1}{(-15 \text{ cm})}.$$

$\boxed{\textbf{E40-21}}$ The image location can be found from Eq. 40-15,

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{(-30\text{ cm})} - \frac{1}{(20\text{ cm})} = \frac{1}{-12\text{ cm}},$$

so the image is located 12 cm from the thin lens, *on the same side as the object.*

E40-22 For a double convex lens $r_1 > 0$ and $r_2 < 0$ (see Fig. 40-21 and the accompanying text). Then the problem states that $r_2 = -r_1/2$. The lens maker's equation can be applied to get

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{3(n-1)}{r_1},$$

so $r_1 = 3(n-1)f = 3(1.5-1)(60\,\text{mm}) = 90\,\text{mm}$, and $r_2 = -45\,\text{mm}$.

E40-23 The object distance is essentially $o = \infty$, so $1/f = 1/o + 1/i$ implies $f = i$, and the image forms at the focal point. In reality, however, the object distance is not infinite, so the magnification is given by $m = -i/o \approx -f/o$, where o is the Earth/Sun distance. The size of the image is then

$$h_i = h_o f/o = 2(6.96\times10^8\text{m})(0.27\,\text{m})/(1.50\times10^{11}\text{m}) = 2.5\,\text{mm}.$$

The factor of two is because the sun's radius is given, and we need the diameter!

E40-24 (a) The flat side has $r_2 = \infty$, so $1/f = (n-1)/r$, where r is the curved side. Then $f = (0.20\,\text{m})/(1.5-1) = 0.40\,\text{m}$.

(b) $1/i = 1/f - 1/o = 1/(0.40\,\text{m}) - 1/(0.40\,\text{m}) = 0$. Then i is ∞.

E40-25 (a) $1/f = (1.5-1)[1/(0.4\,\text{m}) - 1/(-0.4\,\text{m})] = 1/(0.40\,\text{m})$.

(b) $1/f = (1.5-1)[1/(\infty) - 1/(-0.4\,\text{m})] = 1/(0.80\,\text{m})$.

(c) $1/f = (1.5-1)[1/(0.4\,\text{m}) - 1/(0.6\,\text{m})] = 1/(2.40\,\text{m})$.

(d) $1/f = (1.5-1)[1/(-0.4\,\text{m}) - 1/(0.4\,\text{m})] = 1/(-0.40\,\text{m})$.

(e) $1/f = (1.5-1)[1/(\infty) - 1/(0.8\,\text{m})] = 1/(-0.80\,\text{m})$.

(f) $1/f = (1.5-1)[1/(0.6\,\text{m}) - 1/(0.4\,\text{m})] = 1/(-2.40\,\text{m})$.

E40-26 (a) $1/f = (n-1)[1/(-r) - 1/r]$, so $1/f = 2(1-n)/r$. $1/i = 1/f - 1/o$ so if $o = r$, then

$$1/i = 2(1-n)/r - 1/r = (1-2n)/r,$$

so $i = r/(1-2n)$. For $n > 0.5$ the image is virtual.

(b) For $n > 0.5$ the image is virtual; the magnification is

$$m = -i/o = -r/(1-2n)/r = 1/(2n-1).$$

$\boxed{\textbf{E40-27}}$ According to the definitions, $o = f + x$ and $i = f + x'$. Starting with Eq. 40-15,

$$\begin{aligned}
\frac{1}{o} + \frac{1}{i} &= \frac{1}{f},\\
\frac{i+o}{oi} &= \frac{1}{f},\\
\frac{2f + x + x'}{(f+x)(f+x')} &= \frac{1}{f},\\
2f^2 + fx + fx' &= f^2 + fx + fx' + xx',\\
f^2 &= xx'.
\end{aligned}$$

E40-28 (a) You can't determine r_1, r_2, or n. i is found from

$$\frac{1}{i} = \frac{1}{+10} - \frac{1}{+20} = \frac{1}{+20},$$

the image is real and inverted. $m = -(20)/(20) = -1$.

(b) You can't determine r_1, r_2, or n. The lens is converging since f is positive. i is found from

$$\frac{1}{i} = \frac{1}{+10} - \frac{1}{+5} = \frac{1}{-10},$$

the image is virtual and upright. $m = -(-10)/(+5) = 2$.

(c) You can't determine r_1, r_2, or n. Since m is positive and greater than one the lens is converging. Then f is positive. i is found from

$$\frac{1}{i} = \frac{1}{+10} - \frac{1}{+5} = \frac{1}{-10},$$

the image is virtual and upright. $m = -(-10)/(+5) = 2$.

(d) You can't determine r_1, r_2, or n. Since m is positive and less than one the lens is diverging. Then f is negative. i is found from

$$\frac{1}{i} = \frac{1}{-10} - \frac{1}{+5} = \frac{1}{-3.3},$$

the image is virtual and upright. $m = -(-3.3)/(+5) = 0.66$.

(e) f is found from

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{+30} - \frac{1}{-30}\right) = \frac{1}{+30}.$$

The lens is converging. i is found from

$$\frac{1}{i} = \frac{1}{+30} - \frac{1}{+10} = \frac{1}{-15},$$

the image is virtual and upright. $m = -(-15)/(+10) = 1.5$.

(f) f is found from

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-30} - \frac{1}{+30}\right) = \frac{1}{-30}.$$

The lens is diverging. i is found from

$$\frac{1}{i} = \frac{1}{-30} - \frac{1}{+10} = \frac{1}{-7.5},$$

the image is virtual and upright. $m = -(-7.5)/(+10) = 0.75$.

(g) f is found from

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-30} - \frac{1}{-60}\right) = \frac{1}{-120}.$$

The lens is diverging. i is found from

$$\frac{1}{i} = \frac{1}{-120} - \frac{1}{+10} = \frac{1}{-9.2},$$

the image is virtual and upright. $m = -(-9.2)/(+10) = 0.92$.

(h) You can't determine r_1, r_2, or n. Upright images have positive magnification. i is found from

$$i = -(0.5)(10) = -5;$$

f is found from
$$\frac{1}{f} = \frac{1}{+10} + \frac{1}{-5} = \frac{1}{-10},$$
so the lens is diverging.

(h) You can't determine r_1, r_2, or n. Real images have negative magnification. i is found from
$$i = -(-0.5)(10) = 5;$$
f is found from
$$\frac{1}{f} = \frac{1}{+10} + \frac{1}{5} = \frac{1}{+3.33},$$
so the lens is converging.

E40-29 $o + i = 0.44\,\mathrm{m} = L$, so
$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} = \frac{1}{o} + \frac{1}{L-o} = \frac{L}{o(L-o)},$$
which can also be written as $o^2 - oL + fL = 0$. This has solution
$$o = \frac{L \pm \sqrt{L^2 - 4fL}}{2} = \frac{(0.44\,\mathrm{m}) \pm \sqrt{(0.44\,\mathrm{m}) - 4(0.11\,\mathrm{m})(0.44\,\mathrm{m})}}{2} = 0.22\,\mathrm{m}.$$
There is only one solution to this problem, but sometimes there are two, and other times there are none!

E40-30 (a) Real images (from real objects) are only produced by converging lenses.

(b) Since $h_i = -h_0/2$, then $i = o/2$. But $d = i+o = o+o/2 = 3o/2$, so $o = 2(0.40\,\mathrm{m})/3 = 0.267\,\mathrm{m}$, and $i = 0.133\,\mathrm{m}$.

(c) $1/f = 1/o + 1/i = 1/(0.267\,\mathrm{m}) + 1/(0.133\,\mathrm{m}) = 1/(0.0889\,\mathrm{m})$.

$\boxed{\textbf{E40-31}}$ Step through the exercise one lens at a time. The object is 40 cm to the left of a converging lens with a focal length of +20 cm. The image from this first lens will be located by solving
$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{(20\text{ cm})} - \frac{1}{(40\text{ cm})} = \frac{1}{40\text{ cm}},$$
so $i = 40$ cm. Since i is positive it is a real image, and it is located to the right of the converging lens. This image becomes the object for the diverging lens.

The image from the converging lens is located 40 cm - 10 cm from the diverging lens, but it is located on the wrong side: the diverging lens is "in the way" so the rays which would form the image hit the diverging lens before they have a chance to form the image. That means that the real image from the converging lens is a *virtual* object in the diverging lens, so that the object distance for the diverging lens is $o = -30$ cm.

The image formed by the diverging lens is located by solving
$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{(-15\text{ cm})} - \frac{1}{(-30\text{ cm})} = \frac{1}{-30\text{ cm}},$$
or $i = -30$ cm. This would mean the image formed by the diverging lens would be a virtual image, and would be located to the left of the diverging lens.

The image is virtual, so it is upright. The magnification from the first lens is
$$m_1 = -i/o = -(40\text{ cm})/(40\text{ cm})) = -1;$$

the magnification from the second lens is

$$m_2 = -i/o = -(-30 \text{ cm})/(-30 \text{ cm})) = -1;$$

which implies an overall magnification of $m_1 m_2 = 1$.

E40-32 (a) The parallel rays of light which strike the lens of focal length f will converge on the focal point. This point will act like an object for the second lens. If the second lens is located a distance L from the first then the object distance for the second lens will be $L - f$. Note that this will be a negative value for $L < f$, which means the object is virtual. The image will form at a point

$$1/i = 1/(-f) - 1/(L - f) = L/f(f - L).$$

Note that i will be positive if $L < f$, so the rays really do converge on a point.

(b) The same equation applies, except switch the sign of f. Then

$$1/i = 1/(f) - 1/(L - f) = L/f(L - f).$$

This is *negative* for $L < f$, so there is no real image, and no converging of the light rays.

(c) If $L = 0$ then $i = \infty$, which means the rays coming from the second lens are parallel.

E40-33 The image from the converging lens is found from

$$\frac{1}{i_1} = \frac{1}{(0.58\,\text{m})} - \frac{1}{(1.12\,\text{m})} = \frac{1}{1.20\,\text{m}}$$

so $i_1 = 1.20\,\text{m}$, and the image is real and inverted.

This real image is $1.97\,\text{m} - 1.20\,\text{m} = 0.77\,\text{m}$ in front of the plane mirror. It acts as an object for the mirror. The mirror produces a virtual image $0.77\,\text{m}$ behind the plane mirror. This image is upright relative to the object which formed it, which was inverted relative to the original object.

This second image is $1.97\,\text{m} + 0.77\,\text{m} = 2.74\,\text{m}$ away from the lens. This second image acts as an object for the lens, the image of which is found from

$$\frac{1}{i_3} = \frac{1}{(0.58\,\text{m})} - \frac{1}{(2.74\,\text{m})} = \frac{1}{0.736\,\text{m}}$$

so $i_3 = 0.736\,\text{m}$, and the image is real and inverted relative to the object which formed it, which was inverted relative to the original object. So this image is actually upright.

E40-34 (a) The first lens forms a real image at a location given by

$$1/i = 1/f - 1/o = 1/(0.1\,\text{m}) - 1/(0.2\,\text{m}) = 1/(0.2\,\text{m}).$$

The image and object distance are the same, so the image has a magnification of 1. This image is $0.3\,\text{m} - 0.2\,\text{m} = 0.1\,\text{m}$ from the second lens. The second lens forms an image at a location given by

$$1/i = 1/f - 1/o = 1/(0.125\,\text{m}) - 1/(0.1\,\text{m}) = 1/(-0.5\,\text{m}).$$

Note that this puts the final image at the location of the original object! The image is magnified by a factor of $(0.5\,\text{m})/(0.1\,\text{m}) = 5$.

(c) The image is virtual, but inverted.

E40-35 If the two lenses "pass" the same amount of light then the solid angle subtended by each lens as seen from the respective focal points must be the same. If we assume the lenses have the same round shape then we can write this as $d_\mathrm{o}/f_\mathrm{o} = d_\mathrm{e}/f_\mathrm{e}$. Then

$$\frac{d_\mathrm{e}}{d_\mathrm{o}} = \frac{f_\mathrm{o}}{f_\mathrm{e}} = m_\theta,$$

or $d_\mathrm{e} = (72\,\mathrm{mm})/36 = 2\,\mathrm{mm}$.

E40-36 (a) $f = (0.25\,\mathrm{m})/(200) \approx 1.3\,\mathrm{mm}$. Then $1/f = (n-1)(2/r)$ can be used to find r; $r = 2(n-1)f = 2(1.5-1)(1.3\,\mathrm{mm}) = 1.3\,\mathrm{mm}$.

(b) The diameter would be twice the radius. In effect, these were tiny glass balls.

E40-37 (a) In Fig. 40-46(a) the image is at the focal point. This means that in Fig. 40-46(b) $i = f = 2.5$ cm, even though $f' \neq f$. Solving,

$$\frac{1}{f} = \frac{1}{(36 \text{ cm})} + \frac{1}{(2.5 \text{ cm})} = \frac{1}{2.34 \text{ cm}}.$$

(b) The effective radii of curvature must have decreased.

E40-38 (a) $s = (25 \text{ cm}) - (4.2 \text{ cm}) - (7.7 \text{ cm}) = 13.1$ cm.

(b) $i = (25 \text{ cm}) - (7.7 \text{ cm}) = 17.3$ cm. Then

$$\frac{1}{o} = \frac{1}{(4.2 \text{ cm})} - \frac{1}{(17.3 \text{ cm})} = \frac{1}{5.54 \text{ cm.}}$$

The object should be placed $5.5 - 4.2 = 1.34$ cm beyond F_1.

(c) $m = -(17.3)/(5.5) = -3.1$.

(d) $m_\theta = (25 \text{ cm})/(7.7 \text{ cm}) = 3.2$.

(e) $M = mm_\theta = -10$.

$\boxed{\textbf{E40-39}}$ Microscope magnification is given by Eq. 40-33. We need to first find the focal length of the objective lens before we can use this formula. We are told in the text, however, that the microscope is constructed so the at the object is placed just beyond the focal point of the objective lens, then $f_\mathrm{ob} \approx 12.0$ mm. Similarly, the intermediate image is formed at the focal point of the eyepiece, so $f_\mathrm{ey} \approx 48.0$ mm. The magnification is then

$$m = \frac{-s(250 \text{ mm})}{f_\mathrm{ob} f_\mathrm{ey}} = -\frac{(285 \text{ mm})(250 \text{ mm})}{(12.0 \text{ mm})(48.0 \text{ mm})} = 124.$$

A more accurate answer can be found by calculating the *real* focal length of the objective lens, which is 11.4 mm, but since there is a *huge* uncertainty in the near point of the eye, I see no point in trying to be more accurate than this.

P40-1 The old intensity is $I_o = P/4\pi d^2$, where P is the power of the point source. With the mirror in place there is an additional amount of light which needs to travel a total distance of $3d$ in order to get to the screen, so it contributes an additional $P/4\pi(3d)^2$ to the intensity. The new intensity is then

$$I_n = P/4\pi d^2 + P/4\pi(3d)^2 = (10/9)P/4\pi d^2 = (10/9)I_o.$$

P40-2 (a) $v_i = di/dt$; but $i = fo/(o-f)$ and $f = r/2$ so

$$v_i = \frac{d}{dt}\left(\frac{ro}{2o-r}\right) = -\left(\frac{r}{2o-r}\right)^2 \frac{do}{dt} = -\left(\frac{r}{2o-r}\right)^2 v_o.$$

(b) Put in the numbers!

$$v_i = -\left(\frac{(15\text{ cm})}{2(75\text{ cm}) - (15\text{ cm})}\right)^2 (5.0\text{ cm/s}) = -6.2\times10^{-2}\text{ cm/s}.$$

(c) Put in the numbers!

$$v_i = -\left(\frac{(15\text{ cm})}{2(7.7\text{ cm}) - (15\text{ cm})}\right)^2 (5.0\text{ cm/s}) = -70\,\text{m/s}$$

(d) Put in the numbers!

$$v_i = -\left(\frac{(15\text{ cm})}{2(0.15\text{ cm}) - (15\text{ cm})}\right)^2 (5.0\text{ cm/s}) = -5.2\text{ cm/s}.$$

P40-3 (b) There are two ends to the object of length L, one of these ends is a distance o_1 from the mirror, and the other is a distance o_2 from the mirror. The images of the two ends will be located at i_1 and i_2.

Since we are told that the object has a short length L we will assume that a differential approach to the problem is in order. Then

$$L = \Delta o = o_1 - o_2 \text{ and } L' = \Delta i = i_1 - i_2,$$

Finding the ratio of L'/L is then reduced to

$$\frac{L'}{L} = \frac{\Delta i}{\Delta o} \approx \frac{di}{do}.$$

We can take the derivative of Eq. 40-15 with respect to changes in o and i,

$$\frac{di}{i^2} + \frac{do}{o^2} = 0,$$

or

$$\frac{L'}{L} \approx \frac{di}{do} = -\frac{i^2}{o^2} = -m^2,$$

where m is the lateral magnification.

(a) Since i is given by

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{o-f}{of},$$

the fraction i/o can also be written

$$\frac{i}{o} = \frac{of}{o(o-f)} = \frac{f}{o-f}.$$

Then

$$L \approx -\frac{i^2}{o^2} = -\left(\frac{f}{o-f}\right)^2$$

P40-4 The left surface produces an image which is found from $n/i = (n-1)/R - 1/o$, but since the incoming rays are parallel we take $o = \infty$ and the expression simplifies to $i = nR/(n-1)$. This image is located a distance $o = 2R - i = (n-2)R/(n-1)$ from the right surface, and the image produced by this surface can be found from

$$1/i = (1-n)/(-R) - n/o = (n-1)/R - n(n-1)/(n-2)R = 2(1-n)/(n-2)R.$$

Then $i = (n-2)R/2(n-1)$.

P40-5 The "1" in Eq. 40-18 is actually n_{air}; the assumption is that the thin lens is in the air. If that isn't so, then we need to replace "1" with n', so Eq. 40-18 becomes

$$\frac{n'}{o} - \frac{n}{|i'|} = \frac{n-n'}{r_1}.$$

A similar correction happens to Eq. 40-21:

$$\frac{n}{|i'|} + \frac{n'}{i} = -\frac{n-n'}{r_2}.$$

Adding these two equations,

$$\frac{n'}{o} + \frac{n'}{i} = (n-n')\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

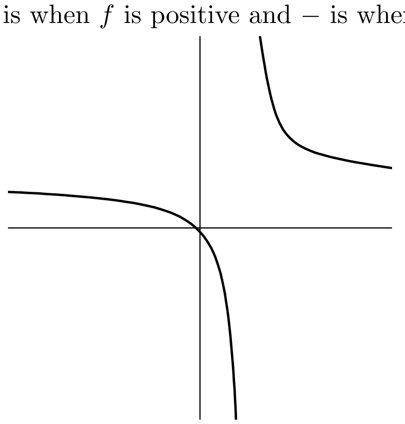
This yields a focal length given by

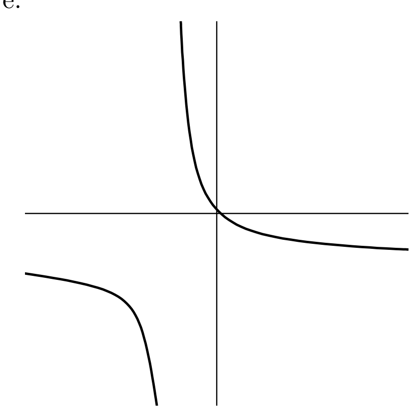
$$\frac{1}{f} = \frac{n-n'}{n}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

P40-6 Start with Eq. 40-4

$$\begin{aligned}\frac{1}{o} + \frac{1}{i} &= \frac{1}{f},\\ \frac{|f|}{o} + \frac{|f|}{i} &= \frac{|f|}{f},\\ \frac{1}{y} + \frac{1}{y'} &= \pm 1,\end{aligned}$$

where + is when f is positive and − is when f is negative.





The plot on the right is for +, that on the left for −.
Real image and objects occur when y or y' is positive.

$\boxed{\textbf{P40-7}}$ (a) The image (which will appear on the screen) and object are a distance $D = o + i$ apart. We can use this information to eliminate one variable from Eq. 40-15,

$$\begin{aligned} \frac{1}{o} + \frac{1}{i} &= \frac{1}{f}, \\ \frac{1}{o} + \frac{1}{D-o} &= \frac{1}{f}, \\ \frac{D}{o(D-o)} &= \frac{1}{f}, \\ o^2 - oD + fD &= 0. \end{aligned}$$

This last expression is a quadratic, and we would expect to get two solutions for o. These solutions will be of the form "something" plus/minus "something else"; the distance between the two locations for o will evidently be twice the "something else", which is then

$$d = o_+ - o_- = \sqrt{(-D)^2 - 4(fD)} = \sqrt{D(D-4f)}.$$

(b) The ratio of the image sizes is m_+/m_-, or i_+o_-/i_-o_+. Now it seems we must find the actual values of o_+ and o_-. From the quadratic in part (a) we have

$$o_\pm = \frac{D \pm \sqrt{D(D-4f)}}{2} = \frac{D \pm d}{2},$$

so the ratio is

$$\frac{o_-}{o_+} = \left(\frac{D-d}{D+d}\right).$$

But $i_- = o_+$, and vice-versa, so the ratio of the image sizes is this quantity squared.

P40-8 $1/i = 1/f - 1/o$ implies $i = fo/(o-f)$. i is only real if $o \geq f$. The distance between the image and object is

$$y = i + o = \frac{of}{o-f} + o = \frac{o^2}{o-f}.$$

This quantity is a minimum when $dy/do = 0$, which occurs when $o = 2f$. Then $i = 2f$, and $y = 4f$.

P40-9 (a) The angular size of each lens is the same when viewed from the *shared* focal point. This means $W_1/f_1 = W_2/f_2$, or

$$W_2 = (f_2/f_1)W_1.$$

(b) Pass the light through the diverging lens first; choose the separation of the lenses so that the focal point of the converging lens is at the same location as the focal point of the diverging lens which is on the opposite side of the diverging lens.

(c) Since $I \propto 1/A$, where A is the area of the beam, we have $I \propto 1/W^2$. Consequently,

$$I_2/I_1 = (W_1/W_2)^2 = (f_1/f_2)^2$$

P40-10 The location of the image in the mirror is given by

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{a+b}.$$

The location of the image in the plate is given by $i' = -a$, which is located at $b - a$ relative to the mirror. Equating,

$$\begin{aligned}
\frac{1}{b-a} + \frac{1}{b+a} &= \frac{1}{f}, \\
\frac{2b}{b^2 - a^2} &= \frac{1}{f}, \\
b^2 - a^2 &= 2bf, \\
a &= \sqrt{b^2 - 2bf}, \\
&= \sqrt{(7.5\text{ cm})^2 - 2(7.5\text{ cm})(-28.2\text{ cm})} = 21.9\text{ cm}.
\end{aligned}$$

$\boxed{\textbf{P40-11}}$ We'll solve the problem by finding out what happens if you put an object in front of the combination of lenses.

Let the object distance be o_1. The first lens will create an image at i_1, where

$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{o_1}$$

This image will act as an object for the second lens.

If the first image is real (i_1 positive) then the image will be on the "wrong" side of the second lens, and as such the real image will act like a virtual object. In short, $o_2 = -i_1$ will give the correct sign to the object distance when the image from the first lens acts like an object for the second lens. The image formed by the second lens will then be at

$$\begin{aligned}
\frac{1}{i_2} &= \frac{1}{f_2} - \frac{1}{o_2}, \\
&= \frac{1}{f_2} + \frac{1}{i_2}, \\
&= \frac{1}{f_2} + \frac{1}{f_1} - \frac{1}{o_1}.
\end{aligned}$$

In this case it appears as if the combination

$$\frac{1}{f_2} + \frac{1}{f_1}$$

is equivalent to the reciprocal of a focal length. We will go ahead and make this connection, and

$$\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1} = \frac{f_1 + f_2}{f_1 f_2}.$$

The rest is straightforward enough.

P40-12 (a) The image formed by the first lens can be found from

$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{2f_1} = \frac{1}{2f_1}.$$

This is a distance $o_2 = 2(f_1 + f_2) = 2f_2$ from the mirror. The image formed by the mirror is at an image distance given by

$$\frac{1}{i_2} = \frac{1}{f_2} - \frac{1}{2f_2} = \frac{1}{2f_2}.$$

Which is at the same point as i_1!. This means it will act as an object o_3 in the lens, and, reversing the first step, produce a final image at O, the location of the original object. There are then three images formed; each is real, same size, and inverted. Three inversions nets an inverted image. The final image at O is therefore inverted.

P40-13 (a) Place an object at o. The image will be at a point i' given by

$$\frac{1}{i'} = \frac{1}{f} - \frac{1}{o},$$

or $i' = fo/(o - f)$.

(b) The lens must be shifted a distance $i' - i$, or

$$i' - i = \frac{fo}{o - f} - 1.$$

(c) The range of motion is

$$\Delta i = \frac{(0.05\,\mathrm{m})(1.2\,\mathrm{m})}{(1.2\,\mathrm{m}) - (0.05\,\mathrm{m})} - 1 = -5.2 \text{ cm}.$$

P40-14 (a) Because magnification is proportional to $1/f$.

(b) Using the results of Problem 40-11,

$$\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1},$$

so $P = P_1 + P_2$.

$\boxed{\textbf{P40-15}}$ We want the maximum linear motion of the train to move no more than 0.75 mm on the film; this means we want to find the size of an object on the train that will form a 0.75 mm image. The object distance is much larger than the focal length, so the image distance is approximately equal to the focal length. The magnification is then $m = -i/o = (3.6 \text{ cm})/(44.5\,\mathrm{m}) = -0.00081$.

The size of an object on the train that would produce a 0.75 mm image on the film is then 0.75 mm/0.00081 = 0.93 m.

How much time does it take the train to move that far?

$$t = \frac{(0.93 \text{ m})}{(135 \text{ km/hr})(1/3600 \text{ hr/s})} = 25 \text{ ms}.$$

P40-16 (a) The derivation leading to Eq. 40-34 depends only on the fact that two converging optical devices are used. Replacing the objective lens with an objective mirror doesn't change anything except the ray diagram.

(b) The image will be located very close to the focal point, so $|m| \approx f/o$, and

$$h_\mathrm{i} = (1.0\,\mathrm{m})\frac{(16.8\,\mathrm{m})}{(2000\,\mathrm{m})} = 8.4\times10^{-3}\mathrm{m}$$

(c) $f_\mathrm{e} = (5\,\mathrm{m})/(200) = 0.025\,\mathrm{m}$. Note that we were given the radius of curvature, not the focal length, of the mirror!

$\boxed{\textbf{E41-1}}$ In this problem we look for the location of the third-order bright fringe, so

$$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(3)(554\times10^{-9}\text{m})}{(7.7\times10^{-6}\text{m})} = 12.5^\circ = 0.22 \text{ rad}.$$

E41-2 $d_1\sin\theta = \lambda$ gives the first maximum; $d_2\sin\theta = 2\lambda$ puts the second maximum at the location of the first. Divide the second expression by the first and $d_2 = 2d_1$. This is a 100% increase in d.

E41-3 $\Delta y = \lambda D/d = (512\times10^{-9}\text{m})(5.4\,\text{m})/(1.2\times10^{-3}\text{m}) = 2.3\times10^{-3}\text{m}$.

E41-4 $d = \lambda/\sin\theta = (592\times10^{-9}\text{m})/\sin(1.00^\circ) = 3.39\times10^{-5}\text{m}$.

$\boxed{\textbf{E41-5}}$ Since the angles are *very* small, we can assume $\sin\theta \approx \theta$ for angles measured in radians.

If the interference fringes are 0.23° apart, then the angular position of the first bright fringe is 0.23° away from the central maximum. Eq. 41-1, written with the small angle approximation in mind, is $d\theta = \lambda$ for this first ($m = 1$) bright fringe. The goal is to find the wavelength which increases θ by 10%. To do this we must increase the right hand side of the equation by 10%, which means increasing λ by 10%. The new wavelength will be $\lambda' = 1.1\lambda = 1.1(589 \text{ nm}) = 650 \text{ nm}$

E41-6 Immersing the apparatus in water will shorten the wavelengths to λ/n. Start with $d\sin\theta_0 = \lambda$; and then find θ from $d\sin\theta = \lambda/n$. Combining the two expressions,

$$\theta = \arcsin[\sin\theta_0/n] = \arcsin[\sin(0.20^\circ)/(1.33)] = 0.15^\circ.$$

E41-7 The third-order fringe for a wavelength λ will be located at $y = 3\lambda D/d$, where y is measured from the central maximum. Then Δy is

$$y_1 - y_2 = 3(\lambda_1 - \lambda_2)D/d = 3(612\times10^{-9}\text{m} - 480\times10^{-9}\text{m})(1.36\,\text{m})/(5.22\times10^{-3}\text{m}) = 1.03\times10^{-4}\text{m}.$$

E41-8 $\theta = \arctan(y/D)$;

$$\lambda = d\sin\theta = (0.120\,\text{m})\sin[\arctan(0.180\,\text{m}/2.0\,\text{m})] = 1.08\times10^{-2}\text{m}.$$

Then $f = v/\lambda = (0.25\,\text{m/s})/(1.08\times10^{-2}\text{m}) = 23$ Hz.

$\boxed{\textbf{E41-9}}$ A variation of Eq. 41-3 is in order:

$$y_m = \left(m + \frac{1}{2}\right)\frac{\lambda D}{d}$$

We are given the distance (on the screen) between the first minima ($m = 0$) and the tenth minima ($m = 9$). Then

$$18 \text{ mm} = y_9 - y_0 = 9\frac{\lambda(50 \text{ cm})}{(0.15 \text{ mm})},$$

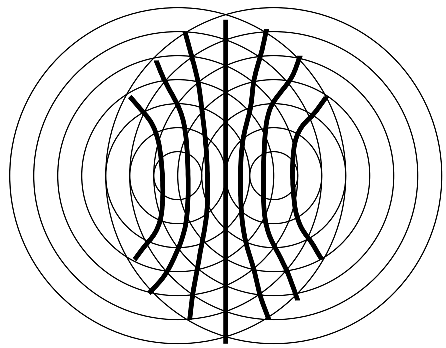
or $\lambda = 6\times10^{-4}$ mm $= 600$ nm.

E41-10 The "maximum" maxima is given by the integer part of

$$m = d\sin(90^\circ)/\lambda = (2.0\,\text{m})/(0.50\,\text{m}) = 4.$$

Since there is no integer part, the "maximum" maxima occurs at 90°. These are point sources radiating in both directions, so there are two central maxima, and four maxima each with $m = 1$, $m = 2$, and $m = 3$. But the $m = 4$ values overlap at 90°, so there are only two. The total is 16.

$\boxed{\textbf{E41-11}}$ This figure should explain it well enough.



E41-12 $\Delta y = \lambda D/d = (589\times10^{-9}\text{m})(1.13\,\text{m})/(0.18\times10^{-3}\text{m}) = 3.70\times10^{-3}\text{m}.$

E41-13 Consider Fig. 41-5, and solve it *exactly* for the information given. For the tenth bright fringe $r_1 = 10\lambda + r_2$. There are two important triangles:

$$r_2^2 = D^2 + (y - d/2)^2$$

and

$$r_1^2 = D^2 + (y + d/2)^2$$

Solving to eliminate r_2,

$$\sqrt{D^2 + (y + d/2)^2} = \sqrt{D^2 + (y - d/2)^2} + 10\lambda.$$

This has solution

$$y = 5\lambda\sqrt{\frac{4D^2 + d^2 - 100\lambda^2}{d^2 - 100\lambda^2}}.$$

The solution predicted by Eq. 41-1 is

$$y' = \frac{10\lambda}{d}\sqrt{D^2 + y'^2},$$

or

$$y' = 5\lambda\sqrt{\frac{4D^2}{d^2 - 100\lambda^2}}.$$

The fractional error is $y'/y - 1$, or

$$\sqrt{\frac{4D^2}{4D^2 + d^2 - 100\lambda^2}} - 1,$$

or

$$\sqrt{\frac{4(40\,\text{mm})^2}{4(40\,\text{mm})^2 + (2\,\text{mm})^2 - 100(589\times10^{-6}\text{mm})^2}} - 1 = -3.1\times10^{-4}.$$

E41-14 (a) $\Delta x = c/\Delta t = (3.00\times10^8\text{m/s})/(1\times10^{-8}\text{s}) = 3\,\text{m}.$
(b) No.

$\boxed{\textbf{E41-15}}$ Leading by 90° is the same as leading by a quarter wavelength, since there are 360° in a circle. The distance from A to the detector is 100 m longer than the distance from B to the detector. Since the wavelength is 400 m, 100 m corresponds to a quarter wavelength.

So a wave peak starts out from source A and travels to the detector. When it has traveled a quarter wavelength a wave peak leaves source B. But when the wave peak from A has traveled a quarter wavelength it is now located at the same distance from the detector as source B, which means the two wave peaks arrive at the detector at the same time.

They are in phase.

E41-16 The first dark fringe involves waves π radians out of phase. Each dark fringe after that involves an additional 2π radians of phase difference. So the mth dark fringe has a phase difference of $(2m+1)\pi$ radians.

E41-17 $I = 4I_0 \cos^2\left(\frac{2\pi d}{\lambda}\sin\theta\right)$, so for this problem we want to plot

$$I/I_0 = \cos^2\left(\frac{2\pi(0.60\,\text{mm})}{(600\times 10^{-9}\text{m})}\sin\theta\right) = \cos^2\left(6280\sin\theta\right).$$

E41-18 The resultant quantity will be of the form $A\sin(\omega t+\beta)$. Solve the problem by looking at $t=0$; then $y_1 = 0$, but $x_1 = 10$, and $y_2 = 8\sin 30° = 4$ and $x_2 = 8\cos 30 = 6.93$. Then the resultant is of length

$$A = \sqrt{(4)^2 + (10+6.93)^2} = 17.4,$$

and has an angle β given by

$$\beta = \arctan(4/16.93) = 13.3°.$$

$\boxed{\textbf{E41-19}}$ (a) We want to know the path length difference of the two sources to the detector. Assume the detector is at x and the second source is at $y = d$. The distance S_1D is x; the distance S_2D is $\sqrt{x^2+d^2}$. The difference is $\sqrt{x^2+d^2} - x$. If this difference is an integral number of wavelengths then we have a maximum; if instead it is a half integral number of wavelengths we have a minimum. For part (a) we are looking for the maxima, so we set the path length difference equal to $m\lambda$ and solve for x_m.

$$\begin{aligned}
\sqrt{x_m^2 + d^2} - x_m &= m\lambda, \\
x_m^2 + d^2 &= (m\lambda + x_m)^2, \\
x_m^2 + d^2 &= m^2\lambda^2 + 2m\lambda x_m + x_m^2, \\
x_m &= \frac{d^2 - m^2\lambda^2}{2m\lambda}
\end{aligned}$$

The first question we need to ask is what happens when $m = 0$. The right hand side becomes indeterminate, so we need to go back to the first line in the above derivation. If $m = 0$ then $d^2 = 0$; since this is *not* true in this problem, there is no $m = 0$ solution.

In fact, we may have even more troubles. x_m needs to be a positive value, so the maximum allowed value for m will be given by

$$\begin{aligned}
m^2\lambda^2 &< d^2, \\
m &< d/\lambda = (4.17\,\text{m})/(1.06\,\text{m}) = 3.93;
\end{aligned}$$

but since m is an integer, $m = 3$ is the maximum value.

The first three maxima occur at $m = 3$, $m = 2$, and $m = 1$. These maxima are located at

$$
\begin{aligned}
x_3 &= \frac{(4.17\,\text{m})^2 - (3)^2(1.06\,\text{m})^2}{2(3)(1.06\,\text{m})} = 1.14 \text{ m},\\
x_2 &= \frac{(4.17\,\text{m})^2 - (2)^2(1.06\,\text{m})^2}{2(2)(1.06\,\text{m})} = 3.04 \text{ m},\\
x_1 &= \frac{(4.17\,\text{m})^2 - (1)^2(1.06\,\text{m})^2}{2(1)(1.06\,\text{m})} = 7.67 \text{ m}.
\end{aligned}
$$

Interestingly enough, as m decreases the maxima get farther away!

(b) The closest maxima to the origin occurs at $x = \pm 6.94$ cm. What then is $x = 0$? It is a local minimum, but the intensity isn't zero. It corresponds to a point where the path length difference is 3.93 wavelengths. It should be half an integer to be a complete minimum.

E41-20 The resultant can be written in the form $A\sin(\omega t + \beta)$. Consider $t = 0$. The three components can be written as

$$
\begin{aligned}
y_1 &= 10\sin 0^\circ = 0,\\
y_2 &= 14\sin 26^\circ = 6.14,\\
y_3 &= 4.7\sin(-41^\circ) = -3.08,\\
y &= 0 + 6.14 - 3.08 = 3.06.
\end{aligned}
$$

and

$$
\begin{aligned}
x_1 &= 10\cos 0^\circ = 10,\\
x_2 &= 14\cos 26^\circ = 12.6,\\
x_3 &= 4.7\cos(-41^\circ) = 3.55,\\
x &= 10 + 12.6 + 3.55 = 26.2.
\end{aligned}
$$

Then $A = \sqrt{(3.06)^2 + (26.2)^2} = 26.4$ and $\beta = \arctan(3.06/26.2) = 6.66^\circ$.

E41-21 The order of the indices of refraction is the same as in Sample Problem 41-4, so

$$d = \lambda/4n = (620 \text{ nm})/4(1.25) = 124 \text{ nm}.$$

E41-22 Follow the example in Sample Problem 41-3.

$$\lambda = \frac{2dn}{m - 1/2} = \frac{2(410 \text{ nm})(1.50)}{m - 1/2} = \frac{1230 \text{ nm}}{m - 1/2}.$$

The result is only in the visible range when $m = 3$, so $\lambda = 492$ nm.

$\boxed{\textbf{E41-23}}$ (a) Light from above the oil slick can be reflected back up from the top of the oil layer or from the bottom of the oil layer. For both reflections the light is reflecting off a substance with a higher index of refraction so both reflected rays pick up a phase change of π. Since both waves have this phase the equation for a maxima is

$$2d + \frac{1}{2}\lambda_n + \frac{1}{2}\lambda_n = m\lambda_n.$$

Remember that $\lambda_n = \lambda/n$, where n is the index of refraction of the thin film. Then $2nd = (m-1)\lambda$ is the condition for a maxima. We know $n = 1.20$ and $d = 460$ nm. We don't know m or λ. It might seem as if there isn't enough information to solve the problem, but we can. We need to find the wavelength in the visible range (400 nm to 700 nm) which has an integer m. Trial and error might work. If $\lambda = 700$ nm, then m is

$$m = \frac{2nd}{\lambda} + 1 = \frac{2(1.20)(460 \text{ nm})}{(700 \text{ nm})} + 1 = 2.58$$

But m needs to be an integer. If we increase m to 3, then

$$\lambda = \frac{2(1.20)(460 \text{ nm})}{(3-1)} = 552 \text{ nm}$$

which is in the visible range. So the oil slick will appear green.

(b) One of the most profound aspects of thin film interference is that wavelengths which are maximally reflected are minimally transmitted, and vice versa. Finding the maximally transmitted wavelengths is the same as finding the minimally reflected wavelengths, or looking for values of m that are half integer.

The most obvious choice is $m = 3.5$, and then

$$\lambda = \frac{2(1.20)(460 \text{ nm})}{(3.5-1)} = 442 \text{ nm}.$$

E41-24 The condition for constructive interference is $2nd = (m - 1/2)\lambda$. Assuming a minimum value of $m = 1$ one finds

$$d = \lambda/4n = (560 \text{ nm})/4(2.0) = 70 \text{ nm}.$$

E41-25 The top surface contributes a phase difference of π, so the phase difference because of the thickness is 2π, or one complete wavelength. Then $2d = \lambda/n$, or $d = (572 \text{ nm})/2(1.33) = 215$ nm.

E41-26 The wave reflected from the first surface picks up a phase shift of π. The wave which is reflected off of the second surface travels an additional path difference of $2d$. The interference will be bright if $2d + \lambda_n/2 = m\lambda_n$ results in m being an integer.

$$m = 2nd/\lambda + 1/2 = 2(1.33)(1.21\times10^{-6}\text{m})/(585\times10^{-9}\text{m}) + 1/2 = 6.00,$$

so the interference is bright.

$\boxed{\textbf{E41-27}}$ As with the oil on the water in Ex. 41-23, both the light which reflects off of the acetone and the light which reflects off of the glass undergoes a phase shift of π. Then the maxima for reflection are given by $2nd = (m - 1)\lambda$. We don't know m, but at some integer value of m we have $\lambda = 700$ nm. If m is increased by exactly $\frac{1}{2}$ then we are at a minimum of $\lambda = 600$ nm. Consequently,

$$2(1.25)d = (m-1)(700 \text{ nm}) \text{ and } 2(1.25)d = (m - 1/2)(600 \text{ nm}),$$

we can set these two expressions equal to each other to find m,

$$(m-1)(700 \text{ nm}) = (m - 1/2)(600 \text{ nm}),$$

so $m = 4$. Then we can find the thickness,

$$d = (4-1)(700 \text{ nm})/2(1.25) = 840 \text{ nm}.$$

E41-28 The wave reflected from the first surface picks up a phase shift of π. The wave which is reflected off of the second surface travels an additional path difference of $2d$. The interference will be bright if $2d + \lambda_n/2 = m\lambda_n$ results in m being an integer. Then $2nd = (m - 1/2)\lambda_1$ is bright, and $2nd = m\lambda_2$ is dark. Divide one by the other and $(m - 1/2)\lambda_1 = m\lambda_2$, so

$$m = \lambda_1/2(\lambda_1 - \lambda_2) = (600\text{ nm})/2(600\text{ nm} - 450\text{ nm}) = 2,$$

then $d = m\lambda_2/2n = (2)(450\text{ nm})/2(1.33) = 338\text{ nm}$.

E41-29 Constructive interference happens when $2d = (m - 1/2)\lambda$. The minimum value for m is $m = 1$; the maximum value is the integer portion of $2d/\lambda + 1/2 = 2(4.8\times10^{-5}\text{m})/(680\times10^{-9}\text{m}) + 1/2 = 141.67$, so $m_{\text{max}} = 141$. There are then 141 bright bands.

E41-30 (a) A half wavelength phase shift occurs for both the air/water interface and the water/oil interface, so if $d = 0$ the two reflected waves are in phase. It will be bright!

(b) $2nd = 3\lambda$, or $d = 3(475\text{ nm})/2(1.20) = 594\text{ nm}$.

E41-31 There is a phase shift on one surface only, so the bright bands are given by $2nd = (m - 1/2)\lambda$. Let the first band be given by $2nd_1 = (m_1 - 1/2)\lambda$. The last bright band is then given by $2nd_2 = (m_1 + 9 - 1/2)\lambda$. Subtract the two equations to get the change in thickness:

$$\Delta d = 9\lambda/2n = 9(630\text{ nm})/2(1.50) = 1.89\,\mu\text{m}.$$

E41-32 Apply Eq. 41-21: $2nd = m\lambda$. In one case we have

$$2n_{\text{air}} = (4001)\lambda,$$

in the other,

$$2n_{\text{vac}} = (4000)\lambda.$$

Equating, $n_{\text{air}} = (4001)/(4000) = 1.00025$.

$\boxed{\textbf{E41-33}}$ (a) We can start with the last equation from Sample Problem 41-5,

$$r = \sqrt{(m - \frac{1}{2})\lambda R},$$

and solve for m,

$$m = \frac{r^2}{\lambda R} + \frac{1}{2}$$

In this exercise $R = 5.0\,\text{m}$, $r = 0.01\,\text{m}$, and $\lambda = 589\text{ nm}$. Then

$$m = \frac{(0.01\,\text{m})^2}{(589\text{ nm})(5.0\,\text{m})} = 34$$

is the number of rings observed.

(b) Putting the apparatus in water effectively changes the wavelength to

$$(589\text{ nm})/(1.33) = 443\text{ nm},$$

so the number of rings will now be

$$m = \frac{(0.01\,\text{m})^2}{(443\text{ nm})(5.0\,\text{m})} = 45.$$

E41-34 $(1.42 \text{ cm}) = \sqrt{(10-\frac{1}{2})R\lambda}$, while $(1.27 \text{ cm}) = \sqrt{(10-\frac{1}{2})R\lambda/n}$. Divide one expression by the other, and $(1.42 \text{ cm})/(1.27 \text{ cm}) = \sqrt{n}$, or $n = 1.25$.

E41-35 $(0.162 \text{ cm}) = \sqrt{(n-\frac{1}{2})R\lambda}$, while $(0.368 \text{ cm}) = \sqrt{(n+20-\frac{1}{2})R\lambda}$. Square both expressions, the divide one by the other, and find

$$(n+19.5)/(n-0.5) = (0.368 \text{ cm}/0.162 \text{ cm})^2 = 5.16$$

which can be rearranged to yield

$$n = \frac{19.5 + 5.16 \times 0.5}{5.16 - 1} = 5.308.$$

Oops! That should be an integer, shouldn't it? The above work is correct, which means that there really aren't bright bands at the specified locations. I'm just going to gloss over that fact and solve for R using the value of $m = 5.308$. Then

$$R = r^2/(m-1/2)\lambda = (0.162 \text{ cm})^2/(5.308-0.5)(546 \text{ nm}) = 1.00\,\text{m}.$$

Well, at least we got the answer which is in the back of the book...

E41-36 Pretend the ship is a two point source emitter, one h above the water, and one h below the water. The one below the water is out of phase by half a wavelength. Then $d\sin\theta = \lambda$, where $d = 2h$, gives the angle for theta for the first minimum.

$$\lambda/2h = (3.43\,\text{m})/2(23\,\text{m}) = 7.46\times10^{-2} = \sin\theta \approx H/D,$$

so $D = (160\,\text{m})/(7.46\times10^{-2}) = 2.14$ km.

E41-37 The phase difference is $2\pi/\lambda_n$ times the path difference which is $2d$, so

$$\phi = 4\pi d/\lambda_n = 4\pi n d/\lambda.$$

We are given that $d = 100\times10^{-9}$m and $n = 1.38$.

(a) $\phi = 4\pi(1.38)(100\times10^{-9}\text{m})/(450\times10^{-9}\text{m}) = 3.85$. Then

$$\frac{I}{I_0} = \cos^2\frac{(3.85)}{2} = 0.12.$$

The reflected ray is diminished by $1 - 0.12 = 88\%$.

(b) $\phi = 4\pi(1.38)(100\times10^{-9}\text{m})/(650\times10^{-9}\text{m}) = 2.67$. Then

$$\frac{I}{I_0} = \cos^2\frac{(2.67)}{2} = 0.055.$$

The reflected ray is diminished by $1 - 0.055 = 95\%$.

E41-38 The change in the optical path length is $2(d - d/n)$, so $7\lambda/n = 2d(1-1/n)$, or

$$d = \frac{7(589\times10^{-9}\text{m})}{2(1.42)-2} = 4.9\times10^{-6}\text{m}.$$

$\boxed{\textbf{E41-39}}$ When M_2 moves through a distance of $\lambda/2$ a fringe has will be produced, destroyed, and then produced again. This is because the light travels twice through any change in distance. The wavelength of light is then

$$\lambda = \frac{2(0.233\text{ mm})}{792} = 588\text{ nm}.$$

E41-40 The change in the optical path length is $2(d - d/n)$, so $60\lambda = 2d(1 - 1/n)$, or

$$n = \frac{1}{1 - 60\lambda/2d} = \frac{1}{1 - 60(500\times10^{-9}\text{m})/2(5\times10^{-2}\text{m})} = 1.00030.$$

$\boxed{\textbf{P41-1}}$ (a) This is a small angle problem, so we use Eq. 41-4. The distance to the screen is $2 \times 20\,\text{m}$, because the light travels to the mirror and back again. Then

$$d = \frac{\lambda D}{\Delta y} = \frac{(632.8\text{ nm})(40.0\,\text{m})}{(0.1\,\text{m})} = 0.253\text{ mm}.$$

(b) Placing the cellophane over one slit will cause the interference pattern to shift to the left or right, but not disappear or change size. How does it shift? Since we are picking up 2.5 waves then we are, in effect, swapping bright fringes for dark fringes.

P41-2 The change in the optical path length is $d - d/n$, so $7\lambda/n = d(1 - 1/n)$, or

$$d = \frac{7(550\times10^{-9}\text{m})}{(1.58) - 1} = 6.64\times10^{-6}\text{m}.$$

P41-3 The distance from S_1 to P is $r_1 = \sqrt{(x + d/2)^2 + y^2}$. The distance from S_2 to P is $r_2 = \sqrt{(x - d/2)^2 + y^2}$. The difference in distances is fixed at some value, say c, so that

$$\begin{aligned}
r_1 - r_2 &= c, \\
r_1^2 - 2r_1r_2 + r_2^2 &= c^2, \\
(r_1^2 + r_2^2 - c^2)^2 &= 4r_1^2r_2^2, \\
(r_1^2 - r_2^2)^2 - 2c^2(r_1^2 + r_2^2) + c^4 &= 0, \\
(2xd)^2 - 2c^2(2x^2 + d^2/2 + 2y^2) + c^4 &= 0, \\
4x^2d^2 - 4c^2x^2 - c^2d^2 - 4c^2y^2 + c^4 &= 0, \\
4(d^2 - c^2)x^2 - 4c^2y^2 &= c^2(d^2 - c^2).
\end{aligned}$$

Yes, that is the equation of a hyperbola.

P41-4 The change in the optical path length for each slit is $nt - t$, where n is the corresponding index of refraction. The net change in the path difference is then $n_2t - n_1t$. Consequently, $m\lambda = t(n_2 - n_1)$, so

$$t = \frac{(5)(480\times10^{-9}\text{m})}{(1.7) - (1.4)} = 8.0\times10^{-6}\text{m}.$$

$\boxed{\textbf{P41-5}}$ The intensity is given by Eq. 41-17, which, in the small angle approximation, can be written as

$$I_\theta = 4I_0\cos^2\left(\frac{\pi d\theta}{\lambda}\right).$$

The intensity will be half of the maximum when

$$\frac{1}{2} = \cos^2\left(\frac{\pi d\Delta\theta/2}{\lambda}\right)$$

or

$$\frac{\pi}{4} = \frac{\pi d\Delta\theta}{2\lambda},$$

which will happen if $\Delta\theta = \lambda/2d$.

P41-6 Follow the construction in Fig. 41-10, except that one of the electric field amplitudes is twice the other. The resultant field will have a length given by

$$\begin{aligned} E' &= \sqrt{(2E_0 + E_0\cos\phi)^2 + (E_0\sin\phi)^2}, \\ &= E_0\sqrt{5 + 4\cos\phi}, \end{aligned}$$

so squaring this yields

$$\begin{aligned} I &= I_0\left(5 + 4\cos\frac{2\pi d\sin\theta}{\lambda}\right), \\ &= I_0\left(1 + 8\cos^2\frac{\pi d\sin\theta}{\lambda}\right), \\ &= \frac{I_\mathrm{m}}{9}\left(1 + 8\cos^2\frac{\pi d\sin\theta}{\lambda}\right). \end{aligned}$$

$\boxed{\textbf{P41-7}}$ We actually did this problem in Exercise 41-27, although slightly differently. One maximum is

$$2(1.32)d = (m - 1/2)(679 \text{ nm}),$$

the other is

$$2(1.32)d = (m + 1/2)(485 \text{ nm}).$$

Set these equations equal to each other,

$$(m - 1/2)(679 \text{ nm}) = (m + 1/2)(485 \text{ nm}),$$

and find $m = 3$. Then the thickness is

$$d = (3 - 1/2)(679 \text{ nm})/2(1.32) = 643 \text{ nm}.$$

P41-8 (a) Since we are concerned with transmission there is a phase shift for two rays, so

$$2d = m\lambda_n$$

The minimum thickness occurs when $m = 1$; solving for d yields

$$d = \frac{\lambda}{2n} = \frac{(525\times10^{-9}\text{m})}{2(1.55)} = 169\times10^{-9}\text{m}.$$

(b) The wavelengths are different, so the other parts have differing phase differences.
(c) The nearest destructive interference wavelength occurs when $m = 1.5$, or

$$\lambda = 2nd = 2(1.55)1.5(169\times10^{-9}\text{m}) = 393\times10^{-9}\text{m}.$$

This is blue-violet.

P41-9 It doesn't matter if we are looking at bright are dark bands. It doesn't even matter if we concern ourselves with phase shifts. All that cancels out. Consider $2\delta d = \delta m\lambda$; then

$$\delta d = (10)(480 \text{ nm})/2 = 2.4\,\mu\text{m}.$$

P41-10 (a) Apply $2d = m\lambda$. Then

$$d = (7)(600\times10^{-9}\text{m})/2 = 2100\times10^{-9}\text{m}.$$

(b) When water seeps in it introduces an extra phase shift. Point A becomes then a bright fringe, and the equation for the number of bright fringes is $2nd = m\lambda$. Solving for m,

$$m = 2(1.33)(2100\times10^{-9}\text{m})/(600\times10^{-9}\text{m}) = 9.3;$$

this means that point B is almost, but not quite, a dark fringe, and there are *nine* of them.

$\boxed{\textbf{P41-11}}$ (a) Look back at the work for Sample Problem 41-5 where it was found

$$r_m = \sqrt{(m - \frac{1}{2})\lambda R},$$

We can write this as

$$r_m = \sqrt{\left(1 - \frac{1}{2m}\right) m\lambda R}$$

and expand the part in parentheses in a binomial expansion,

$$r_m \approx \left(1 - \frac{1}{2}\frac{1}{2m}\right)\sqrt{m\lambda R}.$$

We will do the same with

$$r_{m+1} = \sqrt{(m + 1 - \frac{1}{2})\lambda R},$$

expanding

$$r_{m+1} = \sqrt{\left(1 + \frac{1}{2m}\right) m\lambda R}$$

to get

$$r_{m+1} \approx \left(1 + \frac{1}{2}\frac{1}{2m}\right)\sqrt{m\lambda R}.$$

Then

$$\Delta r \approx \frac{1}{2m}\sqrt{m\lambda R},$$

or

$$\Delta r \approx \frac{1}{2}\sqrt{\lambda R/m}.$$

(b) The area between adjacent rings is found from the difference,

$$A = \pi\left(r_{m+1}^2 - r_m^2\right),$$

and into this expression we will substitute the *exact* values for r_m and r_{m+1},

$$\begin{aligned} A &= \pi\left((m + 1 - \frac{1}{2})\lambda R - (m - \frac{1}{2})\lambda R\right), \\ &= \pi\lambda R. \end{aligned}$$

Unlike part (a), we did not need to assume $m \gg 1$ in order to arrive at this expression; it is exact for all m.

P41-12 The path length shift that occurs when moving the mirror as distance x is $2x$. This means $\phi = 2\pi 2x/\lambda = 4\pi x/\lambda$. The intensity is then

$$I = 4I_0 \cos^2 \frac{2\pi x}{\lambda}$$

E42-1 $\lambda = a \sin\theta = (0.022\,\text{mm})\sin(1.8^\circ) = 6.91\times10^{-7}\text{m}$.

E42-2 $a = \lambda/\sin\theta = (0.10\times10^{-9}\text{m})/\sin(0.12\times10^{-3}\text{ rad}/2) = 1.7\,\mu\text{m}$.

$\boxed{\textbf{E42-3}}$ (a) This is a valid small angle approximation problem: the distance between the points on the screen is much less than the distance to the screen. Then

$$\theta \approx \frac{(0.0162\,\text{m})}{(2.16\,\text{m})} = 7.5\times10^{-3}\text{ rad}.$$

(b) The diffraction minima are described by Eq. 42-3,

$$\begin{aligned} a\sin\theta &= m\lambda, \\ a\sin(7.5\times10^{-3}\text{ rad}) &= (2)(441\times10^{-9}\,\text{m}), \\ a &= 1.18\times10^{-4}\,\text{m}. \end{aligned}$$

E42-4 $a = \lambda/\sin\theta = (633\times10^{-9}\text{m})/\sin(1.97^\circ/2) = 36.8\,\mu\text{m}$.

$\boxed{\textbf{E42-5}}$ (a) We again use Eq. 42-3, but we will need to throw in a few extra subscripts to distinguish between which wavelength we are dealing with. If the angles match, then so will the sine of the angles. We then have $\sin\theta_{a,1} = \sin\theta_{b,2}$ or, using Eq. 42-3,

$$\frac{(1)\lambda_a}{a} = \frac{(2)\lambda_b}{a},$$

from which we can deduce $\lambda_a = 2\lambda_b$.

(b) Will any other minima coincide? We want to solve for the values of m_a and m_b that will be integers and have the same angle. Using Eq. 42-3 one more time,

$$\frac{m_a\lambda_a}{a} = \frac{m_b\lambda_b}{a},$$

and then substituting into this the relationship between the wavelengths, $m_a = m_b/2$. whenever m_b is an even integer m_a is an integer. Then *all* of the diffraction minima from λ_a are overlapped by a minima from λ_b.

E42-6 The angle is given by $\sin\theta = 2\lambda/a$. This is a small angle, so we can use the small angle approximation of $\sin\theta = y/D$. Then

$$y = 2D\lambda/a = 2(0.714\,\text{m})(593\times10^{-9}\text{m})/(420\times10^{-6}\text{m}) = 2.02\,\text{mm}.$$

E42-7 Small angles, so $y/D = \sin\theta = \lambda/a$. Then

$$a = D\lambda/y = (0.823\,\text{m})(546\times10^{-9}\text{m})/(5.20\times10^{-3}\text{m}/2) = 1.73\times10^{-4}\text{m}.$$

E42-8 (b) Small angles, so $\Delta y/D = \Delta m\lambda/a$. Then

$$a = \Delta mD\lambda/\Delta y = (5-1)(0.413\,\text{m})(546\times10^{-9}\text{m})/(0.350\times10^{-3}\text{m}) = 2.58\,\text{mm}.$$

(a) $\theta = \arcsin(\lambda/a) = \arcsin[(546\times10^{-9}\text{m})/(2.58\,\text{mm})] = 1.21\times10^{-2\circ}$.

E42-9 Small angles, so $\Delta y/D = \Delta m\lambda/a$. Then

$$\Delta y = \Delta mD\lambda/a = (2-1)(2.94\,\text{m})(589\times10^{-9}\text{m})/(1.16\times10^{-3}\text{m}) = 1.49\times10^{-3}\text{m}.$$

E42-10 Doubling the width of the slit results in a narrowing of the diffraction pattern. Since the width of the central maximum is effectively cut in half, then there is twice the energy in half the space, producing four times the intensity.

$\boxed{\textbf{E42-11}}$ (a) This is a small angle approximation problem, so

$$\theta = (1.13\text{ cm})/(3.48\,\text{m}) = 3.25 \times 10^{-3}\text{ rad}.$$

(b) A convenient measure of the phase difference, α is related to θ through Eq. 42-7,

$$\alpha = \frac{\pi a}{\lambda}\sin\theta = \frac{\pi(25.2\times 10^{-6}\text{m})}{(538\times 10^{-9}\text{m})}\sin(3.25\times 10^{-3}\text{ rad}) = 0.478\text{ rad}$$

(c) The intensity at a point is related to the intensity at the central maximum by Eq. 42-8,

$$\frac{I_\theta}{I_\text{m}} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin(0.478\text{ rad})}{(0.478\text{ rad})}\right)^2 = 0.926$$

E42-12 Consider Fig. 42-11; the angle with the vertical is given by $(\pi - \phi)/2$. For Fig. 42-10(d) the circle has wrapped once around onto itself so the angle with the vertical is $(3\pi - \phi)/2$. Substitute α into this expression and the angel against the vertical is $3\pi/2 - \alpha$.

Use the result from Problem 42-3 that $\tan\alpha = \alpha$ for the maxima. The lowest non-zero solution is $\alpha = 4.49341$ rad. The angle against the vertical is then 0.21898 rad, or 12.5°.

E42-13 Drawing heavily from Sample Problem 42-4,

$$\theta_x = \arcsin\left(\frac{\alpha_x\lambda}{\pi a}\right) = \arcsin\left(\frac{1.39}{10\pi}\right) = 2.54^\circ.$$

Finally, $\Delta\theta = 2\theta_x = 5.1^\circ$.

E42-14 (a) Rayleigh's criterion for resolving images (Eq. 42-11) requires that two objects have an angular separation of at least

$$\theta_\text{R} = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(540\times 10^{-9})}{(4.90\times 10^{-3}\text{m})}\right) = 1.34\times 10^{-4}\text{ rad}$$

(b) The linear separation is $y = \theta D = (1.34\times 10^{-4}\text{ rad})(163\times 10^3\text{m}) = 21.9\,\text{m}$.

$\boxed{\textbf{E42-15}}$ (a) Rayleigh's criterion for resolving images (Eq. 42-11) requires that two objects have an angular separation of at least

$$\theta_\text{R} = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(562\times 10^{-9})}{(5.00\times 10^{-3}\text{m})}\right) = 1.37\times 10^{-4}\text{ rad}.$$

(b) Once again, this is a small angle, so we can use the small angle approximation to find the distance to the car. In that case $\theta_\text{R} = y/D$, where y is the headlight separation and D the distance to the car. Solving,

$$D = y/\theta_R = (1.42\,\text{m})/(1.37\times 10^{-4}\text{ rad}) = 1.04\times 10^4\text{m},$$

or about six or seven miles.

E42-16 $y/D = 1.22\lambda/a$; or

$$D = (5.20\times10^{-3}\mathrm{m})(4.60\times10^{-3}/m)/1.22(542\times10^{-9}\mathrm{m}) = 36.2\,\mathrm{m}.$$

$\boxed{\textbf{E42-17}}$ The smallest resolvable angular separation will be given by Eq. 42-11,

$$\theta_\mathrm{R} = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(565\times10^{-9}\mathrm{m})}{(5.08\,\mathrm{m})}\right) = 1.36\times10^{-7}\ \mathrm{rad},$$

The smallest objects resolvable on the Moon's surface by this telescope have a size y where

$$y = D\theta_\mathrm{R} = (3.84\times10^{8}\,\mathrm{m})(1.36\times10^{-7}\ \mathrm{rad}) = 52.2\,\mathrm{m}$$

E42-18 $y/D = 1.22\lambda/a$; or

$$y = 1.22(1.57\times10^{-2}\mathrm{m})(6.25\times10^{3}\mathrm{m})/(2.33\,\mathrm{m}) = 51.4\,\mathrm{m}$$

E42-19 $y/D = 1.22\lambda/a$; or

$$D = (4.8\times10^{-2}\mathrm{m})(4.3\times10^{-3}/m)/1.22(0.12\times10^{-9}\mathrm{m}) = 1.4\times10^{6}\,\mathrm{m}.$$

E42-20 $y/D = 1.22\lambda/a$; or

$$d = 1.22(550\times10^{-9}\mathrm{m})(160\times10^{3}\mathrm{m})/(0.30\,\mathrm{m}) = 0.36\,\mathrm{m}.$$

$\boxed{\textbf{E42-21}}$ Using Eq. 42-11, we find the minimum resolvable angular separation is given by

$$\theta_\mathrm{R} = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(475\times10^{-9}\mathrm{m})}{(4.4\times10^{-3}\mathrm{m})}\right) = 1.32\times10^{-4}\ \mathrm{rad}$$

The dots are 2 mm apart, so we want to stand a distance D away such that

$$D > y/\theta_\mathrm{R} = (2\times10^{-3}\mathrm{m})/(1.32\times10^{-4}\ \mathrm{rad}) = 15\,\mathrm{m}.$$

E42-22 $y/D = 1.22\lambda/a$; or

$$y = 1.22(500\times10^{-9}\mathrm{m})(354\times10^{3}\mathrm{m})/(9.14\,\mathrm{m}/2) = 4.73\times10^{-2}\,\mathrm{m}.$$

E42-23 (a) $\lambda = v/f$. Now use Eq. 42-11:

$$\theta = \arcsin\left(1.22\frac{(1450\,\mathrm{m/s})}{(25\times10^{3}\mathrm{Hz})(0.60\,\mathrm{m})}\right) = 6.77^\circ.$$

(b) Following the same approach,

$$\theta = \arcsin\left(1.22\frac{(1450\,\mathrm{m/s})}{(1\times10^{3}\mathrm{Hz})(0.60\,\mathrm{m})}\right)$$

has no real solution, so there is no minimum.

E42-24 (a) $\lambda = v/f$. Now use Eq. 42-11:

$$\theta = \arcsin\left(1.22\frac{(3\times10^8\,\text{m/s})}{(220\times10^9\text{Hz})(0.55\,\text{m})}\right) = 0.173^\circ.$$

This is the angle from the central maximum; the angular width is twice this, or 0.35°.

(b) use Eq. 42-11:

$$\theta = \arcsin\left(1.22\frac{(0.0157\,\text{m})}{(2.33\,\text{m})}\right) = 0.471^\circ.$$

This is the angle from the central maximum; the angular width is twice this, or 0.94°.

$\boxed{\textbf{E42-25}}$ The linear separation of the fringes is given by

$$\frac{\Delta y}{D} = \Delta\theta = \frac{\lambda}{d} \text{ or } \Delta y = \frac{\lambda D}{d}$$

for sufficiently small d compared to λ.

E42-26 (a) $d\sin\theta = 4\lambda$ gives the location of the fourth interference maximum, while $a\sin\theta = \lambda$ gives the location of the first diffraction minimum. Hence, if $d = 4a$ there will be no fourth interference maximum!

(b) Since $d\sin\theta_{m_i} = m_i\lambda$ gives the interference maxima and $a\sin\theta_{m_d} = m_d\lambda$ gives the diffraction minima, and $d = 4a$, then whenever $m_i = 4m_d$ there will be a missing maximum.

E42-27 (a) The central diffraction envelope is contained in the range

$$\theta = \arcsin\frac{\lambda}{a}$$

This angle corresponds to the mth maxima of the interference pattern, where

$$\sin\theta = m\lambda/d = m\lambda/2a.$$

Equating, $m = 2$, so there are three interference bands, since the $m = 2$ band is "washed out" by the diffraction minimum.

(b) If $d = a$ then $\beta = \alpha$ and the expression reduces to

$$\begin{aligned} I\theta &= I_\text{m} cos^2\alpha\frac{\sin^2\alpha}{\alpha^2}, \\ &= I_\text{m}\frac{\sin^2(2\alpha)}{2\alpha^2}, \\ &= 2I_\text{m}\left(\frac{\sin\alpha'}{\alpha'}\right)^2, \end{aligned}$$

where $\alpha = 2\alpha'$, which is the same as replacing a by $2a$.

E42-28 Remember that the central peak has an envelope width twice that of any other peak. Ignoring the central maximum there are $(11-1)/2 = 5$ fringes in any other peak envelope.

$\boxed{\textbf{E42-29}}$ (a) The first diffraction minimum is given at an angle θ such that $a \sin\theta = \lambda$; the order of the interference maximum at that point is given by $d \sin\theta = m\lambda$. Dividing one expression by the other we get $d/a = m$, with solution $m = (0.150)/(0.030) = 5$. The fact that the answer is *exactly* 5 implies that the fifth interference maximum is squelched by the diffraction minimum. Then there are only four complete fringes on either side of the central maximum. Add this to the central maximum and we get nine as the answer.

(b) For the third fringe $m = 3$, so $d \sin\theta = 3\lambda$. Then β is Eq. 42-14 is 3π, while α in Eq. 42-16 is

$$\alpha = \frac{\pi a}{\lambda}\frac{3\lambda}{d} = 3\pi\frac{a}{d},$$

so the relative intensity of the third fringe is, from Eq. 42-17,

$$(\cos 3\pi)^2 \left(\frac{\sin(3\pi a/d)}{(3\pi a/d)}\right)^2 = 0.255.$$

P42-1 $y = m\lambda D/a$. Then

$$y = (10)(632.8\times10^{-9}\text{m})(2.65\,\text{m})/(1.37\times10^{-3}\text{m}) = 1.224\times10^{-2}\text{m}.$$

The separation is twice this, or 2.45 cm.

P42-2 If $a \gg \lambda$ then the diffraction pattern is extremely tight, and there is effectively no light at P. In the event that either shape produces an interference pattern at P then the other shape must produce an equal but opposite electric field vector at that point so that when both patterns from both shapes are superimposed the field cancel.

But the intensity is the field vector squared; hence the two patterns look identical.

$\boxed{\textbf{P42-3}}$ (a) We want to take the derivative of Eq. 42-8 with respect to α, so

$$\begin{aligned}
\frac{dI_\theta}{d\alpha} &= \frac{d}{d\alpha} I_\text{m}\left(\frac{\sin\alpha}{\alpha}\right)^2, \\
&= I_\text{m}2\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\cos\alpha}{\alpha} - \frac{\sin\alpha}{\alpha^2}\right), \\
&= I_\text{m}2\frac{\sin\alpha}{\alpha^3}\left(\alpha\cos\alpha - \sin\alpha\right).
\end{aligned}$$

This equals zero whenever $\sin\alpha = 0$ or $\alpha\cos\alpha = \sin\alpha$; the former is the case for a minima while the latter is the case for the maxima. The maxima case can also be written as

$$\tan\alpha = \alpha.$$

(b) Note that as the order of the maxima increases the solutions get closer and closer to odd integers times $\pi/2$. The solutions are

$$\alpha = 0,\ 1.43\pi,\ 2.46\pi,\ \text{etc.}$$

(c) The m values are $m = \alpha/\pi - 1/2$, and correspond to

$$m = 0.5,\ 0.93,\ 1.96,\ \text{etc.}$$

These values will get closer and closer to integers as the values are increased.

P42-4 The outgoing beam strikes the moon with a circular spot of radius

$$r = 1.22\lambda D/a = 1.22(0.69\times10^{-6}\text{m})(3.82\times10^{8}\text{m})/(2\times1.3\,\text{m}) = 123\,\text{m}.$$

The light is *not* evenly distributed over this circle.

If P_0 is the power in the light, then

$$P_0 = \int I_\theta r\,dr\,d\phi = 2\pi\int_0^R I_\theta r\,dr,$$

where R is the radius of the central peak and I_θ is the angular intensity. For $a \gg \lambda$ we can write $\alpha \approx \pi a r/\lambda D$, then

$$P_0 = 2\pi I_m\left(\frac{\lambda D}{\pi a}\right)^2\int_0^{\pi/2}\frac{\sin^2\alpha}{\alpha}d\alpha \approx 2\pi I_m\left(\frac{\lambda D}{\pi a}\right)^2(0.82).$$

Then the intensity at the center falls off with distance D as

$$I_m = 1.9\,(a/\lambda D)^2\,P_0$$

The fraction of light collected by the mirror on the moon is then

$$P_1/P_0 = 1.9\left(\frac{(2\times1.3\,\text{m})}{(0.69\times10^{-6}\text{m})(3.82\times10^{8}\text{m})}\right)^2\pi(0.10\,\text{m})^2 = 5.6\times10^{-6}.$$

The fraction of light collected by the mirror on the Earth is then

$$P_2/P_1 = 1.9\left(\frac{(2\times0.10\,\text{m})}{(0.69\times10^{-6}\text{m})(3.82\times10^{8}\text{m})}\right)^2\pi(1.3\,\text{m})^2 = 5.6\times10^{-6}.$$

Finally, $P_2/P_0 = 3\times10^{-11}$.

P42-5 (a) The ring is reddish because it occurs at the blue minimum.

(b) Apply Eq. 42-11 for *blue* light:

$$d = 1.22\lambda/\sin\theta = 1.22(400\text{ nm})/\sin(0.375^\circ) = 70\,\mu\text{m}.$$

(c) Apply Eq. 42-11 for *red* light:

$$\theta = \arcsin\left(1.22(700\text{ nm})/(70\,\mu\text{m})\right) \approx 0.7^\circ,$$

which occurs 3 lunar radii from the moon.

P42-6 The diffraction pattern is a property of the speaker, not the interference between the speakers. The diffraction pattern should be unaffected by the phase shift. The interference pattern, however, should shift up or down as the phase of the second speaker is varied.

$\boxed{\textbf{P42-7}}$ (a) The missing fringe at $\theta = 5^\circ$ is a good hint as to what is going on. There should be some sort of *interference* fringe, unless the *diffraction* pattern has a minimum at that point. This would be the first minimum, so

$$a\sin(5^\circ) = (440\times10^{-9}\text{m})$$

would be a good measure of the width of each slit. Then $a = 5.05\times10^{-6}$m.

(b) If the diffraction pattern envelope were not present we could expect that the fourth interference maxima beyond the central maximum would occur at this point, and then

$$d\sin(5^\circ) = 4(440 \times 10^{-9}\mathrm{m})$$

yielding

$$d = 2.02 \times 10^{-5}\mathrm{m}.$$

(c) Apply Eq. 42-17, where $\beta = m\pi$ and

$$\alpha = \frac{\pi a}{\lambda}\sin\theta = \frac{\pi a}{\lambda}\frac{m\lambda}{d} = m\frac{\pi a}{d} = m\pi/4.$$

Then for $m = 1$ we have

$$I_1 = (7)\left(\frac{\sin(\pi/4)}{(\pi/4)}\right)^2 = 5.7;$$

while for $m = 2$ we have

$$I_2 = (7)\left(\frac{\sin(2\pi/4)}{(2\pi/4)}\right)^2 = 2.8.$$

These are in good agreement with the figure.

E43-1 (a) $d = (21.5\times10^{-3}\text{m})/(6140) = 3.50\times10^{-6}\text{m}$.

(b) There are a number of angles allowed:

$$\begin{aligned}\theta &= \arcsin[(1)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 9.7^\circ,\\ \theta &= \arcsin[(2)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 19.5^\circ,\\ \theta &= \arcsin[(3)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 30.3^\circ,\\ \theta &= \arcsin[(4)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 42.3^\circ,\\ \theta &= \arcsin[(5)(589\times10^{-9}\text{m})/(3.50\times10^{-6}\text{m})] = 57.3^\circ.\end{aligned}$$

E43-2 The distance between adjacent rulings is

$$d = \frac{(2)(612\times10^{-9}\text{m})}{\sin(33.2^\circ)} = 2.235\times10^{-6}\text{m}.$$

The number of lines is then

$$N = D/d = (2.86\times10^{-2}\text{m})/(2.235\times10^{-6}\text{m}) = 12,800.$$

$\boxed{\textbf{E43-3}}$ We want to find a relationship between the angle and the order number which is linear. We'll plot the data in this representation, and then use a least squares fit to find the wavelength.

The data to be plotted is

m	θ	$\sin\theta$	m	θ	$\sin\theta$
1	17.6°	0.302	-1	-17.6°	-0.302
2	37.3°	0.606	-2	-37.1°	-0.603
3	65.2°	0.908	-3	-65.0°	-0.906

On my calculator I get the best straight line fit as

$$0.302m + 8.33\times10^{-4} = \sin\theta_m,$$

which means that

$$\lambda = (0.302)(1.73\,\mu\text{m}) = 522 \text{ nm}.$$

E43-4 Although an approach like the solution to Exercise 3 should be used, we'll assume that each measurement is perfect and error free. Then randomly choosing the third maximum,

$$\lambda = \frac{d\sin\theta}{m} = \frac{(5040\times10^{-9}\text{m})\sin(20.33^\circ)}{(3)} = 586\times10^{-9}\text{m}.$$

$\boxed{\textbf{E43-5}}$ (a) The principle maxima occur at points given by Eq. 43-1,

$$\sin\theta_m = m\frac{\lambda}{d}.$$

The difference of the sine of the angle between any two adjacent orders is

$$\sin\theta_{m+1} - \sin\theta_m = (m+1)\frac{\lambda}{d} - m\frac{\lambda}{d} = \frac{\lambda}{d}.$$

Using the information provided we can find d from

$$d = \frac{\lambda}{\sin\theta_{m+1} - \sin\theta_m} = \frac{(600\times10^{-9})}{(0.30)-(0.20)} = 6\,\mu\text{m}.$$

It doesn't take much imagination to recognize that the second and third order maxima were given.

(b) If the fourth order maxima is missing it must be because the diffraction pattern envelope has a minimum at that point. Any fourth order maxima should have occurred at $\sin\theta_4 = 0.4$. If it is a diffraction minima then

$$a\sin\theta_m = m\lambda \text{ where } \sin\theta_m = 0.4$$

We can solve this expression and find

$$a = m\frac{\lambda}{\sin\theta_m} = m\frac{(600\times10^{-9}\text{m})}{(0.4)} = m1.5\,\mu\text{m}.$$

The minimum width is when $m = 1$, or $a = 1.5\,\mu$m.

(c) The visible orders would be integer values of m *except* for when m is a multiple of four.

E43-6 (a) Find the maximum integer value of $m = d/\lambda = (930 \text{ nm})/(615 \text{ nm}) = 1.5$, hence $m = -1, 0, +1$; there are three diffraction maxima.

(b) The first order maximum occurs at

$$\theta = \arcsin(615 \text{ nm})/(930 \text{ nm}) = 41.4^\circ.$$

The width of the maximum is

$$\delta\theta = \frac{(615 \text{ nm})}{(1120)(930 \text{ nm})\cos(41.4^\circ)} = 7.87\times10^{-4} \text{ rad},$$

or 0.0451°.

E43-7 The fifth order maxima will be visible if $d/\lambda \geq 5$; this means

$$\lambda \leq \frac{d}{5} = \frac{(1\times10^{-3}\text{m})}{(315 \text{ rulings})(5)} = 635\times10^{-9}\text{m}.$$

E43-8 (a) The maximum could be the first, and then

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1\times10^{-3}\text{m})\sin(28^\circ)}{(200)(1)} = 2367\times10^{-9}\text{m}.$$

That's not visible. The first visible wavelength is at $m = 4$, then

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1\times10^{-3}\text{m})\sin(28^\circ)}{(200)(4)} = 589\times10^{-9}\text{m}.$$

The next is at $m = 5$, then

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1\times10^{-3}\text{m})\sin(28^\circ)}{(200)(5)} = 469\times10^{-9}\text{m}.$$

Trying $m = 6$ results in an ultraviolet wavelength.

(b) Yellow-orange and blue.

$\boxed{\textbf{E43-9}}$ A grating with 400 rulings/mm has a slit separation of

$$d = \frac{1}{400 \text{ mm}^{-1}} = 2.5 \times 10^{-3} \text{ mm}.$$

To find the number of orders of the entire visible spectrum that will be present we need only consider the wavelength which will be on the outside of the maxima. That will be the longer wavelengths, so we only need to look at the 700 nm behavior. Using Eq. 43-1,

$$d \sin\theta = m\lambda,$$

and using the maximum angle 90°, we find

$$m < \frac{d}{\lambda} = \frac{(2.5 \times 10^{-6}\text{m})}{(700 \times 10^{-9}\text{m})} = 3.57,$$

so there can be at most three orders of the entire spectrum.

E43-10 In this case $d = 2a$. Since interference maxima are given by $\sin\theta = m\lambda/d$ while diffraction minima are given at $\sin\theta = m'\lambda/a = 2m'\lambda/d$ then diffraction minima overlap with interference maxima whenever $m = 2m'$. Consequently, all even m are at diffraction minima and therefore vanish.

$\boxed{\textbf{E43-11}}$ If the second-order spectra overlaps the third-order, it is because the 700 nm second-order line is at a larger angle than the 400 nm third-order line.

Start with the wavelengths multiplied by the appropriate order parameter, then divide both side by d, and finally apply Eq. 43-1.

$$\begin{aligned} 2(700\text{ nm}) &> 3(400\text{ nm}), \\ \frac{2(700\text{ nm})}{d} &> \frac{3(400\text{ nm})}{d}, \\ \sin\theta_{2,\lambda=700} &> \sin\theta_{3,\lambda=400}, \end{aligned}$$

regardless of the value of d.

E43-12 Fig. 32-2 shows the path length difference for the right hand side of the grating as $d\sin\theta$. If the beam strikes the grating at ang angle ψ then there will be an additional path length difference of $d\sin\psi$ on the right hand side of the figure. The diffraction pattern then has two contributions to the path length difference, these add to give

$$d(\sin\theta + \sin psi) = m\lambda.$$

E43-13

E43-14 Let $d\sin\theta_i = \lambda_i$ and $\theta_1 + 20° = \theta_2$. Then

$$\sin\theta_2 = \sin\theta_1\cos(20°) + \cos\theta_1\sin(20°).$$

Rearranging,

$$\sin\theta_2 = \sin\theta_1\cos(20°) + \sqrt{1 - \sin^2\theta_1}\sin(20°).$$

Substituting the equations together yields a rather nasty expression,

$$\frac{\lambda_2}{d} = \frac{\lambda_1}{d}\cos(20°) + \sqrt{1 - (\lambda_1/d)^2}\sin(20°).$$

Rearranging,

$$(\lambda_2 - \lambda_1 \cos(20^\circ))^2 = \left(d^2 - \lambda_1^2\right)\sin^2(20^\circ).$$

Use $\lambda_1 = 430$ nm and $\lambda_2 = 680$ nm, then solve for d to find $d = 914$ nm. This corresponds to 1090 rulings/mm.

E43-15 The shortest wavelength passes through at an angle of

$$\theta_1 = \arctan(50\,\text{mm})/(300\,\text{mm}) = 9.46^\circ.$$

This corresponds to a wavelength of

$$\lambda_1 = \frac{(1\times10^{-3}\text{m})\sin(9.46^\circ)}{(350)} = 470\times10^{-9}\text{m}.$$

The longest wavelength passes through at an angle of

$$\theta_2 = \arctan(60\,\text{mm})/(300\,\text{mm}) = 11.3^\circ.$$

This corresponds to a wavelength of

$$\lambda_2 = \frac{(1\times10^{-3}\text{m})\sin(11.3^\circ)}{(350)} = 560\times10^{-9}\text{m}.$$

E43-16 (a) $\Delta\lambda = \lambda/R = \lambda/Nm$, so

$$\Delta\lambda = (481\text{ nm})/(620\text{ rulings/mm})(5.05\,\text{mm})(3) = 0.0512\text{ nm}.$$

(b) m_m is the largest integer smaller than d/λ, or

$$m_\text{m} \leq 1/(481\times10^{-9}\text{m})(620\text{ rulings/mm}) = 3.35,$$

so $m = 3$ is highest order seen.

$\boxed{\textbf{E43-17}}$ The required resolving power of the grating is given by Eq. 43-10

$$R = \frac{\lambda}{\Delta\lambda} = \frac{(589.0\text{ nm})}{(589.6\text{ nm}) - (589.0\text{ nm})} = 982.$$

Our resolving power is then $R = 1000$.

Using Eq. 43-11 we can find the number of grating lines required. We are looking at the second-order maxima, so

$$N = \frac{R}{m} = \frac{(1000)}{(2)} = 500.$$

E43-18 (a) $N = R/m = \lambda/m\Delta\lambda$, so

$$N = \frac{(415.5\text{ nm})}{(2)(415.496\text{ nm} - 415.487\text{ nm})} = 23100.$$

(b) $d = w/N$, where w is the width of the grating. Then

$$\theta = \arcsin\frac{m\lambda}{d} = \arcsin\frac{(23100)(2)(415.5\times10^{-9}\text{m})}{(4.15\times10^{-2}\text{m})} = 27.6^\circ.$$

E43-19 $N = R/m = \lambda/m\Delta\lambda$, so

$$N = \frac{(656.3 \text{ nm})}{(1)(0.180 \text{ nm})} = 3650$$

E43-20 Start with Eq. 43-9:

$$D = \frac{m}{d\cos\theta} = \frac{d\sin\theta/\lambda}{d\cos\theta} = \frac{\tan\theta}{\lambda}.$$

$\boxed{\textbf{E43-21}}$ (a) We find the ruling spacing by Eq. 43-1,

$$d = \frac{m\lambda}{\sin\theta_m} = \frac{(3)(589 \text{ nm})}{\sin(10.2^\circ)} = 9.98\,\mu\text{m}.$$

(b) The resolving power of the grating needs to be at least $R = 1000$ for the third-order line; see the work for Ex. 43-17 above. The number of lines required is given by Eq. 43-11,

$$N = \frac{R}{m} = \frac{(1000)}{(3)} = 333,$$

so the width of the grating (or at least the part that is being used) is $333(9.98\,\mu\text{m}) = 3.3$ mm.

E43-22 (a) Condition (1) is satisfied if

$$d \geq 2(600 \text{ nm})/\sin(30^\circ) = 2400 \text{ nm}.$$

The dispersion is maximal for the smallest d, so $d = 2400$ nm.

(b) To remove the third order requires $d = 3a$, or $a = 800$ nm.

E43-23 (a) The angles of the first three orders are

$$\begin{aligned}
\theta_1 &= \arcsin\frac{(1)(589\times10^{-9}\text{m})(40000)}{(76\times10^{-3}\text{m})} = 18.1^\circ,\\
\theta_2 &= \arcsin\frac{(2)(589\times10^{-9}\text{m})(40000)}{(76\times10^{-3}\text{m})} = 38.3^\circ,\\
\theta_3 &= \arcsin\frac{(3)(589\times10^{-9}\text{m})(40000)}{(76\times10^{-3}\text{m})} = 68.4^\circ.
\end{aligned}$$

The dispersion for each order is

$$\begin{aligned}
D_1 &= \frac{(1)(40000)}{(76\times10^{6}\text{nm})\cos(18.1^\circ)}\frac{360^\circ}{2\pi} = 3.2\times10^{-2\circ}/\text{nm},\\
D_2 &= \frac{(2)(40000)}{(76\times10^{6}\text{nm})\cos(38.3^\circ)}\frac{360^\circ}{2\pi} = 7.7\times10^{-2\circ}/\text{nm},\\
D_3 &= \frac{(3)(40000)}{(76\times10^{6}\text{nm})\cos(68.4^\circ)}\frac{360^\circ}{2\pi} = 2.5\times10^{-1\circ}/\text{nm}.
\end{aligned}$$

(b) $R = Nm$, so

$$\begin{aligned}
R_1 &= (40000)(1) = 40000,\\
R_2 &= (40000)(2) = 80000,\\
R_3 &= (40000)(3) = 120000.
\end{aligned}$$

E43-24 $d = m\lambda/2\sin\theta$, so

$$d = \frac{(2)(0.122 \text{ nm})}{2\sin(28.1^\circ)} = 0.259 \text{ nm}.$$

$\boxed{\textbf{E43-25}}$ Bragg reflection is given by Eq. 43-12

$$2d\sin\theta = m\lambda,$$

where the angles are measured not against the normal, but against the plane. The value of d depends on the family of planes under consideration, but it is at never larger than a_0, the unit cell dimension.

We are looking for the smallest angle; this will correspond to the largest d and the smallest m. That means $m = 1$ and $d = 0.313$ nm. Then the minimum angle is

$$\theta = \sin^{-1}\frac{(1)(29.3 \times 10^{-12}\,\text{m})}{2(0.313 \times 10^{-9}\,\text{m})} = 2.68^\circ.$$

E43-26 $2d/\lambda = \sin\theta_1$ and $2d/2\lambda = \sin\theta_2$. Then

$$\theta_2 = \arcsin[2\sin(3.40^\circ)] = 6.81^\circ.$$

$\boxed{\textbf{E43-27}}$ We apply Eq. 43-12 to each of the peaks and find the product

$$m\lambda = 2d\sin\theta.$$

The four values are 26 pm, 39 pm, 52 pm, and 78 pm. The last two values are twice the first two, so the wavelengths are 26 pm and 39 pm.

E43-28 (a) $2d\sin\theta = m\lambda$, so

$$d = \frac{(3)(96.7 \text{ pm})}{2\sin(58.0^\circ)} = 171 \text{ pm}.$$

(b) $\lambda = 2(171 \text{ pm})\sin(23.2^\circ)/(1) = 135$ pm.

E43-29 The angle against the face of the crystal is $90^\circ - 51.3^\circ = 38.7^\circ$. The wavelength is

$$\lambda = 2(39.8 \text{ pm})\sin(38.7^\circ)/(1) = 49.8 \text{ pm}.$$

E43-30 If $\lambda > 2d$ then $\lambda/2d > 1$. But

$$\lambda/2d = \sin\theta/m.$$

This means that $\sin\theta > m$, but the sine function can never be greater than one.

E43-31 There are too many unknowns. It is only possible to determine the ratio d/λ.

E43-32 A wavelength will be diffracted if $m\lambda = 2d\sin\theta$. The possible solutions are

$$\begin{aligned}\lambda_3 &= 2(275 \text{ pm})\sin(47.8)/(3) = 136 \text{ pm},\\ \lambda_4 &= 2(275 \text{ pm})\sin(47.8)/(4) = 102 \text{ pm}.\end{aligned}$$

$\boxed{\textbf{E43-33}}$ We use Eq. 43-12 to first find d;

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.261\times10^{-9}\,\mathrm{m})}{2\sin(63.8^\circ)} = 1.45\times10^{-10}\,\mathrm{m}.$$

d is the spacing between the planes in Fig. 43-28; it correspond to half of the diagonal distance between two cell centers. Then

$$(2d)^2 = a_0^2 + a_0^2,$$

or

$$a_0 = \sqrt{2}d = \sqrt{2}(1.45\times10^{-10}\,\mathrm{m}) = 0.205\ \mathrm{nm}.$$

E43-34 Diffraction occurs when $2d\sin\theta = m\lambda$. The angles in this case are then given by

$$\sin\theta = m\frac{(0.125\times10^{-9}\mathrm{m})}{2(0.252\times10^{-9}\mathrm{m})} = (0.248)m.$$

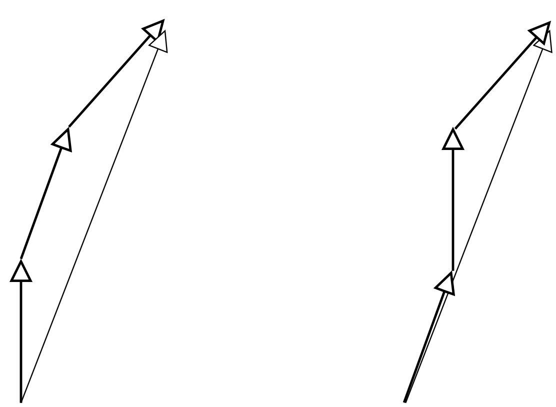
There are four solutions to this equation. They are 14.4°, 29.7°, 48.1°, and 82.7°. They involve rotating the crystal from the original orientation ($90^\circ - 42.4^\circ = 47.6^\circ$) by amounts

$$\begin{aligned}
47.6^\circ - 14.4^\circ &= 33.2^\circ,\\
47.6^\circ - 29.7^\circ &= 17.9^\circ,\\
47.6^\circ - 48.1^\circ &= -0.5^\circ,\\
47.6^\circ - 82.7^\circ &= -35.1^\circ.
\end{aligned}$$

$\boxed{\textbf{P43-1}}$ Since the slits are *so* narrow we only need to consider interference effects, not diffraction effects. There are three waves which contribute at any point. The phase angle between adjacent waves is

$$\phi = 2\pi d\sin\theta/\lambda.$$

We can add the electric field vectors as was done in the previous chapters, or we can do it in a different order as is shown in the figure below.



Then the vectors sum to

$$E(1 + 2\cos\phi).$$

We need to square this quantity, and then normalize it so that the central maximum is the maximum. Then

$$I = I_\mathrm{m}\frac{(1 + 4\cos\phi + 4\cos^2\phi)}{9}.$$

P43-2 (a) Solve ϕ for $I = I_{\rm m}/2$, this occurs when

$$\frac{3}{\sqrt{2}} = 1 + 2\cos\phi,$$

or $\phi = 0.976$ rad. The corresponding angle θ_x is

$$\theta_x \approx \frac{\lambda\phi}{2\pi d} = \frac{\lambda(0.976)}{2\pi d} = \frac{\lambda}{6.44d}.$$

But $\Delta\theta = 2\theta_x$, so

$$\Delta\theta \approx \frac{\lambda}{3.2d}.$$

(b) For the two slit pattern the half width was found to be $\Delta\theta = \lambda/2d$. The half width in the three slit case *is* smaller.

$\boxed{\textbf{P43-3}}$ (a) and (b) A plot of the intensity quickly reveals that there is an alternation of large maximum, then a smaller maximum, etc. The large maxima are at $\phi = 2n\pi$, the smaller maxima are half way between those values.

(c) The intensity at these secondary maxima is then

$$I = I_{\rm m}\frac{(1 + 4\cos\pi + 4\cos^2\pi)}{9} = \frac{I_{\rm m}}{9}.$$

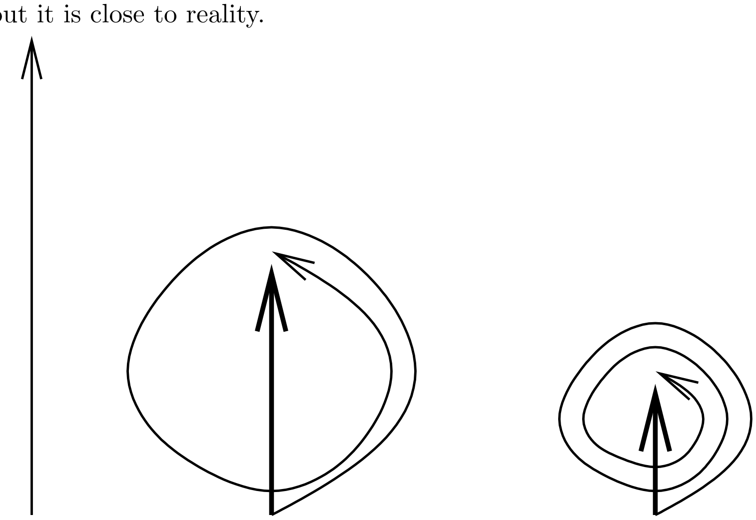
Note that the minima are *not* located half-way between the maxima!

P43-4 Covering up the middle slit will result in a two slit apparatus with a slit separation of $2d$. The half width, as found in Problem 41-5, is then

$$\Delta\theta = \lambda/2(2d), = \lambda/4d,$$

which is *narrower* than before covering up the middle slit by a factor of $3.2/4 = 0.8$.

P43-5 (a) If N is large we can treat the phasors as summing to form a flexible "line" of length $N\delta E$. We then assume (incorrectly) that the secondary maxima occur when the loop wraps around on itself as shown in the figures below. Note that the resultant phasor always points straight up. This isn't right, but it is close to reality.



The length of the resultant depends on how many loops there are. For $k = 0$ there are none. For $k = 1$ there are one and a half loops. The circumference of the resulting circle is $2N\delta E/3$, the diameter is $N\delta E/3\pi$. For $k = 2$ there are two and a half loops. The circumference of the resulting circle is $2N\delta E/5$, the diameter is $N\delta E/5\pi$. The pattern for higher k is similar: the circumference is $2N\delta E/(2k+1)$, the diameter is $N\delta E/(k+1/2)\pi$.

The intensity at this "approximate" maxima is proportional to the resultant squared, or

$$I_k \propto \frac{(N\delta E)^2}{(k+1/2)^2\pi^2}.$$

but I_{m} is proportional to $(N\delta E)^2$, so

$$I_k = I_{\mathrm{m}}\frac{1}{(k+1/2)^2\pi^2}.$$

(b) Near the middle the vectors simply fold back on one another, leaving a resultant of δE. Then

$$I_k \propto (\delta E)^2 = \frac{(N\delta E)^2}{N^2},$$

so

$$I_k = \frac{I_{\mathrm{m}}}{N^2},$$

(c) Let α have the values which result in $\sin\alpha = 1$, and then the two expressions are identical!

P43-6 (a) $v = f\lambda$, so $\delta v = f\delta\lambda + \lambda\delta f$. Assuming $\delta v = 0$, we have $\delta f/f = -\delta\lambda/\lambda$. Ignore the negative sign (we don't need it here). Then

$$R = \frac{\lambda}{\Delta\lambda} = \frac{f}{\Delta f} = \frac{c}{\lambda\Delta f},$$

and then

$$\Delta f = \frac{c}{R\lambda} = \frac{c}{Nm\lambda}.$$

(b) The ray on the top gets there first, the ray on the bottom must travel an additional distance of $Nd\sin\theta$. It takes a time

$$\Delta t = Nd\sin\theta/c$$

to do this.

(c) Since $m\lambda = d\sin\theta$, the two resulting expression can be multiplied together to yield

$$(\Delta f)(\Delta t) = \frac{c}{Nm\lambda}\frac{Nd\sin\theta}{c} = 1.$$

This is almost, but not quite, one of Heisenberg's uncertainty relations!

P43-7 (b) We sketch parallel lines which connect centers to form almost any right triangle similar to the one shown in the Fig. 43-18. The triangle will have two sides which have integer multiple lengths of the lattice spacing a_0. The hypotenuse of the triangle will then have length $\sqrt{h^2+k^2}a_0$, where h and k are the integers. In Fig. 43-18 $h = 2$ while $k = 1$. The number of planes which cut the diagonal is equal to h^2+k^2 if, and only if, h and k are relatively prime. The inter-planar spacing is then

$$d = \frac{\sqrt{h^2+k^2}a_0}{h^2+k^2} = \frac{a_0}{\sqrt{h^2+k^2}}.$$

(a) The next five spacings are then

$$
\begin{aligned}
h = 1, \quad & k = 1, \quad & d &= a_0/\sqrt{2}, \\
h = 1, \quad & k = 2, \quad & d &= a_0/\sqrt{5}, \\
h = 1, \quad & k = 3, \quad & d &= a_0/\sqrt{10}, \\
h = 2, \quad & k = 3, \quad & d &= a_0/\sqrt{13}, \\
h = 1, \quad & k = 4, \quad & d &= a_0/\sqrt{17}.
\end{aligned}
$$

P43-8 The middle layer cells will also diffract a beam, but this beam will be exactly π out of phase with the top layer. The two beams will then cancel out exactly because of destructive interference.

$\boxed{\textbf{E44-1}}$ (a) The direction of propagation is determined by considering the argument of the sine function. As t increases y must decrease to keep the sine function "looking" the same, so the wave is propagating in the negative y direction.

(b) The electric field is orthogonal (perpendicular) to the magnetic field (so $E_x = 0$) and the direction of motion (so $E_y = 0$); Consequently, the only non-zero term is E_z. The magnitude of E will be equal to the magnitude of B times c. Since $\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \vec{\mathbf{B}}/\mu_0$, when $\vec{\mathbf{B}}$ points in the positive x direction then $\vec{\mathbf{E}}$ must point in the negative z direction in order that $\vec{\mathbf{S}}$ point in the negative y direction. Then

$$E_z = -cB\sin(ky + \omega t).$$

(c) The polarization is given by the direction of the electric field, so the wave is linearly polarized in the z direction.

E44-2 Let one wave be polarized in the x direction and the other in the y direction. Then the net electric field is given by $E^2 = E_x^2 + E_y^2$, or

$$E^2 = E_0^2\left(\sin^2(kz - \omega t) + \sin^2(kz - \omega t + \beta)\right),$$

where β is the phase difference. We can consider any point in space, including $z = 0$, and then average the result over a full cycle. Since β merely shifts the integration limits, then the result is independent of β. Consequently, there are no interference effects.

E44-3 (a) The transmitted intensity is $I_0/2 = 6.1{\times}10^{-3}W/\text{m}^2$. The maximum value of the electric field is

$$E_\text{m} = \sqrt{2\mu_0 cI} = \sqrt{2(1.26{\times}10^{-6}\text{H/m})(3.00{\times}10^{8}\text{m/s})(6.1{\times}10^{-3}\text{W/m}^2)} = 2.15\,\text{V/m}.$$

(b) The radiation pressure is caused by the absorbed half of the incident light, so

$$p = I/c = (6.1{\times}10^{-3}\text{W/m}^2)/(3.00{\times}10^{8}\text{m/s}) = 2.03{\times}10^{-11}\text{Pa}.$$

E44-4 The first sheet transmits half the original intensity, the second transmits an amount proportional to $\cos^2\theta$. Then $I = (I_0/2)\cos^2\theta$, or

$$\theta = \arccos\sqrt{2I/I_0} = \arccos\sqrt{2(I_0/3)/I_0}35.3^\circ.$$

$\boxed{\textbf{E44-5}}$ The first sheet polarizes the un-polarized light, half of the intensity is transmitted, so $I_1 = \frac{1}{2}I_0$.

The second sheet transmits according to Eq. 44-1,

$$I_2 = I_1\cos^2\theta = \frac{1}{2}I_0\cos^2(45^\circ) = \frac{1}{4}I_0,$$

and the transmitted light is polarized in the direction of the second sheet.

The third sheet is 45° to the second sheet, so the intensity of the light which is transmitted through the third sheet is

$$I_3 = I_2\cos^2\theta = \frac{1}{4}I_0\cos^2(45^\circ) = \frac{1}{8}I_0.$$

E44-6 The transmitted intensity through the first sheet is proportional to $\cos^2\theta$, the transmitted intensity through the second sheet is proportional to $\cos^2(90° - \theta) = \sin^2\theta$. Then

$$I = I_0 \cos^2\theta \sin^2\theta = (I_0/4)\sin^2 2\theta,$$

or

$$\theta = \frac{1}{2}\arcsin\sqrt{4I/I_0} = \frac{1}{2}\arcsin\sqrt{4(0.100I_0)/I_0} = 19.6°.$$

Note that 70.4° is also a valid solution!

E44-7 The first sheet transmits half of the original intensity; each of the remaining sheets transmits an amount proportional to $\cos^2\theta$, where $\theta = 30°$. Then

$$\frac{I}{I_0} = \frac{1}{2}\left(\cos^2\theta\right)^3 = \frac{1}{2}\left(\cos(30°)\right)^6 = 0.211$$

E44-8 The first sheet transmits an amount proportional to $\cos^2\theta$, where $\theta = 58.8°$. The second sheet transmits an amount proportional to $\cos^2(90° - \theta) = \sin^2\theta$. Then

$$I = I_0 \cos^2\theta \sin^2\theta = (43.3\,\mathrm{W/m^2})\cos^2(58.8°)\sin^2(58.8°) = 8.50\,\mathrm{W/m^2}.$$

$\boxed{\textbf{E44-9}}$ Since the incident beam is unpolarized the first sheet transmits 1/2 of the original intensity. The transmitted beam then has a polarization set by the first sheet: 58.8° to the vertical. The second sheet is horizontal, which puts it 31.2° to the first sheet. Then the second sheet transmits $\cos^2(31.2°)$ of the intensity incident on the second sheet. The final intensity transmitted by the second sheet can be found from the product of these terms,

$$I = (43.3\,\mathrm{W/m^2})\left(\frac{1}{2}\right)\left(\cos^2(31.2°)\right) = 15.8\,\mathrm{W/m^2}.$$

E44-10 $\theta_\mathrm{p} = \arctan(1.53/1.33) = 49.0°$.

$\boxed{\textbf{E44-11}}$ (a) The angle for complete polarization of the reflected ray is Brewster's angle, and is given by Eq. 44-3 (since the first medium is air)

$$\theta_\mathrm{p} = \tan^{-1} n = \tan^{-1}(1.33) = 53.1°.$$

(b) Since the index of refraction depends (slightly) on frequency, then so does Brewster's angle.

E44-12 (b) Since $\theta_\mathrm{r} + \theta_\mathrm{p} = 90°$, $\theta_\mathrm{p} = 90° - (31.8°) = 58.2°$.
(a) $n = \tan\theta_\mathrm{p} = \tan(58.2°) = 1.61$.

E44-13 The angles are between

$$\theta_\mathrm{p} = \tan^{-1} n = \tan^{-1}(1.472) = 55.81°.$$

and

$$\theta_\mathrm{p} = \tan^{-1} n = \tan^{-1}(1.456) = 55.52°.$$

E44-14 The smallest possible thickness t will allow for one half a wavelength phase difference for the o and e waves. Then $\Delta n t = \lambda/2$, or

$$t = (525\times10^{-9}\mathrm{m})/2(0.022) = 1.2\times10^{-5}\mathrm{m}.$$

$\boxed{\textbf{E44-15}}$ (a) The incident wave is at 45° to the optical axis. This means that there are two components; assume they originally point in the $+y$ and $+z$ direction. When they travel through the half wave plate they are now out of phase by 180°; this means that when one component is in the $+y$ direction the other is in the $-z$ direction. In effect the polarization has been rotated by 90°.

(b) Since the half wave plate will delay one component so that it emerges 180° "later" than it should, it will in effect reverse the handedness of the circular polarization.

(c) Pretend that an unpolarized beam can be broken into two orthogonal linearly polarized components. Both are then rotated through 90°; but when recombined it looks like the original beam. As such, there is no apparent change.

E44-16 The quarter wave plate has a thickness of $x = \lambda/4\Delta n$, so the number of plates that can be cut is given by

$$N = (0.250\times10^{-3}\text{m})4(0.181)/(488\times10^{-9}\text{m}) = 371.$$

$\boxed{\textbf{P44-1}}$ Intensity is proportional to the electric field *squared*, so the original intensity reaching the eye is I_0, with components $I_\text{h} = (2.3)^2 I_\text{v}$, and then

$$I_0 = I_\text{h} + I_\text{v} = 6.3 I_\text{v} \text{ or } I_\text{v} = 0.16 I_0.$$

Similarly, $I_\text{h} = (2.3)^2 I_\text{v} = 0.84 I_0$.

(a) When the sun-bather is standing only the vertical component passes, while

(b) when the sun-bather is lying down only the horizontal component passes.

P44-2 The intensity of the transmitted light which was originally unpolarized is reduced to $I_\text{u}/2$, regardless of the orientation of the polarizing sheet. The intensity of the transmitted light which was originally polarized is between 0 and I_p, depending on the orientation of the polarizing sheet. Then the maximum transmitted intensity is $I_\text{u}/2 + I_\text{p}$, while the minimum transmitted intensity is $I_\text{u}/2$. The ratio is 5, so

$$5 = \frac{I_\text{u}/2 + I_\text{p}}{I_\text{u}/2} = 1 + 2\frac{I_\text{p}}{I_\text{u}},$$

or $I_\text{p}/I_\text{u} = 2$. Then the beam is 1/3 unpolarized and 2/3 polarized.

$\boxed{\textbf{P44-3}}$ Each sheet transmits a fraction

$$\cos^2\alpha = \cos^2\left(\frac{\theta}{N}\right).$$

There are N sheets, so the fraction transmitted through the stack is

$$\left(\cos^2\left(\frac{\theta}{N}\right)\right)^N.$$

We want to evaluate this in the limit as $N \to \infty$.

As N gets larger we can use a small angle approximation to the cosine function,

$$\cos x \approx 1 - \frac{1}{2}x^2 \text{ for } x \ll 1$$

The the transmitted intensity is

$$\left(1 - \frac{1}{2}\frac{\theta^2}{N^2}\right)^{2N}.$$

This expression can also be expanded in a binomial expansion to get

$$1 - 2N\frac{1}{2}\frac{\theta^2}{N^2},$$

which in the limit as $N \to \infty$ approaches 1.

The stack then transmits *all* of the light which makes it past the first filter. Assuming the light is originally unpolarized, then the stack transmits half the original intensity.

P44-4 (a) Stack several polarizing sheets so that the angle between any two sheets is sufficiently small, but the total angle is 90°.

(b) The transmitted intensity fraction needs to be 0.95. Each sheet will transmit a fraction $\cos^2\theta$, where $\theta = 90°/N$, with N the number of sheets. Then we want to solve

$$0.95 = \left(\cos^2(90°/N)\right)^N$$

for N. For large enough N, θ will be small, so we can expand the cosine function as

$$\cos^2\theta = 1 - \sin^2\theta \approx 1 - \theta^2,$$

so

$$0.95 \approx \left(1 - (\pi/2N)^2\right)^N \approx 1 - N(\pi/2N)^2,$$

which has solution $N = \pi^2/4(0.05) = 49$.

$\boxed{\textbf{P44-5}}$ Since passing through a quarter wave plate twice can rotate the polarization of a linearly polarized wave by 90°, then if the light passes through a polarizer, through the plate, reflects off the coin, then through the plate, and through the polarizer, it would be possible that when it passes through the polarizer the second time it is 90° to the polarizer and no light will pass. You won't see the coin.

On the other hand if the light passes first through the plate, then through the polarizer, then is reflected, the passes again through the polarizer, *all* the reflected light will pass through he polarizer and eventually work its way out through the plate. So the coin will be visible.

Hence, side A must be the polarizing sheet, and that sheet must be at 45° to the optical axis.

P44-6 (a) The displacement of a ray is given by

$$\tan\theta_k = y_k/t,$$

so the shift is

$$\Delta y = t(\tan\theta_e - \tan\theta_o).$$

Solving for each angle,

$$\begin{aligned}\theta_e &= \arcsin\left(\frac{1}{(1.486)}\sin(38.8°)\right) = 24.94°,\\ \theta_o &= \arcsin\left(\frac{1}{(1.658)}\sin(38.8°)\right) = 22.21°.\end{aligned}$$

The shift is then

$$\Delta y = (1.12\times10^{-2}\text{m})\,(\tan(24.94) - \tan(22.21)) = 6.35\times10^{-4}\text{m}.$$

(b) The e-ray bends less than the o-ray.

(c) The rays have polarizations which are perpendicular to each other; the o-wave being polarized along the direction of the optic axis.

(d) One ray, then the other, would disappear.

P44-7 The method is outline in Sample Problem 44-24; use a polarizing sheet to pick out the o-ray or the e-ray.

$\boxed{\textbf{E45-1}}$ (a) The energy of a photon is given by Eq. 45-1, $E = hf$, so

$$E = hf = \frac{hc}{\lambda}.$$

Putting in "best" numbers

$$hc = \frac{(6.62606876\times10^{-34}\text{J}\cdot\text{s})}{(1.602176462\times10^{-19}\text{ C})}(2.99792458\times10^{8}\text{m/s}) = 1.23984\times10^{-6}\text{ eV}\cdot\text{m}.$$

This means that $hc = 1240$ eV $\cdot$ nm is accurate to almost one part in 8000!

(b) $E = (1240$ eV $\cdot$ nm$)/(589$ nm$) = 2.11$ eV.

E45-2 Using the results of Exercise 45-1,

$$\lambda = \frac{(1240\text{ eV}\cdot\text{nm})}{(0.60\text{ eV})} = 2100\text{ nm},$$

which is in the infrared.

E45-3 Using the results of Exercise 45-1,

$$E_1 = \frac{(1240\text{ eV}\cdot\text{nm})}{(375\text{ nm})} = 3.31\text{ eV},$$

and

$$E_2 = \frac{(1240\text{ eV}\cdot\text{nm})}{(580\text{ nm})} = 2.14\text{ eV},$$

The difference is $\Delta E = (3.307$ eV$) - (2.138$ eV$) = 1.17$ eV.

E45-4 $P = E/t$, so, using the result of Exercise 45-1,

$$P = (100/\text{s})\frac{(1240\text{ eV}\cdot\text{nm})}{(540\text{ nm})} = 230\text{ eV/s}.$$

That's a small 3.68×10^{-17}W.

$\boxed{\textbf{E45-5}}$ When talking about the regions in the sun's spectrum it is more common to refer to wavelengths than frequencies. So we will use the results of Exercise 45-1(a), and solve

$$\lambda = hc/E = (1240\text{ eV}\cdot\text{nm})/E.$$

The energies are between $E = (1.0\times10^{18}\text{J})/(1.6\times10^{-19}\text{C}) = 6.25$ eV and $E = (1.0\times10^{16}\text{J})/(1.6\times10^{-19}\text{C}) = 625$ eV. These energies correspond to wavelengths between 198 nm and 1.98 nm; this is the ultraviolet range.

E45-6 The energy per photon is $E = hf = hc/\lambda$. The intensity is power per area, which is energy per time per area, so

$$I = \frac{P}{A} = \frac{E}{At} = \frac{nhc}{\lambda At} = \frac{hc}{\lambda A}\frac{n}{t}.$$

But $R = n/t$ is the rate of photons per unit time. Since h and c are constants and I and A are equal for the two beams, we have $R_1/\lambda_1 = R_2/\lambda_2$, or

$$R_1/R_2 = \lambda_1/\lambda_2.$$

$\boxed{\textbf{E45-7}}$ (a) Since the power is the same, the bulb with the larger energy per photon will emits *fewer* photons per second. Since longer wavelengths have lower energies, the bulb emitting 700 nm must be giving off more photons per second.

(b) How many more photons per second? If E_1 is the energy per photon for one of the bulbs, then $N_1 = P/E_1$ is the number of photons per second emitted. The difference is then

$$N_1 - N_2 = \frac{P}{E_1} - \frac{P}{E_2} = \frac{P}{hc}(\lambda_1 - \lambda_2),$$

or

$$N_1 - N_2 = \frac{(130\,\text{W})}{(6.63\times10^{-34}\text{J}\cdot\text{s})(3.00\times10^{8}\text{m/s})}((700\times10^{-9}\text{m}) - (400\times10^{-9}\text{m})) = 1.96\times10^{20}.$$

E45-8 Using the results of Exercise 45-1, the energy of one photon is

$$E = \frac{(1240\text{ eV}\cdot\text{nm})}{(630\text{ nm})} = 1.968\text{ eV},$$

The total *light* energy given off by the bulb is

$$E_\text{t} = Pt = (0.932)(70\,\text{W})(730\text{ hr})(3600\text{ s/hr}) = 1.71\times10^{8}\text{J}.$$

The number of photons is

$$n = \frac{E_\text{t}}{E_0} = \frac{(1.71\times10^{8}\text{J})}{(1.968\text{ eV})(1.6\times10^{-19}\text{J/eV})} = 5.43\times10^{26}.$$

E45-9 Apply Wien's law, Eq. 45-4, $\lambda_\text{max}T = 2898\,\mu\text{m}\cdot\text{K}$; so

$$T = \frac{(2898\,\mu\text{m}\cdot\text{K})}{(32\times10^{-12}\text{m})} = 91\times10^{6}\text{K}.$$

Actually, the wavelength was supposed to be $32\,\mu$m. Then the temperature would be 91 K.

E45-10 Apply Wien's law, Eq. 45-4, $\lambda_\text{max}T = 2898\,\mu\text{m}\cdot\text{K}$; so

$$\lambda = \frac{(2898\,\mu\text{m}\cdot\text{K})}{(0.0020\,\text{K})} = 1.45\,\text{m}.$$

This is in the radio region, near 207 on the FM dial.

E45-11 The wavelength of the maximum spectral radiancy is given by Wien's law, Eq. 45-4,

$$\lambda_\text{max}T = 2898\,\mu\text{m}\cdot\text{K}.$$

Applying to each temperature in turn,

(a) $\lambda = 1.06\times10^{-3}$m, which is in the microwave;
(b) $\lambda = 9.4\times10^{-6}$m, which is in the infrared;
(c) $\lambda = 1.6\times10^{-6}$m, which is in the infrared;
(d) $\lambda = 5.0\times10^{-7}$m, which is in the visible;
(e) $\lambda = 2.9\times10^{-10}$m, which is in the x-ray;
(f) $\lambda = 2.9\times10^{-41}$m, which is in a hard gamma ray.

E45-12 (a) Apply Wien's law, Eq. 45-4, $\lambda_{\max}T = 2898\,\mu\text{m}\cdot\text{K}$.; so

$$\lambda = \frac{(2898\,\mu\text{m}\cdot\text{K})}{(5800\,\text{K})} = 5.00\times 10^{-7}\text{m}.$$

That's blue-green.

(b) Apply Wien's law, Eq. 45-4, $\lambda_{\max}T = 2898\,\mu\text{m}\cdot\text{K}$.; so

$$T = \frac{(2898\,\mu\text{m}\cdot\text{K})}{(550\times 10^{-9}\text{m})} = 5270\,\text{K}.$$

E45-13 $I = \sigma T^4$ and $P = IA$. Then

$$P = (5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4)(1900\,\text{K})^4\pi(0.5\times 10^{-3}\text{m})^2 = 0.58\,\text{W}.$$

E45-14 Since $I \propto T^4$, doubling T results in a $2^4 = 16$ times increase in I. Then the new power level is

$$(16)(12.0\,\text{mW}) = 192\,\text{mW}.$$

$\boxed{\textbf{E45-15}}$ (a) We want to apply Eq. 45-6,

$$R(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}.$$

We know the ratio of the spectral radiancies at two different wavelengths. Dividing the above equation at the first wavelength by the same equation at the second wavelength,

$$3.5 = \frac{\lambda_1^5\left(e^{hc/\lambda_1 kT} - 1\right)}{\lambda_2^5\left(e^{hc/\lambda_2 kT} - 1\right)},$$

where $\lambda_1 = 200$ nm and $\lambda_2 = 400$ nm. We can considerably simplify this expression if we let

$$x = e^{hc/\lambda_2 kT},$$

because since $\lambda_2 = 2\lambda_1$ we would have

$$e^{hc/\lambda_1 kT} = e^{2hc/\lambda_2 kT} = x^2.$$

Then we get

$$3.5 = \left(\frac{1}{2}\right)^5 \frac{x^2 - 1}{x - 1} = \frac{1}{32}(x + 1).$$

We will use the results of Exercise 45-1 for the exponents and then rearrange to get

$$T = \frac{hc}{\lambda_1 k\ln(111)} = \frac{(3.10\text{ eV})}{(8.62\times 10^{-5}\text{ eV/K})\ln(111)} = 7640\,\text{K}.$$

(b) The method is the same, except that instead of 3.5 we have 1/3.5; this means the equation for x is

$$\frac{1}{3.5} = \frac{1}{32}(x + 1),$$

with solution $x = 8.14$, so then

$$T = \frac{hc}{\lambda_1 k\ln(8.14)} = \frac{(3.10\text{ eV})}{(8.62\times 10^{-5}\text{ eV/K})\ln(8.14)} = 17200\,\text{K}.$$

E45-16 $hf = \phi$, so

$$f = \frac{\phi}{h} = \frac{(5.32 \text{ eV})}{(4.14\times10^{-15}\text{eV}\cdot\text{s})} = 1.28\times10^{15}\text{Hz}.$$

E45-17 We'll use the results of Exercise 45-1. Visible red light has an energy of

$$E = \frac{(1240 \text{ eV}\cdot\text{nm})}{(650 \text{ nm})} = 1.9 \text{ eV}.$$

The substance must have a work function *less* than this to work with red light. This means that only cesium will work with red light. Visible blue light has an energy of

$$E = \frac{(1240 \text{ eV}\cdot\text{nm})}{(450 \text{ nm})} = 2.75 \text{ eV}.$$

This means that barium, lithium, and cesium will work with blue light.

E45-18 Since $K_\text{m} = hf - \phi$,

$$K_\text{m} = (4.14\times10^{-15}\text{eV}\cdot\text{s})(3.19\times10^{15}\text{Hz}) - (2.33 \text{ eV}) = 10.9 \text{ eV}.$$

$\boxed{\textbf{E45-19}}$ (a) Use the results of Exercise 45-1 to find the energy of the corresponding photon,

$$E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(678 \text{ nm})} = 1.83 \text{ eV}.$$

Since this energy is less than than the minimum energy required to remove an electron then the photo-electric effect will not occur.

(b) The cut-off wavelength is the longest possible wavelength of a photon that will still result in the photo-electric effect occurring. That wavelength is

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{E} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(2.28 \text{ eV})} = 544 \text{ nm}.$$

This would be visible as green.

E45-20 (a) Since $K_\text{m} = hc/\lambda - \phi$,

$$K_\text{m} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(200 \text{ nm})}(4.2 \text{ eV}) = 2.0 \text{ eV}.$$

(b) The minimum kinetic energy is zero; the electron just barely makes it off the surface.

(c) $V_\text{s} = K_\text{m}/q = 2.0\,\text{V}$.

(d) The cut-off wavelength is the longest possible wavelength of a photon that will still result in the photo-electric effect occurring. That wavelength is

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{E} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(4.2 \text{ eV})} = 295 \text{ nm}.$$

E45-21 $K_\text{m} = qV_\text{s} = 4.92$ eV. But $K_\text{m} = hc/\lambda - \phi$, so

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{(4.92 \text{ eV} + 2.28 \text{ eV})} = 172 \text{ nm}.$$

E45-22 (a) $K_{\rm m} = qV_{\rm s}$ and $K_{\rm m} = hc/\lambda - \phi$. We have two different values for $qV_{\rm s}$ and λ, so subtracting this equation from itself yields

$$q(V_{{\rm s},1} - V_{{\rm s},2}) = hc/\lambda_1 - hc/\lambda_2.$$

Solving for λ_2,

$$\begin{aligned}\lambda_2 &= \frac{hc}{hc/\lambda_1 - q(V_{{\rm s},1} - V_{{\rm s},2})},\\ &= \frac{(1240 \text{ eV}\cdot\text{nm})}{(1240 \text{ eV}\cdot\text{nm})/(491 \text{ nm}) - (0.710 \text{ eV}) + (1.43 \text{ eV})},\\ &= 382 \text{ nm}.\end{aligned}$$

(b) $K_{\rm m} = qV_{\rm s}$ and $K_{\rm m} = hc/\lambda - \phi$, so

$$\phi = (1240 \text{ eV}\cdot\text{nm})/(491 \text{ nm}) - (0.710 \text{ eV}) = 1.82 \text{ eV}.$$

$\boxed{\textbf{E45-23}}$ (a) The stopping potential is given by Eq. 45-11,

$$V_0 = \frac{h}{e}f - \frac{\phi}{e},$$

so

$$V_0 = \frac{(1240 \text{ eV}\cdot\text{nm})}{e(410 \text{ nm}} - \frac{(1.85 \text{ eV}}{e} = 1.17\,\text{V}.$$

(b) These are *not* relativistic electrons, so

$$v = \sqrt{2K/m} = c\sqrt{2K/mc^2} = c\sqrt{2(1.17 \text{ eV})/(0.511\times10^6 \text{ eV})} = 2.14\times10^{-3}c,$$

or $v = 64200\,\text{m/s}$.

E45-24 It will have become the stopping potential, or

$$V_0 = \frac{h}{e}f - \frac{\phi}{e},$$

so

$$V_0 = \frac{(4.14\times10^{-15}\text{eV}\cdot\text{m})}{(1.0e)}(6.33\times10^{14}/\text{s}) - \frac{(2.49 \text{ eV})}{(1.0e)} = 0.131\,\text{V}.$$

E45-25

E45-26 (a) Using the results of Exercise 45-1,

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{(20\times10^3 \text{ eV})} = 62 \text{ pm}.$$

(b) This is in the x-ray region.

E45-27 (a) Using the results of Exercise 45-1,

$$E = \frac{(1240 \text{ eV}\cdot\text{nm})}{(41.6\times10^{-3} \text{ nm})} = 29,800 \text{ eV}.$$

(b) $f = c/\lambda = (3\times10^8\text{m/s})/(41.6 \text{ pm}) = 7.21\times10^{18}/\text{s}$.
(c) $p = E/c = 29,800 \text{ eV}/c = 2.98\times10^4 \text{ eV}/c$.

E45-28 (a) $E = hf$, so

$$f = \frac{(0.511\times10^6\text{eV})}{(4.14\times10^{-15}\text{eV}\cdot\text{s})} = 1.23\times10^{20}/\text{s}.$$

(b) $\lambda = c/f = (3\times10^8\text{m/s})/(1.23\times10^{20}/\text{s}) = 2.43$ pm.
(c) $p = E/c = (0.511\times10^6\text{eV})/c$.

$\boxed{\textbf{E45-29}}$ The initial momentum of the system is the momentum of the photon, $p = h/\lambda$. This momentum is imparted to the sodium atom, so the final speed of the sodium is $v = p/m$, where m is the mass of the sodium. Then

$$v = \frac{h}{\lambda m} = \frac{(6.63\times10^{-34}\text{J}\cdot\text{s})}{(589\times10^{-9}\text{m})(23)(1.7\times10^{-27}\text{kg})} = 2.9 \text{ cm/s}.$$

E45-30 (a) $\lambda_\text{C} = h/mc = hc/mc^2$, so

$$\lambda_\text{C} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(0.511\times10^6\text{eV})} = 2.43 \text{ pm}.$$

(c) Since $E_\lambda = hf = hc/\lambda$, and $\lambda = h/mc = hc/mc^2$, then

$$E_\lambda = hc/\lambda = mc^2.$$

(b) See part (c).

E45-31 The change in the wavelength of a photon during Compton scattering is given by Eq. 45-17,

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\phi).$$

We'll use the results of Exercise 45-30 to save some time, and let $h/mc = \lambda_\text{C}$, which is 2.43 pm.
(a) For $\phi = 35°$,

$$\lambda' = (2.17 \text{ pm}) + (2.43 \text{ pm})(1 - \cos 35°) = 2.61 \text{ pm}.$$

(b) For $\phi = 115°$,

$$\lambda' = (2.17 \text{ pm}) + (2.43 \text{ pm})(1 - \cos 115°) = 5.63 \text{ pm}.$$

E45-32 (a) We'll use the results of Exercise 45-1:

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{(0.511\times10^6\text{eV})} = 2.43 \text{ pm}.$$

(b) The change in the wavelength of a photon during Compton scattering is given by Eq. 45-17,

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\phi).$$

We'll use the results of Exercise 45-30 to save some time, and let $h/mc = \lambda_\text{C}$, which is 2.43 pm.

$$\lambda' = (2.43 \text{ pm}) + (2.43 \text{ pm})(1 - \cos 72°) = 4.11 \text{ pm}.$$

(c) We'll use the results of Exercise 45-1:

$$E = \frac{(1240 \text{ eV}\cdot\text{nm})}{(4.11 \text{ pm})} = 302 \text{ keV}.$$

$\boxed{\textbf{E45-33}}$ The change in the wavelength of a photon during Compton scattering is given by Eq. 45-17,

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi).$$

We are *not* using the expression with the form $\Delta\lambda$ because $\Delta\lambda$ and ΔE are *not* simply related.

The wavelength is related to frequency by $c = f\lambda$, while the frequency is related to the energy by Eq. 45-1, $E = hf$. Then

$$\begin{aligned}
\Delta E &= E - E' = hf - hf', \\
&= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right), \\
&= hc\frac{\lambda' - \lambda}{\lambda\,\lambda'}.
\end{aligned}$$

Into this last expression we substitute the Compton formula. Then

$$\Delta E = \frac{h^2}{m}\frac{(1 - \cos\phi)}{\lambda\,\lambda'}.$$

Now $E = hf = hc/\lambda$, and we can divide this on both sides of the above equation. Also, $\lambda' = c/f'$, and we can substitute this into the right hand side of the above equation. Both of these steps result in

$$\frac{\Delta E}{E} = \frac{hf'}{mc^2}(1 - \cos\phi).$$

Note that mc^2 is the rest energy of the scattering particle (usually an electron), while hf' is the energy of the scattered photon.

E45-34 The wavelength is related to frequency by $c = f\lambda$, while the frequency is related to the energy by Eq. 45-1, $E = hf$. Then

$$\begin{aligned}
\Delta E &= E - E' = hf - hf', \\
&= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right), \\
&= hc\frac{\lambda' - \lambda}{\lambda\,\lambda'}, \\
\frac{\Delta E}{E} &= \frac{\Delta\lambda}{\lambda + \Delta\lambda},
\end{aligned}$$

But $\Delta E/E = 3/4$, so

$$3\lambda + 3\Delta\lambda = 4\Delta\lambda,$$

or $\Delta\lambda = 3\lambda$.

E45-35 The maximum shift occurs when $\phi = 180^\circ$, so

$$\Delta\lambda_{\mathrm{m}} = 2\frac{h}{mc} = 2\frac{(1240\ \mathrm{eV\cdot nm})}{938\ \mathrm{MeV})} = 2.64\times10^{-15}\mathrm{m}.$$

E45-36 Since $E = hf$ frequency shifts are identical to energy shifts. Then we can use the results of Exercise 45-33 to get

$$(0.0001) = \frac{(0.9999)(6.2\ \mathrm{keV})}{(511\ \mathrm{keV})}(1 - \cos\phi),$$

which has solution $\phi = 7.4^\circ$.

(b) $(0.0001)(6.2\ \mathrm{keV}) = 0.62\ \mathrm{eV}$.

$\boxed{\textbf{E45-37}}$ (a) The change in wavelength is independent of the wavelength and is given by Eq. 45-17,

$$\Delta\lambda = \frac{hc}{mc^2}(1-\cos\phi) = 2\frac{(1240 \text{ eV}\cdot\text{nm})}{(0.511\times 10^6 \text{ eV})} = 4.85\times 10^{-3} \text{ nm}.$$

(b) The change in energy is given by

$$\begin{aligned}
\Delta E &= \frac{hc}{\lambda_\text{f}} - \frac{hc}{\lambda_\text{i}},\\
&= hc\left(\frac{1}{\lambda_\text{i}+\Delta\lambda} - \frac{1}{\lambda_\text{i}}\right),\\
&= (1240 \text{ eV}\cdot\text{nm})\left(\frac{1}{(9.77 \text{ pm})+(4.85 \text{ pm})} - \frac{1}{(9.77 \text{ pm})}\right) = -42.1 \text{ keV}
\end{aligned}$$

(c) This energy went to the electron, so the final kinetic energy of the electron is 42.1 keV.

E45-38 For $\phi = 90^\circ$ $\Delta\lambda = h/mc$. Then

$$\begin{aligned}
\frac{\Delta E}{E} &= 1 - \frac{hf'}{hf} = 1 - \frac{\lambda}{\lambda+\Delta\lambda},\\
&= \frac{h/mc}{\lambda + h/mc}.
\end{aligned}$$

But $h/mc = 2.43$ pm for the electron (see Exercise 45-30).

(a) $\Delta E/E = (2.43 \text{ pm})/(3.00 \text{ cm} + 2.43 \text{ pm}) = 8.1\times 10^{-11}$.
(b) $\Delta E/E = (2.43 \text{ pm})/(500 \text{ nm} + 2.43 \text{ pm}) = 4.86\times 10^{-6}$.
(c) $\Delta E/E = (2.43 \text{ pm})/(0.100 \text{ nm} + 2.43 \text{ pm}) = 0.0237$.
(d) $\Delta E/E = (2.43 \text{ pm})/(1.30 \text{ pm} + 2.43 \text{ pm}) = 0.651$.

E45-39 We can use the results of Exercise 45-33 to get

$$(0.10) = \frac{(0.90)(215 \text{ keV})}{(511 \text{ keV})}(1-\cos\phi),$$

which has solution $\phi = 42/6^\circ$.

E45-40 (a) A crude estimate is that the photons can't arrive more frequently than once every $10-8$s. That would provide an emission rate of 10^8/s.

(b) The power output would be

$$P = (10^8)\frac{(1240 \text{ eV}\cdot\text{nm})}{(550 \text{ nm})} = 2.3\times 10^8 \text{eV/s},$$

which is 3.6×10^{-11} W!

E45-41 We can follow the example of Sample Problem 45-6, and apply

$$\lambda = \lambda_0(1 - v/c).$$

(a) Solving for λ_0,

$$\lambda_0 = \frac{(588.995 \text{ nm})}{(1-(-300\,\text{m/s})(3\times 10^8 \text{m/s})} = 588.9944 \text{ nm}.$$

(b) Applying Eq. 45-18,

$$\Delta v = -\frac{h}{m\lambda} = -\frac{(6.6\times 10^{-34}\mathrm{J}\cdot\mathrm{s})}{(22)(1.7\times 10^{-27}\mathrm{kg})(590\times 10^{-9}\mathrm{m})} = 3\times 10^{-2}\mathrm{m/s}.$$

(c) Emitting another photon will slow the sodium by about the same amount.

E45-42 (a) $(430\,\mathrm{m/s})/(0.15\,\mathrm{m/s}) \approx 2900$ interactions.

(b) If the argon averages a speed of 220 m/s, then it requires interactions at the rate of

$$(2900)(220\,\mathrm{m/s})/(1.0\,\mathrm{m}) = 6.4\times 10^{5}/\mathrm{s}$$

if it is going to slow down in time.

$\boxed{\textbf{P45-1}}$ The radiant intensity is given by Eq. 45-3, $I = \sigma T^4$. The power that is radiated through the opening is $P = IA$, where A is the area of the opening. But energy goes both ways through the opening; it is the *difference* that will give the net power transfer. Then

$$P_{\mathrm{net}} = (I_0 - I_1)A = \sigma A\left(T_0^4 - T_1^4\right).$$

Put in the numbers, and

$$P_{\mathrm{net}} = (5.67\times 10^{-8}\mathrm{W/m^2{\cdot}K^4})(5.20\times 10^{-4}\mathrm{m^2})\left((488\,\mathrm{K})^4 - (299\,\mathrm{K})^4\right) = 1.44\,\mathrm{W}.$$

P45-2 (a) $I = \sigma T^4$ and $P = IA$. Then $T^4 = P/\sigma A$, or

$$T = \sqrt[4]{\frac{(100\,\mathrm{W})}{(5.67\times 10^{-8}\mathrm{W/m^2\cdot K^4})\pi(0.28\times 10^{-3}\mathrm{m})(1.8\times 10^{-2}\mathrm{m})}} = 3248\,\mathrm{K}.$$

That's 2980°C.

(b) The rate that energy is radiated off is given by $dQ/dt = mC\,dT/dt$. The mass is found from $m = \rho V$, where V is the volume. This can be combined with the power expression to yield

$$\sigma A T^4 = -\rho V C dT/dt,$$

which can be integrated to yield

$$\Delta t = \frac{\rho V C}{3\sigma A}(1/T_2^3 - 1/T_1^3).$$

Putting in numbers,

$$\begin{aligned}\Delta t &= \frac{(19300\mathrm{kg/m^3})(0.28\times 10^{-3}\mathrm{m})(132\mathrm{J/kgC})}{3(5.67\times 10^{-8}\mathrm{W/m^2\cdot K^4})(4)}[1/(2748\,\mathrm{K})^3 - 1/(3248\,\mathrm{K})^3],\\ &= 20\,\mathrm{ms}.\end{aligned}$$

$\boxed{\textbf{P45-3}}$ Light from the sun will "heat-up" the thin black screen. As the temperature of the screen increases it will begin to radiate energy. When the rate of energy radiation from the screen is equal to the rate at which the energy from the sun strikes the screen we will have equilibrium. We need first to find an expression for the rate at which energy from the sun strikes the screen.

The temperature of the sun is T_{S}. The radiant intensity is given by Eq. 45-3, $I_{\mathrm{S}} = \sigma {T_{\mathrm{S}}}^4$. The total power radiated by the sun is the product of this radiant intensity and the surface area of the sun, so

$$P_{\mathrm{S}} = 4\pi {r_{\mathrm{S}}}^2 \sigma {T_{\mathrm{S}}}^4,$$

where r_S is the radius of the sun.

Assuming that the lens is on the surface of the Earth (a reasonable assumption), then we can find the power incident on the lens if we know the intensity of sunlight at the distance of the Earth from the sun. That intensity is

$$I_E = \frac{P_S}{A} = \frac{P_S}{4\pi {R_E}^2},$$

where R_E is the radius of the Earth's orbit. Combining,

$$I_E = \sigma {T_S}^4 \left(\frac{r_S}{R_E}\right)^2$$

The total power incident on the lens is then

$$P_{lens} = I_E A_{lens} = \sigma {T_S}^4 \left(\frac{r_S}{R_E}\right)^2 \pi {r_l}^2,$$

where r_l is the radius of the lens. All of the energy that strikes the lens is focused on the image, so the power incident on the lens is also incident on the image.

The screen radiates as the temperature increases. The radiant intensity is $I = \sigma T^4$, where T is the temperature of the screen. The power radiated is this intensity times the surface area, so

$$P = IA = 2\pi {r_i}^2 \sigma T^4.$$

The factor of "2" is because the screen has two sides, while r_i is the radius of the image. Set this equal to P_{lens},

$$2\pi {r_i}^2 \sigma T^4 = \sigma {T_S}^4 \left(\frac{r_S}{R_E}\right)^2 \pi {r_l}^2,$$

or

$$T^4 = \frac{1}{2} {T_S}^4 \left(\frac{r_S\, r_l}{R_E\, r_i}\right)^2.$$

The radius of the image of the sun divided by the radius of the sun is the magnification of the lens. But magnification is also related to image distance divided by object distance, so

$$\frac{r_i}{r_S} = |m| = \frac{i}{o},$$

Distances should be measured from the lens, but since the sun is so far from the Earth, we won't be far off in stating $o \approx R_E$. Since the object is so far from the lens, the image will be very, very close to the focal point, so we can also state $i \approx f$. Then

$$\frac{r_i}{r_S} = \frac{f}{R_E},$$

so the expression for the temperature of the thin black screen is considerably simplified to

$$T^4 = \frac{1}{2} {T_S}^4 \left(\frac{r_l}{f}\right)^2.$$

Now we can put in some of the numbers.

$$T = \frac{1}{2^{1/4}} (5800\,\mathrm{K}) \sqrt{\frac{(1.9\text{ cm})}{(26\text{ cm})}} = 1300\,\mathrm{K}.$$

P45-4 The derivative of R with respect to λ is

$$-10\,\frac{\pi\,c^2\,h}{\lambda^6\,(e^{(\frac{h\,c}{\lambda\,k\,T})}-1)}+\frac{2\,\pi\,c^3\,h^2\,e^{(\frac{h\,c}{\lambda\,k\,T})}}{\lambda^7\,(e^{(\frac{h\,c}{\lambda\,k\,T})}-1)^2\,k\,T}.$$

Ohh, that's ugly. Setting it equal to zero allows considerable simplification, and we are left with

$$(5-x)e^x = 5,$$

where $x = hc/\lambda kT$. The solution is found numerically to be $x = 4.965114232$. Then

$$\lambda = \frac{(1240\text{ eV}\cdot\text{nm})}{(4.965)(8.62\times 10^{-5}\text{eV/K})T} = \frac{2.898\times 10^{-3}\text{m}\cdot\text{K}}{T}.$$

$\boxed{\textbf{P45-5}}$ (a) If the planet has a temperature T, then the radiant intensity of the planet will be $I\sigma T^4$, and the rate of energy radiation from the planet will be

$$P = 4\pi R^2\sigma T^4,$$

where R is the radius of the planet.

A steady state planet temperature requires that the energy from the sun arrive at the same rate as the energy is radiated from the planet. The intensity of the energy from the sun a distance r from the sun is

$$P_{\text{sun}}/4\pi r^2,$$

and the total power incident on the planet is then

$$P = \pi R^2\frac{P_{\text{sun}}}{4\pi r^2}.$$

Equating,

$$\begin{aligned} 4\pi R^2\sigma T^4 &= \pi R^2\frac{P_{\text{sun}}}{4\pi r^2}, \\ T^4 &= \frac{P_{\text{sun}}}{16\pi\sigma r^2}. \end{aligned}$$

(b) Using the last equation and the numbers from Problem 3,

$$T = \frac{1}{\sqrt{2}}(5800\,\text{K})\sqrt{\frac{(6.96\times 10^8\text{m})}{(1.5\times 10^{11}\text{m})}} = 279\,\text{K}.$$

That's about 43° F.

P45-6 (a) Change variables as suggested, then $\lambda = hc/xkT$ and $d\lambda = -(hc/x^2kT)dx$. Integrate (note the swapping of the variables of integration picks up a minus sign):

$$\begin{aligned} I &= \int\frac{2\pi c^2 h}{(hc/xkT)^5}\frac{(hc/x^2kT)dx}{e^x-1}, \\ &= \frac{2\pi k^4T^4}{h^3c^2}\int\frac{x^3\,dx}{e^x-1}, \\ &= \frac{2\pi^5k^4}{15h^3c^2}T^4. \end{aligned}$$

P45-7 (a) $P = E/t = nhf/t = (hc/\lambda)(n/t)$, where n/t is the rate of photon emission. Then

$$n/t = \frac{(100\,\mathrm{W})(589\times10^{-9}\mathrm{m})}{(6.63\times10^{-34}\mathrm{J\cdot s})(3.00\times10^{8}\mathrm{m/s})} = 2.96\times10^{20}/\mathrm{s}.$$

(b) The flux at a distance r is the rate divided by the area of the sphere of radius r, so

$$r = \sqrt{\frac{(2.96\times10^{20}/\mathrm{s})}{4\pi(1\times10^{4}/\mathrm{m^2\cdot s})}} = 4.8\times10^{7}\mathrm{m}.$$

(c) The photon density is the flux divided by the speed of light; the distance is then

$$r = \sqrt{\frac{(2.96\times10^{20}/\mathrm{s})}{4\pi(1\times10^{6}/\mathrm{m^3})(3\times10^{8}\mathrm{m/s})}} = 280\,\mathrm{m}.$$

(d) The flux is given by

$$\frac{(2.96\times10^{20}/\mathrm{s})}{4\pi(2.0\,\mathrm{m})^2} = 5.89\times10^{18}/\mathrm{m^2\cdot s}.$$

The photon density is

$$(5.89\times10^{18}\mathrm{m^2\cdot s})/(3.00\times10^{8}\mathrm{m/s}) = 1.96\times10^{10}/\mathrm{m^3}.$$

P45-8 Momentum conservation requires

$$p_\lambda = p_e,$$

while energy conservation requires

$$E_\lambda + mc^2 = E_e.$$

Square both sides of the energy expression and

$$\begin{aligned} E_\lambda^2 + 2E_\lambda mc^2 + m^2c^4 &= E_e^2 = p_e^2c^2 + m^2c^4, \\ E_\lambda^2 + 2E_\lambda mc^2 &= p_e^2c^2, \\ p_\lambda^2c^2 + 2E_\lambda mc^2 &= p_e^2c^2. \end{aligned}$$

But the momentum expression can be used here, and the result is

$$2E_\lambda mc^2 = 0.$$

Not likely.

P45-9 (a) Since $qvB = mv^2/r$, $v = (q/m)rB$ The kinetic energy of (non-relativistic) electrons will be

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{q^2(rB)^2}{m},$$

or

$$K = \frac{1}{2}\frac{(1.6\times10^{-19}\mathrm{C})}{(9.1\times10^{-31}\mathrm{kg})}(188\times10^{-6}\mathrm{T\cdot m})^2 = 3.1\times10^{3}\ \mathrm{eV}.$$

(b) Use the results of Exercise 45-1,

$$\phi = \frac{(1240\ \mathrm{eV\cdot nm})}{(71\times10^{-3}\mathrm{nm})} - 3.1\times10^{3}\mathrm{eV} = 1.44\times10^{4}\ \mathrm{eV}.$$

P45-10

$\boxed{\textbf{P45-11}}$ (a) The maximum value of $\Delta\lambda$ is $2h/mc$. The maximum energy lost by the photon is then

$$\begin{aligned}\Delta E &= \frac{hc}{\lambda_\mathrm{f}} - \frac{hc}{\lambda_\mathrm{i}},\\ &= hc\left(\frac{1}{\lambda_\mathrm{i}+\Delta\lambda} - \frac{1}{\lambda_\mathrm{i}}\right),\\ &= hc\frac{-2h/mc}{\lambda(\lambda+2h/mc)},\end{aligned}$$

where in the last line we wrote λ for λ_i. The energy given to the electron is the negative of this, so

$$K_\mathrm{max} = \frac{2h^2}{m\lambda(\lambda+2h/mc)}.$$

Multiplying through by $1^2 = (E\lambda/hc)^2$ we get

$$K_\mathrm{max} = \frac{2E^2}{mc^2(1+2hc/\lambda mc^2)}.$$

or

$$K_\mathrm{max} = \frac{E^2}{mc^2/2+E}.$$

(b) The answer is

$$K_\mathrm{max} = \frac{(17.5\text{ keV})^2}{(511\text{ eV})/2+(17.5\text{ keV})} = 1.12\text{ keV}.$$

$\boxed{\textbf{E46-1}}$ (a) Apply Eq. 46-1, $\lambda = h/p$. The momentum of the bullet is

$$p = mv = (0.041\,\text{kg})(960\,\text{m/s}) = 39\text{kg}\cdot\text{m/s},$$

so the corresponding wavelength is

$$\lambda = h/p = (6.63\times 10^{-34}\text{J}\cdot\text{s})/(39\text{kg}\cdot\text{m/s}) = 1.7\times 10^{-35}\,\text{m}.$$

(b) This length is much too small to be significant. How much too small? If the radius of the galaxy were one meter, this distance would correspond to the diameter of a proton.

E46-2 (a) $\lambda = h/p$ and $p^2/2m = K$, then

$$\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{(1240\text{ eV}\cdot\text{nm})}{\sqrt{2(511\text{ keV})}\sqrt{K}} = \frac{1.226\text{ nm}}{\sqrt{K}}.$$

(b) $K = eV$, so

$$\lambda = \frac{1.226\text{ nm}}{\sqrt{eV}} = \sqrt{\frac{1.5\,\text{V}}{V}}\text{nm}.$$

E46-3 For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$.

(a) For the electron,

$$\lambda = \frac{(1240\text{ eV}\cdot\text{nm})}{\sqrt{2(511\text{ keV})(1.0\text{ keV})}} = 0.0388\text{ nm}.$$

(c) For the neutron,

$$\lambda = \frac{(1240\text{ MeV}\cdot\text{fm})}{\sqrt{2(940\text{ MeV})(0.001\text{ MeV})}} = 904\text{ fm}.$$

(b) For ultra-relativistic particles $K \approx E \approx pc$, so

$$\lambda = \frac{hc}{E} = \frac{(1240\text{ eV}\cdot\text{nm})}{(1000\text{ eV})} = 1.24\text{ nm}.$$

E46-4 For non-relativistic particles $p = h/\lambda$ and $p^2/2m = K$, so $K = (hc)^2/2mc^2\lambda^2$. Then

$$K = \frac{(1240\text{ eV}\cdot\text{nm})^2}{2(5.11\times 10^6\text{eV})(589\text{ nm})^2} = 4.34\times 10^{-6}\text{ eV}.$$

$\boxed{\textbf{E46-5}}$ (a) Apply Eq. 46-1, $p = h/\lambda$. The proton speed would then be

$$v = \frac{h}{m\lambda} = c\frac{hc}{mc^2\lambda} = c\frac{(1240\text{ MeV}\cdot\text{fm})}{(938\text{ MeV})(113\text{ fm})} = 0.0117c.$$

This is good, because it means we were justified in using the non-relativistic equations. Then $v = 3.51\times 10^6\text{m/s}$.

(b) The kinetic energy of this electron would be

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(938\text{ MeV})(0.0117)^2 = 64.2\text{ keV}.$$

The potential through which it would need to be accelerated is 64.2 kV.

E46-6 (a) $K = qV$ and $p = \sqrt{2mK}$. Then

$$p = \sqrt{2(22)(932\text{ MeV}/c^2)(325\text{ eV})} = 3.65\times 10^6\text{ eV}/c.$$

(b) $\lambda = h/p$, so

$$\lambda = \frac{hc}{pc} = \frac{(1240\text{ eV}\cdot\text{nm})}{(3.65\times 10^6\text{ eV}/c)c} = 3.39\times 10^{-4}\text{nm}.$$

E46-7 (a) For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$. For the alpha particle,

$$\lambda = \frac{(1240\text{ MeV}\cdot\text{fm})}{\sqrt{2(4)(932\text{ MeV})(7.5\text{ MeV})}} = 5.2\text{ fm}.$$

(b) Since the wavelength of the alpha is considerably smaller than the distance to the nucleus we can ignore the wave nature of the alpha particle.

E46-8 (a) For non-relativistic particles $p = h/\lambda$ and $p^2/2m = K$, so $K = (hc)^2/2mc^2\lambda^2$. Then

$$K = \frac{(1240\text{ keV}\cdot\text{pm})^2}{2(511\text{keV})(10\text{ pm})^2} = 15\text{ keV}.$$

(b) For ultra-relativistic particles $K \approx E \approx pc$, so

$$E = \frac{hc}{\lambda} = \frac{(1240\text{ keV}\cdot\text{pm})}{(10\text{ pm})} = 124\text{ keV}.$$

$\boxed{\textbf{E46-9}}$ The relativistic relationship between energy and momentum is

$$E^2 = p^2c^2 + m^2c^4,$$

and if the energy is very large (compared to mc^2), then the contribution of the mass to the above expression is small, and

$$E^2 \approx p^2c^2.$$

Then from Eq. 46-1,

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{E} = \frac{(1240\text{ MeV}\cdot fm)}{(50\times 10^3\text{ MeV})} = 2.5\times 10^{-2}\text{ fm}.$$

E46-10 (a) $K = 3kT/2$, $p = \sqrt{2mK}$, and $\lambda = h/p$, so

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{3mkT}} = \frac{hc}{\sqrt{3mc^2kT}},\\ &= \frac{(1240\text{ MeV}\cdot fm)}{\sqrt{3(4)(932\text{MeV})(8.62\times 10^{-11}\text{MeV/K})(291\,\text{K})}} = 74\text{ pm}.\end{aligned}$$

(b) $pV = NkT$; assuming that each particle occupies a cube of volume $d^3 = V_0$ then the inter-particle spacing is d, so

$$d = \sqrt[3]{V/N} = \sqrt[3]{\frac{(1.38\times 10^{-23}\text{J/K})(291\,\text{K})}{(1.01\times 10^5\text{Pa})}} = 3.4\text{ nm}.$$

E46-11 $p = mv$ and $p = h/\lambda$, so $m = h/\lambda v$. Taking the ratio,

$$\frac{m_e}{m} = \frac{\lambda v}{\lambda_e v_e} = (1.813\times10^{-4})(3) = 5.439\times10^{-4}.$$

The mass of the unknown particle is then

$$m = \frac{(0.511 \text{ MeV}/c^2)}{(5.439\times10^{-4})} = 939.5 \text{ MeV}.$$

That would make it a neutron.

E46-12 (a) For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$.

For the electron,

$$\lambda = \frac{(1240 \text{ eV}\cdot\text{nm})}{\sqrt{2(5.11\times10^5 \text{ eV})(1.5 \text{ eV})}} = 1.0 \text{ nm}.$$

For ultra-relativistic particles $K \approx E \approx pc$, so for the photon

$$\lambda = \frac{hc}{E} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(1.5 \text{ eV})} = 830 \text{ nm}.$$

(b) Electrons with energies that high are ultra-relativistic. Both the photon and the electron will then have the same wavelength;

$$\lambda = \frac{hc}{E} = \frac{(1240 \text{ MeV}\cdot\text{fm})}{(1.5 \text{ GeV})} = 0.83 \text{ fm}.$$

$\boxed{\textbf{E46-13}}$ (a) The classical expression for kinetic energy is

$$p = \sqrt{2mK},$$

so

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2K}} = \frac{(1240 \text{ keV}\cdot pm)}{\sqrt{2(511 \text{ keV})(25.0 \text{ keV})}} = 7.76 \text{ pm}.$$

(a) The relativistic expression for momentum is

$$pc = sqrtE^2 - m^2c^4 = \sqrt{(mc^2+K)^2 - m^2c^4} = \sqrt{K^2 + 2mc^2K}.$$

Then

$$\lambda = \frac{hc}{pc} = \frac{(1240 \text{ keV}\cdot pm)}{\sqrt{(25.0 \text{ keV})^2 + 2(511 \text{ keV})(25.0 \text{ keV})}} = 7.66 \text{ pm}.$$

E46-14 We want to match the wavelength of the gamma to that of the electron. For the gamma, $\lambda = hc/E_\gamma$. For the electron, $K = p^2/2m = h^2/2m\lambda^2$. Combining,

$$K = \frac{h^2}{2mh^2c^2}E_\gamma^2 = \frac{E_\gamma^2}{2mc^2}.$$

With numbers,

$$K = \frac{(136\text{keV})^2}{2(511 \text{ keV})} = 18.1 \text{ keV}.$$

That would require an accelerating potential of 18.1 kV.

E46-15 First find the wavelength of the neutrons. For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$. Then

$$\lambda = \frac{(1240\text{ keV}\cdot\text{pm})}{\sqrt{2(940\times10^3\text{ keV})(4.2\times10^{-3}\text{ keV})}} = 14\text{ pm}.$$

Bragg reflection occurs when $2d\sin\theta = \lambda$, so

$$\theta = \arcsin(14\text{ pm})/2(73.2\text{ pm}) = 5.5^\circ.$$

E46-16 This is merely a Bragg reflection problem. Then $2d\sin\theta = m\lambda$, or

$$\begin{aligned}\theta &= \arcsin(1)(11\text{ pm})/2(54.64\text{ pm}) = 5.78^\circ,\\ \theta &= \arcsin(2)(11\text{ pm})/2(54.64\text{ pm}) = 11.6^\circ,\\ \theta &= \arcsin(3)(11\text{ pm})/2(54.64\text{ pm}) = 17.6^\circ.\end{aligned}$$

$\boxed{\textbf{E46-17}}$ (a) Since $\sin 52^\circ = 0.78$, then $2(\lambda/d) = 1.57 > 1$, so there is no diffraction order other than the first.

(b) For an accelerating potential of 54 volts we have $\lambda/d = 0.78$. Increasing the potential will increase the kinetic energy, increase the momentum, and decrease the wavelength. d won't change. The kinetic energy is increased by a factor of $60/54 = 1.11$, the momentum increases by a factor of $\sqrt{1.11} = 1.05$, so the wavelength changes by a factor of $1/1.05 = 0.952$. The new angle is then

$$\theta = \arcsin(0.952\times0.78) = 48^\circ.$$

E46-18 First find the wavelength of the electrons. For non-relativistic particles $\lambda = h/p$ and $p^2/2m = K$, so $\lambda = hc/\sqrt{2mc^2K}$. Then

$$\lambda = \frac{(1240\text{ keV}\cdot\text{pm})}{\sqrt{2(511\text{ keV})(0.380\text{ keV})}} = 62.9\text{ pm}.$$

This is now a Bragg reflection problem. Then $2d\sin\theta = m\lambda$, or

$$\begin{aligned}\theta &= \arcsin(1)(62.9\text{ pm})/2(314\text{ pm}) = 5.74^\circ,\\ \theta &= \arcsin(2)(62.9\text{ pm})/2(314\text{ pm}) = 11.6^\circ,\\ \theta &= \arcsin(3)(62.9\text{ pm})/2(314\text{ pm}) = 17.5^\circ,\\ \theta &= \arcsin(4)(62.9\text{ pm})/2(314\text{ pm}) = 23.6^\circ,\\ \theta &= \arcsin(5)(62.9\text{ pm})/2(314\text{ pm}) = 30.1^\circ,\\ \theta &= \arcsin(6)(62.9\text{ pm})/2(314\text{ pm}) = 36.9^\circ,\\ \theta &= \arcsin(7)(62.9\text{ pm})/2(314\text{ pm}) = 44.5^\circ,\\ \theta &= \arcsin(8)(62.9\text{ pm})/2(314\text{ pm}) = 53.3^\circ,\\ \theta &= \arcsin(9)(62.9\text{ pm})/2(314\text{ pm}) = 64.3^\circ.\end{aligned}$$

But the odd orders vanish (see Chapter 43 for a discussion on this).

E46-19 Since $\Delta f\cdot\Delta t \approx 1/2\pi$, we have

$$\Delta f = 1/2\pi(0.23\,\text{s}) = 0.69/\text{s}.$$

E46-20 Since $\Delta f \cdot \Delta t \approx 1/2\pi$, we have

$$\Delta f = 1/2\pi(0.10\times10^{-9}\text{s}) = 1.6\times10^{10}/\text{s}.$$

The bandwidth wouldn't fit in the frequency allocation!

$\boxed{\textbf{E46-21}}$ Apply Eq. 46-9,

$$\Delta E \geq \frac{h}{2\pi\Delta t} = \frac{4.14\times10^{-15}\text{ eV}\cdot\text{s})}{2\pi(8.7\times10^{-12}\text{s})} = 7.6\times10^{-5}\text{ eV}.$$

This is *much* smaller than the photon energy.

E46-22 Apply Heisenberg twice:

$$\Delta E_1 = \frac{4.14\times10^{-15}\text{ eV}\cdot\text{s})}{2\pi(12\times10^{-9}\text{s})} = 5.49\times10^{-8}\text{ eV}.$$

and

$$\Delta E_2 = \frac{4.14\times10^{-15}\text{ eV}\cdot\text{s})}{2\pi(23\times10^{-9}\text{s})} = 2.86\times10^{-8}\text{ eV}.$$

The sum is $\Delta E_{\text{transition}} = 8.4\times10^{-8}\text{eV}$.

E46-23 Apply Heisenberg:

$$\Delta p = \frac{6.63\times10^{-34}\text{J}\cdot\text{s})}{2\pi(12\times10^{-12}\text{m})} = 8.8\times10^{-24}\text{kg}\cdot\text{m/s}.$$

E46-24 $\Delta p = (0.5\,\text{kg})(1.2\,\text{s}) = 0.6\,\text{kg}\cdot\text{m/s}$. The position uncertainty would then be

$$\Delta x = \frac{(0.6\,\text{J/s})}{2\pi(0.6\,\text{kg}\cdot\text{m/s})} = 0.16\,\text{m}.$$

$\boxed{\textbf{E46-25}}$ We want $v \approx \Delta v$, which means $p \approx \Delta p$. Apply Eq. 46-8, and

$$\Delta x \geq \frac{h}{2\pi\Delta p} \approx \frac{h}{2\pi p}.$$

According to Eq. 46-1, the de Broglie wavelength is related to the momentum by

$$\lambda = h/p,$$

so

$$\Delta x \geq \frac{\lambda}{2\pi}.$$

E46-26 (a) A particle confined in a (one dimensional) box of size L will have a position uncertainty of no more than $\Delta x \approx L$. The momentum uncertainty will then be no less than

$$\Delta p \geq \frac{h}{2\pi\Delta x} \approx \frac{h}{2\pi L}.$$

so

$$\Delta p \approx \frac{(6.63\times10^{-34}\text{J}\cdot\text{s})}{2\pi(\times10^{-10}\,\text{m})} = 1\times10^{-24}\text{kg}\cdot\text{m/s}.$$

(b) Assuming that $p \approx \Delta p$, we have

$$p \geq \frac{h}{2\pi L},$$

and then the electron will have a (minimum) kinetic energy of

$$E \approx \frac{p^2}{2m} \approx \frac{h^2}{8\pi^2 m L^2}.$$

or

$$E \approx \frac{(hc)^2}{8\pi^2 mc^2 L^2} = \frac{(1240 \text{ keV} \cdot \text{pm})^2}{8\pi^2 (511 \text{ keV})(100 \text{ pm})^2} = 0.004 \text{ keV}.$$

$\boxed{\textbf{E46-27}}$ (a) A particle confined in a (one dimensional) box of size L will have a position uncertainty of no more than $\Delta x \approx L$. The momentum uncertainty will then be no less than

$$\Delta p \geq \frac{h}{2\pi \Delta x} \approx \frac{h}{2\pi L}.$$

so

$$\Delta p \approx \frac{(6.63\times10^{-34}\text{J} \cdot \text{s})}{2\pi(\times10^{-14}\text{ m})} = 1\times10^{-20}\text{kg} \cdot \text{m/s}.$$

(b) Assuming that $p \approx \Delta p$, we have

$$p \geq \frac{h}{2\pi L},$$

and then the electron will have a (minimum) kinetic energy of

$$E \approx \frac{p^2}{2m} \approx \frac{h^2}{8\pi^2 m L^2}.$$

or

$$E \approx \frac{(hc)^2}{8\pi^2 mc^2 L^2} = \frac{(1240 \text{ MeV} \cdot \text{fm})^2}{8\pi^2 (0.511 \text{ MeV})(10 \text{ fm})^2} = 381 \text{ MeV}.$$

This is so large compared to the mass energy of the electron that we must consider relativistic effects. It will be very relativistic ($381 \gg 0.5!$), so we can use $E = pc$ as was derived in Exercise 9. Then

$$E = \frac{hc}{2\pi L} = \frac{(1240 \text{ MeV} \cdot \text{fm})}{2\pi(10 \text{ fm})} = 19.7 \text{ MeV}.$$

This is the *total* energy; so we subtract 0.511 MeV to get $K = 19$ MeV.

E46-28 We want to find L when $T = 0.01$. This means solving

$$\begin{aligned}
T &= 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) e^{-2kL}, \\
(0.01) &= 16\frac{(5.0 \text{ eV})}{(6.0 \text{ eV})}\left(1 - \frac{(5.0 \text{ eV})}{(6.0 \text{ eV})}\right) e^{-2k'L}, \\
&= 2.22 e^{-2k'L}, \\
\ln(4.5\times10^{-3}) &= 2(5.12\times10^{9}/\text{m})L, \\
5.3\times10^{-10}\text{m} &= L.
\end{aligned}$$

$\boxed{\textbf{E46-29}}$ The wave number, k, is given by

$$k = \frac{2\pi}{hc}\sqrt{2mc^2(U_0 - E)}.$$

(a) For the proton $mc^2 = 938$ MeV, so

$$k = \frac{2\pi}{(1240 \text{ MeV} \cdot \text{fm})}\sqrt{2(938 \text{ MeV})(10 \text{ MeV} - 3.0 \text{ MeV})} = 0.581 \text{ fm}^{-1}.$$

The transmission coefficient is then

$$T = 16\frac{(3.0 \text{ MeV})}{(10 \text{ MeV})}\left(1 - \frac{(3.0 \text{ MeV})}{(10 \text{ MeV})}\right)e^{-2(0.581 \text{ fm}^{-1})(10 \text{ fm})} = 3.0\times10^{-5}.$$

(b) For the deuteron $mc^2 = 2 \times 938$ MeV, so

$$k = \frac{2\pi}{(1240 \text{ MeV} \cdot \text{fm})}\sqrt{2(2)(938 \text{ MeV})(10 \text{ MeV} - 3.0 \text{ MeV})} = 0.821 \text{ fm}^{-1}.$$

The transmission coefficient is then

$$T = 16\frac{(3.0 \text{ MeV})}{(10 \text{ MeV})}\left(1 - \frac{(3.0 \text{ MeV})}{(10 \text{ MeV})}\right)e^{-2(0.821 \text{ fm}^{-1})(10 \text{ fm})} = 2.5\times10^{-7}.$$

E46-30 The wave number, k, is given by

$$k = \frac{2\pi}{hc}\sqrt{2mc^2(U_0 - E)}.$$

(a) For the proton $mc^2 = 938$ MeV, so

$$k = \frac{2\pi}{(1240 \text{ keV} \cdot \text{pm})}\sqrt{2(938 \text{ MeV})(6.0 \text{ eV} - 5.0 \text{ eV})} = 0.219 \text{ pm}^{-1}.$$

We want to find T. This means solving

$$\begin{aligned} T &= 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)e^{-2kL}, \\ &= 16\frac{(5.0 \text{ eV})}{(6.0 \text{ eV})}\left(1 - \frac{(5.0 \text{ eV})}{(6.0 \text{ eV})}\right)e^{-2(0.219\times10^{12})(0.7\times10^{-9})}, \\ &= 1.6\times10^{-133}. \end{aligned}$$

A current of 1 kA corresponds to

$$N = (1\times10^3\text{C/s})/(1.6\times10^{-19}\text{C}) = 6.3\times10^{21}/\text{s}$$

protons per seconds. The time required for one proton to pass is then

$$t = 1/(6.3\times10^{21}/\text{s})(1.6\times10^{-133}) = 9.9\times10^{110}\text{s}.$$

That's 10^{104} years!

$\boxed{\textbf{P46-1}}$ We will interpret low energy to mean non-relativistic. Then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_{\rm n}K}}.$$

The diffraction pattern is then given by

$$d\sin\theta = m\lambda = mh/\sqrt{2m_{\rm n}K},$$

where m is diffraction order while $m_{\rm n}$ is the neutron mass. We want to investigate the spread by taking the derivative of θ with respect to K,

$$d\cos\theta\, d\theta = -\frac{mh}{2\sqrt{2m_{\rm n}K^3}}\, dK.$$

Divide this by the original equation, and then

$$\frac{\cos\theta}{\sin\theta} d\theta = -\frac{dK}{2K}.$$

Rearrange, change the differential to a difference, and then

$$\Delta\theta = \tan\theta \frac{\Delta K}{2K}.$$

We dropped the negative sign out of laziness; but the angles are in radians, so we need to multiply by $180/\pi$ to convert to degrees.

P46-2

P46-3 We want to solve

$$T = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) e^{-2kL},$$

for E. Unfortunately, E is contained in k since

$$k = \frac{2\pi}{hc}\sqrt{2mc^2(U_0 - E)}.$$

We can do this by iteration. The maximum possible value for

$$\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)$$

is $1/4$; using this value we can get an estimate for k:

$$\begin{aligned} (0.001) &= 16(0.25)e^{-2kL}, \\ \ln(2.5\times 10^{-4}) &= -2k(0.7\text{ nm}), \\ 5.92/\text{ nm} &= k. \end{aligned}$$

Now solve for E:

$$\begin{aligned} E &= U_0 - (hc)^2k^2 8mc^2\pi^2, \\ &= (6.0\text{ eV}) - \frac{(1240\text{ eV}\cdot\text{nm})^2(5.92/\text{nm})^2}{8\pi^2(5.11\times 10^5\text{eV})}, \\ &= 4.67\text{ eV}. \end{aligned}$$

Put this value for E back into the transmission equation to find a new k:

$$\begin{aligned}
T &= 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2kL},\\
(0.001) &= 16\frac{(4.7\text{ eV})}{(6.0\text{ eV})}\left(1-\frac{(4.7\text{ eV})}{(6.0\text{ eV})}\right)e^{-2kL},\\
\ln(3.68\times10^{-4}) &= -2k(0.7\text{ nm}),\\
5.65/\text{ nm} &= k.
\end{aligned}$$

Now solve for E using this new, improved, value for k:

$$\begin{aligned}
E &= U_0-(hc)^2k^28mc^2\pi^2,\\
&= (6.0\text{ eV})-\frac{(1240\text{ eV}\cdot\text{nm})^2(5.65/\text{nm})^2}{8\pi^2(5.11\times10^5\text{eV})},\\
&= 4.78\text{ eV}.
\end{aligned}$$

Keep at it. You'll eventually stop around $E = 5.07$ eV.

P46-4 (a) A one percent increase in the barrier height means $U_0 = 6.06$ eV.
For the electron $mc^2 = 5.11\times10^5$ eV, so

$$k=\frac{2\pi}{(1240\text{ eV}\cdot\text{nm})}\sqrt{2(5.11\times10^5\text{ eV})(6.06\text{ eV}-5.0\text{ eV})}=5.27\text{ nm}^{-1}.$$

We want to find T. This means solving

$$\begin{aligned}
T &= 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2kL},\\
&= 16\frac{(5.0\text{ eV})}{(6.06\text{ eV})}\left(1-\frac{(5.0\text{ eV})}{(6.06\text{ eV})}\right)e^{-2(5.27)(0.7)},\\
&= 1.44\times10^{-3}.
\end{aligned}$$

That's a 16% decrease.
(b) A one percent increase in the barrier thickness means $L = 0.707$ nm.
For the electron $mc^2 = 5.11\times10^5$ eV, so

$$k=\frac{2\pi}{(1240\text{ eV}\cdot\text{nm})}\sqrt{2(5.11\times10^5\text{ eV})(6.0\text{ eV}-5.0\text{ eV})}=5.12\text{ nm}^{-1}.$$

We want to find T. This means solving

$$\begin{aligned}
T &= 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2kL},\\
&= 16\frac{(5.0\text{ eV})}{(6.0\text{ eV})}\left(1-\frac{(5.0\text{ eV})}{(6.0\text{ eV})}\right)e^{-2(5.12)(0.707)},\\
&= 1.59\times10^{-3}.
\end{aligned}$$

That's a 8.1% decrease.
(c) A one percent increase in the incident energy means $E = 5.05$ eV.
For the electron $mc^2 = 5.11\times10^5$ eV, so

$$k=\frac{2\pi}{(1240\text{ eV}\cdot\text{nm})}\sqrt{2(5.11\times10^5\text{ eV})(6.0\text{ eV}-5.05\text{ eV})}=4.99\text{ nm}^{-1}.$$

We want to find T. This means solving

$$\begin{aligned} T &= 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2kL}, \\ &= 16\frac{(5.05 \text{ eV})}{(6.0 \text{ eV})}\left(1-\frac{(5.05 \text{ eV})}{(6.0 \text{ eV})}\right)e^{-2(4.99)(0.7)}, \\ &= 1.97\times 10^{-3}. \end{aligned}$$

That's a 14% increase.

P46-5 First, the rule for exponents

$$e^{i(a+b)} = e^{ia}\,e^{ib}.$$

Then apply Eq. 46-12, $e^{i\theta} = \cos\theta + i\sin\theta$,

$$\cos(a+b) + i\sin(a+b) = (\cos a + i\sin a)(\sin b + i\sin b).$$

Expand the right hand side, remembering that $i^2 = -1$,

$$\cos(a+b) + i\sin(a+b) = \cos a\cos b + i\cos a\sin b + i\sin a\cos b - \sin a\sin b.$$

Since the real part of the left hand side must equal the real part of the right and the imaginary part of the left hand side must equal the imaginary part of the right, we actually have *two* equations. They are

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

and

$$\sin(a+b) = \cos a\sin b + \sin a\cos b.$$

P46-6

$\boxed{\textbf{E47-1}}$ (a) The ground state energy level will be given by

$$E_1 = \frac{h^2}{8mL^2} = \frac{(6.63 \times 10^{-34}\,\mathrm{J\cdot s})^2}{8(9.11 \times 10^{-31}\mathrm{kg})(1.4 \times 10^{-14}\,\mathrm{m})^2} = 3.1 \times 10^{-10}\,\mathrm{J}.$$

The answer is correct, but the units make it almost useless. We can divide by the electron charge to express this in electron volts, and then $E = 1900$ MeV. Note that this is an extremely relativistic quantity, so the energy expression loses validity.

(b) We can repeat what we did above, or we can apply a "trick" that is often used in solving these problems. Multiplying the top and the bottom of the energy expression by c^2 we get

$$E_1 = \frac{(hc)^2}{8(mc^2)L^2}$$

Then

$$E_1 = \frac{(1240\ \mathrm{MeV\cdot fm})^2}{8(940\ \mathrm{MeV})(14\ \mathrm{fm})^2} = 1.0\ \mathrm{MeV}.$$

(c) Finding an neutron inside the nucleus seems reasonable; but finding the electron would not. The energy of such an electron is considerably larger than binding energies of the particles in the nucleus.

E47-2 Solve

$$E_n = \frac{n^2(hc)^2}{8(mc^2)L^2}$$

for L, then

$$\begin{aligned} L &= \frac{nhc}{\sqrt{8mc^2E_n}}, \\ &= \frac{(3)(1240\ \mathrm{eV\cdot nm})}{\sqrt{8(5.11\times10^5\mathrm{eV})(4.7\ \mathrm{eV})}}, \\ &= 0.85\ \mathrm{nm}. \end{aligned}$$

E47-3 Solve for $E_4 - E_1$:

$$\begin{aligned} E_4 - E_1 &= \frac{4^2(hc)^2}{8(mc^2)L^2} - \frac{1^2(hc)^2}{8(mc^2)L^2}, \\ &= \frac{(16-1)(1240\ \mathrm{eV\cdot nm})^2}{8(5.11\times10^5)(0.253\ \mathrm{nm})^2}, \\ &= 88.1\ \mathrm{eV}. \end{aligned}$$

E47-4 Since $E \propto 1/L^2$, doubling the width of the well will lower the ground state energy to $(1/2)^2 = 1/4$, or 0.65 eV.

E47-5 (a) Solve for $E_2 - E_1$:

$$\begin{aligned} E_2 - E_1 &= \frac{2^2h^2}{8mL^2} - \frac{1^2h^2}{8mL^2}, \\ &= \frac{(3)(6.63\times10^{-34}\mathrm{J\cdot s})^2}{8(40)(1.67\times10^{-27}\mathrm{kg})(0.2\,\mathrm{m})^2}, \\ &= 6.2\times10^{-41}\mathrm{J}. \end{aligned}$$

(b) $K = 3kT/2 = 3(1.38\times10^{-23}\mathrm{J/K})(300\,\mathrm{K})/2 = 6.21\times10^{-21}$. The ratio is 1×10^{-20}.
(c) $T = 2(6.2\times10^{-41}\mathrm{J})/3(1.38\times10^{-23}\mathrm{J/K}) = 3.0\times10^{-18}\mathrm{K}$.

E47-6 (a) The fractional difference is $(E_{n+1} - E_n)/E_n$, or

$$\begin{aligned}\frac{\Delta E_n}{E_n} &= \left[(n+1)^2\frac{h^2}{8mL^2} - n^2\frac{h^2}{8mL^2}\right] / \left[n^2\frac{h^2}{8mL^2}\right],\\ &= \frac{(n+1)^2 - n^2}{n^2},\\ &= \frac{2n+1}{n^2}.\end{aligned}$$

(b) As $n \to \infty$ the fractional difference goes to zero; the system behaves as if it is continuous.

$\boxed{\textbf{E47-7}}$ (a) We will take advantage of the "trick" that was developed in part (b) of Exercise 47-1. Then

$$E_n = n^2\frac{(hc)^2}{8mc^2\,L} = (15)^2\frac{(1240\text{ eV}\cdot\text{nm})^2}{8(0.511\times 10^6\text{ eV})(0.0985\text{ nm})^2} = 8.72\text{ keV}.$$

(b) The magnitude of the momentum is *exactly* known because $E = p^2/2m$. This momentum is given by

$$pc = \sqrt{2mc^2E} = \sqrt{2(511\text{ keV})(8.72\text{ keV})} = 94.4\text{ keV}.$$

What we don't know is in which direction the particle is moving. It is bouncing back and forth between the walls of the box, so the momentum could be directed toward the right or toward the left. The uncertainty in the momentum is then

$$\Delta p = p$$

which can be expressed in terms of the box size L by

$$\Delta p = p = \sqrt{2mE} = \sqrt{\frac{n^2h^2}{4L^2}} = \frac{nh}{2L}.$$

(c) The uncertainty in the position is 98.5 pm; the electron could be *anywhere* inside the well.

E47-8 The probability distribution function is

$$P_2 = \frac{2}{L}\sin^2\frac{2\pi x}{L}.$$

We want to integrate over the central third, or

$$\begin{aligned}P &= \int_{-L/6}^{L/6}\frac{2}{L}\sin^2\frac{2\pi x}{L}dx,\\ &= \frac{1}{\pi}\int_{-\pi/3}^{\pi/3}\sin^2\theta\,d\theta,\\ &= 0.196.\end{aligned}$$

E47-9 (a) Maximum probability occurs when the argument of the cosine (sine) function is $k\pi$ ($[k+1/2]\pi$). This occurs when

$$x = NL/2n$$

for odd N.

(b) Minimum probability occurs when the argument of the cosine (sine) function is $[k+1/2]\pi$ ($k\pi$). This occurs when

$$x = NL/2n$$

for even N.

E47-10 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

(a) The Lyman series is the series which ends on E_1. The least energetic state starts on E_2. The transition energy is

$$E_2 - E_1 = (13.6 \text{ eV})(1/1^2 - 1/2^2) = 10.2 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(10.2 \text{ eV})} = 121.6 \text{ nm}.$$

(b) The series limit is

$$0 - E_1 = (13.6 \text{ eV})(1/1^2) = 13.6 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(13.6 \text{ eV})} = 91.2 \text{ nm}.$$

$\boxed{\textbf{E47-11}}$ The ground state of hydrogen, as given by Eq. 47-21, is

$$E_1 = -\frac{me^4}{8\epsilon_0^2 h^2} = -\frac{(9.109 \times 10^{-31}\,\text{kg})(1.602 \times 10^{-19}\,\text{C})^4}{8(8.854 \times 10^{-12}\,\text{F/m})^2(6.626 \times 10^{-34}\,\text{J} \cdot \text{s})^2} = 2.179 \times 10^{-18}\,\text{J}.$$

In terms of electron volts the ground state energy is

$$E_1 = -(2.179 \times 10^{-18}\,\text{J})/(1.602 \times 10^{-19}\,\text{C}) = -13.60 \text{ eV}.$$

E47-12 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

(c) The transition energy is

$$\Delta E = E_3 - E_1 = (13.6 \text{ eV})(1/1^2 - 1/3^2) = 12.1 \text{ eV}.$$

(a) The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(12.1 \text{ eV})} = 102.5 \text{ nm}.$$

(b) The momentum is

$$p = E/c = 12.1 \text{ eV}/c.$$

E47-13 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

(a) The Balmer series is the series which ends on E_2. The least energetic state starts on E_3. The transition energy is

$$E_3 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/3^2) = 1.89 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(1.89 \text{ eV})} = 656 \text{ nm}.$$

(b) The next energetic state starts on E_4. The transition energy is

$$E_4 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/4^2) = 2.55 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(2.55 \text{ eV})} = 486 \text{ nm}.$$

(c) The next energetic state starts on E_5. The transition energy is

$$E_5 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/5^2) = 2.86 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(2.86 \text{ eV})} = 434 \text{ nm}.$$

(d) The next energetic state starts on E_6. The transition energy is

$$E_6 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/6^2) = 3.02 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(3.02 \text{ eV})} = 411 \text{ nm}.$$

(e) The next energetic state starts on E_7. The transition energy is

$$E_7 - E_2 = (13.6 \text{ eV})(1/2^2 - 1/7^2) = 3.12 \text{ eV}.$$

The wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(3.12 \text{ eV})} = 397 \text{ nm}.$$

E47-14 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

The transition energy is

$$\Delta E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(121.6 \text{ nm})} = 10.20 \text{ eV}.$$

This *must* be part of the Lyman series, so the higher state must be

$$E_n = (10.20 \text{ eV}) - (13.6 \text{ eV}) = -3.4 \text{ eV}.$$

That would correspond to $n = 2$.

E47-15 The binding energy is the energy required to remove the electron. If the energy of the electron is negative, then that negative energy is a measure of the energy required to set the electron free.

The first excited state is when $n = 2$ in Eq. 47-21. It is *not* necessary to re-evaluate the constants in this equation every time, instead, we start from

$$E_n = \frac{E_1}{n^2} \text{ where } E_1 = -13.60 \text{ eV}.$$

Then the first excited state has energy

$$E_2 = \frac{(-13.6 \text{ eV})}{(2)^2} = -3.4 \text{ eV}.$$

The binding energy is then 3.4 eV.

E47-16 $r_n = a_0 n^2$, so
$$n = \sqrt{(847 \text{ pm})/(52.9 \text{ pm})} = 4.$$

$\boxed{\textbf{E47-17}}$ (a) The energy of this photon is
$$E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(1281.8 \text{ nm})} = 0.96739 \text{ eV}.$$

The final state of the hydrogen must have an energy of no more than -0.96739, so the largest possible n of the final state is
$$n < \sqrt{13.60 \text{ eV}/0.96739 \text{ eV}} = 3.75,$$

so the final n is 1, 2, or 3. The initial state is only slightly higher than the final state. The jump from $n = 2$ to $n = 1$ is *too* large (see Exercise 15), any other initial state would have a larger energy difference, so $n = 1$ is *not* the final state.

So what level might be above $n = 2$? We'll try
$$n = \sqrt{13.6 \text{ eV}/(3.4 \text{ eV} - 0.97 \text{ eV})} = 2.36,$$

which is *so* far from being an integer that we don't need to look farther. The $n = 3$ state has energy $13.6 \text{ eV}/9 = 1.51$ eV. Then the initial state could be
$$n = \sqrt{13.6 \text{ eV}/(1.51 \text{ eV} - 0.97 \text{ eV})} = 5.01,$$

which is close enough to 5 that we can assume the transition was $n = 5$ to $n = 3$.

(b) This belongs to the Paschen series.

E47-18 In Exercise 47-21 we show that the hydrogen levels can be written as
$$E_n = -(13.6 \text{ eV})/n^2.$$

(a) The transition energy is
$$\Delta E = E_4 - E_1 = (13.6 \text{ eV})(1/1^2 - 1/4^2) = 12.8 \text{ eV}.$$

(b) All transitions $n \to m$ are allowed for $n \leq 4$ and $m < n$. The transition energy will be of the form
$$E_n - E_m = (13.6 \text{ eV})(1/m^2 - 1/n^2).$$
The six possible results are 12.8 eV, 12.1 eV, 10.2 eV, 2.55 eV, 1.89 eV, and 0.66 eV.

E47-19 $\Delta E = h/2\pi\Delta t$, so
$$\Delta E = (4.14\times10^{-15} \text{ eV} \cdot \text{s})/2\pi(1\times10^{-8}\text{s}) = 6.6\times10^{-8}\text{eV}.$$

E47-20 (a) According to electrostatics and uniform circular motion,
$$mv^2/r = e^2/4\pi\epsilon_0 r^2,$$

or
$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}} == \sqrt{\frac{e^4}{4\epsilon_0^2 h^2 n^2}} = \frac{e^2}{2\epsilon_0 hn}.$$

Putting in the numbers,

$$v = \frac{(1.6\times10^{-19}\text{C})^2}{2(8.85\times10^{-12}\text{F/m})(6.63\times10^{-34}\text{J}\cdot\text{s})n} = \frac{2.18\times10^6\text{m/s}}{n}.$$

In this case $n = 1$.

(b) $\lambda = h/mv$,

$$\lambda = (6.63\times10^{-34}\text{J}\cdot\text{s})/(9.11\times10^{-31}\text{kg})(2.18\times10^6\text{m/s}) = 3.34\times10^{-10}\text{m}.$$

(c) $\lambda/a_0 = (3.34\times10^{-10}\text{m})/(5.29\times10^{-11}) = 6.31 \approx 2\pi$. Actually, it is exactly 2π.

E47-21 In order to have an inelastic collision with the 6.0 eV neutron there must exist a transition with an energy difference of less than 6.0 eV. For a hydrogen atom in the ground state $E_1 = -13.6$ eV the nearest state is

$$E_2 = (-13.6 \text{ eV})/(2)^2 = -3.4 \text{ eV}.$$

Since the difference is 10.2 eV, it will *not* be possible for the 6.0 eV neutron to have an inelastic collision with a ground state hydrogen atom.

E47-22 (a) The atom is originally in the state n given by

$$n = \sqrt{(13.6 \text{ eV})/(0.85 \text{ eV})} = 4.$$

The state with an excitation energy of 10.2 eV, is

$$n = \sqrt{(13.6 \text{ eV})/(13.6 \text{ eV} - 10.2 \text{ eV})} = 2.$$

The transition energy is then

$$\Delta E = (13.6 \text{ eV})(1/2^2 - 1/4^2) = 2.55 \text{ eV}.$$

E47-23 According to electrostatics and uniform circular motion,

$$mv^2/r = e^2/4\pi\epsilon_0 r^2,$$

or

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}} == \sqrt{\frac{e^4}{4\epsilon_0^2 h^2 n^2}} = \frac{e^2}{2\epsilon_0 hn}.$$

The de Broglie wavelength is then

$$\lambda = \frac{h}{mv} = \frac{2\epsilon_0 hn}{me^2}.$$

The ratio of λ/r is

$$\frac{\lambda}{r} = \frac{2\epsilon_0 hn}{me^2 a_0 n^2} = kn,$$

where k is a constant. As $n \to \infty$ the ratio goes to zero.

E47-24 In Exercise 47-21 we show that the hydrogen levels can be written as

$$E_n = -(13.6 \text{ eV})/n^2.$$

The transition energy is

$$\Delta E = E_4 - E_1 = (13.6 \text{ eV})(1/1^2 - 1/4^2) = 12.8 \text{ eV}.$$

The momentum of the emitted photon is

$$p = E/c = (12.8 \text{ eV})/c.$$

This is the momentum of the recoiling hydrogen atom, which then has velocity

$$v = \frac{p}{m} = \frac{pc}{mc^2}c = \frac{(12.8 \text{ eV})}{(932 \text{ MeV})}(3.00\times10^8 \text{m/s}) = 4.1 \text{ m/s}.$$

$\boxed{\textbf{E47-25}}$ The first Lyman line is the $n = 1$ to $n = 2$ transition. The second Lyman line is the $n = 1$ to $n = 3$ transition. The first Balmer line is the $n = 2$ to $n = 3$ transition. Since the photon frequency is proportional to the photon energy ($E = hf$) and the photon energy is the energy difference between the two levels, we have

$$f_{n\to m} = \frac{E_m - E_n}{h}$$

where the E_n is the hydrogen atom energy level. Then

$$\begin{aligned}
f_{1\to 3} &= \frac{E_3 - E_1}{h}, \\
&= \frac{E_3 - E_2 + E_2 - E_1}{h} = \frac{E_3 - E_2}{h} + \frac{E_2 - E_1}{h}, \\
&= f_{2\to 3} + f_{1\to 2}.
\end{aligned}$$

E47-26 Use

$$E_n = -Z^2(13.6 \text{ eV})/n^2.$$

(a) The ionization energy of the ground state of He^+ is

$$E_n = -(2)^2(13.6 \text{ eV})/(1)^2 = 54.4 \text{ eV}.$$

(b) The ionization energy of the $n = 3$ state of Li^{2+} is

$$E_n = -(3)^2(13.6 \text{ eV})/(3)^2 = 13.6 \text{ eV}.$$

$\boxed{\textbf{E47-27}}$ (a) The energy levels in the He^+ spectrum are given by

$$E_n = -Z^2(13.6 \text{ eV})/n^2,$$

where $Z = 2$, as is discussed in Sample Problem 47-6. The photon wavelengths for the $n = 4$ series are then

$$\lambda = \frac{hc}{E_n - E_4} = \frac{hc/E_4}{1 - E_n/E_4},$$

which can also be written as

$$
\begin{aligned}
\lambda &= \frac{16hc/(54.4\text{ eV})}{1-16/n^2}, \\
&= \frac{16hcn^2/(54.4\text{ eV})}{n^2-16}, \\
&= \frac{Cn^2}{n^2-16},
\end{aligned}
$$

where $C = hc/(3.4\text{ eV}) = 365$ nm.

(b) The wavelength of the first line is the transition from $n = 5$,

$$\lambda = \frac{(365\text{ nm})(5)^2}{(5)^2-(4)^2} = 1014\text{ nm}.$$

The series limit is the transition from $n = \infty$, so

$$\lambda = 365\text{ nm}.$$

(c) The series starts in the infrared (1014 nm), and ends in the ultraviolet (365 nm). So it must also include some visible lines.

E47-28 We answer these questions out of order!

(a) $n = 1$.

(b) $r = a_0 = 5.29\times10^{-11}$m.

(f) According to electrostatics and uniform circular motion,

$$mv^2/r = e^2/4\pi\epsilon_0 r^2,$$

or

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}} == \sqrt{\frac{e^4}{4\epsilon_0^2 h^2 n^2}} = \frac{e^2}{2\epsilon_0 hn}.$$

Putting in the numbers,

$$v = \frac{(1.6\times10^{-19}\text{C})^2}{2(8.85\times10^{-12}\text{F/m})(6.63\times10^{-34}\text{J}\cdot\text{s})(1)} = 2.18\times10^6\text{m/s}.$$

(d) $p = (9.11\times10^{-31}\text{kg})(2.18\times10^6\text{m/s}) = 1.99\times10^{-24}\text{kg}\cdot\text{m/s}$.

(e) $\omega = v/r = (2.18\times10^6\text{m/s})/(5.29\times10^{-11}\text{m}) = 4.12\times10^{16}\text{rad/s}$.

(c) $l = pr = (1.99\times10^{-24}\text{kg}\cdot\text{m/s})(5.29\times10^{-11}\text{m}) = 1.05\times10^{-34}\text{J}\cdot\text{s}$.

(g) $F = mv^2/r$, so

$$F = (9.11\times10^{-31}\text{kg})(2.18\times10^6\text{m/s})^2/(5.29\times10^{-11}\text{m}) = 8.18\times10^{-8}\text{N}.$$

(h) $a = (8.18\times10^{-8}\text{N})/(9.11\times10^{-31}kg) = 8.98\times10^{22}\text{m/s}^2$.

(i) $K = mv^2/r$, or

$$K = \frac{(9.11\times10^{-31}\text{kg})(2.18\times10^6\text{m/s})^2}{2} = 2.16\times10^{-18}\text{J} = 13.6\text{ eV}.$$

(k) $E = -13.6$ eV.

(j) $P = E - K = (-13.6\text{ eV}) - (13.6\text{ eV}) = -27.2$ eV.

E47-29 For each r in the quantity we have a factor of n^2.

(a) n is proportional to n.
(b) r is proportional to n^2.
(f) v is proportional to $\sqrt{1/r}$, or $1/n$.
(d) p is proportional to v, or $1/n$.
(e) ω is proportional to v/r, or $1/n^3$.
(c) l is proportional to pr, or n.
(g) f is proportional to v^2/r, or $1/n^4$.
(h) a is proportional to F, or $1/n^4$.
(i) K is proportional to v^2, or $1/n^2$.
(j) E is proportional to $1/n^2$.
(k) P is proportional to $1/n^2$.

E47-30 (a) Using the results of Exercise 45-1,

$$E_1 = \frac{(1240 \text{ eV} \cdot \text{nm})}{(0.010 \text{ nm})} = 1.24\times 10^5 \text{ eV}.$$

(b) Using the results of Problem 45-11,

$$K_{\text{max}} = \frac{E^2}{mc^2/2 + E} = \frac{(1.24\times 10^5 \text{ eV})^2}{(5.11\times 10^5 \text{ eV})/2 + (1.24\times 10^5 \text{ eV})} = 40.5\times 10^4 \text{ eV}.$$

(c) This would likely knock the electron way out of the atom.

E47-31 The energy of the photon in the series limit is given by

$$E_{\text{limit}} = (13.6 \text{ eV})/n^2,$$

where $n = 1$ for Lyman, $n = 2$ for Balmer, and $n = 3$ for Paschen. The wavelength of the photon is

$$\lambda_{\text{limit}} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(13.6 \text{ eV})} n^2 = (91.17 \text{ nm})n^2.$$

The energy of the longest wavelength comes from the transition from the nearest level, or

$$E_{\text{long}} = \frac{(-13.6 \text{ eV})}{(n+1)^2} - \frac{(-13.6 \text{ eV})}{n^2} = (13.6 \text{ eV})\frac{2n+1}{[n(n+1)]^2}.$$

The wavelength of the photon is

$$\lambda_{\text{long}} = \frac{(1240 \text{ eV} \cdot \text{nm})[n(n+1)]^2}{(13.6 \text{ eV})n^2} = (91.17 \text{ nm})\frac{[n(n+1)]^2}{2n+1}.$$

(a) The wavelength interval $\lambda_{\text{long}} - \lambda_{\text{limit}}$, or

$$\Delta\lambda = (91.17 \text{ nm})\frac{n^2(n+1)^2 - n^2(2n+1)}{2n+1} = (91.17 \text{ nm})\frac{n^4}{2n+1}.$$

For $n = 1$, $\Delta\lambda = 30.4$ nm. For $n = 2$, $\Delta\lambda = 292$ nm. For $n = 3$, $\Delta\lambda = 1055$ nm.

(b) The frequency interval is found from

$$\Delta f = \frac{E_{\text{limit}} - E_{\text{long}}}{h} = \frac{(13.6 \text{ eV})}{(4.14\times 10^{-15}\text{eV} \cdot \text{s})}\frac{1}{(n+1)^2} = \frac{(3.29\times 10^{15}/\text{s})}{(n+1)^2}.$$

For $n = 1$, $\Delta f = 8.23\times 10^{14}$Hz. For $n = 2$, $\Delta f = 3.66\times 10^{14}$Hz. For $n = 3$, $\Delta f = 2.05\times 10^{14}$Hz.

E47-32

$\boxed{\textbf{E47-33}}$ (a) We'll use Eqs. 47-25 and 47-26. At $r = 0$

$$\psi^2(0) = \frac{1}{\pi a_0^3} e^{-2(0)/a_0} = \frac{1}{\pi a_0^3} = 2150 \text{ nm}^{-3},$$

while

$$P(0) = 4\pi(0)^2\psi^2(0) = 0.$$

(b) At $r = a_0$ we have

$$\psi^2(a_0) = frac1\pi a_0^3 e^{-2(a_0)/a_0} = \frac{e^{-2}}{\pi a_0^3} = 291 \text{ nm}^{-3},$$

and

$$P(a_0) = 4\pi(a_0)^2\psi^2(a_0) = 10.2 \text{ nm}^{-1}.$$

E47-34 Assume that $\psi(a_0)$ is a reasonable estimate for $\psi(r)$ everywhere inside the small sphere. Then

$$\psi^2 = \frac{e^{-2}}{\pi a_0^3} = \frac{0.1353}{\pi a_0^3}.$$

The probability of finding it in a sphere of radius $0.05a_0$ is

$$\int_0^{0.05a_0} \frac{(0.1353)4\pi r^2\, dr}{\pi a_0^3} = \frac{4}{3}(0.1353)(0.05)^3 = 2.26\times 10^{-5}.$$

E47-35 Using Eq. 47-26 the ratio of the probabilities is

$$\frac{P(a_0)}{P(2a_0)} = \frac{(a_0)^2 e^{-2(a_0)/a_0}}{(2a_0)^2 e^{-2(2a_0)/a_0}} = \frac{e^{-2}}{4e^{-4}} = 1.85.$$

E47-36 The probability is

$$\begin{aligned} P &= \int_{a_0}^{1.016a_0} \frac{4r^2 e^{-2r/a_0}}{a_0^3} dr, \\ &= \frac{1}{2}\int_2^{2.032} u^2 e^{-u} du, \\ &= 0.00866. \end{aligned}$$

$\boxed{\textbf{E47-37}}$ If $l = 3$ then m_l can be 0, ± 1, ± 2, or ± 3.

(a) From Eq. 47-30, $L_z = m_l h/2\pi$.. So L_z can equal 0, $\pm h/2\pi$, $\pm h/\pi$, or $\pm 3h/2\pi$.

(b) From Eq. 47-31, $\theta = \arccos(m_l/\sqrt{l(l+1)})$, so θ can equal 90°, 73.2°, 54.7°, or 30.0°.

(c) The magnitude of $\vec{\mathbf{L}}$ is given by Eq. 47-28,

$$L = \sqrt{l(l+1)}\frac{h}{2\pi} = \sqrt{3}h/\pi.$$

E47-38 The maximum possible value of m_l is 5. Apply Eq. 47-31:

$$\theta = \arccos \frac{(5)}{\sqrt{(5)(5+1)}} = 24.1°.$$

E47-39 Use the hint.

$$\begin{aligned}
\Delta p \cdot \Delta x &= \frac{h}{2\pi}, \\
\Delta p \frac{r}{r} \Delta x &= \frac{h}{2\pi}, \\
\Delta p \cdot r \frac{\Delta x}{r} &= \frac{h}{2\pi}, \\
\Delta L \cdot \Delta \theta &= \frac{h}{2\pi}.
\end{aligned}$$

E47-40 Note that there is a typo in the formula; $P(r)$ must have dimensions of one over length. The probability is

$$\begin{aligned}
P &= \int_0^\infty \frac{r^4 e^{-r/a_0}}{24a_0^5} dr, \\
&= \frac{1}{24} \int_0^\infty u^4 e^{-u} du, \\
&= 1.00
\end{aligned}$$

What does it mean? It means that if we look for the electron, we will find it somewhere.

E47-41 (a) Find the maxima by taking the derivative and setting it equal to zero.

$$\frac{dP}{dr} = \frac{r(2a - r)(4a^2 - 6ra + r^2)}{8a_0^6} e^{-r} = 0.$$

The solutions are $r = 0$, $r = 2a$, and $4a^2 - 6ra + r^2 = 0$. The first two correspond to minima (see Fig. 47-14). The other two are the solutions to the quadratic, or $r = 0.764a_0$ and $r = 5.236a_0$.

(b) Substitute these two values into Eq. 47-36. The results are

$$P(0.764a_0) = 0.981 \text{ nm}^{-1}.$$

and

$$P(5.236a_0) = 3.61 \text{ nm}^{-1}.$$

E47-42 The probability is

$$\begin{aligned}
P &= \int_{5.00a_0}^{5.01a_0} \frac{r^2(2 - r/a_0)^2 e^{-r/a_0}}{8a_0^3} dr, \\
&= 0.01896.
\end{aligned}$$

E47-43 $n = 4$ and $l = 3$, while m_l can be any of

$$-3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,$$

while m_s can be either $-1/2$ or $1/2$. There are 14 possible states.

E47-44 n must be greater than l, so $n \geq 4$. $|m_l|$ must be less than or equal to l, so $|m_l| \leq 3$. m_s is $-1/2$ or $1/2$.

$\boxed{\textbf{E47-45}}$ If $m_l = 4$ then $l \geq 4$. But $n \geq l + 1$, so $n > 4$. We only know that $m_s = \pm 1/2$.

E47-46 There are $2n^2$ states in a shell n, so if $n = 5$ there are 50 states.

E47-47 Each is in the $n = 1$ shell, the $l = 0$ angular momentum state, and the $m_l = 0$ state. But one is in the state $m_s = +1/2$ while the other is in the state $m_s = -1/2$.

E47-48 Apply Eq. 47-31:

$$\theta = \arccos \frac{(+1/2)}{\sqrt{(1/2)(1/2+1)}} = 54.7^\circ$$

and

$$\theta = \arccos \frac{(-1/2)}{\sqrt{(1/2)(1/2+1)}} = 125.3^\circ.$$

E47-49 All of the statements are true.

E47-50 There are n possible values for l (start at 0!). For each value of l there are $2l+1$ possible values for m_l. If $n = 1$, the sum is 1. If $n = 2$, the sum is $1+3 = 4$. If $n = 3$, the sum is $1+3+5 = 9$. The pattern is clear, the sum is n^2. But there are two spin states, so the number of states is $2n^2$.

$\boxed{\textbf{P47-1}}$ We can simplify the energy expression as

$$E = E_0\left(n_x^2 + n_y^2 + n_z^2\right) \text{ where } E_0 = \frac{h^2}{8mL^2}.$$

To find the lowest energy levels we need to focus on the values of n_x, n_y, and n_z.

It doesn't take much imagination to realize that the set $(1, 1, 1)$ will result in the smallest value for $n_x^2 + n_y^2 + n_z^2$. The next choice is to set one of the values equal to 2, and try the set $(2, 1, 1)$.

Then it starts to get harder, as the next lowest might be either $(2, 2, 1)$ or $(3, 1, 1)$. The only way to find out is to try. I'll tabulate the results for you:

n_x	n_y	n_z	$n_x^2+n_y^2+n_z^2$	Mult.	n_x	n_y	n_z	$n_x^2+n_y^2+n_z^2$	Mult.
1	1	1	3	1	3	2	1	14	6
2	1	1	6	3	3	2	2	17	3
2	2	1	9	3	4	1	1	18	3
3	1	1	11	3	3	3	1	19	3
2	2	2	12	1	4	2	1	21	6

We are now in a position to state the five lowest energy levels. The fundamental quantity is

$$E_0 = \frac{(hc)^2}{8mc^2L^2} = \frac{(1240 \text{ eV}\cdot\text{nm})^2}{8(0.511\times10^6 \text{ eV})(250 \text{ nm})^2} = 6.02\times10^{-6} \text{ eV}.$$

The five lowest levels are found by multiplying this fundamental quantity by the numbers in the table above.

P47-2 (a) Write the states between 0 and L. Then all states, odd or even, can be written with probability distribution function

$$P(x) = \frac{2}{L}\sin^2\frac{n\pi x}{L},$$

we find the probability of finding the particle in the region $0 \leq x \leq L/3$ is

$$\begin{aligned} P &= \int_0^{L/3} \frac{2}{L}\cos^2\frac{n\pi x}{L}dx, \\ &= \frac{1}{3}\left(1 - \frac{sin(2n\pi/3)}{2n\pi/3}\right). \end{aligned}$$

(b) If $n = 1$ use the formula and $P = 0.196$.
(c) If $n = 2$ use the formula and $P = 0.402$.
(d) If $n = 3$ use the formula and $P = 0.333$.
(e) Classically the probability distribution function is uniform, so there is a 1/3 chance of finding it in the region 0 to L/3.

P47-3 The region of interest is small compared to the variation in $P(x)$; as such we can approximate the probability with the expression $P = P(x)\Delta x$.
(b) Evaluating,

$$\begin{aligned} P &= \frac{2}{L}\sin^2\frac{4\pi x}{L}\Delta x, \\ &= \frac{2}{L}\sin^2\frac{4\pi(L/8)}{L}(0.0003L), \\ &= 0.0006. \end{aligned}$$

(b) Evaluating,

$$\begin{aligned} P &= \frac{2}{L}\sin^2\frac{4\pi x}{L}\Delta x, \\ &= \frac{2}{L}\sin^2\frac{4\pi(3L/16)}{L}(0.0003L), \\ &= 0.0003. \end{aligned}$$

P47-4 (a) $P = \Psi^*\Psi$, or

$$P = A_0^2 e^{-2\pi m\omega x^2/h}.$$

(b) Integrating,

$$\begin{aligned} 1 &= A_0^2\int_{-\infty}^{\infty} e^{-2\pi m\omega x^2/h}dx, \\ &= A_0^2\sqrt{\frac{h}{2\pi m\omega}}\int_{-\infty}^{\infty} e^{-u^2}du, \\ &= A_0^2\sqrt{\frac{h}{2\pi m\omega}}\sqrt{pi}, \\ \sqrt[4]{\frac{2m\omega}{h}} &= A_0. \end{aligned}$$

(c) $x = 0$.

$\boxed{\textbf{P47-5}}$ We will want an expression for

$$\frac{d^2}{dx^2}\psi_0.$$

Doing the math one derivative at a time,

$$\begin{aligned}
\frac{d^2}{dx^2}\psi_0 &= \frac{d}{dx}\left(\frac{d}{dx}\psi_0\right),\\
&= \frac{d}{dx}\left(A_0(-2\pi m\omega x/h)e^{-\pi m\omega x^2/h}\right),\\
&= A_0(-2\pi m\omega x/h)^2 e^{-\pi m\omega x^2/h} + A_0(-2\pi m\omega/h)e^{-\pi m\omega x^2/h},\\
&= \left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right) A_0 e^{-\pi m\omega x^2/h},\\
&= \left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right)\psi_0.
\end{aligned}$$

In the last line we factored out ψ_0. This will make our lives easier later on.

Now we want to go to Schrödinger's equation, and make some substitutions.

$$\begin{aligned}
-\frac{h^2}{8\pi^2 m}\frac{d^2}{dx^2}\psi_0 + U\psi_0 &= E\psi_0,\\
-\frac{h^2}{8\pi^2 m}\left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right)\psi_0 + U\psi_0 &= E\psi_0,\\
-\frac{h^2}{8\pi^2 m}\left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right) + U &= E,
\end{aligned}$$

where in the last line we divided through by ψ_0. Now for some algebra,

$$\begin{aligned}
U &= E + \frac{h^2}{8\pi^2 m}\left((2\pi m\omega x/h)^2 - (2\pi m\omega/h)\right),\\
&= E + \frac{m\omega^2 x^2}{2} - \frac{h\omega}{4\pi}.
\end{aligned}$$

But we are given that $E = h\omega/4\pi$, so this simplifies to

$$U = \frac{m\omega^2 x^2}{2}$$

which looks like a harmonic oscillator type potential.

P47-6 Assume the electron is originally in the state n. The classical frequency of the electron is f_0, where

$$f_0 = v/2\pi r.$$

According to electrostatics and uniform circular motion,

$$mv^2/r = e^2/4\pi\epsilon_0 r^2,$$

or

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 m r}} == \sqrt{\frac{e^4}{4\epsilon_0^2 h^2 n^2}} = \frac{e^2}{2\epsilon_0 h n}.$$

Then

$$f_0 = \frac{e^2}{2\epsilon_0 h n}\frac{1}{2\pi}\frac{\pi m e^2}{\epsilon_0 h^2 n^2} = \frac{me^4}{4\epsilon_0^2 h^3 n^3} = \frac{-2E_1}{hn^3}$$

Here $E_1 = -13.6$ eV.

Photon frequency is related to energy according to $f = \Delta E_{nm}/h$, where ΔE_{nm} is the energy of transition from state n down to state m. Then

$$f = \frac{E_1}{h}\left(\frac{1}{n^2} - \frac{1}{m^2}\right),$$

where $E_1 = -13.6$ eV. Combining the fractions and letting $m = n - \delta$, where δ is an integer,

$$\begin{aligned} f &= \frac{E_1}{h}\frac{m^2 - n_2}{m^2 n^2}, \\ &= \frac{-E_1}{h}\frac{(n-m)(m+n)}{m^2 n^2}, \\ &= \frac{-E_1}{h}\frac{\delta(2n+\delta)}{(n+\delta)^2 n^2}, \\ &\approx \frac{-E_1}{h}\frac{\delta(2n)}{(n)^2 n^2}, \\ &= \frac{-2E_1}{hn^3}\delta = f_0\delta. \end{aligned}$$

P47-7 We need to use the reduced mass of the muon since the muon and proton masses are so close together. Then

$$m = \frac{(207)(1836)}{(207)+(1836)} m_\mathrm{e} = 186 m_\mathrm{e}.$$

(a) Apply Eq. 47-20 1/2:

$$a_\mu = a_0/(186) = (52.9 \text{ pm})/(186) = 0.284 \text{ pm}.$$

(b) Apply Eq. 47-21:

$$E_\mu = E_1(186) = (13.6 \text{ eV})(186) = 2.53 \text{ keV}.$$

(c) $\lambda = (1240 \text{ keV}\cdot\text{pm})/(2.53 \text{ pm}) = 490$ pm.

P47-8 (a) The reduced mass of the electron is

$$m = \frac{(1)(1)}{(1)+(1)} m_\mathrm{e} = 0.5 m_\mathrm{e}.$$

The spectrum is similar, except for this additional factor of 1/2; hence

$$\lambda_\mathrm{pos} = 2\lambda_\mathrm{H}.$$

(b) $a_\mathrm{pos} = a_0/(186) = (52.9 \text{ pm})/(1/2) = 105.8$ pm. This is the distance between the particles, but they are both revolving about the center of mass. The radius is then half this quantity, or 52.9 pm.

$\boxed{\textbf{P47-9}}$ This problem isn't really that much of a problem. Start with the magnitude of a vector in terms of the components,

$$L_x^2 + L_y^2 + L_z^2 = L^2,$$

and then rearrange,

$$L_x^2 + L_y^2 = L^2 - L_z^2.$$

According to Eq. 47-28 $L^2 = l(l+1)h^2/4\pi^2$, while according to Eq. 47-30 $L_z = m_l h/2\pi$. Substitute that into the equation, and

$$L_x^2 + L_y^2 = l(l+1)h^2/4\pi^2 - m_l^2 h^2/4\pi^2 = \left(l(l+1) - m_l^2\right)\frac{h^2}{4\pi^2}.$$

Take the square root of both sides of this expression, and we are done.

The maximum value for m_l is l, while the minimum value is 0. Consequently,

$$\sqrt{L_x^2 + L_y^2} = \sqrt{l(l+1) - m_l^2}\, h/2\pi \leq \sqrt{l(l+1)}\, h/2\pi,$$

and

$$\sqrt{L_x^2 + L_y^2} = \sqrt{l(l+1) - m_l^2}\, h/2\pi \geq \sqrt{l}\, h/2\pi.$$

P47-10 Assume that $\psi(0)$ is a reasonable estimate for $\psi(r)$ everywhere inside the small sphere. Then

$$\psi^2 = \frac{e^{-0}}{\pi a_0^3} = \frac{1}{\pi a_0^3}.$$

The probability of finding it in a sphere of radius 1.1×10^{-15}m is

$$\int_0^{1.1\times10^{-15}\mathrm{m}} \frac{4\pi r^2\, dr}{\pi a_0^3} = \frac{4}{3}\frac{(1.1\times10^{-15}\mathrm{m})^3}{(5.29\times10^{-11}\mathrm{m})^3} = 1.2\times10^{-14}.$$

P47-11 Assume that $\psi(0)$ is a reasonable estimate for $\psi(r)$ everywhere inside the small sphere. Then

$$\psi^2 = \frac{(2)^2 e^{-0}}{32\pi a_0^3} = \frac{1}{8\pi a_0^3}.$$

The probability of finding it in a sphere of radius 1.1×10^{-15}m is

$$\int_0^{1.1\times10^{-15}\mathrm{m}} \frac{4\pi r^2\, dr}{8\pi a_0^3} = \frac{1}{6}\frac{(1.1\times10^{-15}\mathrm{m})^3}{(5.29\times10^{-11}\mathrm{m})^3} = 1.5\times10^{-15}.$$

P47-12 (a) The wave function squared is

$$\psi^2 = \frac{e^{-2r/a_0}}{\pi a_0^3}$$

The probability of finding it in a sphere of radius $r = xa_0$ is

$$\begin{aligned} P &= \int_0^{xa_0} \frac{4\pi r^2 e^{-2r/a_0}\, dr}{\pi a_0^3}, \\ &= \int_0^{x} 4x^2 e^{-2x}\, dx, \\ &= 1 - e^{-2x}(1 + 2x + 2x^2). \end{aligned}$$

(b) Let $x = 1$, then

$$P = 1 - e^{-2}(5) = 0.323.$$

$\boxed{\textbf{P47-13}}$ We want to evaluate the difference between the values of P at $x = 2$ and $x = 2$. Then

$$\begin{aligned} P(2) - P(1) &= \left(1 - e^{-4}(1 + 2(2) + 2(2)^2)\right) - \left(1 - e^{-2}(1 + 2(1) + 2(1)^2)\right), \\ &= 5e^{-2} - 13e^{-4} = 0.439. \end{aligned}$$

P47-14 Using the results of Problem 47-12,

$$0.5 = 1 - e^{-2x}(1 + 2x + 2x^2),$$

or

$$e{-}2x = 1 + 2x + 2x^2.$$

The result is $x = 1.34$, or $r = 1.34a_0$.

P47-15 The probability of finding it in a sphere of radius $r = xa_0$ is

$$\begin{aligned} P &= \int_0^{xa_0} \frac{r^2(2 - r/a_0)^2 e^{-r/a_0}\, dr}{8a_0^3} \\ &= \frac{1}{8}\int_0^x x^2(2-x)^2 e^{-x}\, dx \\ &= 1 - e^{-x}(y^4/8 + y^2/2 + y + 1). \end{aligned}$$

The minimum occurs at $x = 2$, so

$$P = 1 - e^{-2}(2 + 2 + 2 + 1) = 0.0527.$$

$\boxed{\textbf{E48-1}}$ The highest energy x-ray photon will have an energy equal to the bombarding electrons, as is shown in Eq. 48-1,

$$\lambda_{\min} = \frac{hc}{eV}$$

Insert the appropriate values into the above expression,

$$\lambda_{\min} = \frac{(4.14 \times 10^{-15}\ \text{eV} \cdot \text{s})(3.00 \times 10^{8}\,\text{m/s})}{eV} = \frac{1240 \times 10^{-9}\ \text{eV} \cdot \text{m}}{eV}.$$

The expression is then

$$\lambda_{\min} = \frac{1240 \times 10^{-9}\ \text{V} \cdot \text{m}}{V} = \frac{1240\ \text{kV} \cdot \text{pm}}{V}.$$

So long as we are certain that the "V" will be measured in units of kilovolts, we can write this as

$$\lambda_{\min} = 1240\ \text{pm}/V.$$

E48-2 $f = c/\lambda = (3.00\times10^{8}\text{m/s})/(31.1\times10^{-12}\text{m}) = 9.646\times10^{18}/\text{s}$. Planck's constant is then

$$h = \frac{E}{f} = \frac{(40.0\ \text{keV})}{(9.646\times10^{18}/\text{s})} = 4.14\times10^{-15}\ \text{eV} \cdot \text{s}.$$

E48-3 Applying the results of Exercise 48-1,

$$\Delta V = \frac{(1240\text{kV} \cdot \text{pm})}{(126\ \text{pm})} = 9.84\ \text{kV}.$$

E48-4 (a) Applying the results of Exercise 48-1,

$$\lambda_{\min} = \frac{(1240\text{kV} \cdot \text{pm})}{(35.0\ \text{kV})} = 35.4\ \text{pm}.$$

(b) Applying the results of Exercise 45-1,

$$\lambda_{K_\beta} = \frac{(1240\text{keV} \cdot \text{pm})}{(25.51\ \text{keV}) - (0.53\ \text{keV})} = 49.6\ \text{pm}.$$

(c) Applying the results of Exercise 45-1,

$$\lambda_{K_\alpha} = \frac{(1240\text{keV} \cdot \text{pm})}{(25.51\ \text{keV}) - (3.56\ \text{keV})} = 56.5\ \text{pm}.$$

$\boxed{\textbf{E48-5}}$ (a) Changing the accelerating potential of the x-ray tube will decrease $\lambda_{\min}$. The new value will be (using the results of Exercise 48-1)

$$\lambda_{\min} = 1240\ \text{pm}/(50.0) = 24.8\ \text{pm}.$$

(b) λ_{K_β} doesn't change. It is a property of the atom, not a property of the accelerating potential of the x-ray tube. The only way in which the accelerating potential might make a difference is if $\lambda_{K_\beta} < \lambda_{\min}$ for which case there would not be a λ_{K_β} line.

(c) λ_{K_α} doesn't change. See part (b).

E48-6 (a) Applying the results of Exercise 45-1,

$$\Delta E = \frac{(1240\text{keV} \cdot \text{pm})}{(19.3 \text{ pm})} = 64.2 \text{ keV}.$$

(b) This is the transition $n = 2$ to $n = 1$, so

$$\Delta E = (13.6 \text{ eV})(1/1^2 - 1/2^2) = 10.2 \text{ eV}.$$

E48-7 Applying the results of Exercise 45-1,

$$\Delta E_\beta = \frac{(1240\text{keV} \cdot \text{pm})}{(62.5 \text{ pm})} = 19.8 \text{ keV}.$$

and

$$\Delta E_\alpha = \frac{(1240\text{keV} \cdot \text{pm})}{(70.5 \text{ pm})} = 17.6 \text{ keV}.$$

The difference is

$$\Delta E = (19.8 \text{ keV}) - (17.6 \text{ keV}) = 2.2 \text{ eV}.$$

E48-8 Since $E_\lambda = hf = hc/\lambda$, and $\lambda = h/mc = hc/mc^2$, then

$$E_\lambda = hc/\lambda = mc^2.$$

or $\Delta V = E_\lambda/e = mc^2/e = 511$ kV.

$\boxed{\textbf{E48-9}}$ The 50.0 keV electron makes a collision and loses half of its energy to a photon, then the photon has an energy of 25.0 keV. The electron is now a 25.0 keV electron, and on the next collision again loses loses half of its energy to a photon, then this photon has an energy of 12.5 keV. On the third collision the electron loses the remaining energy, so this photon has an energy of 12.5 keV. The wavelengths of these photons will be given by

$$\lambda = \frac{(1240 \text{ keV} \cdot \text{pm})}{E},$$

which is a variation of Exercise 45-1.

E48-10 (a) The x-ray will need to knock free a K shell electron, so it must have an energy of at least 69.5 keV.

(b) Applying the results of Exercise 48-1,

$$\lambda_{\text{min}} = \frac{(1240\text{kV} \cdot \text{pm})}{(69.5 \text{ kV})} = 17.8 \text{ pm}.$$

(c) Applying the results of Exercise 45-1,

$$\lambda_{K_\beta} = \frac{(1240\text{keV} \cdot \text{pm})}{(69.5 \text{ keV}) - (2.3 \text{ keV})} = 18.5 \text{ pm}.$$

Applying the results of Exercise 45-1,

$$\lambda_{K_\alpha} = \frac{(1240\text{keV} \cdot \text{pm})}{(69.5 \text{ keV}) - (11.3 \text{ keV})} = 21.3 \text{ pm}.$$

E48-11 (a) Applying the results of Exercise 45-1,

$$E_{K_\beta} = \frac{(1240\text{keV} \cdot \text{pm})}{(63 \text{ pm})} = 19.7 \text{ keV}.$$

Again applying the results of Exercise 45-1,

$$E_{K_\beta} = \frac{(1240\text{keV} \cdot \text{pm})}{(71 \text{ pm})} = 17.5 \text{ keV}.$$

(b) Zr or Nb; the others will not significantly absorb either line.

E48-12 Applying the results of Exercise 45-1,

$$\lambda_{K_\alpha} = \frac{(1240\text{keV} \cdot \text{pm})}{(8.979 \text{ keV}) - (0.951 \text{ keV})} = 154.5 \text{ pm}.$$

Applying the Bragg reflection relationship,

$$d = \frac{\lambda}{2\sin\theta} = \frac{(154.5 \text{ pm})}{2\sin(15.9°)} = 282 \text{ pm}.$$

E48-13 Plot the data. The plot should look just like Fig 48-4. Note that the vertical axis is $\sqrt{f}$, which is related to the wavelength according to $\sqrt{f} = \sqrt{c/\lambda}$.

E48-14 Remember that the m in Eq. 48-4 refers to the electron, not the nucleus. This means that the constant C in Eq. 48-5 is the same for all elements. Since $\sqrt{f} = \sqrt{c/\lambda}$, we have

$$\frac{\lambda_1}{\lambda_2} = \left(\frac{Z_2 - 1}{Z_1 - 1}\right)^2.$$

For Ga and Nb the wavelength ratio is then

$$\frac{\lambda_{\text{Nb}}}{\lambda_{\text{Ga}}} = \left(\frac{(31) - 1}{(41) - 1}\right)^2 = 0.5625.$$

E48-15 (a) The ground state question is fairly easy. The $n = 1$ shell is completely occupied by the first two electrons. So the third electron will be in the $n = 2$ state. The lowest energy angular momentum state in any shell is the s sub-shell, corresponding to $l = 0$. There is only one choice for m_l in this case: $m_l = 0$. There is no way at this level of coverage to distinguish between the energy of either the spin up or spin down configuration, so we'll arbitrarily pick spin up.

(b) Determining the configuration for the first excited state will require some thought. We could assume that one of the K shell electrons ($n = 1$) is promoted to the L shell ($n = 2$). Or we could assume that the L shell electron is promoted to the M shell. Or we could assume that the L shell electron remains in the L shell, but that the angular momentum value is changed to $l = 1$. The question that we would need to answer is which of these possibilities has the lowest energy.

The answer is the last choice: increasing the l value results in a small increase in the energy of multi-electron atoms.

E48-16 Refer to Sample Problem 47-6:

$$r_1 = \frac{a_0(1)^2}{Z} = \frac{(5.29\times10^{-11}\text{m})}{(92)} = 5.75\times10^{-13}\,\text{m}.$$

$\boxed{\textbf{E48-17}}$ We will assume that the ordering of the energy of the shells and sub-shells is the same. That ordering is

$$1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p$$
$$< 6s < 4f < 5d < 6p < 7s < 5f < 6d < 7p < 8s.$$

If there is no spin the s sub-shell would hold 1 electron, the p sub-shell would hold 3, the d sub-shell 5, and the f sub-shell 7. inert gases occur when a p sub-shell has filled, so the first three inert gases would be element 1 (Hydrogen), element $1 + 1 + 3 = 5$ (Boron), and element $1 + 1 + 3 + 1 + 3 = 9$ (Fluorine).

Is there a pattern? Yes. The new inert gases have *half* of the atomic number of the original inert gases. The factor of one-half comes about because there are no longer two spin states for each set of n, l, m_l quantum numbers.

We can save time and simply divide the atomic numbers of the remaining inert gases in half: element 18 (Argon), element 27 (Cobalt), element 43 (Technetium), element 59 (Praseodymium).

E48-18 The pattern is

$$2 + 8 + 8 + 18 + 18 + 32 + 32+?$$

or

$$2(1^2 + 2^2 + 2^2 + 3^3 + 3^3 + 4^2 + 4^2 + x^2)$$

The unknown is probably $x = 5$, the next noble element is probably

$$118 + 2 \cdot 5^2 = 168.$$

E48-19 (a) Apply Eq. 47-23, which can be written as

$$E_n = \frac{(-13.6 \text{ eV})Z^2}{n^2}.$$

For the valence electron of sodium $n = 3$,

$$Z = \sqrt{\frac{(5.14 \text{ eV})(3)^2}{(13.6 \text{ eV})}} = 1.84,$$

while for the valence electron of potassium $n = 4$,

$$Z = \sqrt{\frac{(4.34 \text{ eV})(4)^2}{(13.6 \text{ eV})}} = 2.26,$$

(b) The ratios with the actual values of Z are 0.167 and 0.119, respectively.

E48-20 (a) There are three m_l states allowed, and two m_s states. The first electron can be in any one of these six combinations of M_1 and m_2. The second electron, given no exclusion principle, could also be in any one of these six states. The total is 36. Unfortunately, this is wrong, because we can't distinguish electrons. Of this total of 36, six involve the electrons being in the same state, while 30 involve the electron being in different states. But if the electrons are in different states, then they could be swapped, and we won't know, so we must divide this number by two. The total number of distinguishable states is then

$$(30/2) + 6 = 21.$$

(b) Six. See the above discussion.

$\boxed{\textbf{E48-21}}$ (a) The Bohr orbits are circular orbits of radius $r_n = a_0 n^2$ (Eq. 47-20). The electron is orbiting where the force is

$$F_n = \frac{e^2}{4\pi\epsilon_0 r_n^2},$$

and this force is equal to the centripetal force, so

$$\frac{mv^2}{r_n} = \frac{e^2}{4\pi\epsilon_0 r_n^2}.$$

where v is the velocity of the electron. Rearranging,

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}}.$$

The time it takes for the electron to make one orbit can be used to calculate the current,

$$i = \frac{q}{t} = \frac{e}{2\pi r_n/v} = \frac{e}{2\pi r_n}\sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}}.$$

The magnetic moment of a current loop is the current times the area of the loop, so

$$\mu = iA = \frac{e}{2\pi r_n}\sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}}\pi r_n^2,$$

which can be simplified to

$$\mu = \frac{e}{2}\sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}} r_n.$$

But $r_n = a_0 n^2$, so

$$\mu = n\frac{e}{2}\sqrt{\frac{a_0 e^2}{4\pi\epsilon_0 m}}.$$

This might not look right, but $a_0 = \epsilon_0 h^2/\pi m e^2$, so the expression can simplify to

$$\mu = n\frac{e}{2}\sqrt{\frac{h^2}{4\pi^2 m^2}} = n\left(\frac{eh}{4\pi m}\right) = n\mu_{\mathrm{B}}.$$

(b) In reality the magnetic moments depend on the angular momentum quantum number, not the principle quantum number. Although the Bohr theory correctly predicts the magnitudes, it does not correctly predict when these values would occur.

E48-22 (a) Apply Eq. 48-14:

$$F_z = \mu_z \frac{dB_z}{dz} = (9.27\times10^{-24}\,\mathrm{J/T})(16\times10^{-3}\,\mathrm{T/m}) = 1.5\times10^{-25}\,\mathrm{N}.$$

(b) $a = F/m$, $\Delta z = at^2/2$, and $t = y/v_y$. Then

$$\Delta z = \frac{Fy^2}{2mv_y^2} = \frac{(1.5\times10^{-25}\,\mathrm{N})(0.82\,\mathrm{m})^2}{2(1.67\times10^{-27}\mathrm{kg})(970\,\mathrm{m/s})^2} = 3.2\times10^{-5}\mathrm{m}.$$

E48-23 $a = (9.27\times10^{-24}\mathrm{J/T})(1.4\times10^3\mathrm{T/m})/(1.7\times10^{-25}\mathrm{kg}) = 7.6\times10^4\mathrm{m/s^2}$.

E48-24 (a) $\Delta U = 2\mu B$, or

$$\Delta U = 2(5.79\times10^{-5}\text{eV/T})(0.520\,\text{T}) = 6.02\times10^{-5}\text{eV}.$$

(b) $f = E/h = (6.02\times10^{-5}\text{eV})(4.14\times10^{-15}\text{eV}\cdot\text{s}) = 1.45\times10^{10}\text{Hz}.$
(c) $\lambda = c/f = (3\times10^{8}\text{m/s})/(1.45\times10^{10}\text{Hz}) = 2.07\times10^{-2}\text{m}.$

E48-25 The energy change can be derived from Eq. 48-13; we multiply by a factor of 2 because the spin is completely flipped. Then

$$\Delta E = 2\mu_z B_z = 2(9.27\times10^{-24}\,\text{J/T})(0.190\,\text{T}) = 3.52\times10^{-24}\text{J}.$$

The corresponding wavelength is

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{J}\cdot\text{s})(3.00\times10^{8}\text{m/s})}{(3.52\times10^{-24}\text{J})} = 5.65\times10^{-2}\text{m}.$$

This is somewhere near the microwave range.

E48-26 The photon has an energy $E = hc/\lambda$. This energy is related to the magnetic field in the vicinity of the electron according to

$$E = 2\mu B,$$

so

$$B = \frac{hc}{2\mu\lambda} = \frac{(1240\ \text{eV}\cdot\text{nm})}{2(5.79\times10^{-5}\text{J/T})(21\times10^{7}\text{nm})} = 0.051\,\text{T}.$$

E48-27 Applying the results of Exercise 45-1,

$$E = \frac{(1240\ \text{eV}\cdot\text{nm})}{(800\text{nm})} = 1.55\ \text{eV}.$$

The production rate is then

$$R = \frac{(5.0\times10^{-3}\text{W})}{(1.55\ \text{eV})(1.6\times10^{-19}\text{J/eV})} = 2.0\times10^{16}/\text{s}.$$

E48-28 (a) $x = (3\times10^{8}\text{m/s})(12\times10^{-12}\text{s}) = 3.6\times10^{-3}\text{m}.$
(b) Applying the results of Exercise 45-1,

$$E = \frac{(1240\ \text{eV}\cdot\text{nm})}{(694.4\text{nm})} = 1.786\ \text{eV}.$$

The number of photons in the pulse is then

$$N = (0.150\text{J})/(1.786\ \text{eV})(1.6\times10^{-19}\text{J/eV}) = 5.25\times10^{17}.$$

E48-29 We need to find out how many 10 MHz wide signals can fit between the two wavelengths. The lower frequency is

$$f_1 = \frac{c}{\lambda_1} = \frac{(3.00\times10^{8}\ \text{m/s})}{700\times10^{-9}\ \text{m})} = 4.29\times10^{14}\ \text{Hz}.$$

The higher frequency is

$$f_1 = \frac{c}{\lambda_1} = \frac{(3.00 \times 10^8\,\mathrm{m/s})}{400 \times 10^{-9}\,\mathrm{m})} = 7.50 \times 10^{14}\ \mathrm{Hz}.$$

The number of signals that can be sent in this range is

$$\frac{f_2 - f_1}{(10\ \mathrm{MHz})} = \frac{(7.50 \times 10^{14}\ \mathrm{Hz}) - (4.29 \times 10^{14}\ \mathrm{Hz})}{(10 \times 10^6\ \mathrm{Hz})} = 3.21 \times 10^7.$$

That's quite a number of television channels.

E48-30 Applying the results of Exercise 45-1,

$$E = \frac{(1240\ \mathrm{eV\cdot nm})}{(632.8\mathrm{nm})} = 1.960\ \mathrm{eV}.$$

The number of photons emitted in one minute is then

$$N = \frac{(2.3\times 10^{-3}\mathrm{W})(60\,\mathrm{s})}{(1.960\ \mathrm{eV})(1.6\times 10^{-19}\mathrm{J/eV})} = 4.4\times 10^{17}.$$

E48-31 Apply Eq. 48-19. $E_13 - E_11 = 2(1.2\ \mathrm{eV})$.. The ratio is then

$$\frac{n_13}{n_11} = e^{-(2.4\ \mathrm{eV})/(8.62\times 10^{-5}\mathrm{eV/K})(2000\ \mathrm{K})} = 9\times 10^{-7}.$$

E48-32 (a) Population inversion means that the higher energy state is more populated; this can only happen if the ratio in Eq. 48-19 is greater than one, which can only happen if the argument of the exponent is positive. That would require a negative temperature.

(b) If $n_2 = 1.1n_1$ then the ratio is 1.1, so

$$T = \frac{(-2.26\ \mathrm{eV}}{(8.62\times 10^{-5}\mathrm{eV/K})\ln(1.1)} = -2.75\times 10^5\mathrm{K}.$$

$\boxed{\textbf{E48-33}}$ (a) At thermal equilibrium the population ratio is given by

$$\frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-\Delta E/kT}.$$

But ΔE can be written in terms of the transition photon wavelength, so this expression becomes

$$N_2 = N_1 e^{-hc/\lambda kT}.$$

Putting in the numbers,

$$N_2 = (4.0\times 10^{20})e^{-(1240\ \mathrm{eV\cdot nm})/\left(582\ \mathrm{nm})(8.62\times 10^{-5}\ \mathrm{eV/K})(300\,\mathrm{K})\right)} = 6.62\times 10^{-16}.$$

That's effectively *none.*

(b) If the population of the upper state were 7.0×10^{20}, then in a single laser pulse

$$E = N\frac{hc}{\lambda} = (7.0\times 10^{20})\frac{(6.63\times 10^{-34}\mathrm{J\cdot s})(3.00\times 10^8\mathrm{m/s})}{(582\times 10^{-9}\mathrm{m})} = 240\,\mathrm{J}.$$

E48-34 The allowed wavelength in a standing wave chamber are $\lambda_n = 2L/n$. For large n we can write

$$\lambda_{n+1} = \frac{2L}{n+1} \approx \frac{2L}{n} - \frac{2L}{n^2}.$$

The wavelength difference is then

$$\Delta\lambda = \frac{2L}{n^2} = \frac{\lambda_n^2}{2L},$$

which in this case is

$$\Delta\lambda = \frac{(533\times10^{-9}\text{m})^2}{2(8.3\times10^{-2}\text{m})} = 1.7\times10^{-12}\text{m}.$$

E48-35 (a) The central disk will have an angle as measured from the center given by

$$d\sin\theta = (1.22)\lambda,$$

and since the parallel rays of the laser are focused on the screen in a distance f, we also have $R/f = \sin\theta$. Combining, and rearranging,

$$R = \frac{1.22 f\lambda}{d}.$$

(b) $R = 1.22(3.5\text{ cm})(515\text{ nm})/(3\text{ mm}) = 7.2\times10^{-6}\text{m}$.
(c) $I = P/A = (5.21\,\text{W})/\pi(1.5\text{ mm})^2 = 7.37\times10^5\text{W/m}^2$.
(d) $I = P/A = (0.84)(5.21\,\text{W})/\pi(7.2\,\mu\text{m})^2 = 2.7\times10^{10}\text{W/m}^2$.

E48-36

P48-1 Let λ_1 be the wavelength of the first photon. Then $\lambda_2 = \lambda_1 + 130$ pm. The total energy transfered to the two photons is then

$$E_1 + E_2 = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = 20.0\text{ keV}.$$

We can solve this for λ_1,

$$\begin{aligned}\frac{20.0\text{ keV}}{hc} &= \frac{1}{\lambda_1} + \frac{1}{\lambda_1 + 130\text{ pm}},\\ &= \frac{2\lambda_1 + 130\text{ pm}}{\lambda_1(\lambda_1 + 130\text{ pm})},\end{aligned}$$

which can also be written as

$$\begin{aligned}\lambda_1(\lambda_1 + 130\text{ pm}) &= (62\text{ pm})(2\lambda_1 + 130\text{ pm}),\\ \lambda_1^2 + (6\text{ pm})\lambda_1 - (8060\text{ pm}^2) &= 0.\end{aligned}$$

This equation has solutions

$$\lambda_1 = 86.8\text{ pm and } -92.8\text{ pm}.$$

Only the positive answer has physical meaning. The energy of this first photon is then

$$E_1 = \frac{(1240\text{ keV}\cdot\text{pm})}{(86.8\text{ pm})} = 14.3\text{ keV}.$$

(a) After this first photon is emitted the electron still has a kinetic energy of

$$20.0\text{ keV} - 14.3\text{ keV} = 5.7\text{ keV}.$$

(b) We found the energy and wavelength of the first photon above. The energy of the second photon *must* be 5.7 keV, with wavelength

$$\lambda_2 = (86.8\text{ pm}) + 130\text{ pm} = 217\text{ pm}.$$

P48-2 Originally,

$$\gamma = \frac{1}{\sqrt{1-(2.73\times10^8\text{m/s})^2/(3\times10^8\text{m/s})^2}} = 2.412.$$

The energy of the electron is

$$E_0 = \gamma mc^2 = (2.412)(511\text{ keV}) = 1232\text{ keV}.$$

Upon emitting the photon the new energy is

$$E = (1232\text{ keV}) - (43.8\text{ keV}) = 1189\text{ keV},$$

so the new gamma factor is

$$\gamma = (1189\text{ keV})/(511\text{ keV}) = 2.326,$$

and the new speed is

$$v = c\sqrt{1 - 1/(2.326)^2} = (0.903)c.$$

P48-3 Switch to a reference frame where the electron is originally at rest.

Momentum conservation requires

$$0 = p_\lambda + p_e = 0,$$

while energy conservation requires

$$mc^2 = E_\lambda + E_e.$$

Rearrange to

$$E_e = mc^2 - E_\lambda.$$

Square both sides of this energy expression and

$$\begin{aligned}
E_\lambda^2 - 2E_\lambda mc^2 + m^2c^4 &= E_e^2 = p_e^2c^2 + m^2c^4,\\
E_\lambda^2 - 2E_\lambda mc^2 &= p_e^2c^2,\\
p_\lambda^2c^2 - 2E_\lambda mc^2 &= p_e^2c^2.
\end{aligned}$$

But the momentum expression can be used here, and the result is

$$-2E_\lambda mc^2 = 0.$$

Not likely.

P48-4 (a) In the Bohr theory we can assume that the K shell electrons "see" a nucleus with charge Z. The L shell electrons, however, are shielded by the one electron in the K shell and so they "see" a nucleus with charge $Z-1$. Finally, the M shell electrons are shielded by the one electron in the K shell and the eight electrons in the K shell, so they "see" a nucleus with charge $Z-9$.

The transition wavelengths are then

$$\begin{aligned}
\frac{1}{\lambda_\alpha} &= \frac{\Delta E}{hc} = \frac{E_0(Z-1)^2}{hc}\left(\frac{1}{2^2} - \frac{1}{1^2}\right),\\
&= \frac{E_0(Z-1)^2}{hc}\frac{3}{4}.
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{\lambda_\beta} &= \frac{\Delta E}{hc} = \frac{E_0}{hc}\left(\frac{1}{3^2} - \frac{1}{1^2}\right),\\
&= \frac{E_0(Z-9)^2}{hc}\frac{-8}{9}.
\end{aligned}$$

The ratio of these two wavelengths is

$$\frac{\lambda_\beta}{\lambda_\alpha} = \frac{27}{32}\frac{(Z-1)^2}{(Z-9)^2}.$$

Note that the formula in the text has the square in the wrong place!

P48-5 (a) $E = hc/\lambda$; the energy difference is then

$$\begin{aligned}\Delta E &= hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right), \\ &= hc\frac{\lambda_2 - \lambda_1}{\lambda_2\lambda_1}. \\ &= \frac{hc}{\lambda_2\lambda_1}\Delta\lambda.\end{aligned}$$

Since λ_1 and λ_2 are *so* close together we can treat the product $\lambda_1\lambda_2$ as being either λ_1^2 or λ_2^2. Then

$$\Delta E = \frac{(1240 \text{ eV}\cdot\text{nm})}{(589 \text{ nm})^2}(0.597 \text{ nm}) = 2.1\times10^{-3}\text{eV}.$$

(b) The same energy difference exists in the $4s \to 3p$ doublet, so

$$\Delta\lambda = \frac{(1139 \text{ nm})^2}{(1240 \text{ eV}\cdot\text{nm})}(2.1\times10^{-3}\text{eV}) = 2.2 \text{ nm}.$$

P48-6 (a) We can assume that the K shell electron "sees" a nucleus of charge $Z-1$, since the other electron in the shell screens it. Then, according to the derivation leading to Eq. 47-22,

$$r_K = a_0/(Z-1).$$

(b) The outermost electron "sees" a nucleus screened by all of the other electrons; as such $Z = 1$, and the radius is

$$r = a_0$$

$\boxed{\textbf{P48-7}}$ We assume in this crude model that one electron moves in a circular orbit attracted to the helium nucleus but repelled from the other electron. Look back to Sample Problem 47-6; we need to use some of the results from that Sample Problem to solve this problem.

The factor of e^2 in Eq. 47-20 (the expression for the Bohr radius) and the factor of $(e^2)^2$ in Eq. 47-21 (the expression for the Bohr energy levels) was from the Coulomb force between the single electron and the single proton in the nucleus. This force is

$$F = \frac{e^2}{4\pi\epsilon_0 r^2}.$$

In our approximation the force of attraction between the one electron and the helium nucleus is

$$F_1 = \frac{2e^2}{4\pi\epsilon_0 r^2}.$$

The factor of two is because there are two protons in the helium nucleus.

There is also a repulsive force between the one electron and the other electron,

$$F_2 = \frac{e^2}{4\pi\epsilon_0 (2r)^2},$$

where the factor of $2r$ is because the two electrons are on opposite side of the nucleus.

The net force on the first electron in our approximation is then

$$F_1 - F_2 = \frac{2e^2}{4\pi\epsilon_0 r^2} - \frac{e^2}{4\pi\epsilon_0 (2r)^2},$$

which can be rearranged to yield

$$F_{\text{net}} = \frac{e^2}{4\pi\epsilon_0 r^2}\left(2 - \frac{1}{4}\right) = \frac{e^2}{4\pi\epsilon_0 r^2}\left(\frac{7}{4}\right).$$

It is apparent that we need to substitute $7e^2/4$ for every occurrence of e^2.

(a) The ground state radius of the helium atom will then be given by Eq. 47-20 with the appropriate substitution,

$$r = \frac{\epsilon_0 h^2}{\pi m(7e^2/4)} = \frac{4}{7}a_0.$$

(b) The energy of *one* electron in this ground state is given by Eq. 47-21 with the substitution of $7e^2/4$ for every occurrence of e^2, then

$$E = -\frac{m(7e^2/4)^2}{8\epsilon_0^4 h^2} = -\frac{49}{16}\frac{me^4}{8\epsilon_0^4 h^2}.$$

We already evaluated all of the constants to be 13.6 eV.

One last thing. There are *two* electrons, so we need to double the above expression. The ground state energy of a helium atom in this approximation is

$$E_0 = -2\frac{49}{16}(13.6\text{ eV}) = -83.3eV.$$

(c) Removing one electron will allow the remaining electron to move closer to the nucleus. The energy of the remaining electron is given by the Bohr theory for He^+, and is

$$E_{\text{He}^+} = (4)(-13.60\text{ eV}) = 54.4\text{ eV},$$

so the ionization energy is 83.3 eV - 54.4 eV = 28.9 eV. This compares well with the accepted value.

P48-8 Applying Eq. 48-19:

$$T = \frac{(-3.2\text{ eV})}{(8.62\times10^{-5}\text{eV/K})\ln(6.1\times10^{13}/2.5\times10^{15})} = 1.0\times10^4\text{K}.$$

P48-9 $\sin\theta \approx r/R$, where r is the radius of the beam on the moon and R is the distance to the moon. Then

$$r = \frac{1.22(600\times10^{-9}\text{m})(3.82\times10^8\text{m})}{(0.118\,\text{m})} = 2360\,\text{m}.$$

The beam diameter is twice this, or 4740 m.

P48-10 (a) $N = 2L/\lambda_n$, or

$$N = \frac{2(6\times10^{-2}\text{m})(1.75)}{(694\times10^{-9})} = 3.03\times10^5.$$

(b) $N = 2nLf/c$, so

$$\Delta f = \frac{c}{2nL} = \frac{(3\times 10^8 \mathrm{m/s})}{2(1.75)(6\times 10^{-2}\mathrm{m})} = 1.43\times 10^9/\mathrm{s}.$$

Note that the travel time to and fro is $\Delta t = 2nL/c$!

(c) $\Delta f/f$ is then

$$\frac{\Delta f}{f} = \frac{\lambda}{2nL} = \frac{(694\times 10^{-9})}{2(1.75)(6\times 10^{-2}\mathrm{m})} = 3.3\times 10^{-6}.$$

$\boxed{\textbf{E49-1}}$ (a) Equation 49-2 is

$$n(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2} = \frac{8\sqrt{2}\pi (mc^2)^{3/2}}{(hc)^3}E^{1/2}.$$

We can evaluate this by substituting in all known quantities,

$$n(E) = \frac{8\sqrt{2}\pi (0.511 \times 10^6 \text{ eV})^{3/2}}{(1240 \times 10^{-9} \text{ eV} \cdot \text{m})^3}E^{1/2} = (6.81 \times 10^{27}\,\text{m}^{-3} \cdot \text{eV}^{-3/2})E^{1/2}.$$

Once again, we simplified the expression by writing hc wherever we could, and then using $hc = 1240 \times 10^{-9}$ eV $\cdot$ m.

(b) Then, if $E = 5.00$ eV,

$$n(E) = (6.81 \times 10^{27}\,\text{m}^{-3} \cdot \text{eV}^{-3/2})(5.00 \text{ eV})^{1/2} = 1.52 \times 10^{28}\,\text{m}^{-3} \cdot \text{eV}^{-1}.$$

E49-2 Apply the results of Ex. 49-1:

$$n(E) = (6.81 \times 10^{27}\,\text{m}^{-3} \cdot \text{eV}^{-3/2})(8.00 \text{ eV})^{1/2} = 1.93 \times 10^{28}\,\text{m}^{-3} \cdot \text{eV}^{-1}.$$

E49-3 Monovalent means only one electron is available as a conducting electron. Hence we need only calculate the density of atoms:

$$\frac{N}{V} = \frac{\rho N_A}{A_r} = \frac{(19.3\times 10^3\text{kg/m}^3)(6.02\times 10^{23}\text{mol}^{-1})}{(0.197\,\text{kg/mol})} = 5.90\times 10^{28}/\text{m}^3.$$

E49-4 Use the ideal gas law: $pV = NkT$. Then

$$p = \frac{N}{V}kT = (8.49\times 10^{28}\text{m}^3)(1.38\times 10^{-23}\text{J}/\cdot\text{K})(297\,\text{K}) = 3.48\times 10^8\text{Pa}.$$

$\boxed{\textbf{E49-5}}$ (a) The approximate volume of a single sodium atom is

$$V_1 = \frac{(0.023\,\text{kg/mol})}{(6.02\times 10^{23}\text{part/mol})(971\,\text{kg/m}^3)} = 3.93\times 10^{-29}\text{m}^3.$$

The volume of the sodium ion sphere is

$$V_2 = \frac{4\pi}{3}(98\times 10^{-12}\,\text{m})^3 = 3.94\times 10^{-30}\,\text{m}^3.$$

The fractional volume available for conduction electrons is

$$\frac{V_1 - V_2}{V_1} = \frac{(3.93\times 10^{-29}\text{m}^3) - (3.94\times 10^{-30}\,\text{m}^3)}{(3.93\times 10^{-29}\text{m}^3)} = 90\%.$$

(b) The approximate volume of a single copper atom is

$$V_1 = \frac{(0.0635\,\text{kg/mol})}{(6.02\times 10^{23}\text{part/mol})(8960\,\text{kg/m}^3)} = 1.18\times 10^{-29}\text{m}^3.$$

The volume of the copper ion sphere is

$$V_2 = \frac{4\pi}{3}(96\times 10^{-12}\,\text{m})^3 = 3.71\times 10^{-30}\,\text{m}^3.$$

The fractional volume available for conduction electrons is

$$\frac{V_1 - V_2}{V_1} = \frac{(1.18\times 10^{-29}\text{m}^3) - (3.71\times 10^{-30}\,\text{m}^3)}{(1.18\times 10^{-29}\text{m}^3)} = 69\%.$$

(c) Sodium, since more of the volume is available for the conduction electron.

E49-6 (a) Apply Eq. 49-6:

$$p = 1/\left[e^{(0.0730\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(0\,\text{K})} + 1\right] = 0.$$

(b) Apply Eq. 49-6:

$$p = 1/\left[e^{(0.0730\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(320\,\text{K})} + 1\right] = 6.62\times10^{-2}.$$

E49-7 Apply Eq. 49-6, remembering to use the energy *difference*:

$$p = 1/\left[e^{(-1.1)\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 1.00,$$

$$p = 1/\left[e^{(-0.1)\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 0.986,$$

$$p = 1/\left[e^{(0.0)\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 0.5,$$

$$p = 1/\left[e^{(0.1)\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 0.014,$$

$$p = 1/\left[e^{(1.1)\ \text{eV})/(8.62\times10^{-5}\text{eV/K})(273\,\text{K})} + 1\right] = 0.0.$$

(b) Inverting the equation,

$$T = \frac{\Delta E}{k\ln(1/p-1)},$$

so

$$T = \frac{(0.1\ \text{eV})}{(8.62\times10^{-5}\text{eV/K})\ln(1/(0.16)-1)} = 700\,\text{K}$$

E49-8 The energy differences are equal, except for the sign. Then

$$\begin{aligned}
\frac{1}{e^{+\Delta E/kt}+1} + \frac{1}{e^{-\Delta E/kt}+1} &= ,\\
\frac{e^{-\Delta E/2kt}}{e^{+\Delta E/2kt}+e^{-\Delta E/2kt}} + \frac{e^{+\Delta E/2kt}}{e^{-\Delta E/2kt}+e^{+\Delta E/2kt}} &= ,\\
\frac{e^{-\Delta E/2kt}+e^{+\Delta E/2kt}}{e^{-\Delta E/2kt}+e^{+\Delta E/2kt}} &= 1.
\end{aligned}$$

$\boxed{\textbf{E49-9}}$ The Fermi energy is given by Eq. 49-5,

$$E_\text{F} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3},$$

where n is the density of conduction electrons. For gold we have

$$n = \frac{(19.3\ \text{g/cm}^3)(6.02\times10^{23}\text{part/mol})}{(197\ \text{g/mol})} = 5.90\times10^{22}\ \text{elect./cm}^3 = 59\ \text{elect./nm}^3$$

The Fermi energy is then

$$E_\text{F} = \frac{(1240\ \text{eV}\cdot\text{nm})^2}{8(0.511\times10^6\ \text{eV})}\left(\frac{3(59\ \text{electrons/nm}^3)}{\pi}\right)^{2/3} = 5.53\ \text{eV}.$$

E49-10 Combine the results of Ex. 49-1 and Eq. 49-6:

$$n_\mathrm{o} = \frac{C\sqrt{E}}{e^{\Delta E/kt}+1}.$$

Then for each of the energies we have

$$n_\mathrm{o} = \frac{(6.81\times 10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(4\,\mathrm{eV})}}{e^{(-3.06\,\mathrm{eV})/(8.62\times 10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 1.36\times 10^{28}/\mathrm{m}^3\cdot\mathrm{eV},$$

$$n_\mathrm{o} = \frac{(6.81\times 10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(6.75\,\mathrm{eV})}}{e^{(-0.31\,\mathrm{eV})/(8.62\times 10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 1.72\times 10^{28}/\mathrm{m}^3\cdot\mathrm{eV},$$

$$n_\mathrm{o} = \frac{(6.81\times 10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(7\,\mathrm{eV})}}{e^{(-0.06\,\mathrm{eV})/(8.62\times 10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 9.02\times 10^{27}/\mathrm{m}^3\cdot\mathrm{eV},$$

$$n_\mathrm{o} = \frac{(6.81\times 10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(7.25\,\mathrm{eV})}}{e^{(0.19\,\mathrm{eV})/(8.62\times 10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 1.82\times 10^{27}/\mathrm{m}^3\cdot\mathrm{eV},$$

$$n_\mathrm{o} = \frac{(6.81\times 10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})\sqrt{(9\,\mathrm{eV})}}{e^{(1.94\,\mathrm{eV})/(8.62\times 10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})}+1} = 3.43\times 10^{18}/\mathrm{m}^3\cdot\mathrm{eV}.$$

E49-11 Solve

$$E_n = \frac{n^2(hc)^2}{8(mc^2)L^2}$$

for $n = 50$, since there are two electrons in each level. Then

$$E_\mathrm{f} = \frac{(50)^2(1240\ \mathrm{eV}\cdot\mathrm{nm})^2}{8(5.11\times 10^5\mathrm{eV})(0.12\ \mathrm{nm})^2} = 6.53\times 10^4\mathrm{eV}.$$

E49-12 We need to be much higher than $T = (7.06\,\mathrm{eV})/(8.62\times 10^{-5}\mathrm{eV/K}) = 8.2\times 10^4\mathrm{K}$.

$\boxed{\textbf{E49-13}}$ Equation 49-5 is

$$E_\mathrm{F} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3},$$

and if we collect the constants,

$$E_\mathrm{F} = \frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3} n^{3/2} = An^{3/2},$$

where, if we multiply the top and bottom by c^2

$$A = \frac{(hc)^2}{8mc^2}\left(\frac{3}{\pi}\right)^{2/3} = \frac{(1240\times 10^{-9}\ \mathrm{eV}\cdot\mathrm{m})^2}{8(0.511\times 10^6\ \mathrm{eV})}\left(\frac{3}{\pi}\right)^{2/3} = 3.65\times 10^{-19}\,\mathrm{m}^2\cdot\mathrm{eV}.$$

E49-14 (a) Inverting Eq. 49-6,

$$\Delta E = kT\ln(1/p - 1),$$

so

$$\Delta E = (8.62\times 10^{-5}\mathrm{eV/K})(1050\,\mathrm{K})\ln(1/(0.91)-1) = -0.209\ \mathrm{eV}.$$

Then $E = (-0.209\,\mathrm{eV}) + (7.06\,\mathrm{eV}) = 6.85\,\mathrm{eV}$.

(b) Apply the results of Ex. 49-1:

$$n(E) = (6.81\times 10^{27}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-3/2})(6.85\ \mathrm{eV})^{1/2} = 1.78\times 10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}.$$

(c) $n_\mathrm{o} = np = (1.78\times 10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1})(0.910) = 1.62\times 10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}$.

E49-15 Equation 49-5 is

$$E_\mathrm{F} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3},$$

and if we rearrange,

$$E_\mathrm{F}{}^{3/2} = \frac{3h^3}{16\sqrt{2}\pi m^{3/2}}n,$$

Equation 49-2 is then

$$n(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2} = \frac{3}{2}nE_\mathrm{F}{}^{-3/2}E^{1/2}.$$

E49-16 $p_\mathrm{h} = 1 - p$, so

$$\begin{aligned} p_\mathrm{h} &= 1 - \frac{1}{e^{\Delta E/kT}+1}, \\ &= \frac{e^{\Delta E/kT}}{e^{\Delta E/kT}+1}, \\ &= \frac{1}{1+e^{-\Delta E/kT}}. \end{aligned}$$

$\boxed{\textbf{E49-17}}$ The steps to solve this exercise are equivalent to the steps for Exercise 49-9, except now the iron atoms each contribute 26 electrons and we have to find the density.

First, the density is

$$\rho = \frac{m}{4\pi r^3/3} = \frac{(1.99\times10^{30}\mathrm{kg})}{4\pi(6.37\times10^6\mathrm{m})^3/3} = 1.84\times10^9\mathrm{kg/m}^3$$

Then

$$\begin{aligned} n &= \frac{(26)(1.84\times10^6\ \mathrm{g/cm}^3)(6.02\times10^{23}\mathrm{part/mol})}{(56\ \mathrm{g/mol})} = 5.1\times10^{29}\ \mathrm{elect./cm}^3, \\ &= 5.1\times10^8\ \mathrm{elect./nm}^3 \end{aligned}$$

The Fermi energy is then

$$E_\mathrm{F} = \frac{(1240\ \mathrm{eV\cdot nm})^2}{8(0.511\times10^6\ \mathrm{eV})}\left(\frac{3(5.1\times10^8\ \mathrm{elect./nm}^3)}{\pi}\right)^{2/3} = 230\ \mathrm{keV}.$$

E49-18 First, the density is

$$\rho = \frac{m}{4\pi r^3/3} = \frac{2(1.99\times10^{30}\mathrm{kg})}{4\pi(10\times10^3\mathrm{m})^3/3} = 9.5\times10^{17}\mathrm{kg/m}^3$$

Then

$$n = (9.5\times10^{17}\mathrm{kg/m}^3)/(1.67\times10^{-27}\mathrm{kg}) = 5.69\times10^{44}/\mathrm{m}^3.$$

The Fermi energy is then

$$E_\mathrm{F} = \frac{(1240\ \mathrm{MeV\cdot fm})^2}{8(940\ \mathrm{MeV})}\left(\frac{3(5.69\times10^{-1}/\mathrm{fm}^3)}{\pi}\right)^{2/3} = 137\ \mathrm{MeV}.$$

E49-19

E49-20 (a) $E_{\rm F} = 7.06\,{\rm eV}$, so

$$f = \frac{3(8.62\times10^{-5}{\rm eV\cdot K})(0\,{\rm K})}{2(7.06\,{\rm eV})} = 0,$$

(b) $f = 3(8.62\times10^{-5}{\rm eV\cdot K})(300\,{\rm K})/2(7.06\,{\rm eV}) = 0.0055.$
(c) $f = 3(8.62\times10^{-5}{\rm eV\cdot K})(1000\,{\rm K})/2(7.06\,{\rm eV}) = 0.0183.$

$\boxed{\textbf{E49-21}}$ Using the results of Exercise 19,

$$T = \frac{2fE_{\rm F}}{3k} = \frac{2(0.0130)(4.71\ {\rm eV})}{3(8.62\times10^{-5}{\rm eV\cdot K})} = 474\,{\rm K}.$$

E49-22 $f = 3(8.62\times10^{-5}{\rm eV\cdot K})(1235\,{\rm K})/2(5.5\,{\rm eV}) = 0.029.$

E49-23 (a) Monovalent means only one electron is available as a conducting electron. Hence we need only calculate the density of atoms:

$$\frac{N}{V} = \frac{\rho N_A}{A_r} = \frac{(10.5\times10^3{\rm kg/m^3})(6.02\times10^{23}{\rm mol^{-1}})}{(0.107\,{\rm kg/mol})} = 5.90\times10^{28}/{\rm m^3}.$$

(b) Using the results of Ex. 49-13,

$$E_{\rm F} = (3.65\times10^{-19}\,{\rm m^2\cdot eV})(5.90\times10^{28}/{\rm m^3})^{2/3} = 5.5\,{\rm eV}.$$

(c) $v = \sqrt{2K/m}$, or

$$v = \sqrt{2(5.5\,{\rm eV})(5.11\times10^5{\rm eV}/c^2)} = 1.4\times10^8{\rm m/s}.$$

(d) $\lambda = h/p$, or

$$\lambda = \frac{(6.63\times10^{-34}{\rm J\cdot s})}{(9.11\times10^{-31}{\rm kg})(1.4\times10^8{\rm m/s})} = 5.2\times10^{-12}{\rm m}.$$

E49-24 (a) Bivalent means two electrons are available as a conducting electron. Hence we need to double the calculation of the density of atoms:

$$\frac{N}{V} = \frac{\rho N_A}{A_r} = \frac{2(7.13\times10^3{\rm kg/m^3})(6.02\times10^{23}{\rm mol^{-1}})}{(0.065\,{\rm kg/mol})} = 1.32\times10^{29}/{\rm m^3}.$$

(b) Using the results of Ex. 49-13,

$$E_{\rm F} = (3.65\times10^{-19}\,{\rm m^2\cdot eV})(1.32\times10^{29}/{\rm m^3})^{2/3} = 9.4\,{\rm eV}.$$

(c) $v = \sqrt{2K/m}$, or

$$v = \sqrt{2(9.4\,{\rm eV})(5.11\times10^5{\rm eV}/c^2)} = 1.8\times10^8{\rm m/s}.$$

(d) $\lambda = h/p$, or

$$\lambda = \frac{(6.63\times10^{-34}{\rm J\cdot s})}{(9.11\times10^{-31}{\rm kg})(1.8\times10^8{\rm m/s})} = 4.0\times10^{-12}{\rm m}.$$

$\boxed{\textbf{E49-25}}$ (a) Refer to Sample Problem 49-5 where we learn that the mean free path λ can be written in terms of Fermi speed v_F and mean time between collisions τ as

$$\lambda = v_\mathrm{F}\tau.$$

The Fermi speed is

$$v_\mathrm{F} = c\sqrt{2E_F/mc^2} = c\sqrt{2(5.51\text{ eV})/(5.11\times10^5\text{ eV})} = 4.64\times10^{-3}c.$$

The time between collisions is

$$\tau = \frac{m}{ne^2\rho} = \frac{(9.11\times10^{-31}\mathrm{kg})}{(5.86\times10^{28}\mathrm{m}^{-3})(1.60\times10^{-19}\mathrm{C})^2(1.62\times10^{-8}\Omega\cdot\mathrm{m})} = 3.74\times10^{-14}\mathrm{s}.$$

We found n by looking up the answers from Exercise 49-23 in the back of the book. The mean free path is then

$$\lambda = (4.64\times10^{-3})(3.00\times10^8\mathrm{m/s})(3.74\times10^{-14}\mathrm{s}) = 52\text{ nm}.$$

(b) The spacing between the ion cores is approximated by the cube root of volume per atom. This atomic volume for silver is

$$V = \frac{(108\text{ g/mol})}{(6.02\times10^{23}\text{part/mol})(10.5\text{ g/cm}^3)} = 1.71\times10^{-23}\mathrm{cm}^3.$$

The distance between the ions is then

$$l = \sqrt[3]{V} = 0.257\text{ nm}.$$

The ratio is

$$\lambda/l = 190.$$

E49-26 (a) For $T = 1000\,\mathrm{K}$ we can use the approximation, so for diamond

$$p = e^{-(5.5\,\mathrm{eV})/2(8.62\times10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})} = 1.4\times10^{-14},$$

while for silicon,

$$p = e^{-(1.1\,\mathrm{eV})/2(8.62\times10^{-5}\mathrm{eV/K})(1000\,\mathrm{K})} = 1.7\times10^{-3},$$

(b) For $T = 4\,\mathrm{K}$ we can use the same approximation, but now $\Delta E \gg kT$ and the exponential function goes to zero.

E49-27 (a) $E - E_\mathrm{F} \approx 0.67\,\mathrm{eV}/2 = 0.34\,\mathrm{eV}$.. The probability the state is occupied is then

$$p = 1/\left[e^{(0.34)\ \mathrm{eV})/(8.62\times10^{-5}\mathrm{eV/K})(290\,\mathrm{K})} + 1\right] = 1.2\times10^{-6}.$$

(b) $E - E_\mathrm{F} \approx -0.67\,\mathrm{eV}/2 = -0.34\,\mathrm{eV}$.. The probability the state is *unoccupied* is then $1-p$, or

$$p = 1 - 1/\left[e^{(-0.34)\ \mathrm{eV})/(8.62\times10^{-5}\mathrm{eV/K})(290\,\mathrm{K})} + 1\right] = 1.2\times10^{-6}.$$

E49-28 (a) $E - E_\mathrm{F} \approx 0.67\,\mathrm{eV}/2 = 0.34\,\mathrm{eV}$.. The probability the state is occupied is then

$$p = 1/\left[e^{(0.34)\ \mathrm{eV})/(8.62\times10^{-5}\mathrm{eV/K})(289\,\mathrm{K})} + 1\right] = 1.2\times10^{-6}.$$

$\boxed{\textbf{E49-29}}$ (a) The number of silicon atoms per unit volume is

$$n = \frac{(6.02\times10^{23}\text{part/mol})(2.33\text{ g/cm}^3)}{(28.1\text{ g/mol})} = 4.99\times10^{22}\text{ part./cm}^3.$$

If one out of 1.0eex7 are replaced then there will be an additional charge carrier density of

$$4.99\times10^{22}\text{ part./cm}^3/1.0\times10^7 = 4.99\times10^{15}\text{ part./cm}^3 = 4.99\times10^{21}\text{m}^{-3}.$$

(b) The ratio is

$$(4.99\times10^{21}\text{m}^{-3})/(2\times1.5\times10^{16}\text{m}^{-3}) = 1.7\times10^5.$$

The extra factor of two is because *all* of the charge carriers in silicon (holes and electrons) are charge carriers.

E49-30 Since one out of every 5×10^6 silicon atoms needs to be replaced, then the mass of phosphorus would be

$$m = \frac{1}{5\times10^6}\frac{30}{28} = 2.1\times10^{-7}\text{ g}.$$

E49-31 $l = \sqrt[3]{1/10^{22}/\text{m}^3} = 4.6\times10^{-8}\text{m}.$

E49-32 The atom density of germanium is

$$\frac{N}{V} = \frac{\rho N_A}{A_r} = \frac{(5.32\times10^3\text{kg/m}^3)(6.02\times10^{23}\text{mol}^{-1})}{(0.197\,\text{kg/mol})} = 1.63\times10^{28}/\text{m}^3.$$

The atom density of the impurity is

$$(1.63\times10^{28}/\text{m}^3)/(1.3\times10^9) = 1.25\times10^{19}.$$

The average spacing is

$$l = \sqrt[3]{1/1.25\times10^{19}/\text{m}^3} = 4.3\times10^{-7}\text{m}.$$

$\boxed{\textbf{E49-33}}$ The first one is an insulator because the lower band is filled and band gap is so large; there is no impurity.

The second one is an extrinsic n-type semiconductor: it is a semiconductor because the lower band is filled and the band gap is small; it is extrinsic because there is an impurity; since the impurity level is close to the top of the band gap the impurity is a donor.

The third sample is an intrinsic semiconductor: it is a semiconductor because the lower band is filled and the band gap is small.

The fourth sample is a conductor; although the band gap is large, the lower band is *not* completely filled.

The fifth sample is a conductor: the Fermi level is above the bottom of the upper band.

The sixth one is an extrinsic p-type semiconductor: it is a semiconductor because the lower band is filled and the band gap is small; it is extrinsic because there is an impurity; since the impurity level is close to the bottom of the band gap the impurity is an acceptor.

E49-34 $6.62\times10^5\text{eV}/1.1\,\text{eV} = 6.0\times10^5$ electron-hole pairs.

E49-35 (a) $R = (1\,\text{V})/(50\times10^{-12}\text{A}) = 2\times10^{10}\Omega.$

(b) $R = (0.75\,\text{V})/(8\,\text{mA}) = 90\Omega.$

E49-36 (a) A region with some potential difference exists that has a gap between the charged areas.

(b) $C = Q/\Delta V$. Using the results in Sample Problem 49-9 for q and ΔV,

$$C = \frac{n_0 e A d/2}{n_0 e d^2/4\kappa\epsilon_0} = 2\kappa\epsilon_0 A/d.$$

E49-37 (a) Apply that ever so useful formula

$$\lambda = \frac{hc}{E} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(5.5 \text{ eV})} = 225 \text{ nm}.$$

Why is this a *maximum?* Because longer wavelengths would have *lower* energy, and so not enough to cause an electron to jump across the band gap.

(b) Ultraviolet.

E49-38 Apply that ever so useful formula

$$E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(295 \text{ nm})} = 4.20\,\text{eV}.$$

E49-39 The photon energy is

$$E = \frac{hc}{\lambda} = \frac{(1240 \text{ eV}\cdot\text{nm})}{(140 \text{ nm})} = 8.86\,\text{eV}.$$

which is enough to excite the electrons through the band gap. As such, the photon will be absorbed, which means the crystal is opaque to this wavelength.

E49-40

P49-1 We can calculate the electron density from Eq. 49-5,

$$\begin{aligned} n &= \frac{\pi}{3}\left(\frac{8mc^2E_\text{F}}{(hc)^2}\right)^{3/2}, \\ &= \frac{\pi}{3}\left(\frac{8(0.511\times10^6 \text{ eV})(11.66 \text{ eV})}{(1240 \text{ eV}\cdot\text{nm})^2}\right)^{3/2}, \\ &= 181 \text{ electrons/nm}^3. \end{aligned}$$

From this we calculate the number of electrons per particle,

$$\frac{(181 \text{ electrons/nm}^3)(27.0 \text{ g/mol})}{(2.70 \text{ g/cm}^3)(6.02\times10^{23} \text{ particles/mol})} = 3.01,$$

which we can reasonably approximate as 3.

P49-2 At absolute zero all states below E_F are filled, an none above. Using the results of Ex. 49-15,

$$\begin{aligned} E_\mathrm{av} &= \frac{1}{n}\int_0^{E_\mathrm{F}} En(E)\,dE, \\ &= \frac{3}{2}{E_\mathrm{F}}^{-3/2}\int_0^{E_\mathrm{F}} E^{3/2}\,dE, \\ &= \frac{3}{2}{E_\mathrm{F}}^{-3/2}\frac{2}{5}{E_\mathrm{F}}^{5/2}, \\ &= \frac{3}{5}E_\mathrm{F}. \end{aligned}$$

P49-3 (a) The total number of conduction electron is

$$n = \frac{(0.0031\,\mathrm{kg})(6.02\times 10^{23}\mathrm{mol}^{-1})}{(0.0635\,\mathrm{kg/mol})} = 2.94\times 10^{22}.$$

The total energy is

$$E = \frac{3}{5}(7.06\,\mathrm{eV})(2.94\times 10^{22}) = 1.24\times 10^{23}\mathrm{eV} = 2\times 10^4\mathrm{J}.$$

(b) This will light a 100 W bulb for

$$t = (2\times 10^4\ \mathrm{J})/(100\,\mathrm{W}) = 200\,\mathrm{s}.$$

P49-4 (a) First do the easy part: $n_\mathrm{c} = N_\mathrm{c}p(E_\mathrm{c})$, so

$$\frac{N_\mathrm{c}}{e^{(E_\mathrm{c}-E_\mathrm{F})/kT}+1}.$$

Then use the results of Ex. 49-16, and write

$$n_\mathrm{v} = N_\mathrm{v}[1-p(E_\mathrm{v})] = \frac{N_\mathrm{v}}{e^{-(E_\mathrm{v}-E_\mathrm{F})/kT}+1}.$$

Since each electron in the conduction band must have left a hole in the valence band, then these two expressions must be equal.

(b) If the exponentials dominate then we can drop the +1 in each denominator, and

$$\begin{aligned} \frac{N_\mathrm{c}}{e^{(E_\mathrm{c}-E_\mathrm{F})/kT}} &= \frac{N_\mathrm{v}}{e^{-(E_\mathrm{v}-E_\mathrm{F})/kT}}, \\ \frac{N_\mathrm{c}}{N_\mathrm{v}} &= e^{(E_\mathrm{c}-2E_\mathrm{F}+E_\mathrm{v})/kT}, \\ E_\mathrm{F} &= \frac{1}{2}\left(E_\mathrm{c}+E_\mathrm{v}+kT\ln(N_\mathrm{c}/N_\mathrm{v})\right). \end{aligned}$$

$\boxed{\textbf{P49-5}}$ (a) We want to use Eq. 49-6; although we don't know the Fermi energy, we do know the differences between the energies in question. In the un-doped silicon $E - E_\mathrm{F} = 0.55$ eV for the bottom of the conduction band. The quantity

$$kT = (8.62\times 10^{-5}\ \mathrm{eV/K})(290\,\mathrm{K}) = 0.025\ \mathrm{eV},$$

which is a good number to remember— at room temperature kT is 1/40 of an electron-volt.

Then

$$p = \frac{1}{e^{(0.55 \text{ eV})/(0.025 \text{ eV})} + 1} = 2.8\times10^{-10}.$$

In the doped silicon $E - E_\text{F} = 0.084$ eV for the bottom of the conduction band. Then

$$p = \frac{1}{e^{(0.084 \text{ eV})/(0.025 \text{ eV})} + 1} = 3.4\times10^{-2}.$$

(b) For the donor state $E - E_\text{F} = -0.066$ eV, so

$$p = \frac{1}{e^{(-0.066 \text{ eV})/(0.025 \text{ eV})} + 1} = 0.93.$$

P49-6 (a) Inverting Eq. 49-6,

$$E - E_\text{F} = kT \ln(1/p - 1),$$

so

$$E_\text{F} = (1.1\,\text{eV} - 0.11\,\text{eV}) - (8.62\times10^{-5}\ \text{eV/K})(290\,\text{K})\ln(1/(4.8\times10^{-5}) - 1) = 0.74\,\text{eV}$$

above the valence band.

(b) $E - E_\text{F} = (1.1\,\text{eV}) - (0.74\,\text{eV}) = 0.36\,\text{eV}$, so

$$p = \frac{1}{e^{(0.36 \text{ eV})/(0.025 \text{ eV})} + 1} = 5.6\times10^{-7}.$$

P49-7 (a) Plot the graph with a spreadsheet. It should look like Fig. 49-12.

(b) $kT = 0.025\,\text{eV}$ when $T = 290\,\text{K}$. The ratio is then

$$\frac{i_\text{f}}{i_\text{r}} = \frac{e^{(0.5\,\text{eV})/(0.025\,\text{eV})} + 1}{e^{(-0.5\,\text{eV})/(0.025\,\text{eV})} + 1} = 4.9\times10^{8}.$$

P49-8

$\boxed{\textbf{E50-1}}$ We want to follow the example set in Sample Problem 50-1. The distance of closest approach is given by

$$\begin{aligned} d &= \frac{qQ}{4\pi\epsilon_0 K_\alpha}, \\ &= \frac{(2)(29)(1.60\times10^{-19}\mathrm{C})^2}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{Nm}^2)(5.30\mathrm{MeV})(1.60\times10^{-13}\mathrm{J/MeV})}, \\ &= 1.57\times10^{-14}\,\mathrm{m}. \end{aligned}$$

That's pretty close.

E50-2 (a) The gold atom can be treated as a point particle:

$$\begin{aligned} F &= \frac{q_1 q_2}{4\pi\epsilon_0 r^2}, \\ &= \frac{(2)(79)(1.60\times10^{-19}\mathrm{C})^2}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{Nm}^2)(0.16\times10^{-9}\mathrm{m})^2}, \\ &= 1.4\times10^{-6}\mathrm{N}. \end{aligned}$$

(b) $W = Fd$, so

$$d = \frac{(5.3\times10^6\mathrm{eV})(1.6\times10^{-19}\mathrm{J/eV})}{(1.4\times10^{-6}\mathrm{N})} = 6.06\times10^{-7}\mathrm{m}.$$

That's 1900 gold atom diameters.

E50-3 Take an approach similar to Sample Problem 50-1:

$$\begin{aligned} K &= \frac{qQ}{4\pi\epsilon_0 d}, \\ &= \frac{(2)(79)(1.60\times10^{-19}\mathrm{C})^2}{4\pi(8.85\times10^{-12}\mathrm{C}^2/\mathrm{Nm}^2)(8.78\times10^{-15}\mathrm{m})(1.60\times10^{-19}\mathrm{J/eV})}, \\ &= 2.6\times10^{7}\mathrm{eV}. \end{aligned}$$

E50-4 All are stable except ^{88}Rb and ^{239}Pb.

$\boxed{\textbf{E50-5}}$ We can make an estimate of the mass number A from Eq. 50-1,

$$R = R_0 A^{1/3},$$

where $R_0 = 1.2$ fm. If the measurements indicate a radius of 3.6 fm we would have

$$A = (R/R_0)^3 = ((3.6\text{ fm})/(1.2\text{ fm}))^3 = 27.$$

E50-6

E50-7 The mass number of the sun is

$$A = (1.99\times10^{30}\mathrm{kg})/(1.67\times10^{-27}\mathrm{kg}) = 1.2\times10^{57}.$$

The radius would be

$$R = (1.2\times10^{-15}\mathrm{m})\sqrt[3]{1.2\times10^{57}} = 1.3\times10^{4}\mathrm{m}.$$

E50-8 ^{239}Pu is composed of 94 protons and $239 - 94 = 145$ neutrons. The combined mass of the free particles is

$$M = Zm_{\rm p} + Nm_{\rm n} = (94)(1.007825\ {\rm u}) + (145)(1.008665\ {\rm u}) = 240.991975\ {\rm u}.$$

The binding energy is the difference

$$E_{\rm B} = (240.991975\ {\rm u} - 239.052156\ {\rm u})(931.5\ {\rm MeV/u}) = 1806.9\ {\rm MeV},$$

and the binding energy per nucleon is then

$$(1806.9\ {\rm MeV})/(239) = 7.56\ {\rm MeV}.$$

$\boxed{\textbf{E50-9}}$ ^{62}Ni is composed of 28 protons and $62 - 28 = 34$ neutrons. The combined mass of the free particles is

$$M = Zm_{\rm p} + Nm_{\rm n} = (28)(1.007825\ {\rm u}) + (34)(1.008665\ {\rm u}) = 62.513710\ {\rm u}.$$

The binding energy is the difference

$$E_{\rm B} = (62.513710\ {\rm u} - 61.928349\ {\rm u})(931.5\ {\rm MeV/u}) = 545.3\ {\rm MeV},$$

and the binding energy per nucleon is then

$$(545.3\ {\rm MeV})/(62) = 8.795\ {\rm MeV}.$$

E50-10 (a) Multiply each by 1/1.007825, so

$$m_{^1{\rm H}} = 1.00000,$$

$$m_{^{12}{\rm C}} = 11.906829,$$

and

$$m_{^{238}{\rm U}} = 236.202500.$$

E50-11 (a) Since the binding energy per nucleon is fairly constant, the energy must be proportional to A.

(b) Coulomb repulsion acts between pairs of protons; there are Z protons that can be chosen as first in the pair, and $Z-1$ protons remaining that can make up the partner in the pair. That makes for $Z(Z-1)$ pairs. The electrostatic energy must be proportional to this.

(c) Z^2 grows faster than A, which is roughly proportional to Z.

E50-12 Solve

$$(0.7899)(23.985042) + x(24.985837) + (0.2101 - x)(25.982593) = 24.305$$

for x. The result is $x = 0.1001$, and then the amount ^{26}Mg is 0.1100.

$\boxed{\textbf{E50-13}}$ The neutron confined in a nucleus of radius R will have a position uncertainty on the order of $\Delta x \approx R$. The momentum uncertainty will then be no less than

$$\Delta p \geq \frac{h}{2\pi\Delta x} \approx \frac{h}{2\pi R}.$$

Assuming that $p \approx \Delta p$, we have

$$p \geq \frac{h}{2\pi R},$$

and then the neutron will have a (minimum) kinetic energy of

$$E \approx \frac{p^2}{2m} \approx \frac{h^2}{8\pi^2 m R^2}.$$

But $R = R_0 A^{1/3}$, so

$$E \approx \frac{(hc)^2}{8\pi^2 mc^2 R_0^2 A^{2/3}}.$$

For an atom with $A = 100$ we get

$$E \approx \frac{(1240 \text{ MeV} \cdot \text{fm})^2}{8\pi^2 (940 \text{ MeV})(1.2 \text{ fm})^2 (100)^{2/3}} = 0.668 \text{ MeV}.$$

This is about a factor of 5 or 10 less than the binding energy per nucleon.

E50-14 (a) To remove a proton,

$$E = [(1.007825) + (3.016049) - (4.002603)]\,(931.5 \text{ MeV}) = 19.81 \text{ MeV}.$$

To remove a neutron,

$$E = [(1.008665) + (2.014102) - (3.016049)]\,(931.5 \text{ MeV}) = 6.258 \text{ MeV}.$$

To remove a proton,

$$E = [(1.007825) + (1.008665) - (2.014102)]\,(931.5 \text{ MeV}) = 2.224 \text{ MeV}.$$

(b) $E = (19.81 + 6.258 + 2.224)\text{MeV} = 28.30$ MeV.
(c) $(28.30 \text{ MeV})/4 = 7.07$ MeV.

E50-15 (a) $\Delta = [(1.007825) - (1)](931.5 \text{ MeV}) = 7.289$ MeV.
(b) $\Delta = [(1.008665) - (1)](931.5 \text{ MeV}) = 8.071$ MeV.
(c) $\Delta = [(119.902197) - (120)](931.5 \text{ MeV}) = -91.10$ MeV.

E50-16 (a) $E_\text{B} = (Zm_\text{H} + Nm_\text{N} - m)c^2$. Substitute the definition for mass excess, $mc^2 = Ac^2 + \Delta$, and

$$\begin{aligned} E_\text{B} &= Z(c^2 + \Delta_\text{H}) + N(c^2 + \Delta_\text{N}) - Ac^2 - \Delta, \\ &= Z\Delta_\text{H} + N\Delta_\text{N} - \Delta. \end{aligned}$$

(b) For ^{197}Au,

$$E_\text{B} = (79)(7.289 \text{ MeV}) + (197 - 79)(8.071 \text{ MeV}) - (-31.157 \text{ MeV}) = 1559 \text{ MeV},$$

and the binding energy per nucleon is then

$$(1559 \text{ MeV})/(197) = 7.92 \text{ MeV}.$$

E50-17 The binding energy of ^{63}Cu is given by

$$M = Zm_\mathrm{p} + Nm_\mathrm{n} = (29)(1.007825\ \mathrm{u}) + (34)(1.008665\ \mathrm{u}) = 63.521535\ \mathrm{u}.$$

The binding energy is the difference

$$E_\mathrm{B} = (63.521535\ \mathrm{u} - 62.929601\ \mathrm{u})(931.5\ \mathrm{MeV/u}) = 551.4\ \mathrm{MeV}.$$

The number of atoms in the sample is

$$n = \frac{(0.003\,\mathrm{kg})(6.02\times10^{23}\mathrm{mol}^{-1})}{(0.0629\,\mathrm{kg/mol})} = 2.87\times10^{22}.$$

The total energy is then

$$(2.87\times10^{22})(551.4\ \mathrm{MeV})(1.6\times10^{-19}\mathrm{J/eV}) = 2.53\times10^{12}\mathrm{J}.$$

E50-18 (a) For ultra-relativistic particles $E = pc$, so

$$\lambda = \frac{(1240\ \mathrm{MeV}\cdot\mathrm{fm})}{(480\ \mathrm{MeV})} = 2.59\ \mathrm{fm}.$$

(b) Yes, since the wavelength is smaller than nuclear radii.

E50-19 We will do this one the easy way because we can. This method won't work except when there is an integer number of half-lives. The activity of the sample will fall to one-half of the initial decay rate after one half-life; it will fall to one-half of one-half (one-fourth) after two half-lives. So two half-lives have elapsed, for a total of (2)(140 d) = 280 d.

E50-20 $N = N_0(1/2)^{t/t_{1/2}}$, so

$$N = (48\times10^{19})(0.5)^{(26)/(6.5)} = 3.0\times10^{19}.$$

E50-21 (a) $t_{1/2} = \ln 2/(0.0108/\mathrm{h}) = 64.2$ h.

(b) $N = N_0(1/2)^{t/t_{1/2}}$, so

$$N/N_0 = (0.5)^{(3)} = 0.125.$$

(c) $N = N_0(1/2)^{t/t_{1/2}}$, so

$$N/N_0 = (0.5)^{(240)/(64.2)} = 0.0749.$$

E50-22 (a) $\lambda = (-dN/dt)/N$, or

$$\lambda = (12/\mathrm{s})/(2.5\times10^{18}) = 4.8\times10^{-18}/\mathrm{s}.$$

(b) $t_{1/2} = \ln 2/\lambda$, so

$$t_{1/2} = \ln 2/(4.8\times10^{-18}/\mathrm{s}) = 1.44\times10^{17}\mathrm{s},$$

which is 4.5 billion years.

E50-23 (a) The decay constant for ^{67}Ga can be derived from Eq. 50-8,

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(2.817\times 10^5\,\mathrm{s})} = 2.461\times 10^{-6}\mathrm{s}^{-1}.$$

The activity is given by $R = \lambda N$, so we want to know how many atoms are present. That can be found from

$$3.42\ \mathrm{g}\left(\frac{1\ \mathrm{u}}{1.6605\times 10^{-24}\ \mathrm{g}}\right)\left(\frac{1\ \mathrm{atom}}{66.93\ \mathrm{u}}\right) = 3.077\times 10^{22}\ \mathrm{atoms}.$$

So the activity is

$$R = (2.461\times 10^{-6}/\mathrm{s}^{-1})(3.077\times 10^{22}\ \mathrm{atoms}) = 7.572\times 10^{16}\ \mathrm{decays/s}.$$

(b) After 1.728×10^5 s the activity would have decreased to

$$R = R_0 e^{-\lambda t} = (7.572\times 10^{16}\ \mathrm{decays/s})e^{-(2.461\times 10^{-6}/\mathrm{s}^{-1})(1.728\times 10^5\ \mathrm{s})} = 4.949\times 10^{16}\ \mathrm{decays/s}.$$

E50-24 $N = N_0 e^{-\lambda t}$, but $\lambda = \ln 2/t_{1/2}$, so

$$N = N_0 e^{-\ln 2t/t_{1/2}} = N_0(2)^{-t/t_{1/2}} = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}}.$$

E50-25 The remaining ^{223} is

$$N = (4.7\times 10^{21})(0.5)^{(28)/(11.43)} = 8.6\times 10^{20}.$$

The number of decays, each of which produced an alpha particle, is

$$(4.7\times 10^{21}) - (8.6\times 10^{20}) = 3.84\times 10^{21}.$$

E50-26 The amount remaining after 14 hours is

$$m = (5.50\ \mathrm{g})(0.5)^{(14)/(12.7)} = 2.562\ \mathrm{g}.$$

The amount remaining after 16 hours is

$$m = (5.50\ \mathrm{g})(0.5)^{(16)/(12.7)} = 2.297\ \mathrm{g}.$$

The difference is the amount which decayed during the two hour interval:

$$(2.562\ \mathrm{g}) - (2.297\ \mathrm{g}) = 0.265\ \mathrm{g}.$$

E50-27 (a) Apply Eq. 50-7,

$$R = R_0 e^{-\lambda t}.$$

We first need to know the decay constant from Eq. 50-8,

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(1.234\times 10^6\,\mathrm{s})} = 5.618\times 10^{-7}\mathrm{s}^{-1}.$$

And the the time is found from

$$\begin{aligned} t &= -\frac{1}{\lambda}\ln\frac{R}{R_0}, \\ &= -\frac{1}{(5.618\times 10^{-7}\mathrm{s}^{-1})}\ln\frac{(170\ \mathrm{counts/s})}{(3050\ \mathrm{counts/s})}, \\ &= 5.139\times 10^6\,\mathrm{s} \approx 59.5\ \mathrm{days}. \end{aligned}$$

Note that counts/s is *not* the same as decays/s. Not all decay events will be picked up by a detector and recorded as a count; we are assuming that whatever scaling factor which connects the initial count rate to the initial decay rate is valid at later times as well. Such an assumption is a reasonable assumption.

(b) The purpose of such an experiment would be to measure the amount of phosphorus that is taken up in a leaf. But the activity of the tracer decays with time, and so without a correction factor we would record the wrong amount of phosphorus in the leaf. That correction factor is R_0/R; we need to multiply the measured counts by this factor to correct for the decay.

In this case

$$\frac{R}{R_0} = e^{\lambda t} = e^{(5.618\times10^{-7}\mathrm{s}^{-1})(3.007\times10^{5}\,\mathrm{s})} = 1.184.$$

E50-28 The number of particles of ^{147}Sm is

$$n = (0.15)\frac{(0.001\,\mathrm{kg})(6.02\times10^{23}\mathrm{mol}^{-1})}{(0.147\,\mathrm{kg/mol})} = 6.143\times10^{20}.$$

The decay constant is

$$\lambda = (120/\mathrm{s})/(6.143\times10^{20}) = 1.95\times10^{-19}/\mathrm{s}.$$

The half-life is

$$t_{1/2} = \ln 2/(1.95\times10^{-19}/\mathrm{s}) = 3.55\times10^{18}\mathrm{s},$$

or 110 Gy.

E50-29 The number of particles of ^{239}Pu is

$$n_0 = \frac{(0.012\,\mathrm{kg})(6.02\times10^{23}\mathrm{mol}^{-1})}{(0.239\,\mathrm{kg/mol})} = 3.023\times10^{22}.$$

The number which decay is

$$n_0 - n = (3.025\times10^{22})\left[1-(0.5)^{(20000)/(24100)}\right] = 1.32\times10^{22}.$$

The mass of helium produced is then

$$m = \frac{(0.004\,\mathrm{kg/mol})(1.32\times10^{22})}{(6.02\times10^{23}\mathrm{mol}^{-1})} = 8.78\times10^{-5}\mathrm{kg}.$$

E50-30 Let $R_{33}/(R_{33}+R32) = x$, where $x_0 = 0.1$ originally, and we want to find out at what time $x = 0.9$. Rearranging,

$$(R_{33}+R32)/R_{33} = 1/x,$$

so

$$R32/R_{33} = 1/x - 1.$$

Since $R = R_0(0.5)^{t/t_{1/2}}$ we can write a ratio

$$\frac{1}{x} - 1 = \left(\frac{1}{x_0} - 1\right)(0.5)^{t/t_{32} - t/t_{33}}.$$

Put in some of the numbers, and

$$\ln[(1/9)/(9)] = \ln[0.5]t\left(\frac{1}{14.3} - \frac{1}{25.3}\right),$$

which has solution $t = 209$ d.

E50-31

E50-32 (a) $N/N_0 = (0.5)^{(4500)/(82)} = 3.0\times10^{-17}$.
(b) $N/N_0 = (0.5)^{(4500)/(0.034)} = 0$.

$\boxed{\textbf{E50-33}}$ The Q values are

$$\begin{aligned} Q_3 &= (235.043923 - 232.038050 - 3.016029)(931.5\text{ MeV}) = -9.46\text{ MeV}, \\ Q_4 &= (235.043923 - 231.036297 - 4.002603)(931.5\text{ MeV}) = 4.68\text{ MeV}, \\ Q_5 &= (235.043923 - 230.033127 - 5.012228)(931.5\text{ MeV}) = -1.33\text{ MeV}. \end{aligned}$$

Only reactions with positive Q values are energetically possible.

E50-34 (a) For the ^{14}C decay,

$$Q = (223.018497 - 208.981075 - 14.003242)(931.5\text{ MeV}) = 31.84\text{ MeV}.$$

For the ^{4}He decay,

$$Q = (223.018497 - 219.009475 - 4.002603)(931.5\text{ MeV}) = 5.979\text{ MeV}.$$

E50-35 $Q = (136.907084 - 136.905821)(931.5\text{ MeV}) = 1.17\text{ MeV}$.

E50-36 $Q = (1.008665 - 1.007825)(931.5\text{ MeV}) = 0.782\text{ MeV}$.

$\boxed{\textbf{E50-37}}$ (a) The kinetic energy of this electron is significant compared to the rest mass energy, so we *must* use relativity to find the momentum. The total energy of the electron is $E = K + mc^2$, the momentum will be given by

$$\begin{aligned} pc &= \sqrt{E^2 - m^2c^4} = \sqrt{K^2 + 2Kmc^2}, \\ &= \sqrt{(1.00\text{ MeV})^2 + 2(1.00\text{ MeV})(0.511\text{ MeV})} = 1.42\text{ MeV}. \end{aligned}$$

The de Broglie wavelength is then

$$\lambda = \frac{hc}{pc} = \frac{(1240\text{ MeV}\cdot\text{fm})}{(1.42\text{ MeV})} = 873\text{ fm}.$$

(b) The radius of the emitting nucleus is

$$R = R_0 A^{1/3} = (1.2\text{ fm})(150)^{1/3} = 6.4\text{ fm}.$$

(c) The longest wavelength standing wave on a string fixed at each end is twice the length of the string. Although the rules for standing waves in a box are slightly more complicated, it is a fair assumption that the electron could not exist as a standing a wave in the nucleus.
(d) See part (c).

E50-38 The electron is relativistic, so

$$\begin{aligned} pc &= \sqrt{E^2 - m^2c^4}, \\ &= \sqrt{(1.71 \text{ MeV} + 0.51 \text{ MeV})^2 - (0.51 \text{ MeV})^2}, \\ &= 2.16 \text{ MeV}. \end{aligned}$$

This is also the magnitude of the momentum of the recoiling ^{32}S. Non-relativistic relations are $K = p^2/2m$, so

$$K = \frac{(2.16 \text{ MeV})^2}{2(31.97)(931.5 \text{ MeV})} = 78.4\,\text{eV}.$$

E50-39 $N = mN_A/M_r$ will give the number of atoms of ^{198}Au; $R = \lambda N$ will give the activity; $\lambda = \ln 2/t_{1/2}$ will give the decay constant. Combining,

$$m = \frac{NM_r}{N_A} = \frac{Rt_{1/2}M_r}{\ln 2 N_A}.$$

Then for the sample in question

$$m = \frac{(250)(3.7\times10^{10}/\text{s})(2.693)(86400\,\text{s})(198 \text{ g/mol})}{\ln 2(6.02\times10^{23}/\text{mol})} = 1.02\times10^{-3}\text{g}.$$

E50-40 $R = (8722/60\,\text{s})/(3.7\times10^{10}/\text{s}) = 3.93\times10^{-9}\text{Ci}.$

$\boxed{\textbf{E50-41}}$ The radiation absorbed dose (rad) is related to the roentgen equivalent man (rem) by the quality factor, so for the chest x-ray

$$\frac{(25 \text{ mrem})}{(0.85)} = 29 \text{ mrad}.$$

This is well beneath the annual exposure average.

Each rad corresponds to the delivery of 10^{-5} J/g, so the energy absorbed by the patient is

$$(0.029)(10^{-5} \text{ J/g})\left(\frac{1}{2}\right)(88\,\text{kg}) = 1.28\times10^{-2} \text{ J}.$$

E50-42 (a) $(75\,kg)(10^{-2}\text{J/kg})(0.024 \text{ rad}) = 1.8\times10^{-2}\text{J}.$

(b) $(0.024 \text{ rad})(12) = 0.29$ rem.

E50-43 $R = R_0(0.5)^{t/t_{1/2}}$, so

$$R_0 = (3.94\,\mu\text{Ci})(2)^{(6.048\text{x}10^5\text{s})/(1.82\text{x}10^5\text{s})} = 39.4\,\mu\text{Ci}.$$

E50-44 (a) $N = mN_A/M_R$, so

$$N = \frac{(2\times10^{-3}\text{g})(6.02\times10^{23}/\text{mol})}{(239 \text{ g/mol})} = 5.08\times10^{18}.$$

(b) $R = \lambda N = \ln 2N/t_{1/2}$, so

$$R = \ln 2(5.08\times10^{18})/(2.411\times10^4\text{y})(3.15\times10^7\text{s/y}) = 4.64\times10^6/\text{s}.$$

(c) $R = (4.64\times10^6/\text{s})/(3.7\times10^{10} \text{ decays/s}\cdot\text{Ci}) = 1.25\times10^{-4}\text{Ci}.$

$\boxed{\textbf{E50-45}}$ The hospital uses a 6000 Ci source, and that is all the information we need to find the number of disintegrations per second:

$$(6000 \text{ Ci})(3.7\times10^{10} \text{ decays/s}\cdot\text{Ci}) = 2.22\times10^{14} \text{ decays/s}.$$

We are told the half life, but to find the number of radioactive nuclei present we want to know the decay constant. Then

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(1.66\times10^{8}\,\text{s})} = 4.17\times10^{-9}\,\text{s}^{-1}.$$

The number of ^{60}Co nuclei is then

$$N = \frac{R}{\lambda} = \frac{(2.22\times10^{14} \text{ decays/s})}{(4.17\times10^{-9}\,\text{s}^{-1})} = 5.32\times10^{22}.$$

E50-46 The annual equivalent does is

$$(12\times10^{-4}\text{rem/h})(20 \text{ h/week})(52 \text{ week/y}) = 1.25 \text{ rem}.$$

E50-47 (a) $N = mN_A/M_R$ and $M_R = (226) + 2(35) = 296$, so

$$N = \frac{(1\times10^{-1}\text{g})(6.02\times10^{23}/\text{mol})}{(296 \text{ g/mol})} = 2.03\times10^{20}.$$

(b) $R = \lambda N = \ln 2N/t_{1/2}$, so

$$R = \ln 2(2.03\times10^{20})/(1600 \text{ y})(3.15\times10^{7}\text{s/y}) = 2.8\times10^{9}\text{Bq}.$$

(c) $(2.8\times10^{9})/(3.7\times10^{10}) = 76$ mCi.

E50-48 $R = \lambda N = \ln 2N/t_{1/2}$, so

$$N = \frac{(4.6\times10^{-6})(3.7\times10^{10}/\text{s})(1.28\times10^{9}\text{y})(3.15\times10^{7}\text{s/y})}{\ln 2} = 9.9\times10^{21},$$

$N = mN_A/M_R$, so

$$m = \frac{(40 \text{ g/mol})(9.9\times10^{21})}{(6.02\times10^{23}/\text{mol})} = 0.658 \text{ g}.$$

$\boxed{\textbf{E50-49}}$ We can apply Eq. 50-18 to find the age of the rock,

$$\begin{aligned} t &= \frac{t_{1/2}}{\ln 2}\ln\left(1+\frac{N_F}{N_I}\right), \\ &= \frac{(4.47\times10^{9}\text{y})}{\ln 2}\ln\left(1+\frac{(2.00\times10^{-3}\text{g})/(206 \text{ g/mol})}{(4.20\times10^{-3}\text{g})/(238 \text{ g/mol})}\right), \\ &= 2.83\times10^{9}\text{y}. \end{aligned}$$

E50-50 The number of atoms of ^{238}U originally present is

$$N = \frac{(3.71\times10^{-3}\text{g})(6.02\times10^{23}/\text{mol})}{(238\text{ g/mol})} = 9.38\times10^{18}.$$

The number remaining after 260 million years is

$$N = (9.38\times10^{18})(0.5)^{(260\text{ My})/(4470\text{ My})} = 9.01\times10^{18}.$$

The difference decays into lead (eventually), so the mass of lead present should be

$$m = \frac{(206\text{ g/mol})(0.37\times10^{18})}{(6.02\times10^{23}/\text{mol})} = 1.27\times10^{-4}\text{ g}.$$

E50-51 We can apply Eq. 50-18 to find the age of the rock,

$$\begin{aligned} t &= \frac{t_{1/2}}{\ln 2}\ln\left(1+\frac{N_F}{N_I}\right), \\ &= \frac{(4.47\times10^{9}\text{y})}{\ln 2}\ln\left(1+\frac{(150\times10^{-6}\text{g})/(206\text{ g/mol})}{(860\times10^{-6}\text{g})/(238\text{ g/mol})}\right), \\ &= 1.18\times10^{9}\text{y}. \end{aligned}$$

Inverting Eq. 50-18 to find the mass of ^{40}K originally present,

$$\frac{N_\text{F}}{N_\text{I}} = 2^{t/t_{1/2}} - 1,$$

so (since they have the same atomic mass) the mass of ^{40}K is

$$m = \frac{(1.6\times10^{-3}\text{ g})}{2^{(1.18)/(1.28)}-1} = 1.78\times10^{-3}\text{g}.$$

E50-52 (a) There is an excess proton on the left and an excess neutron, so the unknown must be a deuteron, or d.

(b) We've added two protons but only one (net) neutron, so the element is Ti and the mass number is 43, or ^{43}Ti.

(c) The mass number doesn't change but we swapped one proton for a neutron, so ^{7}Li.

E50-53 Do the math:

$$Q = (58.933200 + 1.007825 - 58.934352 - 1.008665)(931.5\text{ MeV}) = -1.86\text{ MeV}.$$

E50-54 The reactions are

$$\begin{gathered} {}^{201}\text{Hg}(\gamma,\alpha)^{197}\text{Pt}, \\ {}^{197}\text{Au}(\text{n},\text{p})^{197}\text{Pt}, \\ {}^{196}\text{Pt}(\text{n},\gamma)^{197}\text{Pt}, \\ {}^{198}\text{Pt}(\gamma,\text{n})^{197}\text{Pt}, \\ {}^{196}\text{Pt}(\text{d},\text{p})^{197}\text{Pt}, \\ {}^{198}\text{Pt}(\text{p},\text{d})^{197}\text{Pt}. \end{gathered}$$

$\boxed{\textbf{E50-55}}$ We will write these reactions in the same way as Eq. 50-20 represents the reaction of Eq. 50-19. It is helpful to work backwards before proceeding by asking the following question: what nuclei will we have if we subtract one of the allowed projectiles?

The goal is ^{60}Co, which has 27 protons and $60 - 27 = 33$ neutrons.

1. Removing a proton will leave 26 protons and 33 neutrons, which is ^{59}Fe; but that nuclide is unstable.
2. Removing a neutron will leave 27 protons and 32 neutrons, which is ^{59}Co; and that nuclide is stable.
3. Removing a deuteron will leave 26 protons and 32 neutrons, which is ^{58}Fe; and that nuclide is stable.

It looks as if only ^{59}Co(n)^{60}Co and ^{58}Fe(d)^{60}Co are possible. If, however, we allow for the possibility of other daughter particles we should also consider some of the following reactions.

1. Swapping a neutron for a proton: ^{60}Ni(n,p)^{60}Co.
2. Using a neutron to knock out a deuteron: ^{61}Ni(n,d)^{60}Co.
3. Using a neutron to knock out an alpha particle: ^{63}Cu(n,α)^{60}Co.
4. Using a deuteron to knock out an alpha particle: ^{62}Ni(d,α)^{60}Co.

E50-56 (a) The possible results are ^{64}Zn, ^{66}Zn, ^{64}Cu, ^{66}Cu, ^{61}Ni, ^{63}Ni, ^{65}Zn, and ^{67}Zn.

(b) The stable results are ^{64}Zn, ^{66}Zn, ^{61}Ni, and ^{67}Zn.

E50-57

E50-58 The resulting reactions are ^{194}Pt(d,α)^{192}Ir, ^{196}Pt(d,α)^{194}Ir, and ^{198}Pt(d,α)^{196}Ir.

E50-59

E50-60 Shells occur at numbers 2, 8, 20, 28, 50, 82. The shells occur separately for protons and neutrons. To answer the question you need to know both Z and $N = A - Z$ of the isotope.

(a) Filled shells are ^{18}O, ^{60}Ni, ^{92}Mo, ^{144}Sm, and ^{207}Pb.

(b) One nucleon outside a shell are ^{40}K, ^{91}Zr, ^{121}Sb, and ^{143}Nd.

(c) One vacancy in a shell are ^{13}C, ^{40}K, ^{49}Ti, ^{205}Tl, and ^{207}Pb.

$\boxed{\textbf{E50-61}}$ (a) The binding energy of this neutron can be found by considering the Q value of the reaction ^{90}Zr(n)^{91}Zr which is

$$(89.904704 + 1.008665 - 90.905645)(931.5\text{ MeV}) = 7.19\text{ MeV}.$$

(b) The binding energy of this neutron can be found by considering the Q value of the reaction ^{89}Zr(n)^{90}Zr which is

$$(88.908889 + 1.008665 - 89.904704)(931.5\text{ MeV}) = 12.0\text{ MeV}.$$

This neutron is bound more tightly that the one in part (a).

(c) The binding energy per nucleon is found by dividing the binding energy by the number of nucleons:

$$\frac{(40\times 1.007825 + 51\times 1.008665 - 90.905645)(931.5\text{ MeV})}{91} = 8.69\text{ MeV}.$$

The neutron in the outside shell of ^{91}Zr is less tightly bound than the average nucleon in ^{91}Zr.

$\boxed{\textbf{P50-1}}$ Before doing anything we need to know whether or not the motion is relativistic. The rest mass energy of an α particle is

$$mc^2 = (4.00)(931.5\text{ MeV}) = 3.73\text{ GeV},$$

and since this is much greater than the kinetic energy we can assume the motion is non-relativistic, and we can apply non-relativistic momentum and energy conservation principles. The initial velocity of the α particle is then

$$v = \sqrt{2K/m} = c\sqrt{2K/mc^2} = c\sqrt{2(5.00\text{ MeV})/(3.73\text{ GeV})} = 5.18\times10^{-2}c.$$

For an elastic collision where the second particle is at originally at rest we have the final velocity of the first particle as

$$v_{1,\mathrm{f}} = v_{1,\mathrm{i}}\frac{m_2 - m_1}{m_2 + m_1} = (5.18\times10^{-2}c)\frac{(4.00\mathrm{u}) - (197\mathrm{u})}{(4.00\mathrm{u}) + (197\mathrm{u})} = -4.97\times10^{-2}c,$$

while the final velocity of the second particle is

$$v_{2,\mathrm{f}} = v_{1,\mathrm{i}}\frac{2m_1}{m_2 + m_1} = (5.18\times10^{-2}c)\frac{2(4.00\mathrm{u})}{(4.00\mathrm{u}) + (197\mathrm{u})} = 2.06\times10^{-3}c.$$

(a) The kinetic energy of the recoiling nucleus is

$$\begin{aligned} K = \frac{1}{2}mv^2 &= \frac{1}{2}m(2.06\times10^{-3}c)^2 = (2.12\times10^{-6})mc^2 \\ &= (2.12\times10^{-6})(197)(931.5\text{ MeV}) = 0.389\text{ MeV}. \end{aligned}$$

(b) Energy conservation is the fastest way to answer this question, since it is an elastic collision. Then

$$(5.00\text{ MeV}) - (0.389\text{ MeV}) = 4.61\text{ MeV}.$$

P50-2 The gamma ray carries away a mass equivalent energy of

$$m_\gamma = (2.2233\text{ MeV})/(931.5\text{ MeV/u}) = 0.002387\text{ u}.$$

The neutron mass would then be

$$m_\mathrm{N} = (2.014102 - 1.007825 + 0.002387)\mathrm{u} = 1.008664\text{ u}.$$

P50-3 (a) There are four substates: m_j can be +3/2, +1/2, -1/2, and -3/2.
(b) $\Delta E = (2/3)(3.26)(3.15\times10^{-8}\mathrm{eV/T})(2.16\,\mathrm{T}) = 1.48\times10^{-7}\mathrm{eV}$.
(c) $\lambda = (1240\,\mathrm{eV\cdot nm})/(1.48\times10^{-7}\mathrm{eV}) = 8.38\,\mathrm{m}$.
(d) This is in the radio region.

P50-4 (a) The charge density is $\rho = 3Q/4\pi R^3$. The charge on the shell of radius r is $dq = 4\pi r^2\rho\, dr$. The potential at the surface of a solid sphere of radius r is

$$V = \frac{q}{4\pi\epsilon_0 r} = \frac{\rho r^2}{3\epsilon_0}.$$

The energy required to add a layer of charge dq is

$$dU = V\,dq = \frac{4\pi\rho^2 r^4}{3\epsilon_0}dr,$$

which can be integrated to yield

$$U = \frac{4\pi\rho^2 R^5}{3\epsilon_0} = \frac{3Q^2}{20\pi\epsilon_0 R}.$$

(b) For ^{239}Pu,

$$U = \frac{3(94)^2(1.6\times10^{-19}\mathrm{C})}{20\pi(8.85\times10^{-12}\mathrm{F/m})(7.45\times10^{-15}\mathrm{m})} = 1024\times10^6\mathrm{eV}.$$

(c) The electrostatic energy is 10.9 MeV per proton.

$\boxed{\textbf{P50-5}}$ The decay rate is given by $R = \lambda N$, where N is the number of radioactive nuclei present. If R exceeds P then nuclei will decay faster than they are produced; but this will cause N to decrease, which means R will decrease until it is equal to P. If R is less than P then nuclei will be produced faster than they are decaying; but this will cause N to increase, which means R will increase until it is equal to P. In either case equilibrium occurs when $R = P$, and it is a stable equilibrium because it is approached no matter which side is larger. Then

$$P = R = \lambda N$$

at equilibrium, so $N = P/\lambda$.

P50-6 (a) $A = \lambda N$; at equilibrium $A = P$, so $P = 8.88\times10^{10}$/s.
(b) $(8.88\times10^{10}/\mathrm{s})(1-e^{-0.269t})$, where t is in hours. The factor 0.269 comes from $\ln(2)/(2.58) = \lambda$.
(c) $N = P/\lambda = (8.88\times10^{10}/\mathrm{s})(3600\ \mathrm{s/h})/(0.269/\mathrm{h}) = 1.19\times10^{15}$.
(d) $m = NM_r/N_A$, or

$$m = \frac{(1.19\times10^{15})(55.94\ \mathrm{g/mol})}{(6.02\times10^{23}/\mathrm{mol})} = 1.10\times10^{-7}\mathrm{g}.$$

P50-7 (a) $A = \lambda N$, so

$$A = \frac{\ln 2 m N_A}{t_{1/2}M_r} = \frac{\ln 2(1\times10^{-3}\mathrm{g})(6.02\times10^{23}/\mathrm{mol})}{(1600)(3.15\times10^7\mathrm{s})(226\ \mathrm{g/mol})} = 3.66\times10^7/\mathrm{s}.$$

(b) The rate *must* be the same if the system is in secular equilibrium.
(c) $N = P/\lambda = t_{1/2}P/\ln 2$, so

$$m = \frac{(3.82)(86400\,\mathrm{s})(3.66\times10^7/\mathrm{s})(222\ \mathrm{g/mol})}{(6.02\times10^{23}/\mathrm{mol})\ln 2} = 6.43\times10^{-9}\mathrm{g}.$$

P50-8 The number of water molecules in the body is

$$N = (6.02\times10^{23}/\mathrm{mol})(70\times10^3\mathrm{g})/(18\ \mathrm{g/mol}) = 2.34\times10^{27}.$$

There are ten protons in each water molecule. The activity is then

$$A = (2.34\times10^{27})\ln 2/(1\times10^{32}\mathrm{y}) = 1.62\times10^{-5}/\mathrm{y}.$$

The time between decays is then

$$1/A = 6200\ \mathrm{y}.$$

$\boxed{\textbf{P50-9}}$ Assuming the ^{238}U nucleus is originally at rest the total initial momentum is zero, which means the magnitudes of the final momenta of the α particle and the ^{234}Th nucleus are equal.

The α particle has a final velocity of

$$v = \sqrt{2K/m} = c\sqrt{2K/mc^2} = c\sqrt{2(4.196 \text{ MeV})/(4.0026\times 931.5 \text{ MeV})} = 4.744\times 10^{-2}c.$$

Since the magnitudes of the final momenta are the same, the ^{234}Th nucleus has a final velocity of

$$(4.744\times 10^{-2}c)\left(\frac{(4.0026 \text{ u})}{(234.04 \text{ u})}\right) = 8.113\times 10^{-4}c.$$

The kinetic energy of the ^{234}Th nucleus is

$$\begin{aligned} K &= \frac{1}{2}mv^2 = \frac{1}{2}m(8.113\times 10^{-4}c)^2 = (3.291\times 10^{-7})mc^2 \\ &= (3.291\times 10^{-7})(234.04)(931.5 \text{ MeV}) = 71.75 \text{ keV}. \end{aligned}$$

The Q value for the reaction is then

$$(4.196 \text{ MeV}) + (71.75 \text{ keV}) = 4.268 \text{ MeV},$$

which agrees well with the Sample Problem.

P50-10 (a) The Q value is

$$Q = (238.050783 - 4.002603 - 234.043596)(931.5 \text{ MeV}) = 4.27 \text{ MeV}.$$

(b) The Q values for each step are

$$Q = (238.050783 - 237.048724 - 1.008665)(931.5 \text{ MeV}) = -6.153 \text{ MeV},$$

$$Q = (237.048724 - 236.048674 - 1.007825)(931.5 \text{ MeV}) = -7.242 \text{ MeV},$$

$$Q = (236.048674 - 235.045432 - 1.008665)(931.5 \text{ MeV}) = -5.052 \text{ MeV},$$

$$Q = (235.045432 - 234.043596 - 1.007825)(931.5 \text{ MeV}) = -5.579 \text{ MeV}.$$

(c) The total Q for part (b) is -24.026 MeV. The difference between (a) and (b) is 28.296 MeV. The binding energy for the alpha particle is

$$E = [2(1.007825) + 2(1.008665) - 4.002603](931.5 \text{ MeV}) = 28.296 \text{ MeV}.$$

P50-11 (a) The emitted positron leaves the atom, so the mass must be subtracted. But the daughter particle now has an extra electron, so that must also be subtracted. Hence the factor $-2m_{\text{e}}$.

(b) The Q value is

$$Q = [11.011434 - 11.009305 - 2(0.0005486)](931.5 \text{ MeV}) = 0.961 \text{ MeV}.$$

P50-12 (a) Capturing an electron is equivalent to negative beta decay in that the total number of electrons is accounted for on both the left and right sides of the equation. The loss of the K shell electron, however, must be taken into account as this energy may be significant.

(b) The Q value is

$$Q = (48.948517 - 48.947871)(931.5 \text{ MeV}) - (0.00547 \text{ MeV}) = 0.596 \text{ MeV}.$$

P50-13 The decay constant for ^{90}Sr is

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(9.15\times 10^8\,\mathrm{s})} = 7.58\times 10^{-10}\mathrm{s}^{-1}.$$

The number of nuclei present in 400 g of ^{90}Sr is

$$N = (400\ \mathrm{g})\frac{(6.02\times 10^{23}/\mathrm{mol})}{(89.9\ \mathrm{g/mol})} = 2.68\times 10^{24},$$

so the overall activity of the 400 g of ^{90}Sr is

$$R = \lambda N = (7.58\times 10^{-10}\mathrm{s}^{-1})(2.68\times 10^{24})/(3.7\times 10^{10}/\mathrm{Ci}\cdot\mathrm{s}) = 5.49\times 10^4\ \mathrm{Ci}.$$

This is spread out over a 2000 km^2 area, so the "activity surface density" is

$$\frac{(5.49\times 10^4\ \mathrm{Ci})}{(20006\,\mathrm{m}^2)} = 2.74\times 10^{-5}\ \mathrm{Ci/m}^2.$$

If the allowable limit is 0.002 mCi, then the area of land that would contain this activity is

$$\frac{(0.002\times 10^{-3}\ \mathrm{Ci})}{(2.74\times 10^{-5}\ \mathrm{Ci/m}^2)} = 7.30\times 10^{-2}\mathrm{m}^2.$$

P50-14 (a) $N = mN_A/M_r$, so

$$N = (2.5\times 10^{-3}\mathrm{g})(6.02\times 10^{23}/\mathrm{mol})/(239\ \mathrm{g/mol}) = 6.3\times 10^{18}.$$

(b) $A = \ln 2N/t_{1/2}$, so the number that decay in 12 hours is

$$\frac{\ln 2(6.3\times 10^{18})(12)(3600\,\mathrm{s})}{(24100)(3.15\times 10^7\mathrm{s})} = 2.5\times 10^{11}.$$

(c) The energy absorbed by the body is

$$E = (2.5\times 10^{11})(5.2\ \mathrm{MeV})(1.6\times 10^{-19}\mathrm{J/eV}) = 0.20\,\mathrm{J}.$$

(d) The dose in rad is $(0.20\,\mathrm{J})/(87\,\mathrm{kg}) = 0.23$ rad.
(e) The biological dose in rem is $(0.23)(13) = 3$ rem.

P50-15 (a) The amount of ^{238}U per kilogram of granite is

$$N = \frac{(4\times 10^{-6}\mathrm{kg})(6.02\times 10^{23}/\mathrm{mol})}{(0.238\ \mathrm{kg/mol})} = 1.01\times 10^{19}.$$

The activity is then

$$A = \frac{\ln 2(1.01\times 10^{19})}{(4.47\times 10^9\mathrm{y})(3.15\times 10^7\mathrm{s/y})} = 49.7/\mathrm{s}.$$

The energy released in one second is

$$E = (49.7/\mathrm{s})(51.7\ \mathrm{MeV}) = 4.1\times 10^{-10}\mathrm{J}.$$

The amount of ^{232}Th per kilogram of granite is

$$N = \frac{(13\times 10^{-6}\mathrm{kg})(6.02\times 10^{23}/\mathrm{mol})}{(0.232\ \mathrm{kg/mol})} = 3.37\times 10^{19}.$$

The activity is then

$$A = \frac{\ln 2(3.37\times 10^{19})}{(1.41\times 10^{10}\text{y})(3.15\times 10^{7}\text{s/y})} = 52.6/\text{s}.$$

The energy released in one second is

$$E = (52.6/\text{s})(42.7 \text{ MeV}) = 3.6\times 10^{-10}\text{J}.$$

The amount of ^{40}K per kilogram of granite is

$$N = \frac{(4\times 10^{-6}\text{kg})(6.02\times 10^{23}/\text{mol})}{(0.040 \text{ kg/mol})} = 6.02\times 10^{19}.$$

The activity is then

$$A = \frac{\ln 2(6.02\times 10^{19})}{(1.28\times 10^{9}\text{y})(3.15\times 10^{7}\text{s/y})} = 1030/\text{s}.$$

The energy released in one second is

$$E = (1030/\text{s})(1.32 \text{ MeV}) = 2.2\times 10^{-10}\text{J}.$$

The total of the three is 9.9×10^{-10}W per kilogram of granite.

(b) The total for the Earth is 2.7×10^{13} W.

P50-16 (a) Since only a is moving originally then the velocity of the center of mass is

$$V = \frac{m_a v_a + m_X(0)}{m_X + m_a} = v_a \frac{m_a}{m_a + m_X}.$$

No, since momentum is conserved.

(b) Moving to the center of mass frame gives the velocity of X as V, and the velocity of a as $v_a - V$. The kinetic energy is now

$$\begin{aligned}
K_{\text{cm}} &= \frac{1}{2}\left(m_X V^2 + m_a(v_a - V)^2\right), \\
&= \frac{v_a^2}{2}\left(m_X \frac{m_a^2}{(m_a + m_X)^2} + m_a \frac{m_X^2}{(m_a + m_X)^2}\right), \\
&= \frac{m_a v_a^2}{2} \frac{m_a m_X + m_X^2}{(m_a + m_X)^2}, \\
&= K_{\text{lab}} \frac{m_X}{m_a + m_X}.
\end{aligned}$$

Yes; kinetic energy is not conserved.

(c) $v_a = \sqrt{2K/m}$, so

$$v_a = \sqrt{2(15.9 \text{ MeV})/(1876 \text{ MeV})}c = 0.130c.$$

The center of mass velocity is

$$V = (0.130c)\frac{(2)}{(2) + (90)} = 2.83\times 10^{-3}c.$$

Finally,

$$K_{\text{cm}} = (15.9 \text{ MeV})\frac{(90)}{(2) + (90)} = 15.6 \text{ MeV}.$$

P50-17 Let $Q = K_{\text{cm}}$ in the result of Problem 50-16, and invert, solving for K_{lab}.

P50-18 (a) Removing a proton from ^{209}Bi:

$$E = (207.976636 + 1.007825 - 208.980383)(931.5 \text{ MeV}) = 3.80 \text{ MeV}.$$

Removing a proton from ^{208}Pb:

$$E = (206.977408 + 1.007825 - 207.976636)(931.5 \text{ MeV}) = 8.01 \text{ MeV}.$$

(b) Removing a neutron from ^{209}Pb:

$$E = (207.976636 + 1.008665 - 208.981075)(931.5 \text{ MeV}) = 3.94 \text{ MeV}.$$

Removing a neutron from ^{208}Pb:

$$E = (206.975881 + 1.008665 - 207.976636)(931.5 \text{ MeV}) = 7.37 \text{ MeV}.$$

E51-1 (a) For the coal,

$$m = (1\times10^9\text{J})/(2.9\times10^7\text{J/kg}) = 34\,\text{kg}.$$

(b) For the uranium,

$$m = (1\times10^9\text{J})/(8.2\times10^{13}\text{J/kg}) = 1.2\times10^{-5}\text{kg}.$$

E51-2 (a) The energy from the coal is

$$E = (100\,\text{kg})(2.9\times10^7\text{J/kg}) = 2.9\times10^9\text{J}.$$

(b) The energy from the uranium in the ash is

$$E = (3\times10^{-6})(100\,\text{kg})(8.2\times10^{13}\text{J}) = 2.5\times10^{10}\text{J}.$$

$\boxed{\textbf{E51-3}}$ (a) There are

$$\frac{(1.00\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(235\text{g/mol})} = 2.56\times10^{24}$$

atoms in 1.00 kg of ^{235}U.

(b) If each atom releases 200 MeV, then

$$(200\times10^6\text{ eV})(1.6\times10^{-19}\text{J/ eV})(2.56\times10^{24}) = 8.19\times10^{13}\,\text{J}$$

of energy could be released from 1.00 kg of ^{235}U.

(c) This amount of energy would keep a 100-W lamp lit for

$$t = \frac{(8.19\times10^{13}\text{ J})}{(100\,\text{W})} = 8.19\times10^{11}\text{s} \approx 26,000\text{ y!}$$

E51-4 $2\,\text{W} = 1.25\times10^{19}\text{eV/s}$. This requires

$$(1.25\times10^{19}\text{eV/s})/(200\times10^6\text{eV}) = 6.25\times10^{10}/\text{s}$$

as the fission rate.

E51-5 There are

$$\frac{(1.00\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(235\text{g/mol})} = 2.56\times10^{24}$$

atoms in 1.00 kg of ^{235}U. If each atom releases 200 MeV, then

$$(200\times10^6\text{ eV})(1.6\times10^{-19}\text{J/ eV})(2.56\times10^{24}) = 8.19\times10^{13}\,\text{J}$$

of energy could be released from 1.00 kg of ^{235}U. This amount of energy would keep a 100-W lamp lit for

$$t = \frac{(8.19\times10^{13}\text{ J})}{(100\,\text{W})} = 8.19\times10^{11}\text{s} \approx 30,000\text{ y!}$$

E51-6 There are

$$\frac{(1.00\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(239\text{g/mol})} = 2.52\times10^{24}$$

atoms in 1.00 kg of ^{239}Pu. If each atom releases 180 MeV, then

$$(180\times10^6\text{ eV})(1.6\times10^{-19}\text{J/ eV})(2.52\times10^{24}) = 7.25\times10^{13}\,\text{J}$$

of energy could be released from 1.00 kg of ^{239}Pu.

$\boxed{\textbf{E51-7}}$ When the ^{233}U nucleus absorbs a neutron we are given a total of 92 protons and 142 neutrons. Gallium has 31 protons and around 39 neutrons; chromium has 24 protons and around 28 neutrons. There are then 37 protons and around 75 neutrons left over. This would be rubidium, but the number of neutrons is *very* wrong. Although the elemental identification is correct, because we must conserve proton number, the isotopes are *wrong* in our above choices for neutron numbers.

E51-8 Beta decay is the emission of an electron from the nucleus; one of the neutrons changes into a proton. The atom now needs one more electron in the electron shells; by using atomic masses (as opposed to nuclear masses) then the beta electron is accounted for. This is *only* true for negative beta decay, not for positive beta decay.

E51-9 (a) There are

$$\frac{(1.0\text{ g})(6.02\times10^{23}\text{mol}^{-1})}{(235\text{g/mol})} = 2.56\times10^{21}$$

atoms in 1.00 g of ^{235}U. The fission rate is

$$A = \ln 2N/t_{1/2} = \ln 2(2.56\times10^{21})/(3.5\times10^{17}\text{y})(365\text{d/y}) = 13.9/\text{d}.$$

(b) The ratio is the inverse ratio of the half-lives:

$$(3.5\times10^{17}\text{y})/(7.04\times10^{8}\text{y}) = 4.97\times10^{8}.$$

E51-10 (a) The atomic number of Y must be $92-54=38$, so the element is Sr. The mass number is $235+1-140-1=95$, so Y is ^{95}Sr.

(b) The atomic number of Y must be $92-53=39$, so the element is Y. The mass number is $235+1-139-2=95$, so Y is ^{95}Y.

(c) The atomic number of X must be $92-40=52$, so the element is Te. The mass number is $235+1-100-2=134$, so X is ^{134}Te.

(d) The mass number difference is $235+1-141-92=3$, so $b=3$.

E51-11 The Q value is

$$Q = [51.94012 - 2(25.982593)](931.5\text{ MeV}) = -23\text{ MeV}.$$

The negative value implies that this fission reaction is not possible.

E51-12 The Q value is

$$Q = [97.905408 - 2(48.950024)](931.5\text{ MeV}) = 4.99\text{ MeV}.$$

The two fragments would have a very large Coulomb barrier to overcome.

$\boxed{\textbf{E51-13}}$ The energy released is

$$(235.043923 - 140.920044 - 91.919726 - 2\times1.008665)(931.5\text{ MeV}) = 174\text{ MeV}.$$

E51-14 Since $E_n > E_b$ fission is possible by thermal neutrons.

E51-15 (a) The uranium starts with 92 protons. The two end products have a total of $58+44=102$. This means that there must have been ten beta decays.

(b) The Q value for this process is

$$Q = (238.050783 + 1.008665 - 139.905434 - 98.905939)(931.5\text{ MeV}) = 231\text{ MeV}.$$

E51-16 (a) The other fragment has $92 - 32 = 60$ protons and $235 + 1 - 83 = 153$ neutrons. That element is ^{153}Nd.

(b) Since $K = p^2/2m$ and momentum is conserved, then $2m_1K_1 = 2m_2K_2$. This means that $K_2 = (m_1/m_2)K_1$. But $K_1 + K_2 = Q$, so

$$K_1\frac{m_2 + m_1}{m_2} = Q,$$

or

$$K_1 = \frac{m_2}{m_1 + m_2}Q,$$

with a similar expression for K_2. Then for ^{83}Ge

$$K = \frac{(153)}{(83 + 153)}(170\text{ MeV}) = 110\text{ MeV},$$

while for ^{153}Nd

$$K = \frac{(83)}{(83 + 153)}(170\text{ MeV}) = 60\text{ MeV},$$

(c) For ^{83}Ge,

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(110\text{ MeV})}{(83)(931\text{ MeV})}}c = 0.053c,$$

while for ^{153}Nd

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(60\text{ MeV})}{(153)(931\text{ MeV})}}c = 0.029c.$$

$\boxed{\textbf{E51-17}}$ Since ^{239}Pu is one nucleon heavier than ^{238}U only one neutron capture is required. The atomic number of Pu is two *more* than U, so two beta decays will be required. The reaction series is then

$$\begin{aligned}
{}^{238}\mathrm{U} + \mathrm{n} &\rightarrow {}^{239}\mathrm{U}, \\
{}^{239}\mathrm{U} &\rightarrow {}^{239}\mathrm{Np} + \beta^- + \bar{\nu}, \\
{}^{239}\mathrm{Np} &\rightarrow {}^{239}\mathrm{Pu} + \beta^- + \bar{\nu}.
\end{aligned}$$

E51-18 Each fission releases 200 MeV. The total energy released over the three years is

$$(190\times10^6\mathrm{W})(3)(3.15\times10^7\mathrm{s}) = 1.8\times10^{16}\mathrm{J}.$$

That's

$$(1.8\times10^{16}\mathrm{J})/(1.6\times10^{-19}\mathrm{J/eV})(200\times10^6\mathrm{eV}) = 5.6\times10^{26}$$

fission events. That requires

$$m = (5.6\times10^{26})(0.235\text{ kg/mol})/(6.02\times10^{23}/\mathrm{mol}) = 218\,\mathrm{kg}.$$

But this is only half the original amount, or 437 kg.

E51-19 According to Sample Problem 51-3 the rate at which non-fission thermal neutron capture occurs is one quarter that of fission. Hence the mass which undergoes non-fission thermal neutron capture is one quarter the answer of Ex. 51-18. The total is then

$$(437\,\mathrm{kg})(1 + 0.25) = 546\,\mathrm{kg}.$$

E51-20 (a) $Q_{\text{eff}} = E/\Delta N$, where E is the total energy released and ΔN is the number of decays. This can also be written as

$$Q_{\text{eff}} = \frac{P}{A} = \frac{Pt_{1/2}}{\ln 2N} = \frac{Pt_{1/2}M_r}{\ln 2N_A m},$$

where A is the activity and P the power output from the sample. Solving,

$$Q_{\text{eff}} = \frac{(2.3\,\text{W})(29\text{ y})(3.15\times10^7\text{s})(90\text{ g/mol})}{\ln 2(6.02\times10^{23}/\text{mol})(1\text{ g})} = 4.53\times10^{-13}\text{J} = 2.8\text{ MeV}.$$

(b) $P = (0.05)m(2300\,\text{W/kg})$, so

$$m = \frac{(150\,\text{W})}{(0.05)(2300\,\text{W/kg})} = 1.3\,\text{kg}.$$

E51-21 Let the energy released by one fission be E_1. If the average time to the next fission event is t_{gen}, then the "average" power output from the one fission is $P_1 = E_1/t_{\text{gen}}$. If every fission event results in the release of k neutrons, each of which cause a later fission event, then after every time period t_{gen} the number of fission events, and hence the average power output from *all* of the fission events, will increase by a factor of k.

For long enough times we can write

$$P(t) = P_0 k^{t/t_{\text{gen}}}.$$

E51-22 Invert the expression derived in Exercise 51-21:

$$k = \left(\frac{P}{P_0}\right)^{t_{\text{gen}}/t} = \left(\frac{(350)}{(1200)}\right)^{(1.3\times10^{-3}\text{s})/(2.6\,\text{s})} = 0.99938.$$

E51-23 Each fission releases 200 MeV. Then the fission rate is

$$(500\times10^6\text{W})/(200\times10^6\text{eV})(1.6\times10^{-19}\text{J/eV}) = 1.6\times10^{19}/s$$

The number of neutrons in "transit" is then

$$(1.6\times10^{19}/\text{s})(1.0\times10^{-3}\text{s}) = 1.6\times10^{16}.$$

E51-24 Using the results of Exercise 51-21:

$$P = (400\text{ MW})(1.0003)^{(300\,\text{s})/(0.03\,\text{s})} = 8030\text{ MW}.$$

E51-25 The time constant for this decay is

$$\lambda = \frac{\ln 2}{(2.77\times10^9\,\text{s})} = 2.50\times10^{-10}\text{s}^{-1}.$$

The number of nuclei present in 1.00 kg is

$$N = \frac{(1.00\,\text{kg})(6.02\times10^{23}\text{ mol}^{-1})}{(238\text{ g/mol})} = 2.53\times10^{24}.$$

The decay rate is then

$$R = \lambda N = (2.50\times10^{-10}\text{s}^{-1})(2.53\times10^{24}) = 6.33\times10^{14}\text{s}^{-1}.$$

The power generated is the decay rate times the energy released per decay,

$$P = (6.33\times10^{14}\text{s}^{-1})(5.59\times10^6\text{ eV})(1.6\times10^{-19}\text{ J/eV}) = 566\,\text{W}.$$

E51-26 The detector detects only a fraction of the emitted neutrons. This fraction is

$$\frac{A}{4\pi R^2} = \frac{(2.5\,\mathrm{m}^2)}{4\pi(35\,\mathrm{m})^2} = 1.62\times10^{-4}.$$

The total flux out of the warhead is then

$$(4.0/\mathrm{s})/(1.62\times10^{-4}) = 2.47\times10^4/\mathrm{s}.$$

The number of ^{239}Pu atoms is

$$N = \frac{A}{\lambda} = \frac{(2.47\times10^4/\mathrm{s})(1.34\times10^{11}\mathrm{y})(3.15\times10^7\mathrm{s/y})}{\ln 2(2.5)} = 6.02\times10^{22}.$$

That's one tenth of a mole, so the mass is $(239)/10 = 24$ g.

E51-27 Using the results of Sample Problem 51-4,

$$t = \frac{\ln[R(0)/R(t)]}{\lambda_5 - \lambda_8},$$

so

$$t = \frac{\ln[(0.03)/(0.0072)]}{(0.984 - 0.155)(1\times10^{-9}/\mathrm{y})} = 1.72\times10^9\mathrm{y}.$$

E51-28 (a) $(15\times10^9\mathrm{W}\cdot\mathrm{y})(2\times10^5\mathrm{y}) = 7.5\times10^4\mathrm{W}$.
(b) The number of fissions required is

$$N = \frac{(15\times10^9\mathrm{W}\cdot\mathrm{y})(3.15\times10^7\mathrm{s/y})}{(200\ \mathrm{MeV})(1.6\times10^{-19}\mathrm{J/eV})} = 1.5\times10^{28}.$$

The mass of ^{235}U consumed is

$$m = (1.5\times10^{28})(0.235\mathrm{kg/mol})/(6.02\times10^{-23}/\mathrm{mol}) = 5.8\times10^3\mathrm{kg}.$$

$\boxed{\textbf{E51-29}}$ If ^{238}U absorbs a neutron it becomes ^{239}U, which will decay by beta decay to first ^{239}Np and then ^{239}Pu; we looked at this in Exercise 51-17. This can decay by alpha emission according to

$$^{239}\mathrm{Pu} \rightarrow ^{235}\mathrm{U} + \alpha.$$

E51-30 The number of atoms present in the sample is

$$N = (6.02\times10^{23}/\mathrm{mol})(1000\,\mathrm{kg})/(2.014\mathrm{g/mol}) = 2.99\times10^{26}.$$

It takes two to make a fusion, so the energy released is

$$(3.27\ \mathrm{MeV})(2.99\times10^{26})/2 = 4.89\times10^{26}\mathrm{MeV}.$$

That's 7.8×10^{13}J, which is enough to burn the lamp for

$$t = (7.8\times10^{13}\mathrm{J})/(100\,\mathrm{W}) = 7.8\times10^{11}\mathrm{s} = 24800\ \mathrm{y}.$$

E51-31 The potential energy at closest approach is

$$U = \frac{(1.6\times10^{-19}\mathrm{C})^2}{4\pi(8.85\times10^{-12}\mathrm{F/m})(1.6\times10^{-15}\mathrm{m})} = 9\times10^5\mathrm{eV}.$$

E51-32 The ratio can be written as

$$\frac{n(K_1)}{n(K_2)} = \sqrt{\frac{K_1}{K_2}} e^{(K_2-K_1)/kT},$$

so the ratio is

$$\sqrt{\frac{(5000\,\text{eV})}{(1900\,\text{eV})}} e^{(-3100\,\text{eV})/(8.62\times10^{-5}\text{eV/K})(1.5\times10^{7}\text{K})} = 0.15.$$

E51-33 (a) See Sample Problem 51-5.

E51-34 Add up *all* of the Q values in the cycle of Fig. 51-10.

$\boxed{\textbf{E51-35}}$ The energy released is

$$(3\times4.002603 - 12.0000000)(931.5\text{ MeV}) = 7.27\text{ MeV}.$$

E51-36 (a) The number of particle of hydrogen in $1\,\text{m}^3$ is

$$N = (0.35)(1.5\times10^5\text{kg})(6.02\times10^{23}/\text{mol})/(0.001\text{ kg/mol}) = 3.16\times10^{31}$$

(b) The density of particles is $N/V = p/kT$; the ratio is

$$\frac{(3.16\times10^{31})(1.38\times10^{-23}\text{J/K})(298\,\text{K})}{(1.01\times10^5\text{Pa})} = 1.2\times10^6.$$

E51-37 (a) There are

$$\frac{(1.00\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(1\text{g/mol})} = 6.02\times10^{26}$$

atoms in 1.00 kg of ^{1}H. If four atoms fuse to releases 26.7 MeV, then

$$(26.7\text{ MeV})(6.02\times10^{26})/4 = 4.0\times10^{27}\text{MeV}$$

of energy could be released from 1.00 kg of ^{1}H.

(b) There are

$$\frac{(1.00\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(235\text{g/mol})} = 2.56\times10^{24}$$

atoms in 1.00 kg of ^{235}U. If each atom releases 200 MeV, then

$$(200\text{ MeV})(2.56\times10^{24}) = 5.1\times10^{26}\text{MeV}$$

of energy could be released from 1.00 kg of ^{235}U.

E51-38 (a) $E = \Delta mc^2$, so

$$\Delta m = \frac{(3.9\times10^{26}\text{J/s})}{(3.0\times10^8\text{m/s})^2} = 4.3\times10^9\text{kg/s}.$$

(b) The fraction of the Sun's mass "lost" is

$$\frac{(4.3\times10^9\text{kg/s})(3.15\times10^7\text{s/y})(4.5\times10^9\text{y})}{(2.0\times10^{30}\text{kg})} = 0.03\,\%.$$

$\boxed{\textbf{E51-39}}$ The rate of consumption is 6.2×10^{11}kg/s, the core has 1/8 the mass but only 35% is hydrogen, so the time remaining is

$$t = (0.35)(1/8)(2.0\times10^{30}\text{kg})/(6.2\times10^{11}\text{kg/s}) = 1.4\times10^{17}\text{s},$$

or about 4.5×10^{9} years.

E51-40 For the first two reactions into one:

$$Q = [2(1.007825) - (2.014102)](931.5\text{ MeV}) = 1.44\text{ MeV}.$$

For the second,

$$Q = [(1.007825) + (2.014102) - (3.016029)](931.5\text{ MeV}) = 5.49\text{ MeV}.$$

For the last,

$$Q = [2(3.016029) - (4.002603) - 2(1.007825)](931.5\text{ MeV}) = 12.86\text{ MeV}.$$

E51-41 (a) Use $mN_A/M_r = N$, so

$$(3.3\times10^{7}\text{J/kg})\frac{(0.012\text{ kg/mol})}{(6.02\times10^{23}/\text{mol})}\frac{1}{(1.6\times10^{-19}\text{J/eV})} = 4.1\text{ eV}.$$

(b) For every 12 grams of carbon we require 32 grams of oxygen, the total is 44 grams. The total mass required is then 40/12 that of carbon alone. The energy production is then

$$(3.3\times10^{7}\text{J/kg})(12/44) = 9\times10^{6}\text{J/kg}.$$

(c) The sun would burn for

$$\frac{(2\times10^{30}\text{kg})(9\times10^{6}\text{J/kg})}{(3.9\times10^{26}\text{W})} = 4.6\times10^{10}\text{s}.$$

That's only 1500 years!

E51-42 The rate of fusion events is

$$\frac{(5.3\times10^{30}\text{W})}{(7.27\times10^{6}\text{eV})(1.6\times10^{-19}\text{J/eV})} = 4.56\times10^{42}/\text{s}.$$

The carbon is then produced at a rate

$$(4.56\times10^{42}/\text{s})(0.012\text{ kg/mol})/(6.02\times10^{23}/\text{mol}) = 9.08\times10^{16}\text{kg/s}.$$

The process will be complete in

$$\frac{(4.6\times10^{32}\text{kg})}{(9.08\times10^{16}\text{kg/s})(3.15\times10^{7}\text{s/y})} = 1.6\times10^{8}\text{y}.$$

E51-43 (a) For the reaction d-d,

$$Q = [2(2.014102) - (3.016029) - (1.008665)](931.5\text{ MeV}) = 3.27\text{ MeV}.$$

(b) For the reaction d-d,

$$Q = [2(2.014102) - (3.016029) - (1.007825)](931.5\text{ MeV}) = 4.03\text{ MeV}.$$

(c) For the reaction d-t,

$$Q = [(2.014102) + (3.016049) - (4.002603) - (1.008665)](931.5\text{ MeV}) = 17.59\text{ MeV}.$$

E51-44 One liter of water has a mass of one kilogram. The number of atoms of ^{2}H is

$$(0.00015\,\text{kg})\frac{(6.02\times10^{23}/\text{mol})}{(0.002\ \text{kg/mol})} = 4.5\times10^{22}.$$

The energy available is

$$(3.27\times10^6\text{eV})(1.6\times10^{-19}\text{J/eV})(4.5\times10^{22})/2 = 1.18\times10^{10}\text{J}.$$

The power output is then

$$\frac{(1.18\times10^{10}\text{J})}{(86400\,\text{s})} = 1.4\times10^5\text{W}$$

E51-45 Assume momentum conservation, then

$$p_\alpha = p_\text{n} \text{ or } v_\text{n}/v_\alpha = m_\alpha/m_\text{n}.$$

The ratio of the kinetic energies is then

$$\frac{K_\text{n}}{K_\alpha} = \frac{m_\text{n}v_\text{n}^2}{m_\alpha v_\alpha^2} = \frac{m_\alpha}{m_\text{n}} \approx 4.$$

Then $K_\text{n} = 4Q/5 = 14.07$ MeV while $K_\alpha = Q/5 = 3.52$ MeV.

E51-46 The Q value is

$$Q = (6.015122 + 1.008665 - 3.016049 - 4.002603)(931.5\ \text{MeV}) = 4.78\ \text{MeV}.$$

Combine the two reactions to get a net $Q = 22.37$ MeV. The amount of ^{6}Li required is

$$N = (2.6\times10^{28}\text{MeV})/(22.37\ \text{MeV}) = 1.16\times10^{27}.$$

The mass of LiD required is

$$m = \frac{(1.16\times10^{27})(0.008\ \text{kg/mol})}{(6.02\times10^{23}/\text{mol})} = 15.4\,\text{kg}.$$

P51-1 (a) Equation 50-1 is

$$R = R_0A^{1/3},$$

where $R_0 = 1.2$ fm. The distance between the two nuclei will be the sum of the radii, or

$$R_0\left((140)^{1/3} + (94)^{1/3}\right).$$

The potential energy will be

$$\begin{aligned}
U &= \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r},\\
&= \frac{e^2}{4\pi\epsilon_0R_0}\frac{(54)(38)}{\left((140)^{1/3} + (94)^{1/3}\right)},\\
&= \frac{(1.60\times10^{-19}\text{C})^2}{4\pi(8.85\times10^{-12}\text{C}^2/\text{Nm}^2)(1.2\ \text{fm})}211,\\
&= 253\ \text{MeV}.
\end{aligned}$$

(b) The energy will eventually appear as thermal energy.

P51-2 (a) Since $R = R_0\sqrt[3]{A}$, the surface area a is proportional to $A^{2/3}$. The fractional change in surface area is

$$\frac{(a_1 + a_2) - a_0}{a_0} = \frac{(140)^{2/3} + (96)^{2/3} - (236)^{2/3}}{(236)^{2/3}} = 25\,\%.$$

(b) Nuclei have a constant density, so there is no change in volume.

(c) Since $U \propto Q^2/R$, $U \propto Q^2/\sqrt[3]{A}$. The fractional change in the electrostatic potential energy is

$$\frac{U_1 + U_2 - U_0}{U_0} = \frac{(54)^2(140)^{-1/3} + (38)^2(96)^{-1/3} - (92)^2(236)^{-1/3}}{(92)^2(236)^{-1/3}} = -36\,\%.$$

P51-3 (a) There are

$$\frac{(2.5\,\text{kg})(6.02\times10^{23}\text{mol}^{-1})}{(239\text{g/mol})} = 6.29\times10^{24}$$

atoms in 2.5 kg of ^{239}Pu. If each atom releases 180 MeV, then

$$(180\text{ MeV})(6.29\times10^{24})/(2.6\times10^{28}\text{MeV/megaton}) = 44\text{ kiloton}$$

is the bomb yield.

P51-4 (a) In an elastic collision the nucleus moves forward with a speed of

$$v = v_0\frac{2m_n}{m_n + m},$$

so the kinetic energy when it moves forward is

$$\Delta K = \frac{m}{2}v_0^2\frac{4m_n^2}{(m + m_n)^2} = K\frac{m_n m}{(m_n + m)^2},$$

where we can write ΔK because in an elastic collision whatever energy kinetic energy the nucleus carries off had to come from the neutron.

(b) For hydrogen,

$$\frac{\Delta K}{K} = \frac{4(1)(1)}{(1+1)^2} = 1.00.$$

For deuterium,

$$\frac{\Delta K}{K} = \frac{4(1)(2)}{(1+2)^2} = 0.89.$$

For carbon,

$$\frac{\Delta K}{K} = \frac{4(1)(12)}{(1+12)^2} = 0.28.$$

For lead,

$$\frac{\Delta K}{K} = \frac{4(1)(206)}{(1+206)^2} = 0.019.$$

(c) If each collision reduces the energy by a factor of $1 - 0.89 = 0.11$, then the number of collisions required is the solution to

$$(0.025\,\text{eV}) = (1\times10^6\text{eV})(0.11)^N,$$

which is $N = 8$.

P51-5 The radii of the nuclei are

$$R = (1.2 \text{ fm})\sqrt[3]{7} = 2.3 \text{ fm}.$$

The using the derivation of Sample Problem 51-5,

$$K = \frac{(3)^2(1.6\times10^{-19}\text{C})^2}{16\pi(8.85\times10^{-12}\text{F/m})(2.3\times10^{-15}\text{m})} = 1.4\times10^6\text{eV}.$$

P51-6 (a) Add up the six equations to get

$$\begin{aligned} {}^{12}\text{C} \quad & + {}^1\text{H} + {}^{13}\text{N} + {}^{13}\text{C} + {}^1\text{H} + {}^{14}\text{N} + {}^1\text{H} + {}^{15}\text{O} + {}^{15}\text{N} + {}^1\text{H} \rightarrow \\ & {}^{13}\text{N} + \gamma + {}^{13}\text{C} + \text{e}^+ + \nu + {}^{14}\text{N} + \gamma + {}^{15}\text{O} + \gamma + {}^{15}\text{N} + \text{e}^+ + \nu + {}^{12}\text{C} + {}^4\text{He}. \end{aligned}$$

Cancel out things that occur on both sides and get

$${}^1\text{H} + {}^1\text{H} + {}^1\text{H} + {}^1\text{H} \rightarrow \gamma + \text{e}^+ + \nu + \gamma + \gamma + \text{e}^+ + \nu + {}^4\text{He}.$$

(b) Add up the Q values, and then add on 4(0.511 MeV for the annihilation of the two positrons.

$\boxed{\textbf{P51-7}}$ (a) Demonstrating the consistency of this expression is considerably easier than deriving it from first principles. From Problem 50-4 we have that a uniform sphere of charge Q and radius R has potential energy

$$U = \frac{3Q^2}{20\pi\epsilon_0 R}.$$

This expression was derived from the fundamental expression

$$dU = \frac{1}{4\pi\epsilon_0}\frac{q\,dq}{r}.$$

For gravity the fundamental expression is

$$dU = \frac{Gm\,dm}{r},$$

so we replace $1/4\pi\epsilon_0$ with G and Q with M. But like charges repel while all masses attract, so we pick up a negative sign.

(b) The initial energy would be zero if $R = \infty$, so the energy released is

$$U = \frac{3GM^2}{5R} = \frac{3(6.7\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\text{kg})^2}{5(7.0\times10^8\text{m})} = 2.3\times10^{41}\text{J}.$$

At the current rate (see Sample Problem 51-6), the sun would be

$$t = \frac{(2.3\times10^{41}\text{J})}{(3.9\times10^{26}\text{W})} = 5.9\times10^{14}\,\text{s},$$

or 187 million years old.

P51-8 (a) The rate of fusion events is

$$\frac{(3.9\times10^{26}\mathrm{W})}{(26.2\times10^{6}\mathrm{eV})(1.6\times10^{-19}\mathrm{J/eV})} = 9.3\times10^{37}/\mathrm{s}.$$

Each event produces two neutrinos, so the rate is

$$1.86\times10^{38}/\mathrm{s}.$$

(b) The rate these neutrinos impinge on the Earth is proportional to the solid angle subtended by the Earth as seen from the Sun:

$$\frac{\pi r^2}{4\pi R^2} = \frac{(6.37\times10^{6}\mathrm{m})^2}{4(1.50\times10^{11}\mathrm{m})^2} = 4.5\times10^{-10},$$

so the rate of neutrinos impinging on the Earth is

$$(1.86\times10^{38}/\mathrm{s})(4.5\times10^{-10}) = 8.4\times10^{28}/\mathrm{s}.$$

P51-9 (a) Reaction A releases, for each d

$$(1/2)(4.03\ \mathrm{MeV}) = 2.02\ \mathrm{MeV},$$

Reaction B releases, for each d

$$(1/3)(17.59\ \mathrm{MeV}) + (1/3)(4.03\ \mathrm{MeV}) = 7.21\ \mathrm{MeV}.$$

Reaction B is better, and releases

$$(7.21\ \mathrm{MeV}) - (2/02\ \mathrm{MeV}) = 5.19\ \mathrm{MeV}$$

more for each N.

P51-10 (a) The mass of the pellet is

$$m = \frac{4}{3}\pi(2.0\times10^{-5}\mathrm{m})^3(200\,\mathrm{kg/m^3}) = 6.7\times10^{-12}\mathrm{kg}.$$

The number of d-t pairs is

$$N = \frac{(6.7\times10^{-12}\mathrm{kg})(6.02\times10^{23}/\mathrm{mol})}{(0.005\ \mathrm{kg/mol})} = 8.06\times10^{14},$$

and if 10% fuse then the energy release is

$$(17.59\ \mathrm{MeV})(0.1)(8.06\times10^{14})(1.6\times10^{-19}\mathrm{J/eV}) = 230\,\mathrm{J}.$$

(b) That's

$$(230\,\mathrm{J})/(4.6\times10^{6}\mathrm{J/kg}) = 0.05\,\mathrm{kg}$$

of TNT.

(c) The power released would be $(230\,\mathrm{J})(100/\mathrm{s}) = 2.3\times10^{4}\mathrm{W}$.

$\boxed{\textbf{E52-1}}$ (a) The gravitational force is given by Gm^2/r^2, while the electrostatic force is given by $q^2/4\pi\epsilon_0 r^2$. The ratio is

$$\begin{aligned}
\frac{4\pi\epsilon_0 Gm^2}{q^2} &= \frac{4\pi(8.85\times10^{-12}\mathrm{C^2/Nm^2})(6.67\times10^{-11}\mathrm{Nm^2/kg^2})(9.11\times10^{-31}\mathrm{kg})^2}{(1.60\times10^{-19}\mathrm{C})^2}, \\
&= 2.4\times10^{-43}.
\end{aligned}$$

Gravitational effects would be swamped by electrostatic effects at *any* separation.

(b) The ratio is

$$\begin{aligned}
\frac{4\pi\epsilon_0 Gm^2}{q^2} &= \frac{4\pi(8.85\times10^{-12}\mathrm{C^2/Nm^2})(6.67\times10^{-11}\mathrm{Nm^2/kg^2})(1.67\times10^{-27}\mathrm{kg})^2}{(1.60\times10^{-19}\mathrm{C})^2}, \\
&= 8.1\times10^{-37}.
\end{aligned}$$

E52-2 (a) $Q = 938.27$ MeV $-$ 0.511 MeV) = 937.76 MeV.

(b) $Q = 938.27$ MeV $-$ 135 MeV) = 803 MeV.

E52-3 The gravitational force from the lead sphere is

$$\frac{Gm_eM}{R^2} = \frac{4\pi G\rho m_e R}{3}.$$

Setting this equal to the electrostatic force from the proton and solving for R,

$$R = \frac{3e^2}{16\pi^2\epsilon_0 G\rho m_e a_0^2},$$

or

$$\frac{3(1.6\times10^{-19}\mathrm{C})^2}{16\pi^2(8.85\times10^{-12}\mathrm{F/m})(6.67\times10^{-11}\mathrm{Nm^2/kg^2})(11350\,\mathrm{kg/m^3})(9.11\times10^{-31}\mathrm{kg})(5.29\times10^{-11}\mathrm{m})^2}$$

which means $R = 2.85\times10^{28}$m.

E52-4 Each γ takes half the energy of the pion, so

$$\lambda = \frac{(1240\text{ MeV}\cdot\text{fm})}{(135\text{ MeV})/2} = 18.4\text{ fm}.$$

$\boxed{\textbf{E52-5}}$ The energy of one of the pions will be

$$E = \sqrt{(pc)^2 + (mc^2)^2} = \sqrt{(358.3\text{ MeV})^2 + (140\text{ MeV})^2} = 385\text{ MeV}.$$

There are two of these pions, so the rest mass energy of the ρ_0 is 770 MeV.

E52-6 $E = \gamma mc^2$, so

$$\gamma = (1.5\times10^6\mathrm{eV})/(20\,\mathrm{eV}) = 7.5\times10^4.$$

The speed is given by

$$v = c\sqrt{1 - 1/\gamma^2} \approx c - c/2\gamma^2,$$

where the approximation is true for large γ. Then

$$\Delta v = c/2(7.5\times10^4)^2 = 2.7\times10^{-2}\mathrm{m/s}.$$

E52-7 $d = c\Delta t = hc/2\pi\Delta E$. Then

$$d = \frac{(1240\ \text{MeV}\cdot\text{fm})}{2\pi(91200\ \text{MeV})} = 2.16\times10^{-3}\text{fm}.$$

E52-8 (a) Electromagnetic.
(b) Weak, since neutrinos are present.
(c) Strong.
(d) Weak, since strangeness changes.

$\boxed{\textbf{E52-9}}$ (a) Baryon number is conserved by having two "p" on one side and a "p" and a Δ^0 on the other. Charge will only be conserved if the particle x is positive. Strangeness will only be conserved if x is strange. Since it can't be a baryon it must be a meson. Then x is K^+.

(b) Baryon number on the left is 0, so x must be an anti-baryon. Charge on the left is zero, so x must be neutral because "n" is neutral. Strangeness is everywhere zero, so the particle must be $\bar{\text{n}}$.

(c) There is one baryon on the left and one on the right, so x has baryon number 0. The charge on the left adds to zero, so x is neutral. The strangeness of x must also be 0, so it must be a π^0.

E52-10 There are two positive on the left, and two on the right. The anti-neutron must then be neutral. The baryon number on the right is one, that on the left would be two, unless the anti-neutron has a baryon number of minus one. There is no strangeness on the right or left, except possible the anti-neutron, so it must also have strangeness zero.

E52-11 (a) Annihilation reactions are electromagnetic, and this involves s$\bar{\text{s}}$.
(b) This is neither weak nor electromagnetic, so it must be strong.
(c) This is strangeness changing, so it is weak.
(d) Strangeness is conserved, so this is neither weak nor electromagnetic, so it must be strong.

E52-12 (a) $\text{K}^0 \to \text{e}^+ + \nu_\text{e}$,
(b) $\text{K}^0 \to \pi^+ + \pi^0$,
(c) $\text{K}^0 \to \pi^+ + \pi^+ + \pi^-$,
(d) $\text{K}^0 \to \pi^+ + \pi^0 + \pi^0$,

E52-13 (a) $\bar{\Delta}^0 \to \bar{\text{p}} + \pi^+$.
(b) $\bar{\text{n}} \to \bar{\text{p}} + \text{e}^+ + \nu_\text{e}$.
(c) $\tau^+ \to \mu^+ + \nu_\mu + \bar{\nu_\tau}$.
(d) $\text{K}^- \to \mu^- + \bar{\nu_\mu}$.

E52-14

E52-15 From top to bottom, they are Δ^{*++}, Δ^{*+}, Δ^{*0}, Σ^{*+}, Ξ^{*0}, Σ^{*0}, Δ^{*-}, Σ^{*-}, Ξ^{*-}, and Ω^-.

E52-16 (a) This is not possible.
(b) uuu works.

$\boxed{\textbf{E52-17}}$ A strangeness of +1 corresponds to the existence of an $\bar{\text{s}}$ anti-quark, which has a charge of +1/3. The only quarks that can combine with this anti-quark to form a meson will have charges of -1/3 or +2/3. It is only possible to have a net charge of 0 or +1. The reverse is true for strangeness -1.

E52-18 Put bars over everything. For the anti-proton, $\bar{\mathrm{u}}\bar{\mathrm{u}}\bar{\mathrm{d}}Z$, for the anti-neutron, $\bar{\mathrm{u}}\bar{\mathrm{d}}\bar{\mathrm{d}}$.

E52-19 We'll construct a table:

quarks	Q	S	C	particle
$\mathrm{u}\bar{\mathrm{c}}$	0	0	-1	D^0
$\mathrm{d}\bar{\mathrm{c}}$	-1	0	-1	D^-
$\mathrm{s}\bar{\mathrm{c}}$	-1	-1	-1	D_s^-
$\mathrm{c}\bar{\mathrm{c}}$	0	0	0	η_c
$\mathrm{c}\bar{\mathrm{u}}$	0	0	1	D^0
$\mathrm{c}\bar{\mathrm{d}}$	1	0	1	D^+
$\mathrm{c}\bar{\mathrm{s}}$	1	1	1	D_s^+

E52-20 (a) Write the quark content out then cancel out the parts which are the same on both sides:

$$\mathrm{dds} \rightarrow \mathrm{udd} + \mathrm{d}\bar{\mathrm{u}},$$

so the fundamental process is

$$\mathrm{s} \rightarrow \mathrm{u} + \mathrm{d} + \bar{\mathrm{u}}.$$

(b) Write the quark content out then cancel out the parts which are the same on both sides:

$$\mathrm{d}\bar{\mathrm{s}} \rightarrow \mathrm{u}\bar{\mathrm{d}} + \mathrm{d}\bar{\mathrm{u}},$$

so the fundamental process is

$$\bar{\mathrm{s}} \rightarrow \mathrm{u} + \bar{\mathrm{d}} + \bar{\mathrm{u}}.$$

(c) Write the quark content out then cancel out the parts which are the same on both sides:

$$\mathrm{u}\bar{\mathrm{d}} + \mathrm{uud} \rightarrow \mathrm{uus} + \mathrm{u}\bar{\mathrm{s}},$$

so the fundamental process is

$$\bar{\mathrm{d}} + \mathrm{d} \rightarrow \mathrm{s} + \bar{\mathrm{s}}.$$

(d) Write the quark content out then cancel out the parts which are the same on both sides:

$$\gamma + \mathrm{udd} \rightarrow \mathrm{uud} + \mathrm{d}\bar{\mathrm{u}},$$

so the fundamental process is

$$\gamma \rightarrow \mathrm{u} + \bar{\mathrm{u}}.$$

E52-21 The slope is

$$\frac{(7000\text{ km/s})}{(100\text{ Mpc})} = 70\text{ km/s}\cdot\text{Mpc}.$$

E52-22 $c = Hd$, so

$$d = (3\times10^5\text{ km/s})/(72\text{ km/s}\cdot\text{Mpc}) = 4300\text{ Mpc}.$$

E52-23 The question should read "What is the..."

The speed of the galaxy is

$$v = Hd = (72\text{ km/s}\cdot\text{Mpc})(240\text{ Mpc}) = 1.72\times10^7\text{m/s}.$$

The red shift of this would then be

$$\lambda = (656.3\text{nm})\frac{\sqrt{1-(1.72\times10^7\text{m/s})^2/(3\times10^8\text{m/s})^2}}{1-(1.72\times10^7\text{m/s})/(3\times10^8\text{m/s})} = 695\text{ nm}.$$

E52-24 We can approximate the red shift as

$$\lambda = \lambda_0/(1 - u/c),$$

so

$$u = c\left(1 - \frac{\lambda_0}{\lambda}\right) = c\left(1 - \frac{(590\text{ nm})}{(602\text{ nm})}\right) = 0.02c.$$

The distance is

$$d = v/H = (0.02)(3\times10^8\text{m/s})/(72\text{ km/s}\cdot\text{Mpc}) = 83\text{ Mpc}.$$

$\boxed{\textbf{E52-25}}$ The minimum energy required to produce the pairs is through the collision of two 140 MeV photons. This corresponds to a temperature of

$$T = (140\text{ MeV})/(8.62\times10^{-5}\text{eV/K}) = 1.62\times10^{12}\text{K}.$$

This temperature existed at a time

$$t = \frac{(1.5\times10^{10}\text{s}^{1/2}\text{K})^2}{(1.62\times10^{12}\text{K})^2} = 86\,\mu\text{s}.$$

E52-26 (a) $\lambda \approx 0.002\,\text{m}$.
(b) $f = (3\times10^8\text{m/s})/(0.002\,\text{m}) = 1.5\times10^{11}\text{Hz}$.
(c) $E = (1240\text{ eV}\cdot\text{nm})/(2\times10^6\text{nm}) = 6.2\times10^{-4}\text{eV}$.

E52-27 (a) Use Eq. 52-3:

$$t = \frac{(1.5\times10^{10}\sqrt{s}\text{K})^2}{(5000\,\text{K})^2} = 9\times10^{12}\text{s}.$$

That's about 280,000 years.
(b) $kT = (8.62\times10^{-5}\text{eV/K})(5000\,\text{K}) = 0.43\,\text{eV}$.
(c) The ratio is

$$\frac{(10^9)(0.43\text{ eV})}{(940\times10^6\text{eV})} = 0.457.$$

$\boxed{\textbf{P52-1}}$ The total energy of the pion is $135 + 80 = 215$ MeV. The gamma factor of relativity is

$$\gamma = E/mc^2 = (215\text{ MeV})/(135\text{ MeV}) = 1.59,$$

so the velocity parameter is

$$\beta = \sqrt{1 - 1/\gamma^2} = 0.777.$$

The lifetime of the pion as measured in the laboratory is

$$t = (8.4\times10^{-17}\text{ s})(1.59) = 1.34\times10^{-16}\text{s},$$

so the distance traveled is

$$d = vt = (0.777)(3.00\times10^8\text{m/s})(1.34\times10^{-16}\text{s}) = 31\text{ nm}.$$

P52-2 (a) $E = K + mc^2$ and $pc = \sqrt{E^2 - (mc^2)^2}$, so

$$pc = \sqrt{(2200 \text{ MeV} + 1777 \text{ MeV})^2 - (1777 \text{ MeV})^2} = 3558 \text{ MeV}.$$

That's the same as

$$p = \frac{(3558\times 10^6\text{eV})}{(3\times 10^8\text{m/s})}(1.6\times 10^{-19}\text{J/eV}) = 1.90\times 10^{-18}\text{kg}\cdot\text{m/s}$$

.

(b) $qvB = mv^2/r$, so $p/qB = r$. Then

$$r = \frac{(1.90\times 10^{-18}\text{kg}\cdot\text{m/s})}{(1.6\times 10^{-19}\text{C})(1.2\,\text{T})} = 9.9\,\text{m}.$$

P52-3 (a) Apply the results of Exercise 45-1:

$$E = \frac{(1240 \text{ MeV}\cdot\text{fm})}{(2898\,\mu\text{m}\cdot\text{K})T} = (4.28\times 10^{-10}\text{MeV/K})T.$$

(b) $T = 2(0.511 \text{ MeV})/(4.28\times 10^{-10}\text{MeV/K}) = 2.39\times 10^9\text{K}.$

P52-4 (a) Since

$$\lambda = \lambda_0 \frac{\sqrt{1-\beta^2}}{1-\beta},$$

we have

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\sqrt{1-\beta^2}}{1-\beta} - 1,$$

or

$$z = \frac{\sqrt{1-\beta^2}+\beta-1}{1-\beta}.$$

Now invert,

$$\begin{aligned}
z(1-\beta)+1-\beta &= \sqrt{1-\beta^2},\\
(z+1)^2(1-\beta)^2 &= 1-\beta^2,\\
(z^2+2z+1)(1-2\beta+\beta^2) &= 1-\beta^2,\\
(z^2+2z+2)\beta^2 - 2(z^2+2z+1)\beta + (z^2+2z) &= 0.
\end{aligned}$$

Solve this quadratic for β, and

$$\beta = \frac{z^2+2z}{z^2+2z+2}.$$

(b) Using the result,

$$\beta = \frac{(4.43)^2+2(4.43)}{(4.43)^2+2(4.43)+2} = 0.934.$$

(c) The distance is

$$d = v/H = (0.934)(3\times 10^8\text{m/s})/(72 \text{ km/s}\cdot\text{Mpc}) = 3893 \text{ Mpc}.$$

P52-5 (a) Using Eq. 48-19,

$$\Delta E = -kT \ln \frac{n_1}{n_2}.$$

Here $n_1 = 0.23$ while $n_2 = 1 - 0.23$, then

$$\Delta E = -(8.62\times10^{-5}\text{eV/K})(2.7\,\text{K})\ln(0.23/0.77) = 2.8\times10^{-4}\text{eV}.$$

(b) Apply the results of Exercise 45-1:

$$\lambda = \frac{(1240\ \text{eV}\cdot\text{nm})}{(2.8\times10^{-4}\text{eV})} = 4.4\,\text{mm}.$$

P52-6 (a) Unlimited expansion means that $v \geq Hr$, so we are interested in $v = Hr$. Then

$$\begin{aligned} Hr &= \sqrt{2GM/r}, \\ H^2r^3 &= 2G(4\pi r^3\rho/3), \\ 3H^2/8\pi G &= \rho. \end{aligned}$$

(b) Evaluating,

$$\frac{3[72\times10^3\text{m/s}\cdot(3.084\times10^{22}\text{m})]^2}{8\pi(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)}\frac{(6.02\times10^{23}/\text{mol})}{(0.001\ \text{kg/mol})} = 5.9/\text{m}^3.$$

P52-7 (a) The force on a particle in a spherical distribution of matter depends only on the matter contained in a sphere of radius *smaller* than the distance to the center of the spherical distribution. And then we can treat all that relevant matter as being concentrated at the center. If M is the total mass, then

$$M' = M\frac{r^3}{R^3},$$

is the fraction of matter contained in the sphere of radius $r < R$. The force on a star of mass m a distance r from the center is

$$F = GmM'/r^2 = GmMr/R^3.$$

This force is the source of the centripetal force, so the velocity is

$$v = \sqrt{ar} = \sqrt{Fr/m} = r\sqrt{GM/R^3}.$$

The time required to make a revolution is then

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{R^3/GM}.$$

Note that this means that the system rotates as if it were a solid body!

(b) If, instead, all of the mass were concentrated at the center, then the centripetal force would be

$$F = GmM/r^2,$$

so

$$v = \sqrt{ar} = \sqrt{Fr/m} = \sqrt{GM/r},$$

and the period would be

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{r^3/GM}.$$

P52-8 We will need to integrate Eq. 45-6 from 0 to $\lambda_{\min}$, divide this by $I(T)$, and set it equal to $z = 0.2\times 10^{-9}$. Unfortunately, we need to know T to perform the integration. Writing what we do know and then letting $x = hc/\lambda kT$,

$$\begin{aligned} z &= \frac{15c^2h^3}{2\pi^5k^4T^4}\int_0^{\lambda_\mathrm{m}} \frac{2\pi c^2 h}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}, \\ &= \frac{15c^2h^3}{2\pi^5k^4T^4}\frac{2\pi k^4T^4}{h^3c^2}\int_\infty^{x_\mathrm{m}} \frac{x^3\,dx}{e^x-1}, \\ &= \frac{15}{\pi^4}\int_{x_\mathrm{m}}^\infty \frac{x^3\,dx}{e^x-1}. \end{aligned}$$

The result is a small number, so we expect that x_m is fairly large. We can then ignore the -1 in the denominator and then write

$$z\pi^4/15 = \int_{x_\mathrm{m}}^\infty x^3 e^{-x} dx$$

which easily integrates to

$$z\pi^4/15 \approx {x_\mathrm{m}}^3 e^{x_\mathrm{m}}.$$

The solution is

$$x \approx 30,$$

so

$$T = \frac{(2.2\times 10^6\mathrm{eV})}{(8.62\times 10^{-5}\mathrm{eV/K})(30)} = 8.5\times 10^8\mathrm{K}.$$