

Research on Hidden Variable Theories: a review of recent progresses.

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Abstract

Quantum Mechanics (QM) is one of the pillars of modern physics: an impressive amount of experiments have confirmed this theory and many technological applications are based on it. Nevertheless, at one century since its development, various aspects concerning its very foundations still remain to be clarified. Among them, the transition from a microscopic probabilistic world into a macroscopic deterministic one and quantum non-locality. A possible way out from these problems would be if QM represents a statistical approximation of an unknown deterministic theory.

This review is addressed to present the most recent progresses on the studies related to Hidden Variable Theories (HVT), both from an experimental and a theoretical point of view, giving a larger emphasis to results with a direct experimental application.

More in details, the first part of the review is a historical introduction to this problem. The Einstein-Podolsky-Rosen argument and the first discussions about HVT are introduced, describing the fundamental Bell's proposal for a general experimental test of every Local HVT and the first attempts to realise it.

The second part of the review is devoted to elucidate the recent progresses toward a conclusive Bell inequalities experiment obtained with entangled photons and other physical systems.

Finally, the last sections are targeted to shortly discuss Non-Local HVT.

Table of contents

1 - Introduction

2 - Historical introduction

2.1) The EPR paradox.

2.2) Hints on the macro-objectivation problem.

2.3) The von Neumann theorem.

2.4) The Bell inequalities.

2.5) Quantum states violating Bell inequalities.

2.6) Theorem about no faster than light transmission by using EPR correlations.

2.7) Toward an experimental test of Local Realism: introductory remarks.

2.8) First experimental tests of Bell inequalities.

2.9) The Orsay experiment.

3 - PDC experiments on Local Realism.

3.1) The Parametric Down Conversion.

3.2) PDC experiments with interferometers.

3.3) First tests of local realism by using PDC polarisation entanglement.

3.4) Bright sources of polarisation entangled states.

3.5) Applications of bright sources of entangled photon pairs.

3.6) Tests of local realism by equalities.

3.7) LHVT built for surviving PDC experiments.

3.8) Test of local realism in Hilbert spaces with dimension larger than 2.

3.9) Tests of non-contextuality.

3.10) Other Quantum Optics experiments connected with local realism and quantum non-locality.

4 - Tests of local realism with other physical systems than photons

4.1) Tests of Bell inequalities with mesons.

4.2) Other tests of local realism with mesons.

4.3) Bell inequalities experiments with ions.

5 -Some conclusions about Local Hidden Variable Theories

6 - Non-Local Hidden Variable Models

6.1) The de Broglie - Bohm theory.

6.2) Nelson Stochastic model.

6.3) Experimental tests of NLHVT against SQM.

7 - Determinism at Planck scale?

8 - Conclusions

1 Introduction

Quantum Mechanics (QM) represents nowadays one of the pillars of modern physics: so far a huge amount of theoretical predictions deriving from this theory has been confirmed by very accurate experimental data. No doubts can be raised on the validity of this theory. Nevertheless, even at one century since its birth, many problems related to the interpretation of this theory persist: non-local effects of entangled states; wave function reduction and the concept of measurement in Quantum Mechanics; the transition from a microscopic probabilistic world to a macroscopic deterministic world¹ perfectly described by classical mechanics (macro-objectivation) and so on. A possible way out from these problems would be if QM represents a statistical approximation of an unknown deterministic theory, where all observables have defined values fixed by unknown variables, the so called Hidden Variable Theories (HVT), a suggestion dating since the celebrated Einstein-Podolsky-Rosen paper of 1935.

Even though, in the past decades, many interesting theoretical works were focused on the solution of this problem originating a large debate, nevertheless they could not lead to any conclusive solution [1,2,3,4].

However, in the last years the experimental know-how concerning creation and manipulation of quantum systems has hugely increased, permitting the realisation of several experiments originally thought as Gedanken Experiment and the conceiving of new ones.

The dream of testing the theoretical ideas proposed in connection with foundation of quantum mechanics is now becoming reality.

Furthermore, the improved knowledge about quantum systems is now leading to the growth of new areas of physics as quantum information theory, which could lead to the realisation of many different interesting technological applications as quantum cryptography [5], quantum computers [6,7,8,9], quantum metrology [10,11,12,13,14], ...

Considering the large conceptual relevance of the problem of completeness of QM and the fundamental role of entanglement in quantum information and metrology, the purpose of this work is to give the basic concepts related to this sector of research (without the impossible pretension of being exhaustive in a field where hundreds of papers appear every year) and a review of recent experiments that have permitted to throw new light on it, with hints to possible future developments.

¹ Leaving out of consideration classical chaotic system, where, non the less, non determinism is due to an imperfect knowledge of initial conditions (whilst a perfect one is, at least in principle, possible in classical mechanics).

In detail, the outline of the paper is the following: after describing how this research field was born with the celebrated Einstein-Podolsky-Rosen paper [15] and how possible experimental investigations on every Local Hidden Variable Theory (LHV) became possible after the Bell work in 1964 [16], we will review many interesting experiments (largely based on entangled photon pairs) devoted to test Bell inequalities [17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41] leading to a substantial agreement with standard quantum mechanics (SQM) and strongly disfavouring LHV theories. We will then consider why, nevertheless, none of these experiments has yet been conclusive since, so far, one has always been forced to introduce a further additional hypothesis [52,53], due to the low total detection efficiency, stating that the observed sample of particle pairs is a faithful subsample of the whole: a problem known as *detection loophole*.

After discussing the pioneer experiments in 70's, we will delineate in some detail the celebrated experiment of Aspect et al. [27], which represented a miliary stone in the field since it allowed for the first time to realise well space-like separated measurements, albeit with a very low collection efficiency.

In the following section are described the big progresses in the direction of a conclusive test of HVT that have been obtained in 90's by using parametric down conversion (PDC) process for generating entangled photon pairs with high angular correlation. The first experiments of this kind had, by construction, a limited total efficiency [28,29,30,31] and were far from eliminating the detection loophole [52]. Anyway, more recent experiment have drawn nearer to a conclusive experiment [32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51] (approaching an efficiency about 20%).

In section 4, after a general discussion about alternative systems for testing local realism as, in specific, entangled mesons, a recent experiment [54] performed using Be ions will be presented. Here very high efficiencies (around 98 %) were reached, but the two subsystems (the two ions) were not really separated systems during measurement.

Finally, we will outline non-local hidden variable models (NLHVT), which are not excluded by Bell inequalities tests, and on the possibility of testing them against SQM.

On the whole in this review we have tried to give a general panorama on the research about local realism (with particular emphasis to recent experimental progresses), endeavouring to be as complete as possible with the hope that this work could be useful both to the neophytes who want to approach this field and to experts who can find a useful summary and a rich source of references.

2 Historical introduction

2.1 The EPR paradox

The debate whether Quantum Mechanics is a complete theory and probabilities have a non-epistemic character (i.e. nature is intrinsically probabilistic) or whether it is a statistical approximation of a deterministic theory and probabilities are due to our ignorance of some parameters (i.e. they are epistemic) dates to the beginning of the theory itself.

The fundamental paper where this problem clearly emerged appeared in 1935 when Einstein, Podolsky and Rosen risen this question by considering an explicit example [15].

For this purpose, they introduced the concept of *element of reality* according to the following definition: if, without disturbing in any way a system, one can predict without any uncertainty the value of a physical quantity, then there is an element of physical reality corresponding to this quantity (i.e. introducing the hypothesis of realism). They formulated also the reasonable hypothesis (at the light of special relativity) that any non-local action was forbidden. A theory is complete when it describes any element of reality.

They concluded that either one of their premises was wrong or Quantum Mechanics was not a complete theory, in the sense that not every element of physical reality had a counterpart in the theory.

More in details, they considered a system consisting of two particles prepared in a state such that the sum of their momenta ($p_1 + p_2$) and the difference of their positions ($x_1 - x_2$) were both defined at the same time (situation possible in QM since they correspond to commuting operators). The case where ($x_1 - x_2$) has eigenvalue a and ($p_1 + p_2$) has eigenvalue 0 is described in QM by a Dirac delta function $\delta(x_1 - x_2 - a)$. This is an example of an entangled state: a state of two or more particles which cannot be factorised in single-particle states, namely (as stated in original Schrödinger definition [55]) a compound state whose subsystems are not probabilistically independent².

By measuring the position of particle 1 (2) one can predict with certainty the position of particle 2 (1), without disturbing in any way this second particle (which at the moment of the measurement can be very far from the other). Position of particle 2 (1) is therefore an element of reality according to the previous definition. On the other hand, a measurement of momentum of particle 1 (2) allows one to infer momentum of particle 2 (1) without acting on it,

² See [56] for an in deep discussion on the definition of entanglement.

thus also momentum of particle 2 (1) is an element of reality. But according to QM position and momentum are conjugate variables and perfect knowledge of one of them implies complete ignorance on the value of the other. Therefore, in this case QM cannot predict all the element of realities and thus QM is an incomplete theory.

The same argument has then been presented by Bohm for a spin system, allowing a very clear physical understanding and avoiding the use of a delta of Dirac wave function.

Let us thus consider a singlet state of two spin 1/2 particles

$$|\psi_0\rangle = \frac{|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}} \quad (1)$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ represent a single particle of spin up and down respectively (in the following we will often use the general quantum information notation for two-level systems, qubits, $|0\rangle, |1\rangle$). This state is manifestly entangled. Let us notice that the complete wave function for the system is obtained by multiplying the spin wave functions by appropriate space wave functions, depending on the space co-ordinates of both particles. When the two space wave functions, e.g. gaussian, are widely separated at the moment of measurement, one can therefore realise two wave packets entangled in spin, but well separated in space.

Repeating the EPR argument, let us suppose that the two particles separate (i.e. the spatial wave functions of the two particles do not overlap anymore), being addressed to two remote measurement apparatuses. Usually the two experimenters are called Alice and Bob, a convention that we will sometimes use in the following. Let us then suppose that Alice measures the z component of the spin of the first particle; this permits to immediately know the z spin component of the particle 2 (which is opposite) without disturbing in any way the second particle. Thus the z component of the spin of the second particle is an element of reality according to the previous definition.

But, since the singlet state is invariant under rotations, we could refer to any other axis (as x or y etc.): thus we can argue that any other spin component of particle 2 is an element of reality. However, spin components on different axes are incompatible variables in Quantum Mechanics, for which one cannot assign definite values at the same time: thus this implies that Quantum Mechanics cannot be a complete theory (according to the EPR definition), since it does not allow a prediction of all elements of reality.

The interesting property of an entangled system in QM is that before measurement the values of some specific observables of the two (or more) subsystems

are not defined. The measurement of the observable for one of the subsystems (e.g. the z component of spin of particle 1 in the previous example) instantaneously fixes the value of the correlated observable of the second subsystem (e.g. the z component of spin of particle 2 in the previous example), independently of the distance of the second particle. An entangled system must be considered as a whole, independently of separation of its parts. This seems to introduce a sort of instantaneous effect which is often called "quantum non-locality" and which in the past caused a strong discussion about the compatibility of QM and special relativity. A subsequent paragraph will be devoted to this point.

Following the EPR presentation, this property of entangled systems and its implications suscitated a large debate [1,2,3,4,57,58]. The position of many authors can be summarised by quoting Ballentine [57], saying that "[to deny] the reality of both spin values until the measurement has actually been performed" represents "an extreme positivist philosophy" and "entails the unreasonable, essentially solipsist position that the reality of particle 2 depends upon some measurement which is not connected to it by any physical interaction".

Within this debate emerged the development of the so-called Local Hidden Variable Theories³: namely of the proposal that it exists a deterministic (and local) theory describing nature, where the precise value of all observables of a physical system are fixed by some unknown variables (the hidden variables). Quantum Mechanics would only be a statistical approximation of this theory. This situation resembles statistical thermodynamics, which describes in a probabilistic way systems composed of many particles, each behaving in a perfectly deterministic way according to the classical equations of motion [59]. Let us emphasize the deep philosophical difference between the two cases: in Standard Quantum Mechanics nature is intrinsically probabilistic, whilst in a hidden variable theory quantum probabilities become epistemic, they are due to our ignorance about hidden variables whose knowledge would give us precise information about every physical quantity.

As we will discuss later, also non-local HVT (where the action on a subsystem can have immediate effect on the other subsystem independently of the distance) or contextual HVT (where the verity value of a statement about a physical property depends not only on hidden variables but on the whole experimental condition as well) have been considered later.

In general, in any hidden variable theory every particle has an assigned value for each observable, determined by a hidden variable (or a set of hidden variables) x . A statistical ensemble of particles has a certain distribution $\rho(x)$ of the hidden variable and thus the average value of an observable A is given by:

³ often denoted with the term Local Realistic Theories (LRT) as well.

$$\langle A \rangle = \int dx \rho(x) A(x) \quad (2)$$

Of course, considering the great success of QM in predicting many different experimental data, a judicious HVT should be built such that the average $\langle A \rangle$ given by (2) reproduces the QM predictions.

2.2 Hints on the macro-objectivation problem

Before proceeding further in discussing the history of studies on Hidden Variable models, let us just give a rapid hint to another largely debated problem of the very foundations of QM, which would find a natural solution in the HVT framework: the macro-objectivation problem.

The problem of macro-objectivation derives from the fact that Schrödinger equation is linear and thus requires that a macroscopic system interacting with an entangled state gets entangled as well.

For example let us consider a macroscopic measurement apparatus described by the state $|\chi_0\rangle$ (i.e. a wave function as complicated as necessary) which interacts with the (microscopic) states $|\phi_1\rangle$ and $|\phi_2\rangle$. The interaction, representing the measurement and lasting a time interval Δt , can be described by a linear evolution operator $U(\Delta t)$. The results of the measurement are then

$$|\chi_0\rangle|\phi_1\rangle \rightarrow U(\Delta t)[|\chi_0\rangle|\phi_1\rangle] = |\chi_1\rangle|\phi_1\rangle \quad (3)$$

and

$$|\chi_0\rangle|\phi_2\rangle \rightarrow |\chi_2\rangle|\phi_2\rangle \quad (4)$$

where the states $|\chi_1\rangle$ and $|\chi_2\rangle$ represent the state of macroscopic apparatus after measurement corresponding, for example, to two different directions of a pointer.

If $|\chi_0\rangle$ interacts with the superposition state

$$a|\phi_1\rangle + b|\phi_2\rangle \quad (5)$$

because of linearity of the evolution equation, one would have

$$|\chi_0\rangle[a|\phi_1\rangle + b|\phi_2\rangle] \rightarrow [a|\chi_1\rangle|\phi_1\rangle + b|\chi_2\rangle|\phi_2\rangle] \quad (6)$$

which is an entangled state involving the macroscopic apparatus as well. If one considers intermediate physical systems, as subsystems of the measurement one, they get entangled in the same way. The chain of entangled systems starting with the microscopic state in a superposition and ending with the detection apparatus is often called von Neumann chain.

Of course at a macroscopic level we do not perceive anything which can be thought as a superposition of two macroscopic situations, for example if the measuring apparatus has a pointer which will be up or down according to if it has measured a property 1 or 2, we always observe the pointer in one well defined position and never in an undefined superposition of pointer up and down at the same time.

A very illuminating example of this problem was proposed by Schrödinger. Let us consider a box with a cat inside. In the box there is also a measurement apparatus gauging a property of a quantum system, which is in a superposition state for the measured observable. According to the obtained result the apparatus opens or not a poison bottle. Thus, in this case the von Neumann chain includes the quantum system, the measuring apparatus and the poison bottle. But at the end also the cat is involved: if the poison has been diffused the cat dies, otherwise it survives. The result of this analysis is therefore that we have a superposition of cat alive and dead, which looks rather a paradoxical situation. From this example in the literature a superposition of two macroscopic states is usually dubbed a "Schrödinger cat".

Therefore, measurements in quantum mechanics would seem to require some process breaking the entanglement: among the possible outcomes only one will be realised and observed in the measurement process. Only one state in the superposition survives the measurement process, i.e. in the previous example the measuring apparatus will be found or in the situation described by $|\chi_1\rangle$, with probability $|a|^2$, or in the one described by the state $|\chi_2\rangle$, with probability $|b|^2$ and the measured state (if the measurement is non-destructive) will be, correspondingly, in the state $|\phi_1\rangle$ or $|\phi_2\rangle$ respectively, after the measuring process. This is called the wave function collapse. However, this request must be justified more precisely. We have to understand at which point of the measurement process the collapse occurs and how this collapse happens.

A first answer [2] is to split the world into a macroscopic one following classical mechanics and a microscopic one following QM (substantially the one adopted by the Copenhagen school). However this solution, even if perfectly useful for practical calculation of quantum processes, is weak from a conceptual point of view since it does not permit to identify the border between quantum and classical worlds. How many particles should a body have for being macroscopic? What about "macroscopic" systems as superconductors which exhibit quantum properties? This answer looks to be too over-simplified.

Various different ideas have been considered for explaining/understanding decoherence at macroscopic level, without reaching for any of them a general consensus in the physicists community. Among them (without any purpose to be exhaustive): the many universes models [60], modal interpretations [61,62,63], decoherence and quantum histories schemes [4,64,65,66], transactional interpretation [67], 'informational' interpretation [68,69], dynamical reduction models (where a non-linear modification of Schrödinger equation is introduced) [70,71,72], reduction by consciousness (wave function collapse happens at observer level) [73], and many others (see for example [2,57,74,75,76,77] and Ref.s therein).

On the other hand, this problem simply does not exist in Hidden Variable Models since in this case the specification of the state by using state vectors is insufficient, there are further parameters (the hidden variables) that we ignore for characterizing the physical situation. The physical system is always in a well specified state (corresponding to one of the quantum mechanical states present in the superposition) univocally determined by the value of the hidden variables. However, it must be noticed that for contextual theories one can attribute an objective state only to those variables which are non-contextual (we will come back on this point when considering de Broglie-Bohm model).

2.3 *The Von Neumann Theorem*

The solution of macro-objectivation and measurement problem together with EPR argument motivated therefore the search for hidden variable alternatives to quantum mechanics. Furthermore, an ulterior argument quoted for supporting HVT was the possibility of defining quantum probabilities in the frame of relative-frequency interpretation of probabilities [77]⁴. However, historically, shortly after EPR paper the quest for hidden variable theories stopped because of von Neumann's claim [79] of having demonstrated a theorem asserting the impossibility of constructing a hidden variable theory reproducing all the results of QM. So far the prestige of von Neumann led to an acritical acceptance of this theorem, but it was then discovered that one of his hypotheses was too restrictive and thus the program of the construction of a hidden variable theory was still possible.

In the following we will sketch the von Neumann argument since this discussion gives some hints on the peculiarity of HVT.

He considered the following situation: the measurement of a particular quan-

⁴ More recently HVT have also been suggested as a solution of the problems connected with the definition of a wave function of the universe in quantum cosmology [78].

tity of an ensemble is found to yield different values, although every system is in the same quantum state. We can give two answers: or the systems are different because of some "hidden" variable that our theoretical scheme is incapable of distinguishing, either the systems are really in the same state and the dispersion of measured values is due to nature itself (which is probabilistic).

The first alternative requires that the total ensemble comprises as many sub-ensembles as there are different results and every system in every sub-ensemble is in a dispersion free state, characterised by a particular value of a hidden variable.

Von Neumann proved that, from assumptions he considered plausible and reproducing QM results, no dispersion-free states are possible.

A hypothesis he introduced was that if three (or more) operators A, B, C satisfy the relation $A + B = C$, then the values should satisfy the relation $v(C) = v(A) + v(B)$, assumption that looks rather natural for a classical theory. In quantum mechanics of course the relation for average values $\langle \Psi | C | \Psi \rangle = \langle \Psi | A + B | \Psi \rangle$ is satisfied, but for non commuting operators one cannot assign definite values $v(A)$ and $v(B)$ to observables A, B simultaneously. Thus, von Neumann, by introducing this hypothesis, made impossible the agreement with QM "ab initio" (see for example [80]).

For clarifying this point, let us consider a system of two spin one half particles. Let $A = \sigma_x$ and $B = \sigma_y$: the eigenvalues of Pauli matrices are ± 1 and thus $v(A)$ and $v(B)$ can assume only the values ± 1 . It follows that $v(A) + v(B)$ can assume the values -2, 0 and 2. But $C = A + B$ is just the operator corresponding to $\sqrt{2}$ times the component of σ along the direction bisecting the x and y axes. As a result its allowed values are $\pm\sqrt{2}$ in disagreement with the previous result. It is therefore completely evident that the hypothesis $v(C) = v(A) + v(B)$ prevents to satisfy QM results even in extremely simple examples. This gives also an idea of which properties, not typical of a classical theory, a HVT must satisfy.

Probably, if this theorem would have been formulated by someone less authoritative than von Neumann, it would have been criticised much earlier (a first critics of the theorem remained completely ignored [81]) than the Bell confutation of 1966 [80] (where also similar arguments of Jauch and Piron [82] were confuted). This remains a interesting example of tortuous paths of science development.

2.4 The Bell inequalities

The subsequent step in discussing possible LHV extensions of QM was the Bell's finding [16] that any realistic Local Hidden Variable theory must satisfy certain inequalities that can be violated in SQM, thus allowing an experimental test of the validity of SQM respect to LHV.

More specifically, let us consider a physical system constituted of two separated subsystems sent to two measurement apparatuses that measure the expectation value of two dichotomic observables A_a, B_b , a, b being two parameters describing the setting of measuring apparatus A and B respectively. The two measurements are performed such to be space-like separated events. Bell showed [16] that expectation values of the two observables satisfy some inequality for every LRT, which can be violated in SQM for a specific choice of parameters.

Bell demonstration was based on considering the specific case where A_a, B_b are the results (± 1 in $\hbar/2$ unities) of a measurement of spin component along directions a, b respectively for a singlet spin state, Eq. 1.

Introducing the expectation value:

$$[C(a, b)]_{\psi_0} = \langle \psi_0 | (\sigma_1 \cdot a)(\sigma_2 \cdot b) | \psi_0 \rangle \quad (7)$$

one has for parallel analysers

$$[C(a, a)]_{\psi_0} = -1 \quad (8)$$

Furthermore, the locality condition is requested, stating that measurement of A (B) depends only upon a (b) and x

$$C(a, b) = \int_X A_a(x) B_b(x) \rho(x) dx \quad (9)$$

Therefore, in a LHVT relation 8 requires

$$A_a(x) = -B_a(x) \quad (10)$$

for every value of the hidden variable x (the states defined by the hidden variable $x, \in X$, belong to a space which can be arbitrary. No specific request on dimensionality or on linearity of operations with it is required. Only a set of Borel subsets of X is defined, so that probability measures can be defined upon it).

From Eq. 10 we can calculate the following function involving three different orientations of analysers, since $[A_b(x)]^2 = 1$:

$$|C(a, b) - C(a, c)| = \left| \int_X [A_a(x)A_b(x) - A_a(x)A_c(x)]\rho(x)dx \right| = \left| \int_X A_a(x)A_b(x)[1 - A_b(x)A_c(x)]\rho(x)dx \right| \leq \int_X [1 - A_b(x)A_c(x)]\rho(x)dx \quad (11)$$

where the last inequality follows from $A, B = \pm 1$. By using the normalization

$$\int_X \rho(x)dx = 1 \quad (12)$$

follows the Bell inequality

$$|C(a, b) - C(a, c)| \leq 1 + C(b, c) \quad (13)$$

which is always satisfied in every LHVT but can be violated in SQM, for example for a, b, c coplanar, with c making an angle of $2\pi/3$ with a and b making an angle of $\pi/3$ with both a, c .

Albeit extremely interesting from a conceptual point of view, this inequality has been derived with the request $[C(a, a)]_{\psi_0} = -1$, which cannot be hold in real experiments due to non unity efficiency of real set-ups.

This problem was eliminated by Clauser-Horne-Shimony-Holt (CHSH) who obtained the inequality [83]

$$|C(a, b) - C(a, c)| + C(b', b) + C(b', c) \leq 2 \quad (14)$$

which is one of the most often used Bell inequalities in experiments. Incidentally, this inequality was derived for stochastic LHVT, i.e. for theories where hidden variables do not determine the measurements results completely, so that they still remain random (i.e. one tries to eliminate any "non-locality", but does not look for a completely deterministic theory). However, Fine's theorem [84] stating that "Necessary and sufficient condition for the existence of a deterministic factorisable LHVT is the existence of a stochastic factorisable LHVT for the same experiment" connects the two cases. Furthermore, more recently, it has also been shown [85] that every probabilistic LHVT can be transformed in a deterministic LHVT by using additional hidden variables.

Also in this case the demonstration is very simple. Let us consider the inequality:

$$|C(a, b) - C(a, c)| \leq \int_X |A_a(x)B_b(x) - A_a(x)B_c(x)|\rho(x)dx =$$

$$\int_X |A_a(x)B_b(x)|[1 - B_b(x)B_c(x)]\rho(x)dx = \int_X [1 - B_b(x)B_c(x)]\rho(x)dx \quad (15)$$

Suppose now that for some b, b' one has $C(b', b) = 1 - \delta$ with $0 \leq \delta \leq 1$, avoiding in this way the previous condition of perfect correlation (i.e. $\delta = 0$). By dividing the set X into two regions $X_{\pm} = \{x | A_{b'}(x) = \pm B_b(x)\}$ one has (being $C(b', b) = 1 - \delta = \int_X B_b^2(x)\rho(x)dx - 2 \int_{X_-} B_b^2(x)\rho(x)dx$)

$$\int_{X_-} dx \rho(x) = \delta/2 \quad (16)$$

and hence:

$$\int_X B_b(x)B_c(x)\rho(x)dx \geq \int_X A_{b'}(x)B_c(x)\rho(x)dx - 2 \int_{X_-} |A_{b'}(x)B_c(x)|\rho(x)dx =$$

$$= C(b', c) - \delta \quad (17)$$

From this result and Eq. 15 follows the inequality 14.

Another example is the inequality proposed by Bell in 1971 [86]

$$S = |C(a, b) - C(a, b')| + |C(a', b') + C(a', b)| \leq 2 \quad (18)$$

Also this demonstration was produced by using a generalisation of locality suited to include systems whose evolution is inherently stochastic.

As a further example, let us then consider, still in detail, the derivation of the inequality proposed by Clauser-Horne [87], which also includes inherently stochastic theories and has been often used in experiments.

A source emits entangled particles, where the first particle goes to detector 1 and the second to detector 2. Let us suppose that before the detector i we select a certain property θ_i , for example, if the particles are entangled in spin, then θ_i is the angle defining the direction (respect to the z-axis) along which we are going to measure the spin. The Clauser-Horne sum then reads:

$$CH = P(\theta_1, \theta_2) - P(\theta_1, \theta'_2) + P(\theta'_1, \theta_2) + P(\theta'_1, \theta'_2) - P(\theta'_1) - P(\theta_2) \quad (19)$$

where $P(\theta_1, \theta_2)$ represents the joint probability of observing a particle in 1 with the selection θ_1 and, in coincidence, a particle in 2 with the selection θ_2 (apices denote other angles choices).

On the other hand, $P(\theta_i)$ represents the probability of observing a single particle at i with selection θ_i .

Let us now suppose that these probabilities derive by a LHVT, where $\rho(x)$ describes the probability distribution for the hidden variable x . Then we have

$$P(\theta_i) = \int dx \rho(x) P(\theta_i, x) \quad (20)$$

and

$$P(\theta_i, \theta_j) = \int dx \rho(x) P(\theta_i, \theta_j, x) \quad (21)$$

Again, if the theory is local, then the measurement in 1 does not depend on the choice of θ_2 and viceversa. Thus, we have:

$$P(\theta_1, \theta_2, x) = P(\theta_1, x) * P(\theta_2, x) \quad (22)$$

In order to demonstrate the Clauser Horne inequality let us now consider an algebraic relation for 4 variables: x, x' , which lie between 0 and X ($X \leq 1$) and y, y' , which lie between 0 and Y ($Y \leq 1$).

Then, it follows:

$$xy - xy' + x'y + x'y' - x'Y - yX \leq 0 \quad (23)$$

In fact, for $x < x'$, one can rewrite this equation as

$$x(y - y') + (x' - X)y + (y' - Y)x' \leq (x' - X)y + x(y - Y) \quad (24)$$

which is negative.

On the other hand for $x \geq x'$, one rewrites Eq. 23 as

$$(x - X)y + (y - Y)x' + (x' - x)y' \quad (25)$$

which again is clearly negative.

By substituting $P(\theta_1, x) = x$, $P(\theta'_1, x) = x'$, $P(\theta_2, x) = y$, $P(\theta'_2, x) = y'$

and $X=1$, $Y=1$, we finally obtain the Clauser-Horne inequality valid for every LHVT

$$P(\theta_1, \theta_2) - P(\theta_1, \theta'_2) + P(\theta'_1, \theta_2) + P(\theta'_1, \theta'_2) - P(\theta'_1) - P(\theta_2) \leq 0 \quad (26)$$

Beyond the ones already presented here, many different derivations of Bell inequalities have been proposed, based on slightly different initial hypotheses, but all of them are substantially equivalent [2] from an experimental point of view: in fact from Fine's theorem [84] it follows that all Bell inequalities for two measurements with two possible outcomes (as the ones discussed here) are equivalent to CHSH, Eq. 14, in the sense that states violating one of them violate CHSH as well. Incidentally, the classification of Bell inequalities for a general number of measurements and outcomes is a very difficult, unsolved, task (from a computational point of view it is a hard NP problem [88]). Some very recent results for 2,3 measurements/outcomes can be found in Ref.s [89,90].

Among various Bell inequalities demonstrations we can quote yet ⁵:

- i) Wigner, Belinfante and Holt proof [59,91,92], based on subdividing the space of states of a two-component system into subspaces corresponding to various possible values of the observable of interest and then studying measures on these subspaces.
- ii) Stapp's proof [96,97] (see also [98]), which is very general since it dispenses with all assumptions about the state of the system and about probability measures on the space of states pointing therefore strictly to locality assumption.
- iii) Santos' proof [99] based on comparison between classical and quantum logic: for the classical Boolean lattice of propositions it exists a metric satisfying triangle inequalities from which one derives quadrilateral inequalities that may be violated by a non-Boolean lattice of propositions (as the QM one): these are the Bell inequalities.

Bell inequalities for generic n-level systems were also demonstrated by using inequalities for Shannon entropy derived in classical information theory [100,101,102] (e.g. $H(A|B) \leq H(A|B') + H(B'|A') + H(A'|B)$ where $H(A|B)$ is the conditional information for the two observables A,B).

For other demonstrations (but the list is far from being complete) the reader can refer to [103,104,105,106,107,108,109,110] and Ref.s therein.

⁵ We do not enter in details of these proofs that can be easily find, together with further ones, in earlier reviews [92,93,94,95] and books [2].

2.5 Quantum states violating Bell inequalities

The next point to be clarified is when a quantum state violates Bell inequalities [2]. This point has been largely debated principally due to its large relevance for quantum information: in the following we will only summarize some of the main results.

In general two-particles pure entangled states violate some Bell inequality, as generally demonstrated in Ref. [111,112,113] where it was shown that for every pure entangled state of two quantum systems is possible to find pairs of observables whose correlations violate some specific Bell inequality. For example, the two photons maximally polarisation - entangled state $|\psi\rangle = \frac{|H\rangle|H\rangle + |V\rangle|V\rangle}{\sqrt{2}}$ (where H,V denote Horizontal and Vertical polarisations respectively) maximally violates the former inequality 26 when the parameters θ_i , representing the setting of a polarizer preceding photon detection, are opportunely selected (e.g. $\theta_1 = 67^\circ.5$, $\theta_2 = 45^\circ$, $\theta'_1 = 22^\circ.5$, $\theta'_2 = 0^\circ$). Furthermore, for two particles systems, the amount of violation of the Bell inequalities has an upper bound [114], e.g. $S \leq 2\sqrt{2}$ for Eq. 18.⁶ On the other hand, the situation is not so clear for more than two particles.

In Ref. [118,119] it was proven that no local realistic description is possible for every pure entangled state of an arbitrary number of particles, provided additional manipulations are allowed.

Mermin [120], and then Ardehali [121] and Belinskii and Klyshko [122], have shown that N spin- $\frac{1}{2}$ particles entangled states originate a violation, exponentially growing with N, of the Bell inequality (MABK)

$$F \leq 2^{n/2}, \text{ n even}$$

$$F \leq 2^{(n-1)/2}, \text{ n odd}$$

where F is the average value of the operator (σ are Pauli matrices)

$$\frac{1}{2i} \left(\prod_{j=1}^N (\sigma_x^j + i\sigma_y^j) - \prod_{j=1}^N (\sigma_x^j - i\sigma_y^j) \right) \quad (27)$$

which, for example, has the value $F = 2^n$ for the quantum state $\frac{|\uparrow\uparrow\ldots\uparrow\rangle + i|\downarrow\downarrow\ldots\downarrow\rangle}{\sqrt{2}}$.

⁶ Further limits on quantum correlations, which could eventually be violated by other probabilities sets, can be found in [115,116,117]. For example [117] proved that considering two observables A^a, B^b such that $\langle A^a B^{b'} \rangle = 1 - \epsilon_1$ and $\langle A^{a'} B^b \rangle = 1 - \epsilon_2$, $0 \leq \epsilon_1, \epsilon_2 \leq 2$, then in QM $|\langle A^a B^b \rangle - \langle A^{a'} B^{b'} \rangle| \leq \sqrt{2\epsilon_1} + \sqrt{2\epsilon_2} + 2\sqrt{\epsilon_1\epsilon_2}$.

On the other hand, Scarani and Gisin [123] showed that the pure entangled states (dubbed generalised GHZ states⁷)

$$\cos(\alpha)|0, \dots, 0\rangle + \sin(\alpha)|1, \dots, 1\rangle \quad (28)$$

do not violate the MABK inequality for $\sin(2\alpha) \leq 1/\sqrt{2^{N-1}}$. Other specific cases were discussed in Ref. [124,125,126].

Later, it was shown how exist pure entangled states for $N \geq 2$ that do not violate any Bell inequality for N particle correlation functions in experiments involving two dichotomic observables [127,128]. In more detail, Ref. [128] showed that all the states of Eq. 28 with N odd and $\sin(2\alpha) \leq 1/\sqrt{2^{N-1}}$ satisfy the Bell inequality [127,129]

$$\sum_{s_1, \dots, s_N = -1, 1} \left| \sum_{k_1, \dots, k_N = 1, 2} s_1^{k_1-1} \dots s_N^{k_N-1} E(k_1, \dots, k_N) \right| \leq 2^N \quad (29)$$

where $E(k_1, \dots, k_N)$ is the correlation function that for the local realistic case, implying the existence of two numbers $A_j(\vec{n}_1)$ and $A_j(\vec{n}_2)$ of value ± 1 describing the predetermined result of a measurement by the j th observer of the observable defined by \vec{n}_1 and \vec{n}_2 respectively, is given by the average over many runs of the experiment:

$$E(k_1, \dots, k_N) = \left\langle \prod_{j=1}^N A_j(\vec{n}_{k_j}) \right\rangle_{avg} \quad (30)$$

Inequality 29 (not equivalent to MABK) is equivalent to the full set of 2^{2^N} Bell inequalities for the correlations functions between measurements on N particles involving two alternative dichotomic observables at each local measurement station [127].

On the other hand, it has been shown that inequalities involving more than two alternative measurements are stronger [130,131] and in particular a general Bell inequality for $N > 2$ and many measurements settings was recently derived [132], which is violated by a larger class of states, as for example all the states of Eq. 28.

Very recently some further progress was also done [133] about violation of Bell inequalities for 3 qubits pure entangled states⁸, showing that every 3-particle state of the form 28 violates some Bell inequality and presenting

⁷ GHZ equality will be described in paragraph 3.6.

⁸ Three qubits pure states form a five parameter family, whose representation can be found in [134].

numerical results indicating that all pure 3-qubits entangled states violate a Bell inequality.

This for what concerns pure states. Nevertheless, it was shown that also specific mixed states maximally violate (CHSH) Bell inequalities [135].

In order to proceed further, let us begin giving a more general definition of entanglement valid also for mixed states. We define a state separable if its density matrix ρ is of the form

$$\rho = \sum_i (\rho_i^A \otimes \rho_i^B) w_i \quad (31)$$

where $w_i > 0$ and ρ_i^A, ρ_i^B are the density matrices of subsystems A and B. Every non-separable state is called entangled [136].

A separable state does not violate Bell inequalities. Various criteria for recognising separable states have been established [137,138,139,140,141,142]. Among them, for two-dimensional Hilbert spaces, the Peres-Horodecki one [137,138]: a state is separable iff the transposition of one of the subsystems (partial transposition), with respect to any subsystem is positive. However, for higher dimensional systems this condition is only necessary: there exist entangled systems whose partial transpose is positive [143,144].

A complete characterization of separable states could be given in terms of entanglement witnesses [138], i.e. a state ρ is entangled iff there exists a Hermitian operator W (an "entanglement witness") such that $Tr[W\sigma] \geq 0$ for all separable states σ , but $Tr[W\rho] < 0$. However, albeit some recent progress (e.g. see [145] and Ref.s therein), the characterization of entanglement witnesses is not known.

Even if the problem whether a general mixed system violates or not Bell inequalities is still unsolved, specific cases have been discussed [132,136,146,147,148,149]. For example, already in 1989 in a seminal paper for these researches Werner has shown [136] that exist specific non separable states (i.e. states not of the form of Eq. 31), dubbed Werner states, as (I is the identity matrix)⁹

$$\rho_W = \frac{1-F}{3}I + \frac{4F-1}{3}|\Psi_-\rangle\langle\Psi_-| \quad F \leq 1/2 \quad (32)$$

that do not violate any Bell inequality with two projective measurements: a

⁹ In the following we will use the conventional notation $|\Phi_{+,-}\rangle = \frac{|0\rangle|0\rangle \pm |1\rangle|1\rangle}{\sqrt{2}}$ and $|\Psi_{+,-}\rangle = \frac{|0\rangle|1\rangle \pm |1\rangle|0\rangle}{\sqrt{2}}$ for the so-called Bell states, forming a basis for entangled states of two particles.

result obtained by explicitly building a LHV model for these states (recently these states have also been experimentally realized by using a PDC source of entangled photons [150,151]). This achievement has then been generalised to the case of positive-operator-valued measurements (POVM) by building a LHV model for POVM on a class of generalised Werner states [149]¹⁰. On the other hand, it has been shown that, if several such pairs are tested simultaneously, a violation of the CHSH inequality may occur, and no local hidden variable model is compatible with the results [153]. Also, Popescu has shown that by considering local measurements of the form $P \otimes I$ and $I \otimes P$ on each subsystem of a Werner state defined in a Hilbert space $H = C^d \otimes C^d$, where P is a projector on a two-dimensional subspace of C^d , one gets violation of Bell inequalities for a subensemble when $d \geq 5$: non-locality can therefore be revealed by sequences of local measurements [154]. None the less, since in Popescu's example the observables leading to a violation of Bell inequality commute with the local measurements operators, these time sequences of measurements can be described by a single observable and Werner's LHV model can be applied, but requiring that later measurements influence preceding ones (a "hidden" violation of causality) [152]. A general result on the conjecture [152] of the equivalence between separability and the existence of a "causal" local hidden variable theory, i.e. on the possibility to reveal non-locality when arbitrary long sequences of general measurements and/or measurements on ensemble of states are considered, is still missing.

A very recent discussion of violation of some generalised Bell inequalities by Werner states and mixtures of W states (i.e. states of the form $1/\sqrt{N}[|10\dots 0\rangle + |010\dots 0\rangle + \dots + |0\dots 01\rangle]$) with noise can be found in [132,133]. Also, a sufficient and necessary condition for violation of CHSH inequality for a two-dimensional Hilbert space has been presented in Ref.[146], while a condition for a maximal violation of CHSH can be found in Ref. [147].

Let us also notice that local actions with classical communication and post-selection (rejection of part of original ensemble) on a mixture that does not violate Bell inequality can generate a mixture violating them [155,156,157,158,159], a procedure known as "distillation". It has been shown [160] that this operation is possible for every inseparable two level system. On the other hand, it has also been demonstrated [161] that there are mixed states, called bound entangled, that are not distillable in spite of being entangled.

It can also briefly be acknowledged that the problem of which states violate Bell inequalities can be related to the one of "measurement of entanglement" of large relevance in quantum information [162]. For a pure state entanglement can be easily quantified in terms of the von Neumann entropy ($E = -Tr \rho \ln \rho$, where ρ is the density matrix of the system). The "entropy of entanglement" is

¹⁰ Explicitly showing that a hypothesis suggested in Ref. [152] is wrong.

defined as the von Neumann entropy of the reduced density matrix operator, which for pure states does not depend on which reduced density matrix is used. It satisfies what are reasonable requests for a measurement of entanglement, i.e.

- i) to be zero for separable states,
- ii) to be invariant for local unitary transformations,
- iii) not to increase for local operations, classical communications and subselection,
- iv) the value for two factorised states is the sum of the values for these two states.

For mixed states the situation is more complicated and various measures of entanglement, satisfying former criteria plus the one of reducing to entropy of entanglement for a pure state, have been proposed. Among them: the relative entropy of the state [163,164], defined as the minimum of quantum relative entropy ($S(\rho||\sigma) = Tr(\rho \ln \rho - \rho \ln \sigma)$) taken over the set D of all separable states σ , i.e.

$$E(\rho) = \min_{\sigma \in D} S(\rho||\sigma) \quad (33)$$

or the concurrence [165],

$$C(\rho) = \max[0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4] \quad (34)$$

λ_i being the square roots of eigenvalues of $\rho(\sigma_y \otimes \sigma_y)\rho^T(\sigma_y \otimes \sigma_y)$, where σ_y is the Pauli matrix¹¹. The relevant point for our discussion is that it has been shown [167] how the violation S of CHSH inequality is, in terms of concurrence, limited by $\max[1, \sqrt{2}C] < S < \sqrt{1 + C^2}$.

Finally, in discussing the states violating Bell inequalities, it is interesting to notice that the Quantum Field Theory vacuum maximally violates Bell inequalities, as pointed out by Summers and Werner [170,171]¹² in the frame of algebraic approach to quantum field theory [173]. This violation vanishes exponentially ($\sim e^{-mr}$) with the spatial separation r of measurements with the

¹¹ Concurrence is monotonically related to another measurement of entanglement, entanglement of formation [166]. For other measures of entanglement see [140,168,169] as well.

¹² A later related result states that, by considering that a fully QFT description requires a q-deformed Hopf algebra of Weyl-Heisenberg one, the bosonic vacuum is a, entangled, generalised coherent state of $SU(1,1) \times SU(1,1)$ [172].

length scale determined by the Compton wave length of the lightest particle (of mass m) in the theory (or $\sim 1/r^2$ for the massless case). However, in principle this vacuum entanglement could be 'extracted' to another physical system [174] and to be experimentally verified.

2.6 *Theorem about no faster than light transmission by using EPR correlations*

Another theoretical point worth to be discussed before describing experimental tests of quantum non-locality, is its compatibility with special relativity.

The main motivation of Einstein, Podolsky and Rosen to include the hypothesis concerning the perfect locality of the system under consideration derived by the necessity of having no instantaneous transmission in agreement with special relativity.

However, as we will see, the non-locality of quantum entangled systems does not allow any transmission of information faster than light and thus does not raise any problem of compatibility with relativity, albeit many opposite claims due to many different authors, also very influent as Popper [175].

For the sake of exemplification, before discussing the general theorem, we rapidly examine some specific examples that allow a clear understanding of why quantum non-locality does not permit superluminal communication.

Let us consider two observers (as usual dubbed Alice and Bob) receiving respectively one particle each of an entangled pair like the one described by the state in Eq. 1.

Let us Alice perform a spin test along the z direction: she obtains a perfectly casual sequence of 1 and -1, each outcome with probability $1/2$. Let us now suppose that also Bob performs the same test (in principle they could be separated of a space-like distance and thus in a certain reference frame the Alice's measurement is before Bob's one, whilst in other reference frames Bob's measurement precedes Alice's one). Quantum mechanics predictions tell us that Bob's results are perfectly correlated with Alice's ones: every time Alice observes a 1 Bob has a -1 and viceversa. Anyway, if there is no classical communication between the two, the only result whose Bob disposes is a sequence of 1 and -1 in a perfectly casual order: he cannot determine in any way if Alice has performed a measurement or not, his results being a random sequence in both the cases. Thus, no information can be transmitted between Alice and Bob in this way.

However, more complicated schemes can be conceived [176]. Let us suppose

for example that Bob could "clone" each particle he receives, i.e. he put any particle into an apparatus creating $4N$ photons exact copies of it.

Now let us also imagine that Alice can decide between performing a test along the z axis or along a basis at 45° respect to the z axis. Let us consider the case where she chooses a test along z and obtains 1 (-1), 1 denoting spin up and -1 spin down respectively. Then Bob uses the cloning system on his particle and sends N copies to four different apparatuses measuring the spin along z , $-z$ or the two conjugated directions of the second basis respectively. He will observe, using z basis, N (0) particles with -1 and 0 (N) with 1, while on the other basis he will observe $N/2$ particles for both 1 and -1 outcomes. Exactly the same result would be obtained, *mutatis mutandis*, if Alice had chosen to perform the test in the other basis. Namely, in this case if Alice obtains 1 (-1), Bob would observe, using the second basis, N (0) photons with -1 and 0 (N) with 1, while on the z basis he will observe $N/2$ particles for both outcomes, 1 and -1. Thus by simply looking for which set-up he observes zero events, Bob would be able to know which basis Alice has decided to use. Of course, such a knowledge could be easily used to transmit a signal.

Nevertheless, also this scheme does not work. In fact a general theorem has been demonstrated stating that it does not exist a way of performing a general cloning for a quantum state [177].

The demonstration of this theorem is rather simple, and we report it here for completeness.

Let us suppose to have a cloning machine which acts on an unknown quantum state $|\Psi_1\rangle$ and a second "target" state $|\Psi_0\rangle$ producing a copy of the first. The action of the cloning machine can be described by a unitary operator U through

$$|\Psi_1\rangle \otimes |\Psi_0\rangle \mapsto U(|\Psi_1\rangle \otimes |\Psi_0\rangle) = |\Psi_1\rangle \otimes |\Psi_1\rangle \quad (35)$$

Let us now assume to apply the same cloning machine to an arbitrary second state $|\Psi_2\rangle$, the result would be

$$|\Psi_2\rangle \otimes |\Psi_0\rangle \mapsto U(|\Psi_2\rangle \otimes |\Psi_0\rangle) = |\Psi_2\rangle \otimes |\Psi_2\rangle \quad (36)$$

taking the inner product of the equalities in the two former equation we obtain:

$$\langle \Psi_1 | \Psi_2 \rangle = (\langle \Psi_1 | \Psi_2 \rangle)^2 \quad (37)$$

but this relation requires that either $|\Psi_1\rangle = |\Psi_2\rangle$ or $|\Psi_1\rangle$ is orthogonal to $|\Psi_2\rangle$, namely the cloning machine can clone only orthogonal states and therefore a universal cloning machine is impossible.

The former examples give a clear hint of why quantum non-locality cannot be used for transmitting faster than light communications. Finally, at the beginning of 80's a general theorem was proved [178] demonstrating the absolute impossibility of using quantum non-locality for faster than light transmission, of which in the following we sketch the demonstration .

Let us consider a composed system $S_1 + S_2$ described by a statistical operator W_{12} . Performing a measurement on S_1 corresponds to project on a specific eigenstate $|s\rangle$; this operation is described by a projection operator $P_s^1 = |s\rangle\langle s|$. Thus if Alice performs a non selective (i.e. where one keeps all the outcomes) measurement on S_1 , the statistic operator W_{12} transforms according to:

$$W_{12} \rightarrow W'_{12} = \sum_s P_s^1 W_{12} P_s^1 \quad (38)$$

However, all the information on the sub-system S_2 is contained in the reduced statistical operator W_2 , obtained taking the partial trace on the Hilbert space H_1 (corresponding to subsystem 1):

$$W_2 = Tr_1[W'_{12}] = Tr_1[\sum_k P_k^1 W_{12} P_k^1] = \sum_k Tr_1[P_k^1 W_{12} P_k^1] \quad (39)$$

where we have used the fact that trace is a linear operation.

Then we have, due to the properties of the trace:

$$W_2 = \sum_k Tr_1[P_k^1 W_{12} P_k^1] = \sum_k Tr_1[P_k^1 W_{12}] = Tr_1[\sum_k P_k^1 W_{12}] = Tr_1[W_{12}] \quad (40)$$

which is exactly the same operator we would have obtained without any measurement on the subsystem S_1 : thus there is no way to distinguish where a measurement on system 1 has been made or not, by performing measurements only on the system 2. The theorem has been extended to more general kinds of measurement [179] and to the case of approximate cloning [180] as well, guaranteeing that no faster-than-light communication is possible using QM non-locality.

Incidentally, it is interesting to notice that faster than light communication, on the other hand, would be possible in HVT if one could know hidden variables values (we will show an explicit example when discussing dBB model). This fact substantially forbids to have access to these variables.

2.7 Toward an experimental test of Local Realism: introductory remarks

The obvious utmost relevance of Bell inequalities derives from the fact that they give a completely general demonstration that every local realistic theory cannot reproduce all the results of quantum mechanics. An experimental observation of a violation of these inequalities represents therefore a conclusive test against these kind of theories.

Nevertheless, unluckily, to obtain a conclusive experimental measurement of Bell inequalities is not an easy task.

First of all, referring for example to Eq. 26, experimentally one measures the number of coincidences $N(\theta_1, \theta_2)$ between two detectors, while in Eq. 26 the joint probability $P(\theta_1, \theta_2) = N(\theta_1, \theta_2)/N$ appears, where N is the total number of pairs emitted by the source, which is not really measurable because usually a large fraction of the pairs is lost.

Anyway, by considering the ratio

$$R = \frac{[N(\theta_1, \theta_2) - N(\theta_1, \theta'_2) + N(\theta'_1, \theta_2) + N(\theta'_1, \theta'_2)]}{[N(\theta'_1) + N(\theta_2)]} \quad (41)$$

N cancels between numerator and denominator and for a LHVT, it is always $R \leq 1$, while in SQM it can reach the value 1.207.

However, the fact that only a subsample of the total number of produced entangled systems is really detected leads to the necessity of an additional assumption: we have to ask that the measured sample is a faithful representation of the whole. In fact, in principle, this subsample could contain a distribution in the hidden variables different from the total one, since the hidden variable values can also be related to the larger or smaller probability of the state to be observed. This means that if the observed sample is not a sufficiently large fraction of the total one, the experiment is testing Local Realism plus the additional hypothesis of having an unbiased measured subsample [52,53,181,182], a problem known as **detection loophole**. From an inspection of inequalities 18,14 or 26 one can deduce that a detection-loophole free test of LR requires to observe, for a maximally-entangled state, at least a 82.84% of the total sample.

This loophole, as we will see, remains the main unsolved problem for arriving to a conclusive test of LHVT against SQM: all the experiments performed up to now were unable to solve it (in the few where this did not happen other strong additional hypotheses were needed).

It must also be noticed that it has been discussed in many different forms

specific for the experiment under consideration, as, for example, the request for polarisation entangled photon states that the probability of counts with a polarizer in place is less than or equal to the probability with the polarizer removed [87], known as no-enhancement hypothesis.

In order to show explicitly an example of how this loophole manifests itself, let us consider more in detail the effect of a low detection efficiency for Eq. 41. The presence of a low efficiency η leads to have much less coincidences ($\propto \eta^2 N$) than single counts ($\propto \eta N$) and therefore to a verification of the inequality $R \leq 1$. In order to overcome this problem one usually substitutes in Eq. 41 single counts with coincidences with no selection on the second channel; for polarisation entangled photons set-ups this substantially means to introduce the no-enhancement hypothesis [181]. Once this is done, denoting with ∞ the absence of selection, Eq. 41

becomes

$$R = \frac{[N(\theta_1, \theta_2) - N(\theta_1, \theta'_2) + N(\theta'_1, \theta_2) + N(\theta'_1, \theta'_2)]}{[N(\theta'_1, \infty) + N(\infty, \theta_2)]} \leq 1 \quad (42)$$

that is the form effectively used in experiments. Incidentally, in some of them it is used the simplified form of Eq. 42 obtained by requiring rotational invariance,

$$R_{sym} = \frac{|N(\pi/8) - N(3\pi/8)|}{N(\infty, \infty)} \leq \frac{1}{4} \quad (43)$$

The limit of a 82.84% detection efficiency is rather difficult to be reached. An important theoretical indication for a way of overcoming this problem has been obtain by Eberhardt [183], who showed, by a numerical minimization, that this limit can be lowered to 66.7% for non-maximally entangled states (with a smaller violation of the inequality), namely entangled states where the different components have different weights. As an example, the region where the Clauser-Horne inequality 26 is violated in function of the detection efficiency η and of the degree of entanglement f ($f = 1$ for maximally entangled states, $f = 0$ non entangled states) for the state $|\psi\rangle = \frac{|H\rangle|H\rangle + f|V\rangle|V\rangle}{\sqrt{(1+f^2)}}$ is shown in Fig. 1.

Another request to be implemented for an ultimate experiment on local realism is that the two measurements are really space like separated (locality loophole), a condition that has been recently well realised (see the following).

Finally, some of other conditions, more easily to be fulfilled from an experimental point of view (albeit not always met in the experiments described in

the following), for a conclusive test concern the temporal window of acquisition [184] and the possibility of not having a background to be subtracted [185].

2.8 First experimental tests of Bell inequalities

Many different systems have been considered in the literature (as entangled pairs of ions, $K\bar{K}$, $\Lambda\bar{\Lambda}$ etc.) for realizing tests of Bell inequalities, but up to now most of the experiments have been realized with entangled photons since these systems present various advantages that we will discuss in the following.

In the first experiments, performed in 70s and 80s, a polarisation entangled photon pair was produced using a cascade atom decay or positronium decay.

Denoting with H and V the horizontal and vertical polarisation, respectively, the state is

$$|\Phi_+\rangle = \frac{|H\rangle|H\rangle + |V\rangle|V\rangle}{\sqrt{2}} \quad (44)$$

for the $J = 0 \rightarrow 1 \rightarrow 0$ atom decay.

Whilst for positronium decay (as the positronium ground state has an odd parity) it is:

$$|\Psi_-\rangle = \frac{|H\rangle|V\rangle - |V\rangle|H\rangle}{\sqrt{2}}. \quad (45)$$

Denoting with $a_\theta = a_H \cos(\theta) + a_V \sin(\theta)$ the annihilation operator with polarisation along a direction making an angle θ with horizontal axis (while $a_{H,V}$ denotes annihilation operator for horizontally and vertically polarized photons respectively), the two corresponding coincidence probabilities are, remembering that single detection probabilities are given by $P(\theta) = \langle \psi | a_\theta^\dagger a_\theta | \psi \rangle$ and joint probabilities by $P_{12}(\theta_1, \phi_2) = \langle \psi | a_{\phi_2}^\dagger a_{\theta_1}^\dagger a_{\theta_1} a_{\phi_2} | \psi \rangle$,

$$P(\theta_1, \theta_2) = 1/2 \cos^2(\theta_1 - \theta_2) \quad (46)$$

and

$$P(\theta_1, \theta_2) = 1/2 \sin^2(\theta_1 - \theta_2) \quad (47)$$

respectively.

It can be easily shown that with a suitable choice of parameters probabilities 46, 47 allow a violation of Bell inequalities. For example, the use of the joint probability 46 in Eq. 41 leads to a maximal violation of $R = 1.207$ for $\theta_1 = 67^\circ.5$, $\theta_2 = 45^\circ$, $\theta'_1 = 22^\circ.5$, $\theta'_2 = 0^\circ$.

The main problem concerning positronium decay is the difficulty of selecting polarisation of high-energy (gamma) photons produced in the decay. This problem has substantially limited the results obtained with this source. In short, since no linear polariser exists for gamma rays, one must analyse the polarisation by measuring the scattering distribution by means of the Klein-Nishina formula and introduce the hypothesis that this result can be correctly related by using QM to the one that would have been obtained by using linear polarisers. On the other hand detection efficiency is next to unity for gamma rays.

Experiments with positronium produced in atomic decays (as ^{64}Co or ^{22}Na) were realised by Kasday, Ullman and Wu [17] and then by others groups [18,19,20]. Most of them was in good agreement with SQM prediction (with the caution due to the auxiliary assumptions that we have just discussed), with the exception of Ref. [18].

More clear results (with less demanding additional assumptions) have been obtained by using entangled photons produced in atomic cascade decay, which are in the visible region of the spectrum so that polarisation can be easily selected. However, in this case, the atom takes away part of the momentum and thus photon directions are not well correlated. This leads to the fact that Eq. 46 must be substituted with:

$$P(\theta_1, \theta_2) = 1/4\eta_1\eta_2f_1g_{12}(\epsilon_1^+\epsilon_2^+ + \epsilon_1^-\epsilon_2^- * F * \cos[2(\theta_1 - \theta_2)]) \quad (48)$$

where η_i is quantum efficiency on channel i , f_1 is the probability of the photon on branch 1 to enter the photo-detector, g_{12} the conditional probability of photon 2 to enter the second detector when photon 1 entered the first, F measures the degree of correlation in the initial pair and finally $\epsilon_1^\pm = \epsilon_1^\parallel \pm \epsilon_1^\perp$, where ϵ_1^\parallel and ϵ_1^\perp are the transmission coefficients of the polariser for parallel or orthogonal polarised light to the polariser axis respectively.

Also the single detection probability is modified in the following way:

$$P_i(\theta) = 1/2\eta_i f_i \epsilon_i^+ \quad (49)$$

For atom cascade decay the parameter F in Eq. 48 is rather far from 1. Of the whole set of produced photon pairs, only a small subsample is detected, leading to a large relevance of detection loophole in this kind of experiments.

As discussed in the previous paragraph, due to low detection efficiency one is bound to introduce additional hypotheses, as the no-enhancement one. Because of this loophole, namely of selecting a subsample of the total number of pairs, one is not simply testing quantum mechanics against LHVT, but has also introduced a further hypothesis, which states that the detected sample is a faithful representation of the initially produced set of pairs.

Even if the detection loophole is rather severe for atomic decay experiments, beautiful results were obtained using this technique, giving first clear experimental indications against LHVT.

In general in these experiments an atomic beam emitted by an oven is excited by means of a laser or an electron beam and then crosses a region between two symmetrically placed photo-detection apparatuses where entangled photons produced in cascade decays are observed.

Among them one can quote the ones by Freedman and Clauser [21] (based on photon pairs at 551 and 423 nm produced in $4p^2\ ^1S_0 \rightarrow 4p4s\ ^1P_1 \rightarrow 4s^2\ ^1S_0$ cascade in calcium), of Clauser [23] and of Fry and Thompson [24] (photons at 436 and 254 nm produced in $7^3S_1 \rightarrow 6^3P_1 \rightarrow 6^1S_0$ ^{200}Hg decay), all in agreement with SQM and showing a violation of Bell inequalities (having measured $R_{sym} = 0.300 \pm 0.08$, $R_{sym} = 0.2885 \pm 0.0093$ and $R_{sym} = 0.296 \pm 0.014$ for Eq. 43 respectively). On the other hand the Holt and Pipkin experiment [22] (photons at 567 and, 405 nm produced in $9^1P_1 \rightarrow 7^3S_1 \rightarrow 6^3P_0$ ^{198}Hg decay, where atoms were excited to 9^1P_1 level by a 100 eV electron beam) led to results in disagreement with SQM and to no violation of Bell inequalities ($R_{sym} = 0.216 \pm 0.013$). However, in this case, a systematic error was identified in the form of stresses in the walls of the bulb containing the electron gun and mercury vapour [92] and the experiment of Ref. [23] substantially reproduced this set-up finding a result in agreement with SQM.

Typical detection efficiencies reached in this series of experiments were less than one per cent (e.g. 0.4% in Ref. [21]).

In this list of 70's experiments one can also quote the Laméhi-Rachti and Mittag experiment [25], based on spin correlations in proton pairs prepared by low-energy S-wave scattering. When auxiliary assumptions are introduced, like the ones described in positronium case, the results agree with SQM (within limited statistics). A similar experiment, with indications again favourable to SQM, was later performed by DM2 collaboration [26], following a suggestion of Ref. [186], by using $\Lambda\bar{\Lambda}$ spin correlated pairs: also in this case the spin was measured indirectly (through $\Lambda \rightarrow \pi p$ decay).

For a more detailed description of these pioneer experiments see [92,93,94].

2.9 The Orsay experiment

The season of cascade atomic decays set-ups was closed by the celebrated Orsay experiment [27], where for the first time the two measurements were space-like separated with an observed Bell inequalities violation of more than 5 standard deviations.

In synthesis, in this set-up [27] (developing a former experiment [187]) the polarisation entangled photon pairs with a wave-length of 422.7 and 551.3 nm were generated by $(J = 0) \rightarrow (J = 1) \rightarrow (J = 0)$ cascade in calcium-40 ($4p^2\ ^1S_0 \rightarrow 4p4s\ ^1P_1 \rightarrow 4s^2\ ^1S_0$ as Ref. [21]), which is particularly suited for coincidence experiments since the life time of the intermediate level is rather short ($\tau = 5$ ns).

The optimum signal to noise ratio for coincidences was obtained by reaching an excitation rate of about $1/\tau$ by exciting the atomic beam with a Krypton ion laser (at 406 nm) and a tunable dye laser (at 581 nm) tuned to resonance for the two-photon process, controlled by two feedback loops (on the wavelength of tunable laser and on the angle between lasers polarisations).

The entangled photons were then addressed to detection apparatuses at 6 m from the source, constituted by a large-aperture aspherical lens, followed by an interference filter, a transport optical system, an acousto-optic device, polarisers and photomultiplier tubes.

The space-like separation between the two detections was obtained by using rapid acousto-optic switches operating at 50 MHz, which were selecting different paths for the incident photons in a way that no communication of the selected basis for the polarisation analysis was possible between the two different detection systems. Thus, the two photodetections after polarisation selection were really two non-causally connected events. Anyway, some doubts about this statement have been raised since the switch was not a random one but a periodic one. As we will discuss later, also this objection has recently been overcome by new experiments [37,38,39], finally eliminating in a uncontroversial way locality loophole. The acousto-optic switch was then followed, on each of the two paths, by a polariser and a photo-multiplier whose signal was addressed to a fourfold coincidence system. Typical run time were around 12000 s.

The observed violation of Clauser-Horne inequality, $CH = 0.101 \pm 0.020$, was 83% of the maximal violation and in good agreement with theoretical SQM prediction ($CH = 0.112$) obtained by including polariser efficiencies and collection solid angles. Of course, due to low detection efficiency, this result was obtained by substituting single counts with coincidences without polarisation selection on the second detector.

It is also worth to notice that in a previous version of this set-up [188] no acousto-optic device was inserted, but polarisation selection was performed by a polarising cube followed by two photomultipliers in a way that the photon was always (modulus collection and detector efficiency) observed. Also in this case an evident violation of CHSH inequality, $S = 2.697 \pm 0.015$, was observed.

In summary, this experiment represented the final result of the series of cascade atomic decays ones and allowed to substantially close the space-like loophole. Nevertheless, collection efficiency was very low (with coincidences ranging between $0 - 40s^{-1}$ against a typical rate of production of pairs of $5 \cdot 10^7 s^{-1}$). This low value, even smaller than the previous ones, was mainly due to the necessity of reducing the divergence of the beams in order to get a good switching. Thus, detection loophole was very far from being eliminated.

Epitomizing, the situation at beginning of 80's was that strong indications favouring SQM against LHVT had been obtained, mainly in atomic cascade experiments, arriving also to a substantial closing of locality loophole. However, detection efficiencies were rather low leaving a large space for criticisms based on detection loophole. Furthermore, limitations of the entangled pair production techniques were such to leave no hope for relevant progresses in this sense: since the low detection efficiency was mainly due to low angular correlation of the pair, even an experiment with perfectly efficient detectors would not have been resolute [189].

A new kind of sources was therefore needed for progressing toward a conclusive experiment: this happened in the 90's when spontaneous parametric down conversion in non-linear crystals became largely exploited.

3 PDC experiments on Local Realism

3.1 *The Parametric Down Conversion*

The parametric down-conversion (PDC), or parametric fluorescence, is a quantum effect without classical counterparts and consists of a spontaneous decay, inside a non-linear crystal, of one photon from a pump beam (usually generated by a laser) into a couple of photons conventionally called signal and idler (for an extensive description of this phenomenon see [190]). This decay process obeys (phase matching laws) to energy conservation

$$\omega_0 = \omega_i + \omega_s \tag{50}$$

and momentum conservation

$$\vec{k}_0 = \vec{k}_i + \vec{k}_s \quad (51)$$

where $\omega_0, \omega_i, \omega_s$ are the frequencies and $\vec{k}_0, \vec{k}_i, \vec{k}_s$ the wave vectors of pump, idler and signal photon respectively. Furthermore, the two photons are produced at the same time (within few tens femtoseconds, as measured with an interference technique).

The probability of a spontaneous decay into a pair of correlated photons is usually very low, of the order of 10^{-9} or lower; therefore with typical pump power of the order of some milliwatts, the fluorescence emission lies at the levels of photon counting regime. Since the photons are produced in pairs and because of the energy and momentum conservation restrictions, the detection of one photon in a certain direction and with a given energy indicates the existence of the pair correlated one, with definite energy in a well defined direction.

As a first interesting application this property allows the realisation of a "heralded single photon source", where the observation of one photon of the pairs "certifies" the presence of the correlated one at a specific frequency and in a determined direction.

In the type I PDC both photons are produced with the same polarisation, orthogonal to the pump one. Photons of equal wave length are emitted on concentric cones centred on the pump laser direction (see fig.2), whose diameter depends on the angle between the pump beam and the optical axis of the crystal, the phase matching angle. When projected into a plane conjugated photons are on the same diameter and opposite respect to the centre of the two concentric circles corresponding to their wave-lengths.

In type II, one produced photon has equal polarisation to the pump one, the other an orthogonal polarisation. For an opportune phase matching angle (see later), they are emitted on intersecting circumferences (see fig.3).

From a theoretical point of view, in summary, the process of PDC in a crystal with active region of volume V and susceptibility $\chi^{(2)}$ can be described by the Hamiltonian (where the sum is over modes allowed by energy and momentum conservation):

$$H = \frac{1}{L^3} \sum_{k,s} \sum_{k',s'} E_l \chi_{i,j,l}^{(2)} (\epsilon_{k,s}^*)_i (\epsilon_{k',s'}^*)_j \int_V d^3r [e^{[-i(k_0 - k - k') \cdot r]} a(k, s) a(k', s') + h.c.] \quad (52)$$

where k_0, k, k' are the quadrimomenta of the pump, idler and signal photon respectively, E_l is the vector amplitude of the pump (strong enough to be

treated as a classical field), whilst $a(k, s)$ and $a(k', s')$ are the annihilation operators for the produced photons (with polarisation s, s').

Developing to the first order the evolution operator acting on the vacuum, one obtains a state in the following form

$$\begin{aligned}
|\Psi(t)\rangle &= \exp\left[-\frac{i}{\hbar} \int_0^t H(t') dt'\right] |0\rangle = \\
&|0\rangle + \frac{1}{L^3 i \hbar} \sum_{k,s} \sum_{k',s'} E_l \chi_{i,j,l}^{(2)}(\epsilon_{k,s}^*)_i (\epsilon_{k',s'}^*)_j \prod_{m=1}^3 \frac{\sin[(\vec{k}_0 - \vec{k} - \vec{k}')_m l_m / 2]}{(\vec{k}_0 - \vec{k} - \vec{k}')_m / 2} \\
&\quad e^{[i(\omega + \omega' - \omega_0)t/2]} \frac{\sin[(\omega + \omega' - \omega_0)t/2]}{(\omega + \omega' - \omega_0)/2} |k, s\rangle |k', s'\rangle
\end{aligned} \tag{53}$$

The sum over k, k' does not allow factorisation into a product of signal and idler states: the state described by Eq. 53 is therefore an entangled state, which can be used for Bell inequalities tests.

The specific experimental schemes based on this state will be the argument of next sections; however, we can begin to point out which is the principal advantage of this technique respect to the cascade atomic decay one. In PDC the two correlated photons, due to the conservation of angular momentum, are emitted within very narrow cones whose specific width can be evaluated smaller than 1 mrad [191,192], depending on the kind and dimension of non-linear crystal and on the pump width. Due to this high spatial correlation of entangled photons the collection efficiency can be in principle very high, at variance with the atomic decay case.

3.2 PDC experiments with interferometers

The type I PDC biphoton state described by Eq. 53 presents a phase and momentum entanglement that can be directly applied for a Bell inequality measurement by using two separated interferometers according to the scheme proposed by Ref. [194]¹³ and realized by [28,29,30,31] (some previous experiments based on this scheme [195,196,197,198] observed interference, but visibility was too low for a Bell inequality test).

The original idea of Franson [194] consisted in placing two Mach-Zender interferometers on the path of the two entangled photons (see fig.4). Let us suppose

¹³ A scheme for a Bell inequalities test by exploiting PDC entanglement had also previously been proposed by Klyshko [193].

that the long arm of the interferometers for the idler and signal photon have a tunable phase ϕ_i and ϕ_s respect to the short one, the final state is of the form

$$\Psi_{fr} = \frac{1}{2} \left[|s_1\rangle |s_2\rangle + |l_1\rangle |l_2\rangle e^{i(\phi_1+\phi_2)} + e^{i(\phi_1)} |l_1\rangle |s_2\rangle + e^{i(\phi_2)} |s_1\rangle |l_2\rangle \right] \quad (54)$$

where the subscripts 1, 2 refer to the photon entering the first and the second interferometer, while s, l denote short and long path, respectively.

After the interferometers the two photons are sent to detectors. If both have followed short or long path they arrive in coincidence, otherwise they are lost for the coincidence window. Incidentally, this means that, also neglecting every other inefficiency, only 50% of the pairs is selected.

The photons arriving in coincidence give a coincidence rate

$$R_c \propto \eta_i \eta_s \langle \Psi_{fr} | a_i^\dagger a_s^\dagger a_i a_s | \Psi_{fr} \rangle = \frac{1}{4} \eta_i \eta_s [1 + \cos(\phi_1 + \phi_2)] \quad (55)$$

where $\eta_i \eta_s$ are quantum efficiencies of detectors on idler and signal path respectively. The striking fact about this equation for the coincidence rate is that it can be modulated with 100% certainty using either of the widely separated phase plates. This "non-local" effect can be used for testing Bell inequalities (e.g. as 14,18,26), where now the parameters are the phases $\phi_{i,s}$.

In the experimental realization of [29] a BBO crystal was pumped by an argon ion laser beam in collinear regime producing photons pairs at 916 nm. A beam splitter separated the pair in two orthogonal directions (of course with an efficiency of 50% that further reduce the total detection efficiency), addressed to the two interferometers. Detectors were silicon avalanche photodiodes (cooled at -25°) with a measured efficiency of 16%. A 7 standard deviations violation of Clauser-Horne inequality 26 was observed. A similar scheme with a higher violation, 16 standard deviations (only inferred from a visibility of $80.4 \pm 0.6\%$), was also presented in Ref.s [32,33].

On the other hand, a modified scheme for exploiting momentum - phase entanglement was used in a previous experiment [28]. In this scheme [34] (see fig. 5) one selects four beams (A,B,C,D) corresponding to the direction of two conjugated photon pairs. One can therefore have idler and signal photons (eventually of different wave-length) emitted either in directions A, C or B, D respectively, originating a state of the form (since one is working in low rate regime, here and generally in the other experiments discussed in this review,

the probability of having two pairs emitted at the same time can be neglected)

$$\Psi_{HSZ} = \frac{1}{\sqrt{2}} [|k_A\rangle|k_C\rangle + |k_D\rangle|k_B\rangle] \quad (56)$$

Then both A,D and B,C are recombined, after reflection on mirrors, on a beam splitter. Before recombination a tunable phase shift ϕ_A and ϕ_C is introduced in paths A,C respectively. On each arm after the beam splitter is placed a photon-detector and coincidences are measured among them. Coincidence rate is modulated depending on the phases $\phi_{A,C}$ in an analogous way to the previous case. The advantage of this scheme is that in principle one can select all the produced pairs.

In the set-up of Ref. [28] a krypton-ion laser operating at 413.4 nm pumped a KDP crystal. Photon pairs at 826.8 nm were then selected. The visibility at coincidence level was of 82%, leading to a violation of inequality 18, $S = 2.21 \pm 0.022$, of 10 standard deviations. The single photon detection rates were around $10^5 s^{-1}$ and coincidence rates around $500 s^{-1}$ (with, subtracted, accidental estimated to be $100 s^{-1}$), leading to an overall detection efficiency of 0.5% (detectors were silicon avalanche photodiodes operating in Geiger mode with quantum efficiency larger than 10%). This number is already a little larger than the ones obtained in atomic cascade experiments, nevertheless remains still very far from what needed for a detection loophole free experiment. Furthermore, the two measurements were not really space like separated.

A post-selection free set-up, namely without the problem of eliminating the long-short terms of Franson's scheme, was then realised [35] by using type II PDC and polarising beam splitters in the interferometers. This results in only two different terms: either the horizontally polarised photon in channel 1 passes through the long path, while its vertical twin brother passes through the long path in channel two, or both take the short paths. A $95.0 \pm 1.4\%$ visibility was observed, but no real test of Bell inequalities was made.

The schemes based on energy-phase entanglement have then found an interesting development since they are well suited for long distance fiber transmission of entanglement.

A first experiment on long distance entanglement transmission was realised by Tapster et al. [36] by using the Franson scheme. A pair of entangled photons at 820 nm and $1.3 \mu m$ was produced in a lithium iodate crystal pumped by an argon laser at 501.7 nm. The shorter wave-length photon was immediately addressed to a single mode fiber interferometer, while the other passed through a 4.3 km single-mode communications fiber before reaching the interferometer ($1.3 \mu m$ is a well suited wave-length for propagation in communications fibers). A 86.9% visibility was obtained, sufficient for claiming the possibility

of violating Bell inequalities, even if no results on these was really presented.

A separation of more than 10 km was then obtained in 1998 [37,38]. This experiment is particularly interesting since, together with another based on polarisation entanglement realised at the same time [39], definitively closed locality loophole. In this case a passive¹⁴ coupler randomly selected which interferometer of the two available (and therefore which measurement) analyses the photon. The observed CHSH inequality violation was $S = 2.92 \pm 0.18$ (2.38 ± 0.16 if accidental coincidences are not subtracted).

Since this experiment was bound to realise various experimental high demanding tasks, it may be interesting to rapidly describe its scheme. The PDC source consisted in a 655 nm diode laser with an external cavity (10 mW) and a KNbO₃ crystal cut for type I PDC. The analysers were two temperature controlled Michelson interferometers with Faraday mirrors (for compensating birefringence in the arms) connected to the source by telecom fibers and separated of 10.9 km bee-line. Fibers were equalized at 1 mm level over 18 km (daily thermal expansion of several millimeters had to be kept into account). Finally, photons were detected by germanium avalanche photodiodes (with 5% quantum efficiencies). For this set-up all together the probability for detecting an emitted photon pair was about $8 \cdot 10^{-6}$: a very large separation of detectors was therefore payed by very low detection efficiency.

A later development of this set-up [40] allowed to realise measurements such that it was possible to invert the temporal order of them by changing the reference frame (*before-before* configuration). A condition that requires the temporal, Δt , and spatial separation, d , of the two events to satisfy the relation

$$|\Delta t| < \frac{v}{c^2}d \quad (57)$$

(where v is the relative velocity between the two reference frames) and is stronger than space-like separation request, verified if

$$|\Delta t| < \frac{d}{c} \quad (58)$$

In this scheme with two interferometers, on each side one of them was kept in the absolute future of the other. The first detector was therefore the one determining the collapse of the wave function when the photon is absorbed in the first layers of photo-detectors. Since the data could be collected by the second detector only, in the actual realisation of the experiment (see fig. 6) the "choice device" was a 20 cm diameter black-painted aluminium disk

¹⁴ the use of a passive coupler can still lead same objections to the effective closing of detection loophole.

of 1 cm thickness turning vertically at 10000 rpm. During absorption the rotation provided a good approximation of linear motion defining the moving reference frame. Observed high visibility 83% was sufficient for guaranteeing Bell inequalities violation (even if this were not really evaluated). This result allowed to exclude some specific models of wave function collapse, where this happens in the frame of the measuring apparatus (multisimultaneity model) [199,200].

The same Geneva group realised later a second experiment [41] further testing this theoretical model where a high intensity PDC source based on Periodically Poled Lithium Niobate crystal pumped at 657 nm produced energy-time entangled photons at 1314 nm (with a conversion efficiency 4 orders of magnitude larger than bulk crystals), which were addressed through optical fibers to two Michelson interferometers, where the beam splitters were substituted by acousto-optic modulators (AOM). Since the AOM is equivalent to a moving beam splitter it was possible to realise the *before-before* configuration of Eq. 57. Again the results were at variance with multisimultaneity predictions [199,200].

For the sake of completeness, it is worth to notice that also a second kind of entanglement (time-bin) beyond energy-time was realised with interferometers and PDC in pulsed regime. The scheme is based on placing on the pump beam a Mach-Zender interferometer (whose path length difference is large compared to the pump pulse length) before the non-linear system where a polarisation entangled pair is generated. The pump photon can thus follow the short or the long path originating the superposition [43]:

$$|\Psi_p\rangle = \frac{1}{\sqrt{2}} [|s\rangle + e^{i\phi} |l\rangle] \quad (59)$$

where $|s\rangle$ and $|l\rangle$ denote the photon which has followed the short and the long path, respectively and ϕ the phase difference between the two paths. The PDC process in the non-linear crystal finally transforms the state of Eq. 59 in the entangled one

$$\frac{1}{\sqrt{2}} [|s\rangle |s\rangle + e^{i\phi} |l\rangle |l\rangle] \quad (60)$$

Even if no direct Bell inequalities test was presented [43], the observed visibility, 84%, was sufficiently high for a violation of them, clearly exceeding the upper limit, 71%, for separable states. Later [44] the robustness of this entanglement to decoherence was shown for a 11 km fiber propagation.

In summary, all together this experiments realised very interesting tests of local realism moving toward a conclusive one and allowing very remote trans-

mission of entanglement. Nevertheless, in general, total detection efficiency was relatively small.

Before concluding this section is worth to point out how the propagation of entangled photons to such large distances has found an obvious application for realising remote quantum key distribution (QKD) protocols. The main idea of these protocols [201] consists in transmitting a cryptographic key by exploiting the correlation properties of quantum systems. The use of specific set-ups [202,203,204,205], analogous to the ones we have just discussed, allowed QKD up to 150 km in fiber and of some tens kilometers in open air. This technique is therefore reaching an applicative stage.

3.3 First tests of local realism by using PDC polarisation entanglement

A second possibility of using PDC photons for testing Bell inequalities is to generate polarisation entangled states.

With the purpose of generating such states, a first scheme is to use a beam splitter for recombining idler and signal photons produced in type I PDC after having rotated the polarisation of one of the two photons, see fig.7.

This scheme was the first set-up realised in 1988 by Shih and Alley [206] and Ou and Mandel [207] for testing local realism with PDC.

In Shih and Alley set-up, after having produced degenerate biphoton pairs by pumping a type I non-linear crystal with a fourth harmonic of a Nd-doped yttrium aluminum garnet laser (with 100 ps pulses), both photons crossed a $\lambda/4$ plate, transforming vertical polarisation into left circular one ($|L\rangle$) and were reflected by a mirror originating the state

$$|\Psi\rangle = e^{i\alpha}|R^A\rangle + e^{i\beta}|R^B\rangle \quad (61)$$

where $|R\rangle$ denotes right-hand circular polarisation state and α and β are the phases associated with path A and B, respectively.

The photons were then superimposed on a beam splitter, generating the state:

$$|\Psi_{SH}\rangle = \frac{1}{2}[e^{i(\alpha+\beta)}|R_1\rangle|R_2\rangle - e^{i(\alpha+\beta)}|L_1\rangle|L_2\rangle + e^{i(\alpha_1+\beta_1)}|R_1\rangle|L_1\rangle - e^{i(\alpha_2+\beta_2)}|R_2\rangle|L_2\rangle] \quad (62)$$

where suffixes 1,2 denote the final beam splitter exit and α_i , β_i the phase of

photon from path A,B to detector $i = 1, 2$ ($\alpha_2 + \beta_1 = \alpha_1 + \beta_2 \equiv \alpha + \beta$). A similar state with linear polarisation was also obtained by placing a $\lambda/2$ plate in path A only.

The state post-selected considering the cases when the photons exit differently from beam splitter can be used for a test of Bell inequalities. A violation of 3 standard deviations was obtained ($R_{sym} = 0.34 \pm 0.03$).

A similar set-up was also used by Ou and Mandel [207], with a violation of 6 standard deviations of Clauser-Horne inequality ($CH = (11.5 \pm 2.0)/min$).

Incidentally, this technique was recently used [208] also for entangling two photons coming from independent sources (one PDC "heralded" photon and one from an attenuated laser beam), showing a violation of CHSH inequality, $|S| = 2.44 \pm 0.13$. It must be noticed that entanglement between photons, coming from independent sources (and violation of Bell inequalities), was also previously obtained [209] with quantum swapping by using type II polarisation entangled photons, as described in next paragraph¹⁵.

These set-ups allowed therefore a first realisation of polarisation entanglement with PDC, nevertheless these kind of schemes select unavoidably a 50% of pairs by using a beam splitter for generating the state and therefore is not well suited for approaching a detection loophole free test of local realism (a possible solution of this problem was offered in Ref. [211]. However, it requires photon-number resolving detectors: even if some progress toward realisation of them has been obtained recently [212,213,214,215,216,217], reliable efficient photon-number resolving detectors are still far from being available.

3.4 *Bright sources of polarisation entangled photon states*

In the last years various bright sources of polarisation entangled states of two photons (see fig.8) have been produced by Type II PDC [45] or by superimposing two type I PDC emissions, in this case based on two thin adjacent crystals [50] or two crystals with an optical condenser between them [51] or by inserting them in an interferometer [218,219] (incidentally one can notice that all of them can be used for generating all four Bell states, Φ_{\pm} and Ψ_{\pm}).

The schemes with type II PDC [220] are based on the fact that in this case PDC correlated photons are emitted with orthogonal polarisations.

¹⁵ It must be noticed that in these experiments, where only linear optical elements are used for entangling the two independent photons, only a limited efficiency is obtained. A possible way out would be the use of non-linear interactions as Kerr effect [210].

In the collinear regime the two degenerate photons exit in two tangent cones, if one selects the intersection point of them, the two orthogonally polarised correlated photons exit in the same direction and can be separated by a beam splitter, generating the entangled state of Eq. 45 when one postselects events where photons have left the beam splitter on different paths (taking therefore only a 50% of original pairs). The experimental implementation of this scheme [221] led to a 10 standard deviations violation of Clauser-Horne inequality 26.

In non-collinear regime [222], if the angle θ_{pm} , between pump and crystal optical axis, is decreased, the two cones separate from each other entirely. On the other hand, if θ_{pm} is increased they intersect: therefore along two correlated intersections a,b one superimposes the probabilities of generating a H (V) or V (H) photon in direction a (b). Nevertheless, this biphoton state is not yet entangled, since, due to birefringence in the non-linear crystal, ordinary and extraordinary photons propagate with different velocities and different directions inside it. Longitudinal and transverse walk-offs must therefore be compensated for restoring indistinguishability between the two polarisations and generating an entangled state. This is usually done by inserting some birefringent medium (as quartz) along the optical path of photons.

The first experiment with non-collinear type II PDC has been presented in Ref. [45], where a pump beam at 351 nm (150 mW) pumped a 3mm long BBO crystal. The transverse walk off was estimated to be negligible compared to coherent pump beam width. On the other hand, the longitudinal walk off (385 fs) was larger than the coherence time (determined by interference filters and irises) and was compensated by an additional BBO crystal. All four Bell states were generated. A very strong violation, 102 standard deviations, of CHSH inequality was reached ($S = -2.6489 \pm 0.0064$ with Ψ_+) with a detection efficiency above 10%, representing an important step toward elimination of detection loophole.

A further important progress realised with type II sources was a conclusive elimination of detection loophole (together with the experiment described in subsection 3.2 [37,38]). As we have seen, one had already get rid of this loop-hole in the Orsay experiment, except for the fact that the selection of measurement was driven by a periodic (not random) signal. This last problem has been solved in Ref. [39]: polarisation entangled photons, produced by pumping a type II BBO crystal with a 351 nm argon-ion-laser, were largely separated by propagating in fibers. The orientation of the polariser was determined by a random number generator. The use of an active switch make this experiment even more indisputable than the interferometer one [37,38]. Photons were then detected (as in the other experiments described in this paragraph) by silicon avalanche photo-diodes with dark counts rate lower than few hundred per second (and therefore negligible respect to observed signal counts). A typical observed value of the Bell parameter S was $S = 2.73 \pm 0.02$, i.e. a violation of

30 standard deviations respect to local realistic predictions. Overall collection efficiency was of 5%. This relatively small value testify the difficulty of eliminating at the same time locality and detection loophole in order to reach an ultimate experiment.

Concluding the presentation of type II sources it must be noticed that recently some very bright sources have been obtained [46,47,48,49]. A first example ($360800s^{-1}$ coincidence counts for a 465 mW pump power) was realised in cw regime with a careful maximization of collection efficiency [46]. The total detection efficiency in the polarisation analysis set-up was $\eta = 0.214$. A violation of 204 standard deviation of CHSH inequality 14 was obtained with a measurement time of only 1s per angle setting. A second example [47] pumped at 397 nm a Periodically Poled KTP crystal generating entangled state by splitting the emitted collinear degenerate photons reaching a measured coincidence flux of $300s^{-1}$ for mW of the pump. The CHSH inequality was violated by $S = 2.711 \pm 0.017$. Finally, by using a type II source producing 20000 entangled pairs per second (with as a pump a violet diode laser at 405 nm and 18 mW power) it was possible to transmit entanglement in open air for more than 600m with a clear violation of CHSH inequality, $S = 2.41 \pm 0.10$ [49]. Very recently this result was then extended up to 13km [205] with a CHSH violation $S = 2.45 \pm 0.09$ (the source being a type II BBO crystal pumped by a 300mW cw Argon Ion laser at 351 nm).

An alternative to use type II PDC is offered [223] by superimposing the emission of two type I PDC crystals whose optical axes are rotated of 90° producing two emissions with orthogonal polarisations. If the optical distance between the two crystals is smaller than the coherence length of pump laser, one generates the entangled state:

$$|\psi_{NME}\rangle = \frac{|H\rangle|H\rangle + f|V\rangle|V\rangle}{\sqrt{(1 + |f|^2)}} \quad (63)$$

The explicit value of the parameter f can be fixed according to the specific choices in the set-up. This is an interesting property since, as we have discussed before, this non-maximally entangled states may allow an elimination of detection loophole at a lower detection efficiency (66.7%) than the one needed for maximally entangled ones (82.8%).

A first scheme used for this purpose is based on superimposing the emission of two thin adjacent type I crystals. More in details, in the first example of this kind of source [50], an argon laser beam at 351 nm, pumped two adjacent BBO crystals 0.59 mm long with optical axes oriented orthogonally. A rotatable half-wave plate on the pump beam before the crystals allowed to tune the laser beam polarisation and therefore the parameter f of the generated state, Eq. 63. The source was rather bright, giving a $21.000s^{-1}$ coincidence rate

for 150-mW pump power (an order of magnitude larger than previous type II sources). A large violation of Bell inequalities (for maximally-entangled states), $S = 2.7007 \pm 0.0029$, was observed.¹⁶

The same year a second source was realized [51,224] by using an optical condenser for superimposing the two emissions. This scheme in principle allows a very precise superposition of the whole parametric fluorescence even with long crystals allowing for higher intensities. More in details, in this scheme (see fig.9) there are two crystals of LiIO_3 250 mm apart, a distance smaller than the coherence length of the pumping laser (a 351 nm argon laser beam). This guarantees indistinguishability in the creation of a couple of photons in the first or in the second crystal. A couple of planoconvex lenses with 120 mm focal length, centred in between, focalises the spontaneous emission from the first crystal into the second one, while maintaining exactly, in principle, the angular spread. A hole of 4 mm diameter is drilled into the centre of the lenses to allow transmission of the pump radiation without absorption and, even more important, without adding stray-light, because of fluorescence and diffusion of the UV radiation. The pumping beam at the exit of the first crystal is displaced from its input direction by birefringence: a small quartz plate (5 x5 x5 mm) in front of the first lens of the condenser compensates this displacement, so that the input conditions are prepared to be the same for the two crystals. Finally, a half-wavelength plate immediately after the condenser rotates the polarisation of the Argon laser beam that excites in the second crystal a spontaneous emission which is cross-polarised with respect to the first one.

The coincidence rate was analogous to the previous scheme, while a test of Bell inequalities by using non-maximally entangled states with $f \simeq 0.4$ led to a violation of Eq. 42 $R = 1.082 \pm 0.006$ when the set of polarisers specific for maximising the violation for this non-maximally entangled state was chosen ($\theta_1 = 72^\circ.24$, $\theta_2 = 45^\circ$, $\theta'_1 = 17^\circ.76$ and $\theta'_2 = 0^\circ$).

In the last years, many studies were devoted to produce sources in pulsed regime (which are particularly useful for quantum information for timing reasons). When the pump pulses are very short (typically hundreds of femtoseconds), amplitudes for photon pairs produced at different depth inside the crystal become distinguishable, reducing two-photon interference visibility [225]. This problem required either to use thin ($\approx 100\mu\text{m}$) non-linear crystals [226] or narrow band spectral filters (for increasing coherence length) in front of detectors [227,228,229]. However, these solutions significantly reduce available flux of entangled photon pairs.

Quite recently, a bright source [218] was obtained by pumping with a fem-

¹⁶ A further test of local realism by equalities realised with this scheme by using non-maximally entangled states will be discussed in a subsequent paragraph.

to second mode locked laser two type I BBO crystals inserted in a Mach-Zender interferometer.

A bright source (always in femtosecond pulsed regime) was also obtained by addressing back, both rotated by a $\lambda/4$ wave plate, the PDC emission and the pump beam to the same type I crystal by means of a spherical mirror [230,231]. A 213σ violation of Bell inequality was observed.

A scheme based on an interferometer was also realised for cw regime [219]. The scheme consists in a Mach-Zender interferometer where the pump enters through a first beam splitter pumping two identical type II crystals inserted each in a different arm (A,B) originating the biphoton state

$$\frac{|H_A(\omega_s)\rangle|V_A(\omega_i)\rangle + |H_B(\omega_s)\rangle|V_B(\omega_i)\rangle}{\sqrt{2}} \quad (64)$$

After the polarisation on one of the arms has been rotated by a half-wave plate the two emissions are recombined on a polarizing beam splitter producing the entangled state $\frac{|H_1(\omega_s)\rangle|V_2(\omega_i)\rangle + |V_1(\omega_s)\rangle|H_2(\omega_i)\rangle}{\sqrt{2}}$, where 1,2 refer to the two PBS ports. This source (whose practical implementation was based on a single crystal with counterpropagating pump beams) reached a flux of entangled photons of $12000 \text{ pairs } s^{-1} \text{ mW}^{-1}$ and 100σ violation of CHSH inequality. Detection efficiency was $\sim 18\%$.

Finally, again in cw regime, in the set-up of Ref.[232] two collinear type I PDC emissions (after having rotated one of them by a half-wave plate) were superimposed in a modified Mach-Zender scheme.

Altogether the realisation of these high efficiency collection very bright sources of polarisation entangled photons points out a possible way for reaching an ultimate test of local realism. At the moment the principal problem remains the detection loophole: the highest detection efficiencies reached so far are some tens per cent, whilst they should arrive, as we have seen, to 82.84% for maximally-entangled states or 66.7% for non-maximally entangled ones. The more stringent limitation for reaching highest efficiency is in the quantum efficiency of detectors (at the moment around 70% for commercial avalanche photo-diodes detectors in visible spectrum). Nevertheless, technological progresses, in particular in superconductor based detectors (see for example [217,233,234]), could lead in the next years to a large improvement of these limits.

3.5 Applications of bright sources of entangled photon pairs

For the sake of completeness, it is worth to point out that the bright sources of entangled photon pairs described in the two former subsections have found very important applications in the developing field of Quantum Information and related areas of research (as Quantum Imaging and Quantum Metrology).

Quantum Information is an exponentially growing area of physics with promising technological applications addressed to study codification, elaboration and transmission of information by using specific properties of Quantum States.

A description of this field is largely beyond the purposes of this review and can be found, for example, in [5,6,7,8,9]. Here, we would like only to mention that entanglement and related quantum non-locality are the main resources exploited by these applications.

Among the main results of these studies we may quote the discovery that a quantum computer could efficiently solve problems that do not have efficient algorithms on a classical one (as factorisation in prime numbers), the ideation and practical implementation of absolutely secure protocols of communication (as already mentioned), the realisation of quantum communication schemes without a classical equivalent as teleportation (i.e. remote reconstruction of an unknown quantum state by sharing an entangled state and classical transmission of information of a measurement on the state to be teleported), dense coding (encoding two bits of information by manipulating only one subsystem of a shared entangled state) and quantum swapping (teleportation of entanglement), under shot-noise measurements in interferometers by using entangled states, etcetera¹⁷.

In particular, applications of sources of entangled photons to this field range from quantum cryptography [235,236,237,238,239,240], teleportation protocols [241,242,243,244,209], quantum imaging [245,246,247], linear quantum optical gates [248,249,250,251,252] (which are a fundamental element for building a quantum optical computer), quantum metrology [11,12,13], entanglement manipulation [253,254,255,256,257,258], quantum tomography [259,260], etc.

¹⁷ Of course these results are obtained in SQM, some of them should be revised in HVT framework and in particular if hidden variables were (eventually partially) accessible.

3.6 Tests of local realism by equalities

Beyond Bell inequalities Local Realism can also be tested by measuring some specific product of observables for entangled states for which the results of SQM and LHVT are different.

A first example was given by Greenberger-Horne-Zeilinger [261,262].

Let us consider a three photon entangled state

$$\Psi_{GHZ} = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle|H\rangle + |V\rangle|V\rangle|V\rangle) \quad (65)$$

Rewriting the state by using the bases

$$|45\rangle = \frac{|H\rangle + |V\rangle}{\sqrt{2}} \quad | - 45\rangle = \frac{|H\rangle - |V\rangle}{\sqrt{2}} \quad (66)$$

$$|R\rangle = \frac{|H\rangle + i|V\rangle}{\sqrt{2}} \quad |L\rangle = \frac{|H\rangle - i|V\rangle}{\sqrt{2}} \quad (67)$$

one has

$$\begin{aligned} \Psi_{GHZ} = \frac{1}{2}(&|R\rangle|L\rangle|45\rangle + |L\rangle|R\rangle|45\rangle + \\ &|R\rangle|R\rangle| - 45\rangle + |L\rangle|L\rangle| - 45\rangle) \end{aligned} \quad (68)$$

or

$$\begin{aligned} \Psi_{GHZ} = \frac{1}{2}(&|45\rangle|45\rangle|45\rangle + |45\rangle| - 45\rangle| - 45\rangle + \\ &| - 45\rangle|45\rangle| - 45\rangle + | - 45\rangle| - 45\rangle|45\rangle) \end{aligned} \quad (69)$$

This state has some significant properties. First of all any individual or two-photon joint measurement is maximally random. Secondly, if one attributes the value $+1$ to $R, 45$ measurements and the value -1 to $L, -45$, then the state is such that the product of three measurements is always -1 . Thus once two measurements are known, the third can be inferred with certainty without performing it: it is therefore an element of reality according to EPR definition.

Let us then consider a measurement on the basis $45, -45$ for all the photons in the framework of a local realistic model. From Eq. 68 derives that whenever the result 45 (-45) is obtained for one photon, the other two must carry

opposite (identical) circular polarisations. Let us then consider the specific example where one measures a -45° polarisation both for photons 2 and 3. Since photon 3 has -45° polarisation, in a hidden variable framework photons 2 and 1 must have equal circular polarisations. On the other hand, since photon 2 has -45° polarisation, photons 1 and 3 must have equal circular polarisations as well. If circular polarisations are element of reality fixed by some hidden variable, then all the three photon must have identical circular polarisations, but if photons 2 and 3 have identical circular polarisations it follows that photon 1 has linear polarisation -45° , thus one can simultaneously measure the outcomes $-45^\circ, -45^\circ, -45^\circ$: but this is at variance with SQM result deriving from Eq. 69. Similar results are obtained for other outcomes as well. There is therefore a sharp difference between SQM and LHVT.

This result implies that if one makes three space-like separated measurements on a suitable entangled state on an opportune basis he obtains a completely different result according if SQM or LHVT are valid. Nevertheless, also in this case one cannot obtain a conclusive test if detection efficiency is not sufficiently high, i.e. detection loophole appears here as well [262,263]. The presence of detection loophole is substantially due to the fact that some specific subset of hidden variables can simply correspond to undetected events in presence of a certain measurement. Thus, if detection efficiency is not sufficiently high, the set of hidden variables which would give results at variance with SQM could simply correspond to undetected events. This limit is rather stringent requiring detector efficiency above 90.8% [262] if emission rate of particle triples is known or a ratio between triple and double coincidence rate above 75% in a general case [263].

Even if an experimental realization of GHZ test presents these problems concerning a conclusive test of local realism, it remains very interesting. In the last years GHZ entanglement has been realised¹⁸, both for the original version of three entangled particles [264,265] and for extensions to four photons [266]. Both the experiments gave results in agreement with SQM and at variance with LHVT within experimental uncertainties (see fig. 10).

Let us sketch how GHZ polarisation entangled state of Eq. 65 were generated [264,265]. The scheme consisted in transforming two pairs of polarisation entangled photons produced simultaneously in a type II crystal pumped by a high intensity UV 200 fs pulse into three entangled photons by using postselection [268]. More in details, in some rare event two entangled pairs $\frac{|H\rangle|V\rangle - |V\rangle|H\rangle}{\sqrt{2}}$ were produced by the same pulse. The selection of the desired state was then

¹⁸ An indication of GHZ effect was previously observed in a NMR experiment [267], however in this case the "experiment was performed on thermal states, [thus] though the spins in the experiment mimic the effects of entanglement, they are not in fact entangled."

obtained by inspecting a posteriori the four-fold coincidence recording obtained by the apparatus in fig. 11: the photon registered at detector T is always horizontally (H) polarized and thus its partner in b must be vertically (V) polarised. The photon reflected at the polarising beam splitter in arm *a* is always V, being turned into equal superposition of V and H by a $\lambda/2$ waveplate, and its partner in arm b must be H. Thus if all four detectors click at the same time, the two photons at detectors D_1 and D_2 must either both have been VV or HH. The photon at D_3 was therefore H or V, respectively. The indistinguishability of both cases was obtained by using narrow band filters (4 nm) to increase coherence time to about 500 fs. The observed outcomes (see fig. 10) agree very well with SQM predictions.

Extension of this scheme were then realised for entangling 4 [266,269] or 5 photons [270]. Even if the experimental difficulty in realising this state limits the real possibility of observing a violation of local realistic predictions, nevertheless these achievements are interesting since it has been shown (as hinted before) that violations of local realism become stronger with the increasing number *N* of entangled particles [120,271,272], in the sense that SQM can violate specific Bell inequalities by an amount growing with *N* and with a lower detection efficiency limit for a detection loophole free experiment.

Finally, let us notice that tests of local realism based on equalities were also proposed by Hardy (and others) for two-particle entangled states [273,274,275,276] (a previous demonstration was given for the six dimensional space of spin 1 particles [277]). In synthesis, this result can be obtained by considering a polarisation entangled state of the form $\alpha|H\rangle|H\rangle - \beta|V\rangle|V\rangle$. On photon 1 (2) one then performs polarisation measurements along one of the *n*+1 possible directions A_i (B_i) with $0 \leq i \leq n$, corresponding to project on the states $|A_i\rangle$ ($|B_i\rangle$). Hardy's theorem states that the propositions about joint probabilities, where $A_i = 1$ ($A_i = 0$) means that A_i has been measured with outcome A_i (its orthogonal):

$$\begin{aligned}
P_i &= P(A_i = 1, B_i = 1) \neq 0 \\
P(A_i = 1, B_{i-1} = 0) &= 0 & 1 \leq i \leq n \\
P(A_{i-1} = 0, B_i = 1) &= 0 & 1 \leq i \leq n \\
P(A_0 = 1, B_0 = 1) &= 0
\end{aligned} \tag{70}$$

for some specific choice of A_i, B_i lead to contradiction with local realism, but can be verified in SQM.

The experimental realisation of this scheme has been made by Rochester [278,279] and Rome [280,281,282] groups by using polarisation entangled photons with results in agreement with SQM.

In little more detail, in Ref. [278,279] equalities 70 are directly checked. Since,

due to experimental imperfections, one does not measure exactly zero results where expected, an estimate of the probability P_i is obtained from these data by using EPR arguments, showing how this estimate is fourteen time smaller than the measured one, contradicting local realism of about 45 standard deviations (0.0070 ± 0.0005 respect to 0.099 ± 0.002). A similar comparison is performed in Ref. [280] as well (with 14 standard deviations from the local realistic prediction); whilst in Ref. [281,282], to avoid problems associated with a *nullum experiment*, is tested an inequality on P_i obtained by associating Hardy theorem with Clauser-Horne inequality. Results are at variance with LR predictions (30,37,26 and 21 standard deviations for $i = 4, 5, 10, 20$, respectively [282]).

Nevertheless, also for Hardy's equalities the detection loophole reappears [283,284], requiring a 82.84% collection efficiency for a conclusive test with maximally entangled states (the same as for Bell inequalities). The experimental efficiency was of $\approx 10\%$ in Ref. [280]. A test of these equalities, always in agreement with SQM, was also realised with non-maximally entangled states [285].

In synthesis, experiments about local realism based on equalities represent a very sharp test of SQM against LHVT. None the less also in this case detection loophole appear as the strongest limitation toward a conclusive experiment. This limit is more difficult to overcome for GHZ scheme, whilst a resolute test of LR by Hardy's two particle scheme is in a situation analogous to Bell inequalities case.

3.7 LHVT built for surviving PDC experiments

As we have seen in the former paragraphs, the most recent PDC experiments have posed very strong constraints on the existence of LHVT. Nevertheless, some space for LHVT is still left by the detection loophole.

Let us briefly comment on some specific models that have been built in order to show explicitly how one can still build a LHVT within the limits of quantum optical tests of local realism¹⁹.

A first example of this kind of models appeared in Ref. [182] and was then excluded by more recent experiments. However, it gave a general scheme for building a LHVT exploiting detection loophole. The main idea is to consider not only 2 outcomes (± 1) of the measured variables, but three (± 1 , undetected) and to build a hidden variable distribution able to reproduce SQM

¹⁹ For the sake of completeness, it is worth to notice that recent attempts to build a LHVT non-violating Bell inequalities in general [286,287,288,289] have been shown to be incorrect [290,291,292].

predictions for the events where both the particles have been observed.

This scheme has then be applied, more recently, for building new LHVT models [293,294,295,296], with the specific purpose of giving examples of LHVT not yet excluded by present experiments.

In order to give an idea of how they are built, let us present a very simple one, built for a singlet state of two spin 1/2 particles [293]. Each particle is characterised by a hidden variable $\vec{\lambda}$ with a uniform a priori probability distribution and by its quantum state ρ . For the singlet state the vectors $\vec{\lambda}$ are opposite for the two particles. If the spin is measured along a direction \vec{a} , the outcome ± 1 is determined by the sign of the scalar product $(\langle \vec{\sigma} \rangle_\rho - \vec{\lambda}) \cdot \vec{a}$ where $\langle \vec{\sigma} \rangle_\rho$ denotes the expectation value of the Pauli matrix. Furthermore, one assumes that at one of the measurement apparatuses (A) an outcome is produced only with probability $|\vec{\lambda}_A \cdot \vec{a}|$. The resulting correlation function is

$$E(\vec{a}, \vec{b}) = \int d\vec{\lambda} \rho(\vec{\lambda}) |outcome\ produced| sign(\vec{\lambda} \cdot \vec{a}) sign(-\vec{\lambda} \cdot \vec{b}) = -\vec{b} \cdot \vec{a} \quad (71)$$

as in SQM. Single particle distributions are also correctly reproduced. The model can then be modified [293] for being symmetric for the two measurement apparatuses. Its validity requires an effective efficiency of less than 75% for both of them.

A more ambitious program was started by a British-Spaniard group with the hope to build a real alternative to Quantum Optics [297,298,299,300,301,302,303]. The main idea was that the probability distribution for the hidden variable is given by the Wigner function, which is positive for photons experiments. Furthermore a model of photodetection, which departs from quantum theory, is built in order to reproduce available experimental results.

A great merit of this model is that it gives a number of constraints, which do not follow from the quantum theory and are experimentally testable.

In particular, there is a minimal light signal level that may be reliably detected: a difference from quantum theory is predicted at low detection rates, namely when the single detection rate R_S is lower than

$$R_S < \frac{\eta F^2 R_c^2}{2Ld^2 \lambda \sqrt{\tau T}} \quad (72)$$

where η is the detection quantum efficiency, F is the focal distance of the lens in front of detectors, R_c is the radius of the active area of the non-linear medium where entangled photons are generated, τ is the coherence time of incident photons, d is the distance between the non-linear medium and the

photo-detectors, λ the average wavelength of detected photons. L and T are two free parameters which are less well determined by the theory: L can be interpreted as the active depth of the detector, while T is the time needed for the photon to be absorbed and should be approximately less than 10 ns, being, in a first approximation, the length of the wave packet divided for the velocity of light.

This prediction has recently been tested by an experiment measuring Clauser - Horne inequality with polarisation entangled photons with a strongly negative result for this model [224]. Further recent negative tests of this model, based on other specific predictions, can also be found elsewhere [304,305].

Finally, for the sake of completeness, it can be mentioned that very recently, one of the authors of the previous model presented a new LHV model [306], which does not have the same degree of development of the former one, but in its simplicity allows one to reproduce all Bell inequalities tests performed with polarisation entangled photons. In the same paper it is suggested that a test of the model can be performed by comparing the visibility:

$$V_a = \frac{N(0) - N(\pi/2)}{N(0) + N(\pi/2)} \quad (73)$$

with

$$V_b = \sqrt{2} \frac{N(\pi/8) - N(3\pi/8)}{N(\pi/8) + N(3\pi/8)} \quad (74)$$

where $N(\theta)$ are the coincidence counts measured on a polarisation maximally entangled photon state when the two polarizers are set to two angles differing of θ . In fact, in the model of Ref. [306]

$$V_b/V_a > 1 + \cos^2(\pi\eta/2) \left[V_b - \frac{\sin^2(\pi\eta/2)}{(\pi\eta/2)^2} \right] \quad (75)$$

is expected, result that can be violated in SQM. This prediction could probably be tested in a near future²⁰.

²⁰ Some first tests, based on previous data, did not produce a conclusive answer [306,307].

3.8 Test of local realism in Hilbert spaces with dimension larger than 2

In the last years it emerged that the use of higher dimension Hilbert spaces ($d > 2$, where states are dubbed qudits in analogy to the quantum information word qubit denoting two level systems), instead of the traditional $d = 2$ ones, can lead to a larger violation of Bell-like inequalities [124,125,126,271,308,309,310,311,312,313]. This result is related [314] to the the discovery that quantum communication based on qudits presents a higher security than the traditional qubit schemes [315,316,317,318,319,320].

More in details, concerning Bell inequalities, various studies were addressed to understand the limit quantum efficiency for a loophole-free test of local realism (LR) and the resistance to noise. For example, in Ref. [308] Bell inequalities with enhanced resistance to detector inefficiency were investigated. This is of particular interest since, as we have seen, the loophole due to low detection efficiency η of the detection apparatuses is the last unsolved problem for a conclusive test of local realism. The result was that the limit for the smallest detection efficiency η^* necessary for a loophole free test of LR, decreases for $d > 2$ maximally entangled states of a $1 - 2\%$ respect to the value $\eta^* = 82.84\%$ for $d = 2$ maximally entangled states with 2x2 number of settings of the detection apparatuses.

Later it was then shown [271] that for a specific hidden variable model differences between Quantum Mechanics (QM) and Local Realistic Theories (LRT) are observable up to $\eta^* > \frac{M_A + M_B - 2}{M_A M_B - 1}$, where M_A and M_B are the number of measurements available to the two experimenters sharing two subsystems of a general entangled state. An asymptotic result for large d was obtained in Ref. [309].

On the other hand, the resistance to noise of some specific Bell inequalities tested by using maximally entangled states generated by multiport beam splitters was investigated as well [124,125], showing how it increases with d . More in details, it was shown how considering a mixed state ($0 \leq F \leq 1$)

$$\rho = (1 - F)|\Psi\rangle\langle\Psi| + F\rho_{noise} \quad (76)$$

where ρ_{noise} is a diagonal matrix with entries equal to $1/9$, the threshold value of F for violating a Clauser-Horne inequality grows from 0.2929 for qubits, to 0.30385 for qutrits up to 0.3223 for states in a $d = 16$ Hilbert space.

In Ref. [125] it was also demonstrated that, for maximally entangled states, the limit detection efficiency decreases from 0.8285 for $d = 2$ up to 0.8080 for $d = 16$ (being 0.8209 for qutrits, $d = 3$). A specific Clauser-Horne like inequality was then proposed and investigated for the previous maximally

entangled system [311] (inequality that includes also the ones presented in [126]). Similar results concerning the resistance to noise of LR tests performed with qudits were obtained in Ref. [312] as well.

A further contribution came by showing [313], performing a numerical study of a generalized Bell inequality [311] on two specific examples (qutrits generated by tritter, three arms interferometer, or biphotons as qutrits), how also in the case of qutrits the use of non-maximally entangled states allows a reduction of the detection efficiency for a conclusive test of local realism respect to maximally entangled ones. Also a stronger reduction to noise was found ($F_{th} = 0.3216$ referring to Eq. 76). Nevertheless, this reduction of the requested state detection efficiency is smaller (from 0.8209 to 0.8139 for tritters and from 0.8505 to 0.7413 for biphotons) than what obtained for qubits.

Finally, it must also be acknowledged that recently some papers were addressed to the study of local realism with 3-4 qutrits. In Ref. [321,322] GHZ paradox was generalized to 3-4 qutrits, in Ref. [323] a Bell inequality for 3 qutrits was presented and in Ref. [324] a numerical study on violation of local realism for 3-4 qutrits was performed (indicating a stronger violation than for 3 qubits case).

All these results, beyond the large conceptual interest, have also stimulated new experimental tests based on qutrit photons entangled states.

A first realisation of qutrits was proposed and realised by Moscow group [325,326,327] by exploiting the superposition of three biphoton states produced in PDC (HH, HV, VV), but in this case no test of local realism was performed.

On the other hand, a first test of local realism was made by Bouwmeester and collaborators [328] by using the rotationally-invariant four photon state

$$\frac{1}{\sqrt{3}}(|HH, VV\rangle - |HV, VH\rangle + |VV, HH\rangle) \quad (77)$$

produced in type II PDC for violating Bell inequality for "spin 1" systems. The results $S = 2.27 \pm 0.02$ was obtained, in evident disagreement with LR $S < 2$ prediction (here and in the following always apart from detection loophole).

Another test of LHVt was then presented [329] based on the use of orbital angular momentum ($0, \hbar, -\hbar$) entangled photons generated by sending both correlated down-converted beams through holographic modules consisting of two displaced holograms, which project photons onto a specific superposition of Laguerre-Gaussian modes (describing specific orbital angular momentum states). Detection apparatuses are then made of holograms (suited for projecting into a specific LG mode) preceding photo-detectors. An extension of

Clauser-Horne-Shimony-Holt inequality to qutrits was observed to be violated of 18 standard deviations.

Finally, Geneva group [330] obtained a 24 standard deviations violation of local realistic predictions by measuring violation of an inequality proposed by Collins and others [126]. In this set-up bin-time entangled photon pairs were used (see fig. 12). They were created via a periodically poled waveguide and two tritters, i.e. balanced interferometers with three arms (a generalisation of the time-bin entanglement of Eq. 60). Tritters were then also used for analysing the entangled qutrits.

Some progresses toward realization of qudits in higher dimensional spaces (up to now without tests of local realism) can be found in Ref.s [331,332].

3.9 Tests of non-contextuality

Few years after the proposal of Bell inequalities, Bell [80] and Kochen-Specker [333] posed further limits to HVT showing that every non-contextual HVT cannot reproduce all the results of SQM, where non-contextuality is defined as the request that each observable has a value in an individual system that would give the result of a measurement regardless of which sets of mutually commuting observables we choose to measure it with.

The difference between Bell-Kochen-Specker theorem and Bell inequalities is that the first one rules out the assignment of non-contextual values to an arbitrary observable, whilst Bell inequalities rules out it even when it is restricted to cases in which it can be justified on the basis of locality. Thus, the Bell-Kochen-Specker theorem permits to eliminate non-contextual hidden variable theories (NCHVT) which form a subset of local realistic hidden variable theories, tested by Bell inequalities (a LHVT requires non-contextuality between observables when the measurements are space like separated, but in general can be contextual when they are not causally disconnected).

More in details Bell and Kochen-Specker considered a physical system of spin 1 and an arbitrary choice of three orthogonal directions a, b, c . The eigenvalues of the square spin components are 0, 1. Furthermore, the sum of them satisfy the following equation

$$S_a^2 + S_b^2 + S_c^2 = s(s + 1) = 2 \quad (78)$$

since we are dealing with particle of spin 1 ($s=1$).

Let us then consider a set of directions containing many different orthogonal triads. The three observables consisting of the squared spin components

along orthogonal triads commute and therefore they can be measured simultaneously. The values of such measurements (0 or 1) must satisfy the same constraint 78 as the observables themselves. Therefore, two of the values must be 1 and the other 0. The no hidden variable theorem is based on finding a quantum mechanical state for which the statistics for the results of measuring any three observables associated with orthogonal triads could not be realized by any distribution of assignments of 0 or 1 to any direction of the set, consistent with the constraint.

Bell gave a general demonstration [80], based on Gleason theorem [334], of the impossibility of satisfying this request. Independently, a year later Kochen and Specker [333] explicitly showed a finite set of directions (117) which do not satisfy it.

Therefore, in principle an experimental test of Bell-Kochen-Specker theorem could exclude every non-contextual HVT, requiring at least some observables of the theory to be context-dependent (however, it must be emphasized that for every HVT not all the observables are contextual). For the sake of completeness, it must be noticed that the experimental relevance of Bell-Kochen-Specker theorem was questioned [335,336] due to real imperfect laboratory experiments. A claim later confuted in Ref. [337,338,339]. However, a direct experimental realization of Bell-Kochen-Specker test of NCHVT disclosed to be rather difficult.

Tests of NCHVT were obtained [340] with polarisation entangled photons produced by parametric down conversion either by using a version of Greenberger-Horne-Zeilinger for NCHVT [341] (that reduces to only two particles test) and an "event ready" test of Bell inequality for only one particle (see related theoretical works in Ref. [342,343]). Both tests showed large violation of non-contextuality (of more than 300 and 170 standard deviations respectively). However, the relatively low collection efficiency, about 8%, imposed to invoke fair sampling assumption for these experiments as well.

Another test, directly based on the original theoretical proposal of [344,345], has been recently realised [346] by using polarisation and path of a single photon (a "heralded photon" produced by PDC) to form a two qubits system. Also here the data agree largely more with SQM (80%) than with NCHVT (20% of data sample). None the less, again detection loophole is not eliminated.

3.10 Other Quantum Optical experiments connected with local realism and quantum non-locality

Before concluding this section, we would like just to list some other recent quantum optical experiments, whose results are connected with the studies of

local realism.

A first interesting possibility is to realise a state that substantially reproduces the original EPR one. A first scheme was realized by Ou et al. [347,348], following the theoretical proposal of [349,350], by employing a subthreshold non-degenerate optical parametric oscillator to generate correlated amplitudes for signal and idler beams of light. The role of position and momentum variables is played by quadrature-phase amplitudes: the amplitudes of signal beams (X_s, Y_s) can be inferred from measurements of amplitudes (X_i, Y_i) of the spatially separated idler beam. The observed values of variances of inferred observables $\Delta_{inf}^2 X_s$ and $\Delta_{inf}^2 Y_s$ give $\Delta_{inf}^2 X_s \Delta_{inf}^2 Y_s = 0.70 \pm 0.01$ in agreement with EPR paradox $\Delta_{inf}^2 X_s \Delta_{inf}^2 Y_s < 1$, namely showing an apparent violation of Heisenberg uncertainty principle.

Recently, an EPR state was also realised [351] by producing position (x) - momentum (p) photon entangled states by means of type II collinear PDC. Observed product of variances of inferred x_S and p_S , $\Delta_{inf}^2 x_S \Delta_{inf}^2 p_S = 0.01$, dramatically violates EPR criterium.

Concerning EPR states it can also be mentioned that, after a paper [352] showing how the EPR state can violate Bell inequalities even if its Wigner function is positive (at variance with previous claims [353,354]), an experiment [355] was addressed, following the theoretical scheme of [356], to test Bell inequalities by homodyne measurements on states produced by a pulsed nondegenerate optical parametric amplifier. A violation larger than 7 standard deviation was observed (of course apart from the various loopholes present in this experiment as well [355]).

Another interesting experiment recently realized [357] concerns the demonstration of quantum non-locality at single particle level.

The original idea [358] consisted in generating by a beam splitter an entangled state between a single photon state $|1\rangle$ and the vacuum $|0\rangle$

$$|\Psi_{BS}\rangle = \frac{|1\rangle_1|0\rangle_2 + i|0\rangle_1|1\rangle_2}{\sqrt{2}} \quad (79)$$

where subscripts 1, 2 denote the two exits of the beam splitter, both addressed to a second beam splitter where they are combined with a local oscillator (LO) for performing a homodyne detection, whose phase $\vartheta_{1,2}$ is the local parameter. The exits $c_{1,2}, d_{1,2}$ of the beam splitters on beam 1, 2 are then all measured by photo-detectors. Ref. [358] showed that correlation functions among intensities I

$$E(\vartheta_1, \vartheta_2) = \frac{\langle (I_{d_1} - I_{c_1})(I_{d_2} - I_{c_2}) \rangle}{\langle (I_{d_1} + I_{c_1})(I_{d_2} + I_{c_2}) \rangle} \quad (80)$$

violates Bell inequalities.

After the discussion of Ref.s [359,360,361] pointed out that this original proposal required additional assumptions, a new version of a test of local realism at single particle level, overcoming these problems, was proposed [362].

The experimental set-up of the Russian-Swedish group [357] substantially implements the scheme of [358], but in a version where single photon and LO co-propagate after the beam splitter, eliminating some problem of the original scheme.

In detail, the single photon was produced by type-I PDC in a LBO crystal pumped with a femtosecond-pulsed mode-locked titanium-sapphire laser after frequency doubling at 390 nm. A fraction of the initial laser beam at 780 nm was used as LO. The observed visibility ($91 \pm 3\%$ after background subtraction) is sufficient for Bell inequalities violation.

As a third example, let us consider a recent experiment [363] that realised a photon version of the proposal of Ref. [364] (originally based on electrons and positrons).

The original Hardy's proposal considers two interferometers, one for electrons and the other for positrons, arranged in such a way that two of their arms intersect. If both particles are at this intersection at the same time they annihilate: this implies in a local realistic model, where particles have defined trajectories, that a certain output of the two interferometers can never appear. On the other hand, this output has non-vanishing probability in SQM.

The scheme realised experimentally [363] uses indistinguishable photons produced in PDC as substitute of electron and positron and photon bunching at a beam splitter as the annihilating interaction. An inequality relating output probabilities valid for LHVT is then tested. Since this inequality is violated of 12 standard deviations, always apart from detection loophole (that appears here as well), the experiment disagree with LHVT prediction.

Finally, another interesting possibility, recently investigated, is the one of entangling a relative high number of photons, generating in this way a superposition of two "quasi-classical states" (a "Schrödinger kitten") [365,366,367,368,369,370] with the hope of better understanding the macro-objectivation process. Entangled states of few photons have been effectively obtained [368,369,370,371]. However, recently entanglement of more clearly "macroscopic" systems, as SQUID, has been obtained as well [372]. Here it is also worth to mention recent works where photon entanglement has been transferred to plasmons (involving approximately 10^{10} electrons) and then back to photons [373,374] that have still shown a Bell inequalities violation [373].

Concluding this section, we would like to address the interested reader to the papers [375,376,377,378] for further experiments, and comments on them, somehow connected with quantum non-locality and local realism.

4 Test of local realism with other physical systems than photons

As we have discussed in the previous paragraphs, the largest part of tests of local realism performed up to now have been realised by using photons, since entangled photons pairs are relatively easy to produce, simple measurement schemes are available and photons can be easily propagated for long distances. However, a conclusive experiment has not yet been realised mainly due to insufficient quantum efficiency of single-photon detectors, even if relevant progresses in this sense have been obtained in the last years.

Therefore, it remains the interest for investigating the possibilities of using other physical systems. Among them we will consider in the next paragraphs the ones more discussed in the literature and most interesting according to our opinion: mesons and ions. Nevertheless, it is worth to mention that relevant results were obtained also with other systems as:

i) Polarisation correlation of 1S_0 proton pairs produced in nuclear reactions [25] (discussed in subsection 2.8).

ii) Neutrons interferometry [379], with $S = 2.051 \pm 0.019$. Detectors had high efficiencies, but one had large losses in interferometers that made impossible to eliminate the detection loophole. Furthermore, entanglement was generated between two different degrees of freedom in a single particle (spatial and spinor part of the wave function): thus, obviously, the two measurements were not separated.

iii) Atoms entangled in a superposition involving two circular Rydberg states produced by single photon exchange in high Q cavity [380]. Purity (≈ 0.63) was too small for a test of local realism: further experimental improvements are needed before this scheme could really be interesting for this purpose.

iv) Single atom entangled with single photon [381]. Where atomic, hyperfine levels of a trapped $^{111}\text{Cd}^+$ ion, and photonic, polarisation, degrees of freedom are probabilistically entangled following a spontaneous emission of a photon from an atomic excited state. A value $S = 2.203 \pm 0.028$ was observed. Since photon detection takes place approximately 1.1 meters away from the atom and detection of atomic degree of freedom takes $125\mu\text{s}$, locality loophole is far from being solved. Also detection efficiency is rather low due to small acceptance angle and transmission loss ($\sim 1\%$), low quantum efficiency of

detectors ($\sim 20\%$) and restriction of the excitation probability to $\sim 10\%$ for suppressing multiple excitations.

4.1 Tests of Bell inequalities with mesons

In the last years many papers [382,383,384,385,386,387,388,389,390,391,392,393,394,395,396,397,398,399] have been devoted to study the possibility of making local realism tests by the use of pseudoscalar meson pairs as $K\bar{K}$ or $B\bar{B}$ ²¹. In fact, if the pair is produced by the decay of a particle at rest in the laboratory frame (as the ϕ at *Daφne*), the two particles can be easily separated to a relatively large distance allowing a space-like separation of the two subsystems and permitting an easy elimination of the locality loophole. Furthermore, very efficient particle detectors are available leading to the hope of easily eliminate detection loophole. Finally, a very low noise is expected as well.

These proposals are based on the use of entangled states of the form :

$$|\Psi\rangle = \frac{|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle}{\sqrt{2}} = \frac{|K_L\rangle|K_S\rangle - |K_S\rangle|K_L\rangle}{\sqrt{2}} \quad (81)$$

where $|K^0\rangle$ and $|\bar{K}^0\rangle$ are the particle and antiparticle related by charge conjugation and composed by a quark of flavour d with an anti-strange \bar{s} and a \bar{d} with a s respectively. Whilst mass eigenstates are

$$|K_L\rangle = \frac{p|K^0\rangle + q|\bar{K}^0\rangle}{\sqrt{|p|^2 + |q|^2}} \quad (82)$$

and

$$|K_S\rangle = \frac{p|K^0\rangle - q|\bar{K}^0\rangle}{\sqrt{|p|^2 + |q|^2}} \quad (83)$$

where $p = 1 + \varepsilon$ and $q = 1 - \varepsilon$ in terms of the (small) electroweak CP-violation parameter ε ($|\varepsilon| = (2.26 \pm 0.02)10^{-3}$). The K_L is the long living state, corresponding for $\varepsilon = 0$ to CP=-1 eigenstate ($|K_-^0\rangle$) for which 2 pions decay is forbidden, and the K_S is the short living state, corresponding for $\varepsilon = 0$ to CP=+1 eigenstate ($|K_+^0\rangle$), for which 2 pions decay is allowed²².

²¹ Properties of other neutral pseudoscalar mesons (as D, D_s and B_s) make them less interesting for this kind of studies.

²² see, for example, Ref. [402] for more details on K_0 phenomenology.

In general the small violation of CP symmetry can be neglected in considerations about local realism, except for a class of proposals that suggested to test local realism by measuring CP-violation parameters [382,383,384,385]. The original idea [382] was to build the Bell inequality

$$P(K_S^0, \bar{K}^0) \leq P(K_S^0, \bar{K}_+^0) + P(K_+^0, \bar{K}^0) \quad (84)$$

based on joint measurement probabilities of a K_S^0 , \bar{K}^0 and the (unphysical) K_+^0 with the state of Eq. 81 and (with some additional hypothesis on phases) to transform it in an inequality on the parameter ε ,

$$Re\{\varepsilon\} \leq |\varepsilon|^2 \quad (85)$$

One later work [383] obtained more stringent bounds independent of any phase convention, that are violated by measured data [403]. Similarly, an inequality on the CP-violation parameter ε' (see [403,402] for a definition) was obtained [384,385]. Nevertheless, it must be noticed that all these results require additional assumptions [386], such as the validity of relations derived in SQM in LHVT as well. Therefore, albeit giving an interesting connection between local realism and specific properties (CP violation) of electroweak lagrangian, they do not really represent a conclusive test of local realism.

Concerning other proposals, first of all it must be noticed that a simple hypothesis [404,389] where the state (81) collapses shortly after its production in two factorised states

$$|K_S\rangle|K_L\rangle, |K_L\rangle|K_S\rangle \quad (86)$$

(or similarly for other pseudoscalar mesons) is already excluded, see for example results of Ref. [405].

On the other hand, a simple test of local realism based on a correlation function defined such that it takes the value 1 when two or none \bar{K}^0 are identified and -1 otherwise, would not lead to a violation of Bell inequalities due to the specific values of $K^0\bar{K}^0$ mixing parameters [388].

Anyway, other Bell inequalities and hidden variable schemes can be considered. Nevertheless, the statement about high efficiency in detection of pseudoscalar mesons does not survive to a deeper analysis [406,407,408]. The main concern about this statement derives from the fact that in most experimental tests proposed up to now, one must tag the P or \bar{P} (where P denotes a pseudoscalar particle and \bar{P} its antiparticle) through its decay. This requires the selection of $\Delta S = \Delta Q$ semileptonic decays (i.e. the ones where the strangeness and charge changes of the hadrons are the same), which represent only a (small)

fraction of the total possible decays of the meson. For example, one has [403] the following branching ratios²³:

$$\begin{aligned}
BR(K_S^0 \rightarrow \pi^+ e^- \nu_e) &= (3.5 \pm 0.2)10^{-4} \\
BR(K_L^0 \rightarrow \pi^+ e^- \nu_e) &= 0.1939 \pm 0.0014 \\
BR(K_L^0 \rightarrow \pi^+ \mu^- \nu_\mu) &= 0.1359 \pm 0.0013 \\
BR(B^0 \rightarrow l^+ \nu_l X) &= 0.105 \pm 0.008 \\
BR(B^0 \rightarrow l^+ \nu_l \rho^-) &= (2.6 \pm 0.7)10^{-4} \\
BR(B^0 \rightarrow l^+ \nu_l \pi^-) &= (1.33 \pm 0.22)10^{-4}
\end{aligned} \tag{87}$$

where X means anything, l denotes a generic lepton (as e^- electron, μ^- muon) and ν_l its related neutrino, π denotes the pseudoscalar meson composed of u and/or d quarks and antiquarks.

Besides this problem, one has to consider experimental cuts on the energies of the decay products, which inevitably further reduce this fraction. Moreover, an additional part of the pairs is lost by decays occurring before the region of observation. Finally, most of these proposals involve the regeneration phenomenon²⁴, which introduces further strong losses; on the other hand if no choice of the measurement set up (e.g. the presence of the regeneration slab) is introduced, the space-like loophole cannot be really eliminated.

The result of these considerations is that one is unavoidably led to subselect a fraction of the total events. As one cannot exclude a priori hidden variables related to the decay properties of the meson and losses, one cannot exclude the sample to be biased and thus the detection loophole appears here too. This is in complete analogy with polarisation photons experiments, where the detection loophole derives by the fact that one can envisage losses related to the values of hidden variables that determine if the photon passes or not a polarisation (or another) selection. Namely, in a local realistic model the properties of a particle are completely specified by the hidden variables. Also for mesons decays and losses, in a LRT, can happen according to the values of the hidden variables (both in a deterministic or in a probabilistic way). States with different hidden variables can decay in different channels, with the condition that the branching ratios *averaged* over the hidden variables distribution reproduce the quantum mechanics predictions.

From this discussion follows that, for what concerns the experiments based on Bell inequalities [387,390,391,392,393,394,395,397], the same limits for the

²³ B denotes the pseudoscalar meson analogous to K where the quark s is substituted by the heavier quark b .

²⁴ i.e. the possibility of regenerating K_S from a K_L beam by exploiting different interaction amplitudes of K^0 and \bar{K}^0 with matter [402].

total efficiency previously discussed remain valid. As the total branching ratio in $\Delta S = \Delta Q$ semileptonic decays, Eq.s 87, is much smaller than 0.8284 (and the same happens for any other selection, as far as we know), this unavoidably implies that a loophole free test of Bell inequalities cannot be performed in this class of experiments. The eventual use of non-maximally entangled states, lowering the efficiency threshold to 0.67, does not substantially change the situation.

This problem does not appear in Ref. [387], however other additional hypotheses are needed (see Eq. 15 and discussion after Eq. 18 of [387]), and thus this proposal does not allow a general test of LRT as well.

Finally, it must also be noticed that the only observation of interference between the two terms of the entangled wave function, Eq. 81, as in Ref [409], does not exclude general LHV, for this feature can be reproduced in a general class of local realistic theories.

In summary, we can conclude that the proposed Bell inequalities measurement on pseudoscalar mesons pairs can not allow a conclusive test of local realism.

Nevertheless, they represent a prominent example of studying local realism with other physical systems than photons.

In this sense the recent experimental test of Bell inequalities [410] with a pair of $B^0\bar{B}^0$ mesons is very interesting. Briefly, an entangled state of the form 81 has been produced from $\Upsilon(4S)$ decay, the flavour has then been identified by reconstructing a semileptonic decay for one meson and from lepton tagging (a multidimensional likelihood method) for the other. After having normalised the correlation function to the undecayed pairs, an evident violation of CHSH inequality has been obtained, $S = 2.725 \pm 0.167_{stat} \pm 0.092_{syst}$ ²⁵.

4.2 Other tests of local realism with mesons

Let us then consider other proposals for testing local realism with mesons, not based on a Bell inequalities measurement. Two proposals of this kind have been recently advanced by F. Selleri and others concerning a $K^0\bar{K}^0$ [398] (very lately further developed in Ref. [397]) or a $B^0\bar{B}^0$ [399] system respectively.

In Ref. [398,411] a very general model is proposed, where the $K^0\bar{K}^0$ pair is local-realistically described by means of two hidden variables. One (λ_1) determines a well defined CP value, another λ_3 determines the times when a

²⁵ For a specific discussion of loopholes of this experiment, next to general arguments of Ref.s [406,407], see Ref. [396].

sudden jump between defined valued of strangeness S (i.e. between a K^0 and a \bar{K}^0) happens. These jumps are necessary for explaining the observed K^0 - \bar{K}^0 oscillations (in the model a further parameter λ_2 driven by λ_3 and determining S is introduced as well).

Denoted by K_1 the state with $CP=1$, $S=1$, K_2 the state with $CP=1$, $S=-1$, K_3 the state with $CP=-1$, $S=1$ and K_4 the state with $CP=-1$, $S=-1$, the initial state can be, with probability $1/4$, in anyone of the states $CP = \pm 1$, $S = \pm 1$. Each of these pairs give, in the local-realistic model (LRM), a certain probability of observing a $\bar{K}^0 \bar{K}^0$ pair at proper times t_a and t_b ($\neq t_a$) of the two particles, which are [399]:

$$\begin{aligned} P_1[t_a, t_b] &= [E_S(t_a)Q_-(t_a) - \rho(t_a)] \cdot E_L(t_a)p_{43}(t_b|t_a) \\ P_2[t_a, t_b] &= [E_S(t_a)Q_+(t_a) + \rho(t_a)] \cdot E_L(t_a)p_{43}(t_b|t_a) \\ P_3[t_a, t_b] &= [E_L(t_a)Q_-(t_a) + \rho(t_a)] \cdot E_S(t_a)p_{21}(t_b|t_a) \\ P_4[t_a, t_b] &= [E_L(t_a)Q_+(t_a) - \rho(t_a)] \cdot E_S(t_a)p_{21}(t_b|t_a) \end{aligned} \quad (88)$$

corresponding to an initial state with K_1 on the left and K_4 on the right, K_2 on the left and K_3 on the right, K_3 on the left and K_2 on the right and K_4 on the left and K_1 on the right, respectively.

In Eq. 88, we have introduced $E_S(t) = \exp(-\gamma_S t)$ and $E_L(t) = \exp(-\gamma_L t)$, where (in units $c = \hbar = 1$) $\gamma_S = (1.1163 \pm 0.0007)10^{10}s^{-1}$ and $\gamma_L = (1.9305 \pm 0.0058)10^7s^{-1}$ denote the decay rate of K_S and K_L [403].

Furthermore, the function Q_{\pm} are defined through:

$$Q_{\pm} = \frac{1}{2} \left[1 \pm \frac{2\sqrt{E_L E_S}}{E_L + E_S} \cos(\Delta m t) \right] \quad (89)$$

where $\Delta m = (0.5292 \pm 0.0010)10^{10}s^{-1}$ is the mass difference $M_{K_L} - M_{K_S}$. We have also introduced the symbol $p_{ij}(t_a|t_b)$ for denoting the probability of having a K_j at time t_b conditioned to have had the state K_i at time t_a . From Ref. [398,399] one has:

$$p_{21}(t_b|t_a) = E_S^{-1}(t_a)[p_{21}(t_b|0) - p_{21}(t_a|0) \cdot E_S(t_b - t_a)] \quad (90)$$

and

$$p_{43}(t_b|t_a) = E_L^{-1}(t_a)[p_{43}(t_b|0) - p_{43}(t_a|0) \cdot E_L(t_b - t_a)] \quad (91)$$

where

$$p_{21}(t|0) = E_S(t)Q_-(t) - \rho(t) \quad (92)$$

and

$$p_{43}(t|0) = E_L(t)Q_-(t) + \rho(t). \quad (93)$$

Finally, $\rho(t)$ is a function not perfectly determined in the model (see discussion in Ref. [398,399]), but which is limited by

$$\begin{aligned} -E_S Q_+ &\leq \rho \leq E_S Q_- \\ -E_L Q_- &\leq \rho \leq E_L Q_+ \end{aligned} \quad (94)$$

The LRM probability of observing a $\bar{K}^0 \bar{K}^0$ pair is given by the sum of the four probabilities of Eq. 88 multiplied by 1/4. Since it is rather different from the quantum mechanical prediction,

$$\begin{aligned} P_{QM}[\bar{K}^0(t_a), \bar{K}^0(t_b)] &= \frac{1}{8} [e^{[-(\gamma_S t_a + \gamma_L t_b)]} + e^{[-(\gamma_L t_a + \gamma_S t_b)]} - \\ &\quad 2e^{[-(1/2)(\gamma_S + \gamma_L)(t_a + t_b)]} \cos(\Delta m(t_a - t_b))] \end{aligned} \quad (95)$$

an experimental measurement of this quantity could represent a conclusive test of local realism [398,399].

However, it has been shown [408] how also in this case detection loophole manifests itself. When the total detection efficiency is lower than 1, the different probabilities can contribute in different ways since the hidden variables, which determine the passing or not the test, could also be related to the decay properties of the meson pair and losses. As for the cases previously discussed, the hidden variables values completely characterise the state, and thus, in principle, even its decay properties. If this is the case, different coefficients a_i can multiply the four probabilities. One has therefore:

$$\begin{aligned} P[\bar{K}^0(t_a), \bar{K}^0(t_b)] &= 1/4 \cdot [a_1 P_1[t_a, t_b] + a_2 P_2[t_a, t_b] + \\ &\quad a_3 P_3[t_a, t_b] + a_4 P_4[t_a, t_b]] \end{aligned} \quad (96)$$

The freedom of the choice of these parameters permits therefore to reproduce the quantum mechanical prediction (the same happens for B mesons [407]), as shown by an explicit numerical calculation in Ref. [406]. Thus also this scheme, albeit interesting, is not suitable for a conclusive test of local realism.

Finally, let us consider a recent proposal [412], which seems to overcome the objections of Ref. [406,407]. It is based on generating a non-maximally entan-

gled state by placing a regenerator slab on the path of kaons pairs produced in ϕ decays, where r is the regeneration parameter, or by considering kaons produced in $p\bar{p}$ annihilation at rest, where r measures the relative strength of p to s wave channels and on selecting the K_S surviving after $T = 10\tau_S$

$$|\Phi\rangle = \frac{R|K^0\rangle|K^0\rangle + R|\bar{K}^0\rangle|\bar{K}^0\rangle + (2-R)|\bar{K}^0\rangle|K^0\rangle - (2+R)|K^0\rangle|\bar{K}^0\rangle}{2\sqrt{2+|R|^2}} \quad (97)$$

where $R = -\text{rexp}[-(i\Delta m + (\Gamma_S - \Gamma_L)/2)T]$ and $R' = -r^2/R$, Δm is the difference between K_L and K_S masses, whilst Γ_S , Γ_L their respective decay widths.

Then one selects, with an appropriate choice of parameters, the case $R = -1$, for which SQM predicts the probabilities of joint detection:

$$\begin{aligned} P_{QM}(K^0, \bar{K}^0) &= \eta\eta'/12 \\ P_{QM}(K^0, K_L) &= 0 \\ P_{QM}(K_L, \bar{K}^0) &= 0 \\ P_{QM}(K_S, K_S) &= 0 \end{aligned} \quad (98)$$

where η and η' are the detection efficiencies for identifying K^0 and \bar{K}^0 , respectively.

In a hidden variable model with distribution $\rho(a)$ of the hidden variable (or variables set) a the probability of observing a K^0 to the left and \bar{K}^0 to the right is:

$$P_{LR}(K^0, \bar{K}^0) = \int da \rho(a) p_l(K^0|a) p_r(\bar{K}^0|a) = \eta\eta'/12 \leq \int_{A_{0,\bar{0}}} da \rho(a) \quad (99)$$

where $p_l(K^0|a)$, $p_r(\bar{K}^0|a)$ are the single kaon probabilities of detecting a K^0 to the left and \bar{K}^0 to the right respectively and $A_{0,\bar{0}}$ is the set of hidden variables corresponding to a K^0 to the left and \bar{K}^0 to the right.

In a LHVT the necessity of reproducing Eq. 99 requires that if a K^0 (\bar{K}^0) is observed to the left (right) a K_S propagates to the right (left). Thus one has $p_l(K_S|a) = 1$, $p_r(K_S|a) = 1$ if a belongs to $A_{0,\bar{0}}$. This is at variance with SQM predictions 99, since:

$$P_{LR}(K_S, K_S) = \int da \rho(a) p_l(K_S|a) p_r(K_S|a) \geq \int_{A_{0,\bar{0}}} da \rho(a) \quad (100)$$

This result seems therefore to show that even if the detection efficiency of strangeness eigenstates is small, nevertheless LHVT can be tested without any additional hypothesis if the K_S , K_L can be determined with perfect efficiency. Experimentally, this determination is realised by looking to the decays between time $T_0 = 10\tau_S$ ($\tau_S = 89.35 \pm 0.08$ ps is the mean life of K_S), where the state 97 is produced, and time T_1 such that a negligible K_L contribution to decays used for tagging K_S is still expected in the interval.

However, also for this scheme detection loophole reappears [408], in fact the former discussion does not consider that in a deterministic theory hidden variables could also fix the channel of decay and the precise time of decay.

Therefore, Eq. 100 becomes:

$$P_{LR}(K_S, K_S) = \sum_C \int_{T_0}^{T_1} dt \sum_{C'} \int_{T_0}^{T_1} dt' \int da \rho(a) p_l(K_S|a) p_r(K_S|a) p_{l,C}(t|a) p_{r,C'}(t'|a) \quad (101)$$

where C and C' run over the different decay channels (allowing an identification of K_S) and $p_{i,C}(t|a)$ gives the probability of the $i = l, r$ (left, right) meson to decay into the channel C at time t .

Let us now consider how this modifies the discussion concerning Eq. 100. If the efficiencies η and η' of K^0 and \bar{K}^0 detection were high, the situation would not substantially change. However, unluckily, they are very small. The method for this detection consists [409] in looking to distinct interaction of K^0 and \bar{K}^0 with matter (interaction and therefore identification that in principle could depend on the hidden variables value): this led in Ref. [409] to the identification of 70 unlike-strangeness events and 19 like-strangeness events over $8 \cdot 10^7$ analysed events! Thus, the few K^0 - \bar{K}^0 identified events could easily correspond to K_S which would not have decayed in the temporal window that allows their identification and thus could not contribute to the integral 101 (on the other hand if η and η' were sufficiently large this would not be possible). Furthermore, it must also be considered that $|K_S\rangle$ and $|K_L\rangle$ are not perfectly orthogonal, for $\langle K_S|K_L\rangle = 3.3 \cdot 10^{-3}$ [403]. This means that a fraction of K_S in the LHVT belongs to a hidden variable set corresponding to decays characteristic of K_L (and vice versa), giving a further fraction of states not contributing to $P_{LR}(K_S, K_S)$ as defined in Eq. 101. Albeit very small, this contribution cannot be neglected due to the small value of $P_{LR}(K^0, \bar{K}^0) = \eta\eta'/12$.

Summing up, the small fraction of simultaneously identified left K^0 and right \bar{K}^0 could be easily accounted for, in a LHVT, by a fraction of K_S that does not decay in an identifiable form, since they decay outside of the temporal window $T_0 < T < T_1$ or in an allowed channel for K_L , as three pions or pion,

lepton, neutrino.

The numerical results of Ref. [408] show that a loophole free test of local realism in this scheme requires $\eta \approx \eta' > 9\%$ that seems very difficult to be obtained experimentally. Therefore, also with this scheme an ultimate test of Local Realism will be hardly obtained.

In summary, from the discussion of the two last subparagraphs, we can therefore reach the conclusion that, even if representing an interesting possibility to study local realism with other physical systems beyond photons, at the moment the suggested LHVT tests with pseudoscalar mesons cannot lead to a conclusive result due to specific forms in which detection loophole reappears here as well.

4.3 Bell inequalities experiments with ions

Finally, let us consider experiments based on entangled ions, which are rather interesting since very high detection efficiencies can be reached. On the other hand, as we will see, in this case the elimination of locality loophole is hard.

For the sake of exemplification, we will describe in the following a recent experiment performed at NIST [54]²⁶.

In synthesis, the set-up consisted in generating an entangled state of the form

$$|\Psi_2\rangle = \frac{|0\rangle|0\rangle - |1\rangle|1\rangle}{\sqrt{2}} \quad (102)$$

by coupling two levels of $2S_{1/2}$ ground state of $^9\text{Be}^+$ ions ($|0\rangle = |F=1, m_f=-1\rangle$, $|1\rangle = |F=2, m_f=-2\rangle$) by a coherent stimulated Raman transition. The two laser beams used to drive the transition had a wavelength of 313 nm and a difference frequency near the hyperfine splitting of the states, $\omega_0 \cong 2\pi \cdot 1.25$ GHz. The fidelity $F = \langle \Psi_2 | \rho_{exp} | \Psi_2 \rangle$ of the generated state (described by a density matrix ρ_{exp}) respect to the theoretical one $|\Psi_2\rangle$ was measured to be of 88%.

After having produced the state $|\Psi_2\rangle$ a Raman pulse of short duration (~ 400 ns) was applied again transforming the state of each ion i as

²⁶ Incidentally, also this technique allowed entanglement of more particles (4) [413]. Anyway no test of local realism was performed in this case.

$$\begin{aligned}
|0\rangle_i &\rightarrow \frac{(|0\rangle_i - ie^{i\phi_i}|1\rangle_i)}{\sqrt{2}} \\
|1\rangle_i &\rightarrow \frac{(|1\rangle_i - ie^{i\phi_i}|0\rangle_i)}{\sqrt{2}}
\end{aligned} \tag{103}$$

where the phase ϕ_i is the phase of the field driving the Raman transitions and represents the parameter used in the Bell inequality test. In the experiment this phase was set either by varying the phase of the radio-frequency synthesizer that determines the Raman difference frequency (controlling the total phase) or by motion Δx_j of a ion along the trap axis, which gives a phase change $\Delta k \Delta x_j$ where Δk is the difference wave vector (controlling the differential phase).

Finally, the state of an ion was probed with circularly polarized light from a 'detection laser beam'. During this detection pulse, ions in the state $|1\rangle$ scatter many photons, whilst ions in the state $|0\rangle$ scatter very few photons. For two ions one can have three cases: zero ions bright, one ion bright, two ions bright. In the one-ion-bright case Bell's measurement requires only knowledge that the states of two ions are different and not which one is bright.

The measured CHSH inequality violation was $S = 2.25 \pm 0.03$, in agreement with SQM predictions once imperfections of the experiments are kept into account.

The total detection efficiency was $\approx 98\%$, where the 2% decrease was mainly due to misidentification of a bright ion due to imperfect circular polarisation of detection light that can cause $|1\rangle \rightarrow |0\rangle$ transitions. No fair sampling hypothesis was therefore needed.

Nevertheless, in this set-up the measurements on two ions not only are not space-like separated, but even one has a common measurement on the two ions. Therefore, one cannot absolutely speak of non-locality in this case.

In principle the two ions could be separated, but this looks a very difficult experimental task.

In conclusion, experiments with ions allow a very high detection efficiency, but the realisation of an ultimate experiment with this technique looks not easy because of the difficulty in having space-like separated measurements.

5 Some conclusions about Local Hidden Variable Theories

In summary, in the previous paragraphs we discussed how EPR argument originated the debate about the possible existence of a local realistic theory,

where physical predictions are deterministic and quantum probabilities derive by our ignorance of some hidden variable fixing all the properties of a physical system. The great beauty of Bell theorem resides in allowing a general test of the possible existence of whatever local hidden variable theory against standard quantum mechanics. In 70's various experiments based on cascade atomic decays gave strong indications against LHVT, culminating in the Orsay experiment closing also (with some minor caveats) locality-loophole (i.e. the request of having space-like separated measurements). However, these experiments had very low detection efficiency leaving open the possibility that the selected subsample was not a faithful representation of the whole one (detection loophole).

The use of entangled photons produced by parametric fluorescence allowed in the 90's to approach a detection-loophole free test of LHVT and to close without any objection the locality one. Furthermore, also other interesting experiments about (or connected with) local realism were performed with this source of entangled states.

Even if, on account to these experiments, little space remains for LHVT, nevertheless, due to the very fundamental relevance of a conclusive test, the work for realising new experiments on Bell inequalities is still going on.

Progresses in photon-detector could eventually allow one to close every loophole with PDC entangled photons. On the other hand, entangled ions are detected with very high efficiency, but to obtain separated measurements is hard. Entangled mesons have suscitated a certain interest as well, but detection loophole reappears in a form hard to be eliminated. Furthermore, other physical systems have also been used for experimental realisations of tests of LRT, but for the moment, with less success.

Finally, before concluding this part, for the sake of completeness we would like also to quote some recent theoretical proposals [414,415,416,417,418,419,420] for eliminating the detection loophole that have not yet found an experimental implementation.

Altogether, the hope to reach in the next years a conclusive answer about local realistic alternatives to standard quantum mechanics seems to be rather reasonable.

6 Non-local Realistic Theories

As we have seen, experiments on Bell inequalities indicate that, a part from the detection loophole, no Local Realistic Theory can represent a valid al-

ternative to SQM. However, it is not excluded the possibility of considering non-local (or eventually lacking other classical properties, e.g. counterfactual definiteness [421]) Realistic Theories (NLRT), namely theories where the action on a subsystem influences superluminally (eventually instantaneously) the whole system. Of course, in order to preserve compatibility with special relativity, this influence must be built such not to introduce faster than light communication.

In particular two non-local hidden variable theories (NLHVT) have suscitated a large interest: the Nelson stochastic [422] mechanics and, even more, the de Broglie-Bohm model [423,424]. In the following subsections, we will describe dBB model and then summarize the main ideas of Nelson's one.

For the sake of completeness, it can be noticed that also the model of Ref. [199,200] (quoted in sub-section 4.2), and the Eberhard one [425] are non-local in the sense that they require some superluminal hidden communication. However, it has been shown that for them arises the problem of possibility of faster than light communication [426,427]. In more generality it has been demonstrated that this problem appears for every model where one has a finite speed superluminal hidden communication in a preferred frame without local hidden variables [427].

Also Bohm-Bub [76] model can be interpreted [428,429] as a non-local hidden variables theory where there are both local hidden variables and a superluminal connection (mixed model), but with finite velocity²⁷.

The problem of faster than light communication arises in this case [429] as well. None the less, since it has attracted a certain interest, in the following lines we rapidly sketch, for the sake of exemplification, the way how this model is built for a two-dimensional Hilbert space.

To a general wave function of the form

$$|\psi\rangle = \psi_1|s_1\rangle + \psi_2|s_2\rangle \quad (104)$$

where $|s_1\rangle, |s_2\rangle$ are eigenstates of a certain observable S representing a basis of the two-dimensional Hilbert space, one associates a vector in the dual space

$$\langle\xi| = \xi_1\langle s_1| + \xi_2\langle s_2| \quad (105)$$

The components of $\langle\xi|$ are the hidden variables. They are supposed to be randomly distributed on the hypersphere of unit radius in Hilbert space defined by $\sum_i |\xi_i|^2 = 1$ (assumption that allows us to reproduce usual QM average

²⁷ For some generalisations of Bhom-Bub model see [59,429,430,431].

for all observables). Furthermore, in order to describe wave function collapse, one postulates that, in addition to Schrödinger equation, during a measurement process of the observable S the following evolution equations operate ($R_i = \frac{|\psi_i|^2}{|\xi_i|^2}$) :

$$\frac{d\psi_1}{dt} = \gamma(R_1 - R_2)\psi_1|\psi_2|^2 \quad (106)$$

$$\frac{d\psi_2}{dt} = \gamma(R_2 - R_1)\psi_2|\psi_1|^2 \quad (107)$$

which lead to $|\psi\rangle \rightarrow e^{i\phi_1}|s_1\rangle$ if $R_1 > R_2$ and to $|\psi\rangle \rightarrow e^{i\phi_2}|s_2\rangle$ if $R_2 > R_1$ (i.e. to the wave function collapse). Some indications of possible tests of the model against SQM in repeated measurements have also been given in [76,429] (see [432] for an earliest experiment and [429,430] for criticisms on it).

Before concluding this introduction to NLRT, it is still worth to shortly mention studies concerning what superluminal classical communication could reproduce quantum correlations. Whilst an unlimited instantaneous communication between subsystems allows one to reproduce SQM results, a communication among a limited number of subsystems (the so called hybrid models) does not generally allow this result. For example, Svetlichny has shown that for the three subsystems case superluminal communication between arbitrary pairs cannot reproduce the results of measurements performed on quantum states [433]. This result was then generalised to an arbitrary number of subsystems [272]. Finally, in Ref. [434] this last outcome was extended to a larger class of classical communication (in the Graphs theory language of [434] to partially paired graphs respect to separable graphs considered in [433,272]). On the other hand, concerning bipartite systems, efforts concentrated on studying the number of bits of the superluminal communication needed for reproducing the quantum correlations [435,436,437], reaching the result that one bit suffices for maximally entangled states [438], whilst this resource is insufficient for non-maximally entangled ones [439].

6.1 The de Broglie - Bohm theory

A particularly interesting example of NLHVT is the theory of de Broglie - Bohm [423,424]²⁸ for the , where the hidden variables are the positions of the particles, which evolve according to a Hamilton-Jacobi equation that includes also a quantum potential term that, as we will see, is derived from Schrödinger

²⁸ we do not discuss here the history of this model and which were the precise positions of de Broglie and of Bohm etc., but we adopt, as became conventional, the name of de Broglie-Bohm for the version presented in the following.

equation and allows one to reproduce (for an ensemble of particles) probabilistic predictions of SQM.

In order to derive the explicit form of this equation of motion, let us consider the usual Schrödinger equation

$$i\hbar \frac{\partial \Psi(x, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \Delta + V(x) \right] \Psi(x, t) \quad (108)$$

and write the wave function as:

$$\Psi = R(x, t) \cdot \exp[iS(x, t)/\hbar] \quad (109)$$

separating real and imaginary part one obtains the "Hamilton-Jacobi" equation

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \quad (110)$$

and the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (v\rho) = 0 \quad (111)$$

where

$$\rho(x, t) = |\Psi|^2 = R(x, t)^2 \quad (112)$$

and

$$v = \frac{\nabla S}{m} \quad (113)$$

is the velocity of the particle given by the "guidance equation", which constitutes the real novelty of the dBB model and has no equivalent in SQM.

$$Q = -\frac{\hbar^2}{2m} \frac{\Delta R}{R} \quad (114)$$

can be considered as a "quantum" potential to be added to the usual one, V , in the Hamilton-Jacobi equation. Incidentally, let us notice that, since R appears both in numerator and denominator, Q is unchanged when the wave function is multiplied by an arbitrary constant, i.e. it does not depend on the

strength of Ψ , but only on its *form*: this has relevant consequences for the *interpretation* of the *field* Ψ and of the quantum potential. We do not discuss this point (see [3,440]), but simply notice that in [3] the quantum potential is interpreted as a form of information (*active information*).

The particle can thus be thought to move according to the combined effect of the quantum and external potential:

$$m \frac{d^2x}{dt^2} = -\nabla(V + Q) \quad (115)$$

where the position x is therefore the hidden variable, which, by assumption, cannot be determined experimentally (without inducing a collapse of the wave function) and therefore remains always unknown (as we will detail more precisely at the end of this subsection). The positions of an ensemble of particles are distributed according to $\rho(x, t)$.

Let us notice the ingenuity of this construction: the quantum potential is such that if one assumes that the particles are initially statistically distributed in x according to $\rho(x, 0) = |\Psi(x, 0)|^2$ (the "quantum equilibrium" postulate), then the evolution implies that at the time t they exactly reproduce the statistical distribution predicted by quantum mechanics $\rho(x, t) = |\Psi(x, t)|^2$.

Measuring the properties of a statistical ensemble of particles no difference appears respect to SQM. However, now every single particle has a well defined position and follows a well defined trajectory. The non-epistemic nature of SQM probabilities becomes epistemic and completely related to our ignorance of hidden variable values.

It must be emphasized that one cannot even in principle know the hidden variables values (the coordinates of the single particles): if this would be possible one can show that this would lead to some different prediction from SQM. Furthermore, in this case, one could transmit faster than light signals. In order to understand how, let us consider a singlet state as (1) sent to two Stern-Gerlach apparatuses. In the dBB theory a correct description of spin (generalizable to many body case as well) can be derived by considering Dirac equation and its non relativistic limit. The precise description of this formulation is however rather ample and beyond the purposes of this review: the interested reader can find it in Ref.s [3,440]. However, the main result of this formulation is that once the Pauli equation is derived as non-relativistic limit of Dirac one, the guidance equation 113 must be substituted by

$$v = \frac{j}{\rho} \quad (116)$$

where $j = \Psi^\dagger \alpha \Psi$ is the Dirac current and ρ the probability density. By using this guidance equation follows that the spin can be interpreted as depending on a context dependent circulatory motion of the trajectories. A particle with spin is still regarded as a simple point particle whose only intrinsic property is its position and that has a velocity determined by the guidance condition 116, which in the non-relativistic approach includes a circulatory contribution determining the spin. In the interaction with a Stern-Gerlach apparatus a wave packets splits according to this circulatory contribution; the result in each individual case depends on the precise initial conditions, i.e. on the value of the hidden variable x .

The former discussion rapidly summarises the correct description of spin in dBB model. Nevertheless, in order to have some clear hint about why is even in principle impossible to know the hidden variable, in the following we present the simplified argument of Ref.[176] for a singlet state of two spin one half particles each addressed to a Stern-Gerlach measurement device. Simplifying a bit, the context dependent spin of a particle crossing a Stern-Gerlach apparatus is determined by the hidden variable x and by the direction of the magnetic field B : the particle cannot cross the horizontal plane, if the position of the particle is in the upper (lower) semiplane it is deviated toward the high (low) and one attributes to the particle a spin up (down). If the field is reversed one would arrive to the opposite conclusions concerning the spin. The value of the spin depends therefore on the general context in which the measurement is performed (i.e. dBB theory is a contextual one). Nevertheless, the dBB reproduces, by construction, the results of QM: thus if one particle is found with spin up the other must have spin down. Let us now imagine that Alice can have access to the hidden variable before performing the Stern-Gerlach test, with her measurement preceding the Bob's one. If she observes the particle in the upper semiplane she leaves the magnetic field as it is and the particle (deviated toward the high) is attributed to have spin up, on the other hand if she sees the particle in the lower semiplane, she reverses the gradient of the magnetic field and the particle (deviated toward the low) is again attributed to have spin up. On the other side Bob leaves his apparatus untouched. But now he will receive only spin down particles which will all be deviated in the same semiplane: he immediately knows that Alice is performing a measurement and thus receive an instantaneous information from her. The access to the hidden variable value would therefore allow a super-luminal transmission: thus, compatibility with special relativity requires that hidden variables can not be known even in principle.

The contextuality ²⁹ we have discussed in the former example constitutes the

²⁹ in the more refined description of spin hinted before, the two spin one half EPR particles initially do not have any circular motion corresponding to spin [3], it emerges, for both, only when one of the particles enters a Stern-Gerlach device: a

main problem of the dBB model (but not for his supporters): the value of some observables (as the spin) depends on the global context in which the measure is performed. The quantum potential depends on all the particles included in the wave function. Even an action on an extreme far particle can lead to relevant changes on the measured properties of a particle entangled with this. Let us notice that we can still define isolated systems grouping the particle in sets, such that the members of different sets are non-interacting and non-entangled.

In general, the result of contextuality is that not all the observables are objective, namely completely characterised by the hidden variable of the theory, but most of them depend on the whole context in which the measurement is performed. However, one can always identify a complete system of observables, which is non-contextual: in the case of dBB, this is given by the positions (on the other hand we have seen as spin is contextual).

In order to give some other explicit example of how dBB theory works, let us now consider the double slit Young experiment in the framework of this theory.

Each particle follows a well defined trajectory and crosses one of the slits. The shape of its trajectory varies according to its initial position in the slit (see fig. 13). The trajectories are determined by the quantum potential and give origin, when a statistical ensemble of particles is considered, to the SQM interference figure. Let us notice that the closing or opening of the second slit immediately affects the particle at the first slit: here is another example of the contextuality and non-locality of the theory. Of course, also in SQM the wave function is affected by the opening or closing of the slit, but in this case no trajectory is defined and one cannot assert the particle to cross one or the other of the two slits.

It is worth to remind that contextuality is a common property to every HVT, which completely reproduces the statistical result of QM. As we have seen this assertion is contained in the Bell and Kochen-Specker theorems.

Finally, even without entering into the details, it must also be noticed that for dBB theory Lorentz invariance is not valid for the individual event (even if any violation would not be detectable experimentally, where a statistical sample of events would be necessary) [3]. This is related to the existence of the instantaneous non-local effect of the quantum potential on the particle in a well defined position on its trajectory.

In summary, dBB model has been built for reproducing the results of SQM for a statistical ensemble of particles, but attributing to every single particle tra-

clear example of contextuality and non-locality of dBB theory.

jectories evolving according a Hamilton-Jacobi equation, containing beyond the classical potential also a second (the "quantum") one. This theory has been deeply investigated in various aspects (as many-body systems, interpretation of transition processes, tunnel phenomena, etc.) and large efforts have been devoted to built a relativistic version of it. For a detailed description of all these studies we address to specific books [3,440] and to some recent works (see [441,442,443,444] and references therein). Here, we would like only to hint at a recent relevant progress concerning bosonic fields. Whilst, for fermionic fields, dBB interpretation is of fermionic particles "guided" by fermionic fields, for bosonic fields an analogous interpretation was difficult to be realised and usually they were treated differently from fermions assuming directly bosonic fields as basic elements of the theory [3]. However, recently a coherent theory of bosonic particles guided by bosonic fields has been developed [445], based on Kemmer-Duffin formalism [446], reestablishing parallelism between fermions and bosons. Let us also mention that some variants of dBB model have been presented where the wave function describes a real physical wave propagating in space-time (substantially dating back to the original de Broglie proposal [423,447]), or with "exotic" density distributions [448], or with modified guidance equation (within constraints of relativistic covariance) [449].

6.2 Nelson Stochastic Model

As a further example of NLHVT let us consider the stochastic mechanics introduced by Edward Nelson in 1966 [422]. The foundation of this model is based on two basic hypotheses. The first assumes that dynamical systems follow trajectories (also here positions are therefore the hidden variables) perturbed by an underlying Brownian motion. The second is a particular form of the second principle of dynamics, where the classical acceleration is replaced by a suitable form of stochastic acceleration.

It can be shown that the basic equation of stochastic mechanics can be derived from variational principles, in complete analogy with classical mechanics, based on the same classical action, but exploiting stochastically perturbed trajectories as trial trajectories. With this purpose let us consider (following the presentation of Ref. [450]) a classical Lagrangian

$$L(q, \frac{dq}{dt}) = \frac{1}{2}m \left(\frac{dq}{dt} \right)^2 - V(q) \quad (117)$$

and introduce a diffusion process, with density $\rho(x, t)$, satisfying the forward Ito stochastic equation (for details on stochastic equations see for example the

book [451])

$$dq(t) = v_{(+)}(q(t), t)dt + dw(t) \quad (118)$$

Let us also introduce a diffusion constant ν , pertaining the Brownian motion $dw(t)$, and forward ($v_{(+)}$), backward ($v_{(-)}$) and osmotic velocities (u) defined by

$$2m\nu = \hbar \quad (119)$$

and

$$v = \frac{1}{2}(v_{(+)} + v_{(-)}) \quad (120)$$

$$u = \frac{1}{2}(v_{(+)} - v_{(-)}) = \nu \nabla \log \rho \quad (121)$$

The density ρ satisfies the continuity equation:

$$\partial_t \rho(x, t) = -\nabla \cdot (\rho v) \quad (122)$$

Once the average stochastic action (E denoting an overall average)

$$A(t_0, t_1; q) = \int_{t_0}^{t_1} E \left[\frac{1}{2} m \left(\frac{\Delta q}{\Delta t} \right)^2 - V(q) \right] dt \quad (123)$$

is introduced, it is found that stationarity of it under arbitrary small variations $\delta v_{(+)}$, with the constraint $\delta \rho(., t_1) = 0$, requires the current velocity field to satisfy the Hamilton-Jacobi equation

$$mv = \nabla S \quad (124)$$

where

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \quad (125)$$

and

$$Q = \frac{-\hbar^2}{2m} \frac{\Delta \sqrt{\rho}}{\sqrt{\rho}} \quad (126)$$

a form equivalent to Schrödinger equation 108 (similarly to what described in the former subsection) when the wave function is rewritten as

$$\Psi = \sqrt{\rho(x, t)} \cdot \exp[iS(x, t)/\hbar] \quad (127)$$

Furthermore, introducing the forward and backward transport operators

$$D_{\pm} = \partial_t + v_{\pm} \cdot \nabla \pm \nu \Delta \quad (128)$$

and the Nelson acceleration

$$a(x, t) = \frac{1}{2}(D_+v_- + D_-v_+) \quad (129)$$

one can rewrite the dynamical equation 125 in the Newton form

$$ma = -\nabla V. \quad (130)$$

In summary, Nelson's model is a scheme where particles have well defined trajectories (as in classical mechanics), but with a stochastic component of motion. When this last part is built suitably the equations of motion become equivalent to Schrödinger equation. Thus, in principle, SQM and Nelson's model are equivalent from a predictive point of view, differing only in the interpretation, since in Nelson's model one attributes to a single particle a trajectory and therefore an hidden variable (the position). Finally, it must be emphasized that, due to the form of the term 126, the transition probabilities for a subsystem can depend on the properties of a far away (space-like separated) different subsystem [452], showing the non-locality of this model.

Extension to quantum field theory have been elaborated [453,454,455] (a similar approach can also be found in Ref. [456,457]).

The interested reader can find specific reviews of this model in Ref.s [458,459]. Other HVT where particles follow definite trajectories, as in dBB and Nelson's models, can be found in section 4 of Ref. [77] (where in general various other HVT models are quoted).

6.3 *Experimental tests of NLHVT against SQM*

As we discussed in subsection 2.4, Bell inequalities are based on the locality hypothesis and thus do not concern NLHVT. One can therefore pose the question of how to compare these theories with SQM or whether, at least for

some of them, there is a perfect equivalence from a predictive point of view with SQM.

A first experimental test of non-local HVT was proposed [447] and realised [460,461] for the variant of dBB theory where the wave function is not only assumed to give "guidance condition" by quantum potential, but it is a real physical wave propagating in space-time. This means that if the wave function splits on different paths, even after detection of the particle on one path, the waves on the other paths ("empty waves") can manifest physical effects (such as interference).

This theoretical proposal [447], following and developing some former ones [462,463,464], considered a source producing a pair of photons by PDC that are addressed to a modified Mach-Zender interferometer (see fig. 14).

In little more detail, two identical photons produced at the same time (for example in PDC) are addressed to two different beam splitters part of the same arm of a Mach-Zender interferometer. If one assumes that the photon is composed of a localized particle and a real wave propagating according with d'Alembert equation and that detection of a particle in one of possible channels does not induce collapse of the wave on other possible channels, then for lossless beam splitters with transmission and reflection coefficients t, r respectively, the outgoing waves Ψ_1 and Ψ_2 from the other two beam splitters of the interferometer are, in terms of waves Φ_1 and Φ_2 , associated to signal and idler photon, respectively

$$\begin{aligned}\Psi_1 &= t^2\Phi_1 + r^2t\Phi_2 + tr^2e^{i\alpha}\Phi_2, \\ \Psi_2 &= t^2e^{i\alpha}\Phi_2\end{aligned}\tag{131}$$

where α is the phase difference between the two paths.

The coincidence probability is then given by:

$$P(D_1, D_2) \propto |\Psi_1|^2|\Psi_2|^2 = |t|^4|\Phi_2|^2[|t|^4|\Phi_1|^2 + 2|r|^4|t|^2(1 + \cos \alpha)|\Phi_2|^2]\tag{132}$$

which depends on the phase difference between the two optical lengths of the interferometer: an effect essentially due to overlapping in the path BS₃-D₁ of signal wave function with empty wave generated by the idler photon going through BS₁ and BS₄.

On the other hand, the usual quantum optical results, easily calculated by combining annihilation operators in the interferometer, does not present any dependence on the phase difference between the two optical lengths of the interferometer.

This experiment has been realised by Rochester group [460] with an equivalent scheme showing perfect agreement with Quantum Optics result.

Since some doubts were arisen [465] about the real superposition of wave packets in the interferometer, a second version of the experiment was then realised answering to these objections [461,466]. The perfect agreement with SQM results represented a conclusive negative test of empty wave models.

Here we can also hint at the proposal of a possible experimental test for the "wavelet" version of de Broglie model compared to SQM [467].

Moreover, before concluding this section, we would like to mention possible tests of the conventional dBB model against SQM. As previously described de Broglie-Bohm model was built to be completely equivalent to Standard Quantum Mechanics. Nevertheless, some authors have suggested that the constraints due to the existence of trajectories could lead to differences between the two, which could eventually be investigated by an experimental test (see for example [468,469,470,471,472,473,474]): namely dBB and SQM are not simply different interpretations of the same theory, but really different theories³⁰.

In particular, a recent proposal of two teams [471,472,473,474] of a variant of double slit experiment gave rise to a certain interest. In little more detail, this scheme considers two identical particles crossing at the same time a double slit each at a specific slit: the calculation of trajectories show that they never cross the double slit symmetry axis and therefore no coincidences are expected in the same semiplane at variance with SQM result. Even if the debate about the validity of this theoretical prediction or its specificity for some variant of dBB model (or an application of this scheme to Nelson Stochastic model as well) is not settled yet [472,479,480,481,482,483,484], it is worth to mention that a recent experimental realisation of this scheme [485,486] has obtained results in perfect agreement with SQM (see Fig. 15 for the scheme of this set-up and experimental data).

Finally, discussing possible experimental evidences that could be obtained against dBB model, it is also worth to quote some results [487] showing that in specific cases (as interferometers with which way detectors) Bohm trajectories can be macroscopically at variance with the recorded track (in this sense they are called "surrealistic" in this paper)³¹. Nevertheless, this achievement is not considered really compelling from dBB model supporters [489,490,491],

³⁰ We do not enter here in the complex debate of when two theories can be considered different, of what means different interpretations of the same theory, etc. [2,3,4,475,476,477,478].

³¹ For later developments of this idea, as "weak" (i.e. such to change minimally the measured state) and "protective" (weak and adiabatic) measurements, see [488] and Ref.s therein.

who argue that in these cases a position detector does not really measure the real position of the particle (on the Bohm trajectory) but is affected by the non-local quantum potential.

7 Determinism at Planck scale?

Before concluding this review, it is interesting to mention that in the last years a further revival of interest for hidden variables theories has arisen starting from a paper of the Nobel laureate 't Hooft where it was suggested that determinism could reappear at Planck scale [78]. A point of large relevance into the program of reaching a completely unified theory including also gravity.

The main idea of this proposal is that at Planck scale physical systems (including gravity) are described by a deterministic theory, but at larger scales we have loss of information (for dissipation): "quantum states" are equivalence classes of the deterministic states, the loss information are the hidden variables.

In order to have an idea of how this scheme works, let us present a simple example from Ref. [78].

Let us consider a discrete system with four states e_1, e_2, e_3, e_4 whose deterministic evolution, after every time step, is

$$e_1 \rightarrow e_2, e_2 \rightarrow e_1, e_3 \rightarrow e_3, e_4 \rightarrow e_1, \quad (133)$$

Even if evolution is deterministic, it can be useful to introduce a Hilbert space in order to handle it probabilistically. This evolution is described by the (non-unitary) operator:

$$U = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (134)$$

However, after a short lapse of time only the states e_1, e_2, e_3 survive. Thus one can simply erase the state e_4 and considering e_1, e_2, e_3 as the "quantum" system with a unitary evolution described by the upper 3x3 part of U , Eq. 134.

This system may therefore be described in three equivalence classes:

$$E_1 = \{e_1\}, E_2 = \{e_2, e_4\}, E_3 = \{e_3\}, \quad (135)$$

with unitary evolution operator (H is a Hamiltonian operator)

$$U' = e^{-iH} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (136)$$

This simple model shows how, if information is allowed to dissipate, one has to define quantum states as equivalence classes of states, where two states are equivalent iff, some time in the future, they evolve into one and the same state. Equivalence classes that form a smaller set of the complete set of primordial states that one starts off with. A continuum model is then presented [78] as well.

Summarizing, the main idea presented in Ref. [78] is therefore that a quantum state is defined as an equivalence class of states all having the same future. These equivalence classes are described by observables, that in quantum terminology correspond to a complete set of commuting operators at every time ("beables" following Bell terminology [353]). A beable describes the information available on a system at Planck scale. All other quantum operators are "changeables" (they do not commute with all beables). A physical system can evolve deterministically at Planck scale, but a probabilistic theory can derive at larger (spatial) scales due to loss of information. If this is the case, Bell inequalities experiments with photons, electrons etc. would not be resolute for testing this deterministic theory since photons, electrons etcetera do not represent true degrees of freedom of it (i.e. correspond to "changeables" and not to "beables").

A further indication of how this mechanism could work was given later [492], showing how a quantum harmonic oscillator can emerge from a pair of classical oscillators with dissipation.

On the same line in Ref. [493] it was shown how quantum field theory in (3+1)- dimensional Minkowsky space could emerge as low energy limit of a (4+1)-dimensional classical gauge theory. Here the fifth dimension would be the hidden variable. Incidentally, let us notice that the local dynamical theory in more dimensions would generate fundamentally non-local effects in lower dimensional space: in order to examine such a model a Bell inequalities test should be performed at the scale of the transition between classical dissipative dynamics to quantum dynamics.

Later developments of these proposals and related arguments can be found in Ref.s [494,495,496,497,498,499,500,501,502] (see also the very recent paper [503] where hidden variables are two additional time dimensions).

Finally, it can also be mentioned a further work [504], where it is shown that a orthomodular lattice of propositions characteristic of quantum logic can be constructed for manifolds in Einstein's theory of general relativity, where both state preparation and measurement apparatus constrain results of experiments (future observations represent hidden variables).

Altogether these highly speculative proposals arose a new interest for the search of a deterministic theory beyond quantum mechanics. Only further studies will be able to show if this ideas could have interesting developments.

8 Conclusions

In this review, after a general introduction to the researches on local realistic alternatives to standard quantum mechanics, we have presented the most recent results (with a larger emphasis to experimental ones) about these studies.

This problem is of the utmost importance for our understanding of the nature, namely for clarifying if nature is intrinsically probabilistic or quantum mechanical probabilities derive by our ignorance of some hidden parameters and therefore an underlying deterministic theory is conceivable.

The transition from the XIX century point of view of a perfectly deterministic nature described by classical mechanics to the actual quantum mechanical point of view of a probabilistic world has been difficult and largely debated and still many points at the very foundations of quantum mechanics need a clarification. It should also be noticed that whilst this new paradigm has been largely accepted by physicists community, its assimilation in diffused culture is still rather limited.

Furthermore, as we have seen, a conclusive experiment falsifying in an absolutely uncontroversial way local realism is still missing.

More in details, for what concerns **local** hidden variable theories, since Bell theorem it is known that a general answer about their validity can be given by an experiment. In the last 40 years various experiments have addressed this problem: strong indications favouring standard quantum mechanics have been obtained, but no conclusive experiment has yet been performed, mainly due to low detection efficiencies that demand for additional assumptions. Nevertheless, relevant progresses toward this goal have been made in the last ten

years and in my opinion an ultimate experiment could not be far in the future.

However, we have to acknowledge that this personal opinion is not generally shared: on one side some authors deem that the large amount of experimental data disfavouring Local Hidden Variable Theories is already largely sufficient for excluding them, on the other side other authors (see for example [505]) claim that the lack of a conclusive experiment after 40 years and in particular the "resistance" of detection loophole to be eliminated could point out a practical impossibility of falsifying local realism. These discussion largely involve methodological questions [475,476,477,478] which are amply beyond the purposes of this paper.

Even if Local Realistic Theorem will be excluded by an ultimate Bell inequalities experiment, **non-local** hidden variable theory will still remain a possible alternative to standard quantum mechanics. Following the discussion of last sections, in our opinion a large space still remains for relevant contributions to study this possibility both from a theoretical and an experimental point of view.

In general interesting developments in this area can be expected in the next years, also in connection with the related developing field of quantum information.

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References

- [1] P. Ghose, Testing Quantum Mechanics on a New Ground, (Cambridge Univ. Press, Cambridge 1999).
- [2] G. Auletta, Foundations and interpretation of quantum mechanics, (World Scientific, Singapore 2000)
- [3] D. Bohm and B.J. Hiley, The undivided Universe, (Routledge, London, 1993).
- [4] R. Omn  s, The intepretation of quantum mechanics, (Princeton, 1990);
- [5] H. Zbinden et al., Appl. Phys. 67 (1998) 743 and references therein.
- [6] A. Ekert and R. Jozsa, Rev. Of Mod. Phys. 68 (1996) 733 and references therein.
- [7] M. Keyl, Phys. Rep. 369 (2002) 431 and references therein.
- [8] M.A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information (Cambridge Univ. press, Cambridge, 2000).

- [9] D. Bouwmeester, A. Ekert and A. Zeilinger, *The Physics of Quantum information* (Springer, Berlin, 2000).
- [10] S.F. Huelga et al., *Phys. Rev. Lett.* 79 (1997) 3865;
- [11] A. Migdall, *Physics Today* (1999), 41, and refs. therein.
- [12] G. Brida et al., *Jour. Mod. Opt.* 47 (2000) 2099; *Phys. Rev. A* 70 (2004) 032332; *Journ. of. Opt. Am Soc. B.* in press (February 2005 issue) and ref.s therein.
- [13] A. Valencia, G. Scarcelli and Y. Shih, quant-ph 0407204.
- [14] V. Giovannetti, S. Lloyd and L. Maccone, *Science* 306 (2004) 1330 and ref.s therein.
- [15] A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.* 47 (1935) 77.
- [16] J.S. Bell, *Physics* 1 (1965) 195.
- [17] L.R. Kasday, J.D. Ullman and C.S. Wu, *Nuovo. Cim. B* 25 (1975) 633.
- [18] G. Faraci et al., *Lett. Nuov. Cim.* 9 (1974) 607.
- [19] A.R. Wilson et al., *J. Phys. G.* 2 (1976) 613.
- [20] M. Bruno et al., *Nuovo Cim. B* 40 (1977) 142.
- [21] S.J. Freedman and J.F. Clauser, *Phys. Rev. Lett.* 28 (1972) 938.
- [22] R.A. Holt and F.M. Pipkin, unpublished (1973).
- [23] J.F. Clauser, *Phys. Rev. Lett.* 36 (1976) 1223.
- [24] E.S. Fry and R.C. Thompson, *Phys. Rev. Lett.* 37 (1976) 465.
- [25] M. Laméhi-Rachti and W. Mittig, *Phys. Rev.* 14 (1976) 2543.
- [26] M.H. Tixier et al., *Phys. Lett. B* 212 (1988) 523.
- [27] A. Aspect et al., *Phys. Rev. Lett.* 49 (1982) 1804.
- [28] J. G. Rarity, and P. R. Tapster, *Phys. Rev. Lett.* 64 (1990) 2495.
- [29] J. Brendel et al. *Eur.Phys.Lett.* 20 (1992) 275.
- [30] P. G. Kwiat et al, *Phys. Rev. A* 41 (1990) 2910.
- [31] W. Tittel et al, quant-ph 9806043.
- [32] P.G. Kwiat, A.M. Steinberg and R.Y. Chiao, *Phys. Rev. A* 47 (1993) R2472.
- [33] R.Y. Chiao, P.G. Kwiat and A.M. Steinberg, *Quant. Semicl. Opt.* 7 (1995) 259.
- [34] M.A. Horne, A. Shimony and A. Zeilinger, *Phys. Rev. Lett.* 62 (1989) 2209.
- [35] D.V. Strekalov et al., *Phys. Rev. A* 54 (1996) R1.
- [36] P.R. Tapster, J.G. Rarity and P.C.M. Owens, *Phys. Rev. Lett.* 73 (1994) 1923.

- [37] W. Tittel et al., Phys. Rev. Lett. 81 (1998) 3563.
- [38] W. Tittel et al., Phys. Rev. A 59 (1999) 4150.
- [39] G. Weihs et al., Phys. Rev. Lett. 81 (1998) 5039.
- [40] H. Zbinden et al., J. Phys. A: Math. Gen. 34 (2001) 7103.
- [41] A. Stefanov et al., Phys. Rev. A 63 (2001) 022111.
- [42] A. Stefanov et al., Phys. Rev. A 67 (2003) 042115.
- [43] J. Brendel et al., Phys. Rev. Lett. 82 (1999) 2594.
- [44] R.T. Thew et al., Phys. Rev. A 66 (2002) 062304.
- [45] P.G. Kwiat et al., Phys. Rev. Lett. 75 (1995) 4337.
- [46] C. Kurtsiefer, M. Oberparleiter and H. Weinfurter, Phys. Rev. A 64 (2001) 023802.
- [47] C. E. Kuklewicz et al., Phys. Rev. A 69 (2004) 013807.
- [48] G. Di Giuseppe et al., Phys. Rev. A 66 (2002) 013801.
- [49] M. Aspelmeyer et al., Science 301 (2003) 621.
- [50] A. G. White et al., Phys. Rev. Lett. 83 (1999) 3103.
- [51] G. Brida, M. Genovese, C. Novero and E. Predazzi, Phys. Lett. A 268 (2000) 12; Proc. of QCCM 3 (Capri 2000), eds P. Tombesi and O. Hirota (Kluwer Ac. Press, New York, 2001) p.399.
- [52] E. Santos, Phys. Lett. A 212 (1996) 10.
- [53] L. De Caro and A. Garuccio Phys. Rev. A 54 (1996) 174.
- [54] M.A. Rowe et al., Nature 409 (2001) 791.
- [55] E. Schrödinger, Proc. of Camb. Phyl. Soc. 31 (1935) 555.
- [56] G. Ghirardi, L. Marinatto and T. Weber, J. Stat. Phys. 108 (2002) 49.
- [57] L.E. Ballentine, Rev. Mod. Phys. 42 (1970) 358.
- [58] V. Karakostas, Journ. for Gen. Phyl. of Science 35 (2004) 283.
- [59] F. J. Belinfante, A survey of hidden-variables theories (Pergamon Press, Toronto, 1973).
- [60] H. Everett, Rev. of Mod. Phys. 29 (1957) 454.
- [61] B. Van Fraassen, E. Beltrametti and B. Van Fraassen ed., in Current Issues in Quantum Logic Plenum, New York 1982;
- [62] D. Dieks, Found. of Phys., 19 (1989) 1397,

- [63] D. Dieks, Phys. Lett. A 142 (1989) 439; 265 (2000) 317.
- [64] R. Griffiths, Phys. Rev. Lett. 70 (1993) 2201.
- [65] W.H. Zurek, Phys. Today (1991) 36.
- [66] M. Gell-Mann and J.B. Hartle, Phys. Rev. D 47 (1993) 3345.
- [67] J.C. Cramer, Rev. Mod. Phys. 58 (1986) 647.
- [68] A. Zeilinger, Found. of Phys. 29 (1999) 631.
- [69] C.A. Fuchs, quant-ph 0205039.
- [70] G. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D 34 (1986) 470.
- [71] P. Pearle, quant-ph 9901077 and ref.s therein.
- [72] G.J. Milburn, Phys. Rev. A 44 (1991) 5401.
- [73] E.P. Wigner, The scientist speculates - An anthology of Partly-Baked ideas, ed. I.J. Good (Heinemann, London 1961) p. 284.
- [74] R. Bonifacio, Nuov. Cim. 114 B (1999) 473.
- [75] P. Ghose, Found. of Phys. 32 (2002) 871; ibidem 893.
- [76] D. Bohm and J. Bub, Reviews of Modern Physics 38 (1966) 453.
- [77] D. Home and M.A.B. Whitaker, Phys. Rep. 210 (1992) 223.
- [78] G. 't Hooft, Class. Quant. Grav. 16 (1999) 3263.
- [79] J. von Neumann, Mathematische Grundlage der Quanten-mechanik (1932).
- [80] J.S. Bell, Rev. Mod. Phys. 38 (1966) 447.
- [81] G. Hermann 1935, unpublished.
- [82] J.M. Jauch and C. Piron, Helv. Phys. Acta 36 (1963) 827.
- [83] J. Clauser et al., Phys. Rev. Lett. 23 (1969) 880.
- [84] A. Fine, Phys. Rev. Lett. 48 (1982) 291.
- [85] I. Percival, Phys. Lett. A 244 (1998) 495.
- [86] J.S. Bell, in "Foundations of Quantum Mechanics", ed. by B. D'Espagnat, (New York, academic Press, 1971).
- [87] J.F. Clauser and M.A. Horne, Phys. Rev. D 10 (1974) 526.
- [88] I. Pitowsky, Quantum Probability, Quantum Logic, Lecture Notes in Physics 321, (Heidelberg, and Springer, 1989); Mathematical Programming 50 (1991) 395.
- [89] D. Collins and N. Gisin, J. Phys. A 37 (2004) 1775.

- [90] C. Sliwa, Phys. Lett. A 317 (2003) 165.
- [91] E.P. Wigner, Am. J. Phys. 38 (1970) 1005.
- [92] J.F. Clauser and A. Shimony, Rep. Prog. Phys. 41 (1978) 1881.
- [93] R.A. Bertlmann, Found. of Phys. 20 (1990) 1191.
- [94] F. Selleri and G. Tarozzi, Riv. Nuov. Cim. 4 (1981) 1.
- [95] N.D. Mermin, Rev. Mod. Phys. 65 (1993) 803.
- [96] H.P. Stapp, Phys. Rev. D 3 (1971) 1303.
- [97] H.P. Stapp, Found. of Phys. 10 (1980) 767.
- [98] H.P. Stapp, quant-ph0010047.
- [99] E. Santos, Phys. Lett. A 115 (1986) 363.
- [100] S.L. Braunstein and C. Caves, Phys. Rev. Lett. 61 (1988) 662.
- [101] S.L. Braunstein and C. Caves, Annals of Phys. 202 (1990) 22.
- [102] B. Schumacher, Phys. Rev. A 44 (1991) 7047.
- [103] B. d’Espagnat, Phys. Rev. D 11 (1975) 1424.
- [104] D. Gutkowski and G. Masotto, Nuov. Cim. 22 B (1974) 121.
- [105] F. Selleri, Found. of Phys. 8 (1978) 103.
- [106] A. Fahmi and M. Golshani, Phys. Lett. A 306 (2003) 259.
- [107] A. Afriat, Journ. Mod. Opt. 50 (2002) 1063.
- [108] G. Grasshoff et al., quant-ph 0312176.
- [109] S. Massar et al., Phys. Rev. A 66 (2002) 052112.
- [110] X. Wu and W. Huang, Phys. Lett. A 276 (2000) 12.
- [111] V. Capasso, D. Fortunato and F. Selleri, Int. J. Mod. Phys. 7 (1973) 319.
- [112] N. Gisin, Phys. Lett. A 154 (1991) 201.
- [113] N. Gisin and A. Peres, Phys. Lett. A 162 (1992) 15.
- [114] B.S. Tsirelson, Lett. in Math. Phys. 4 (1980) 93.
- [115] L. Khalfin and B.S. Tsirelson in: Symposium on the Foundations of Modern Physics 1985, ed.s P. Lahti and P. Mittelstaedt (Singapore, World scisntific, 1985), p. 441.
- [116] L. Landau, Found. of Phys. 18 (1988) 449.

- [117] M. Hillery and B. Yurke, *Quant. Opt.* 7 (1995) 215.
- [118] S. Popescu and D. Rohrlich, *Phys. Lett. A* 166 (1992) 293.
- [119] D. Gosal et al., *Phys. Rev. A* 70 (2004) 042106.
- [120] N.D. Mermin, *Phys. Rev. Lett.* 65 (1990) 1838.
- [121] M. Ardehali, *Phys. Rev. A* 46 (1992) 5375.
- [122] A.V. Belinskii and D.N. Klyshko, *Phys. Usp.* 36 (1993) 653.
- [123] V. Scarani and N. Gisin, *Journ. Phys. A* 34 (2001) 6043.
- [124] D. Kaszlikowski et al., *Phys. Rev. Lett.* 85 (2000) 4418.
- [125] D. Kaszlikowski et al., *Phys. Rev. A* 65 (2002) 032118.
- [126] D. Collins et al., *Phys. Rev. Lett.* 88 (2002) 040404.
- [127] M. Zukowsky et al., *Phys. Rev. Lett.* 88 (2002) 210401.
- [128] M. Zukowsky et al., *Phys. Rev. Lett.* 88 (2002) 210402.
- [129] R.F. Werner and M.M. Wolf, *Phys. Rev. A* 64 (2001) 032112.
- [130] X.-H. Wu and H.-S. Zong, *Phys. Lett. A* 307 (2003) 262.
- [131] X.-H. Wu and H.-S. Zong, *Phys. Rev. A* 68 (2003) 032102.
- [132] W. Laskowski et al., *Phys. Rev. Lett.* 93 (2004) 200401.
- [133] J.L. Chen, *Phys. Rev. Lett.* 93 (2004) 140407.
- [134] A. Acin et al., *Journ. Phys. A* 34 (2001) 6725.
- [135] S.L. Braunstein, A. Mann and M. Revzen, *Phys. Rev. Lett.* 68 (1992) 3259.
- [136] R. Werner, *Phys. Rev. A* 40 (1989) 4277.
- [137] A. Peres, *Phys. Rev. Lett.* 77 (1996) 1413.
- [138] M. Horodecki, P. Horodecki and R. Horodecki, *Phys. Lett. A* 223 (1996) 1.
- [139] M. Horodecki and P. Horodecki, *Phys. Rev. A* 59 (1999) 4206.
- [140] R.A. Bertlemann et al., *Phys. Rev. A* 66 (2002) 032319.
- [141] A. C. Doherty et al., *Phys. Rev. A* 69 (2004) 022308.
- [142] L.M. Ioannu and B.C Travaglione, *quant-ph* 0311184.
- [143] P. Horodecki, *Phys. Lett. A* 232 (1997) 333.
- [144] C.H. Bennet et al., *Phys. Rev. Lett.* 82 (1999) 5385.
- [145] M. Lewenstein et al., *Phys. Rev. A* 63 (2001) 044304.

- [146] R. Horodecki et al., Phys. Lett. A 200 (1995) 340.
- [147] P.K. Aravind, Phys. Lett. A 200 (1995) 345.
- [148] A. Sen(De) et al., Phys. Rev. A 68 (2003) 062306.
- [149] J. Barrett, Phys. Rev. A 65 (2002) 042302.
- [150] Y.-S. Zhang et al., Phys. Rev. A 66 (2002) 062315.
- [151] C. Cinelli et al., Physical Review A 70 (2004) 022321.
- [152] S. Teufel et al., Phys. Rev. A 56 (1997) 1217.
- [153] A. Peres, Phys. Rev. A 54 (1996) 2685.
- [154] S. Popescu, Phys. Rev. Lett. 74 (1995) 2619.
- [155] N. Gisin, Phys. Lett. A 210 (1996) 151.
- [156] D. Deutsch et al., Phys. Rev. Lett. 77 (1996) 2818;
- [157] C. Macchiavello, Phys. Lett. A 246 (1998) 385.
- [158] C.H. Bennet et al., Phys. Rev. A 53 (1996) 2046.
- [159] M. Murao et al., Phys. Rev. A 57 (1998) R4075.
- [160] M. Horodecki, P. Horodecki and R. Horodecki, Phys. Rev. Lett. 78 (1997) 574.
- [161] M. Horodecki, P. Horodecki and R. Horodecki, Phys. Rev. Lett. 80 (1998) 5239.
- [162] M.B. Plenio and V. Vedral, Contempor. Phys. 39 (1998) 431.
- [163] V. Vedral et al., Phys. Rev. Lett. 78 (1997) 2275.
- [164] V. Vedral and M.B. Plenio, Phys. Rev. A 57 (1998) 1619.
- [165] W.K. Wootters, Phys. Rev. Lett. 80 (1998) 2245.
- [166] C. Bennet et al., Phys. Rev. A 54 (1998) 3824.
- [167] F. Verstaete and M.M. Wolf, Phys. Rev. Lett. (2002) 170401.
- [168] J. Eisert and M. Plenio, Journ. Mod. Opt. 46 (1999) 145
- [169] A. Di Lisi et al., quant-ph 0407254; A. Osterloh and J. Siewert, quant-ph 0410102 and ref.s therein.
- [170] S.J. Summers and R. Werner, Phys. Lett. 110A (1985) 257.
- [171] S.J. Summers and R. Werner, J. Math. Phys. 28 (1987) 2440.
- [172] A. Iorio et al., quant-ph 0207040; Annals of Phys. 309 (2004) 151 and Ref.s therein.
- [173] R. Haag and D. Kastler, J. Math. Phys. 5 (1964) 848.

- [174] B. Reznik, Found. of Phys. 33 (2003) 167.
- [175] K. Popper, Quantum Theory and the Schism in Physics (Hutchinson, London, 1982), 27-29.
- [176] G.C. Ghirardi, "Sneaking a look at God's Cards", (Princeton University Press, 2004).
- [177] W.K. Wootters and W.H. Zurek, Nature 299 (1982) 802.
- [178] G. C. Ghirardi, A. Rimini and T. Weber, Lett. Nuov. Cim. 27 (1980) 293.
- [179] G. C. Ghirardi et al., Europhys. Lett. 6 (1988) 95.
- [180] D. Bruss et al., Phys. Rev. A 62 (2000) 062302.
- [181] J.F. Clauser and M.A. Horne, Phys. Rev. D 10 (1974) 526.
- [182] P.M. Pearle, Phys. Rev. D 2 (1970) 1418.
- [183] P. H. Eberhard, Phys. Rev. A 47 (1993) R747.
- [184] E. Santos, Phys. Lett. A 200 (1995) 1.
- [185] C. Thompson, quant-ph/9903066
- [186] N.A. Törnqvist, Found. Phys. 11 (1981) 171; Europhys. Lett. 1 (1986) 381; Phys. Lett. A 117 (1986) 1.
- [187] A. Aspect et al., Phys. Rev. Lett. 47 (1981) 460.
- [188] A. Aspect et al., Phys. Rev. Lett. 49 (1981) 91.
- [189] E. Santos Phys. Rev. Lett. 66 (1991) 1388; Phys. Rev. A 46 (1992) 3646.
- [190] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics, (Cambridge Univ. Press, Cambridge, 1985).
- [191] A. Joobeur et al., Phys. Rev. A 54 (1994) 3349.
- [192] C.K. Hong and L. Mandel, Phys. Rev. A 31 (1985) 2409.
- [193] D.N. Klyshko, Phys. Lett. A 132 (1988) 299.
- [194] J. P. Franson, Phys. Rev. Lett. 62 (1989) 2205.
- [195] Z.Y. Ou et al., Phys. Rev. Lett. 65 (1990) 321.
- [196] P.G. Kwiat et al., Phys. Rev. A 41 (1990) 2910.
- [197] J.G. Rarity et al., Phys. Rev. A 65 (1990) 1348.
- [198] J.D. Franson, Phys. Rev. A 44 (1991) 4552.
- [199] A. Suarez and V. Scarani, Phys. Lett. A 232 (1997) 9.
- [200] A. Suarez, Phys. Lett. A 269 (2000) 293.

- [201] N. Gisin et al., Rev. Mod. Phys. 74 (2002) 145.
- [202] I. Marcikic et al., quant-ph 0404124.
- [203] T. Kimura et al., Jpn. J. Appl. Phys. 43 (2004) L1217.
- [204] W. Tittel et al., Phys. Rev. Lett. 84 (2000) 4737.
- [205] C.-Z. Peng et al., quant-ph0412218.
- [206] Y.H. Shih and C.O. Alley, Phys. Rev. Lett. 61 (1988) 2921; Y.H.Shih et al., Phys. Rev. A 47 (1993) 1288.
- [207] Z.J. Ou and L. Mandel, Phys. Rev. Lett. 61 (1988) 50.
- [208] T.B. Pittman and J.D. Franson, Phys. Rev. Lett. 90 (2003) 240401.
- [209] T. Jennewein et al., Phys. Rev. Lett. 88 (2002) 017903.
- [210] M. Genovese, Phys. Rev. A. 63 (2001) 044303.
- [211] S. Popescu, L. Hardy and M. Zukowski, Phys. Rev. A 56 (1997) R4353.
- [212] D. Achilles et al., Journal of Modern Optics 51 (2004) 1499.
- [213] M.J. Fitch et al., Phys. Rev. A 68 (2003) 043814.
- [214] O. Haderka et al., quant-ph 0302154.
- [215] J. Rehacek et al., Phys. Rev. A 67 (2003) 061801.
- [216] K. Banaszek and I.A. Walmsley, Opt. Lett. 28 (2003) 52.
- [217] M. Rajteri et al., Nucl. Inst. and Meth. in Phys. Res. A 444 (2000) 461.
- [218] Y. Kim et al., Phys. Rev. A 63, (2001) 060301(R); Phys. Rev. A 63 (2001) 062301.
- [219] M. Fiorentino, Phys. Rev. A 69 (2004) 041801(R).
- [220] A. Garuccio, in "Fundamental Problems in Quantum Theory", Ed. D. Greenberger , (New York Academy of Sciences, 1995).
- [221] T.E. Kiess et al., Phys. Rev. Lett. 71 (1993) 3893.
- [222] P.G. Kwiat et al., Phys. Rev. A 49 (1994) 3209.
- [223] L. Hardy, Phys. Lett. A 161 (1992) 326.
- [224] G. Brida et al., Phys. Lett. A 299 (2002) 121.
- [225] T.E. Keller and M.H. Rubin, Phys. Rev. A 56 (1997) 1534.
- [226] A. V. Sergienko et al., Phys. Rev. A 60 (1999) R2622.
- [227] W. P. Grice and I.A. Walmsley, Phys. Rev. A 56 (1997) 1627.
- [228] W. P. Grice et al., Phys. Rev. A 57 (1998) R2289.

- [229] G. Di Giuseppe et al., Phys. Rev. A 56 (1997) R21.
- [230] M. Barbieri et al., quant-ph 0303018.
- [231] C. Cinelli et al., quant-ph 0406148.
- [232] A.V. Burlakov et al., Phys. Rev. A 64 (2001) 041803.
- [233] D. Rosenberg et al., Nucl. Inst. and Meth. in Phys. Res. A 520 (2004) 537.
- [234] S. Takeuchi et al., Appl. Phys. Lett. 8 (1999) 1063.
- [235] W.T. Buttler et al., Phys. Rev. Lett. 84 (2000) 5652.
- [236] T. Jennewein et al., Phys. Rev. Lett. 84 (2000) 4729.
- [237] R. Alléaume et al., quant-ph 0402110.
- [238] A. Poppe et al., quant-ph 0404115.
- [239] C. Kurtsiefer et al., Nature 419 (2002) 450.
- [240] T. Kimura, Jpn. J. Appl. Phys. 43 (2004) L217.
- [241] D. Bouwmeester et al., Nature 390 (1997) 575.
- [242] D. Boschi et al., Phys. Rev. Lett. 80 (1996) 1121.
- [243] Y.-H. Kim et al., Phys. Rev. Lett. 86 (2001) 1370.
- [244] R. Ursin et al., Nature 430 (2004) 849.
- [245] A.F. Abouraddy et al., Phys. Rev. Lett. 93 (2004) 213903.
- [246] D.Z. Cao et al., quant-ph 0401109.
- [247] M. D'Angelo, M. V. Chekhova and Y. Shih, Phys. Rev. Lett. 87 (2001) 013602.
- [248] T.B. Pittman, B.C. Jacobs and J.D. Franson, Phys. Rev. A 69 (2004) 042306.
- [249] Z. Zhao et al., quant-ph 0404129.
- [250] S. Gasparoni et al., Phys. Rev. Lett. 93 (2004) 020504.
- [251] M. Fiorentino and F.N.C. Wong, Phys. Rev. Lett. 93 (2004) 070502.
- [252] J.L. O'Brien et al., Nature 426 (2003) 264.
- [253] J.-W. Pan et al., Nature 410 (2001) 1067.
- [254] P.G. Kwiat et al., Nature 409 (2001) 1014.
- [255] Z. Zhao et al., Phys. Rev. Lett. 90 (2003) 207901.
- [256] Y.-H. Kim et al., Phys. Rev. A 67 (2003) 010301.
- [257] P.G. Kwiat et al., Science 290 (2000) 498.

- [258] T. Yamamoto et al., Nature 421 (2003) 343.
- [259] A.I. Lvovsky et al., Phys. Rev. Lett. 87 (2001) 050402.
- [260] A. Zavatta et al., Phys. Rev. A 70 (2004) 053821; M. Bondani et al., quant-ph/0502060.
- [261] D.M.Greenberger, M.A. Horne and A. Zeilinger, in "Bell's Theorem, Quantum Theory and Conceptions of the Universe", ed. M. Kafatos, (Kluwer, Dordrecht, 1989), 73.
- [262] D.M.Greenberger, M.A. Horne, a. Shimony and A. Zeilinger, Am. Journ. of Phys. 58 (1990) 1131.
- [263] J. Larsson, Phys. Rev. A 57 (1998) R3145.
- [264] J.W. Pan et al., Nature 403 (2000) 515.
- [265] D. Bouwmeester et al., Phys. Rev. D 82 (1999) 1345.
- [266] Z. Zhao et al., Phys. Rev. Lett. 91 (2003) 180401.
- [267] R.J. Nelson, D.G. Cory and S. Lloyd, Phys. Rev. A 61 (2000) 022106.
- [268] A. Zeilinger et al., Phys. Rev. Lett. 78 (1997) 3031.
- [269] H. Weinfurter and M. Zukowski, Phys. Rev. A 64 (2001) 010102.
- [270] Z. Zhao et al., quant-ph/0402096.
- [271] S. Massar and S. Pironio, Phys. Rev. A 68 (2004) 062109.
- [272] D. Collins et al., Phys. Rev. Lett. 88 (2002) 170405.
- [273] L. Hardy, Phys. Rev. Lett. 71 (1993) 1665.
- [274] T.F. Jordan, Phys. Rev. A 50 (1994) 62.
- [275] J. Torgerson, D. Branning and L. Mandel, Appl. Phys. B 60 (1994) 267.
- [276] S. Goldstein, Phys. Rev. Lett. 72 (1994) 1951.
- [277] P. Heywood and M.L.G. Redhead, Found. of Phys. 13 (1983) 481.
- [278] J.R. Torgerson et al., Phys. Lett. A 204 (1995) 323.
- [279] J.R. Torgerson et al., Phys. Lett. A 214 (1996) 319.
- [280] G.Di Giuseppe, F.De Martini and D.Boschi, Physical Review A 56 (1997) 176; Phys. Lett. A 228 (1997) 208.
- [281] D.Boschi, S.Branca, F.De Martini and L.Hardy, Phys. Rev. Lett. 79 (1998) 2755.
- [282] M. Barbieri et al., quant-ph 0406156.
- [283] A. Garuccio, Phys. Rev. A 52 (1995) 2535.

- [284] A. Cabello and E. Santos, Phys. Lett. A 214 (1996) 316.
- [285] A.G. White et al., Phys. Rev. Lett. 83 (1999) 3103.
- [286] L. Accardi and M. Regoli, quant-ph0007005, in Proc. of QCCM 3, eds P. Tombesi and O. Hirota (Kluwer Ac. Press, New York, 2001) pag. 313.
- [287] L. Accardi and M. Regoli, quant-ph011086.
- [288] L. Accardi, K. Imafuku and M. Regoli, quant-ph 0112067.
- [289] K. Hess and W. Phillip, Europhys. Lett. 57 (2002) 775.
- [290] R.D. Gill, quant-ph/0110137; published in Mathematical Statistics and Applications: Festschrift for Constance van Eeden. Eds: M. Moore, S. Froda and C. Léger. IMS Lecture Notes – Monograph Series, Volume 42 (2003). Institute of Mathematical Statistics. Beachwood, Ohio.
- [291] R.D. Gill et al., Europhys. Lett. 61 (2003) 282.
- [292] D.M. Appleby, Int. Journ. Quant. Inf. 1 (2003) 29.
- [293] N. Gisin and B. Gisin, Phys. Lett. A 260 (1999) 323.
- [294] J.-A. Larsson et al., quant-ph 9812053.
- [295] S. Mirell, Phys. Rev. A 65 (2002) 032102.
- [296] T. Durt, quant-ph 9907061.
- [297] T.W. Marshal and E. Santos, Found. of Phys. 18 (1988) 185.
- [298] A. Casado et al., J. Opt. Soc. Of Am. B 14 (1997) 494;
- [299] A. Casado et al., Phys. Rev. A 55 (1997) 3879.
- [300] A. Casado et al., Phys. Rev. A 56 (1997) 2477.
- [301] A. Casado et al., J. Opt. Soc. Of Am. B 15 (1998) 1572.
- [302] A. Casado et al., Eur. Phys. Journ. D 11 (2000) 465; D 13 (2001) 109.
- [303] A. Casado et al., quant-ph 0202097.
- [304] G. Brida et al., Journ. Mod. Opt. 11 (2003) 1757.
- [305] G. Brida et al., Phys. Lett. A 328 (2004) 313.
- [306] E. Santos, quant-ph 0401003.
- [307] M. Genovese et al., quant-ph 0409054 to appear in Optics and Spectroscopy.
- [308] S. Massar et al., Phys. Rev. A 66 (2002) 052112.
- [309] S. Massar, Phys. Rev. A 65 (2002) 032121.

- [310] T. Durt et al., Phys. Rev. A 64 (2001) 024101; quant-ph 0101084.
- [311] D. Kaszlikowski et al., Phys. Rev. A 65 (2002) 0321118.
- [312] H. Bechmann-Pasquinucci and N. Gisin, QIC 3 (2003) 157.
- [313] M. Genovese, Phys. Rev. A in press.
- [314] A. Acin et al., Int. J. Quant. Inf. 2 (2004) 23.
- [315] H. Bechmann-Pasquinucci and W. Tittel, Phys. Rev. A 61 (2000) 062308.
- [316] H. Bechmann-Pasquinucci and A. Peres, Phys. Rev. Lett. 85 (2000) 3313.
- [317] M. Bourennane et al., Phys. Rev. A 63 (2001) 062303.
- [318] N.J. Cerf et al., Phys. Rev. Lett. 88 (2002) 127902.
- [319] D. Bruss and C. Macchiavello, Phys. Rev. Lett. 88 (2002) 127901.
- [320] M. Genovese and C. Novero, Eur. Journ. of Phys. D. 21 (2002) 109.
- [321] D. Kaszlikowski and M. Zukowski, Phys. Rev. A 66 (2002) 042107.
- [322] N.J. Cerf and S. Massar, Phys. Rev. Lett. 89 (2002) 080402.
- [323] A. Acin et al., Phys. Rev. Lett. 92 (2004) 250404.
- [324] D. Kaszlikowski et al., Phys. Rev. A 66 (2002) 032103.
- [325] G.A. Maslennikov et al., J. Opt. B 5 (2003) S530.
- [326] A.V. Burlakov et al., Phys. Rev. A 60 (1999) R4209-R4212.
- [327] M.V. Chekhova et al., Phys. Rev. A 70 (2004) 053801.
- [328] J.C. Howell, A. Lamas-Linares, and D. Bouwmeester, Phys. Rev. Lett. 88 (02) 030401.
- [329] A. Vaziri et al., Phys. Rev. Lett. 89 (2002) 240401.
- [330] R.T. Thew et al., Phys. Rev. Lett. 93 (2004) 010503.
- [331] H. de Riedmatten et al., Quant. Inf. and Comp. 2 (2002) 425.
- [332] L. Neves et al., quant-ph 0411054.
- [333] S. Kochen and R.P. Specker, J. Math. Mech. 17 (1967) 59.
- [334] A.M. Gleason, Journ. of Math. and Mech. 6 (1957) 885.
- [335] D.A. Meyer, Phys. Rev. Lett. 83 (1999) 3751.
- [336] A. Kent, Phys. Rev. Lett. 83 (1999) 3755.
- [337] N.D. Mermin, quant-ph 9912081.
- [338] C. Simon, C. Brukner and A. Zeilinger, Phys. Rev. Lett. 86 (2001) 4427.

- [339] A. Cabello, Phys. Rev. A 65 (2002) 052101.
- [340] M. Michler, H. Weinfurter and M. Zukowski, Phys. Rev. Lett. 84 (2000) 5457.
- [341] M. Zukowski, Phys. Lett. A 157 (1991) 198.
- [342] S.M. Roy and V. Singh, Phys. Rev. A 48 (1993) 3379.
- [343] A. Cabello and G. Garcia-Alcaine, Phys. Rev. Lett. 80 (1998) 1797.
- [344] C. Simon et al., Phys. Rev. Lett. 85 (2000) 1783;
- [345] A. Cabello and G. Garcia-Alcaine, Phys. Rev. Lett. 80 (1998) 1797.
- [346] Y.-F. Huang et al., Phys. Rev. Lett. 90 (2003) 250401.
- [347] Z.Y. Ou et al., Phys. Rev. Lett. 68 (1992) 3663.
- [348] Z.Y. Ou et al., Appl. Phys. B 55 (1992) 265.
- [349] M.D. Reid and P.D. Drummond, Phys. Rev. Lett. 60 (1988) 2731.
- [350] M.D. Reid, Phys. Rev. A 40 (1989) 3930.
- [351] J.C. Howell et al., Phys. Rev. Lett. 92 (2004) 210403.
- [352] K. Banaszek and K. Wódkiewicz, Phys. Rev. A 58 (1998) 4345.
- [353] J.S. Bell, Speakable and Unspeakable in Quantum Mechanics (Cambridge Univ. Press, Cambridge, UK, 1988)
- [354] A.M. Cetto et al., Phys. Lett. A 113 (1985) 304.
- [355] A. Kuzmich, I.A. Walmsley and L. Mandel, Phys. Rev. Lett. 85 (2000) 1349.
- [356] P. Grangier, M.J. Potasek and B. Yurke, Phys. Rev. A 38 (1988) R3132.
- [357] B. Hessmo et al., Phys. Rev. Lett. 92 (2004) 180401.
- [358] S.M. Tan et al., Phys. Rev. Lett. 66 (1991) 252.
- [359] E. Santos, Phys. Rev. Lett. 68 (1992) 894.
- [360] S.M. Tan et al., Phys. Rev. Lett. 68 (1992) 895.
- [361] P. Grangier et al., Phys. Rev. A 38 (1988) 3132.
- [362] L. Hardy, Phys. Rev. Lett. 73 (1994) 2279.
- [363] W.T.M. Irvine et al., quant-ph 0410160.
- [364] L. Hardy, Phys. Rev. Lett. 68 (1992) 2981.
- [365] M. Genovese and C. Novero, Phys. Lett. A 271 (2000) 48.
- [366] F. De Martini et al., Phys. Rev. A 60 (1999) 1636.
- [367] M. Dakna et al., Phys. Rev. A 55 (1997) 3184.

- [368] F.A. Bovino, F. De Martini and V. Mussi, quant-ph 9905048.
- [369] A.I. Lvosky and J. Mlynek, Phys. Rev. Lett. 88 (2002) 250401.
- [370] H.S. Eisenberg et al., Phys. Rev. Lett., Vol 93 (2004) 193901.
- [371] F. De Martini and F. Sciarrino, quant-ph 0410225.
- [372] J.R. Friedman et al., Nature 406 (2000) 43.
- [373] E. Altewisher et al., Nature 418 (2002) 304.
- [374] S. Fasel et al., quant-ph 0410064.
- [375] Y.-H. Kim and Y. Shih, Found.Phys. 29 (1999) 1849.
- [376] A.J. Short, quant-ph0005063.
- [377] A.M. Steinberg, P.G. Kwiat and R.Y. Chiao, Phys. Rev. Lett. 71 (1993) 708.
- [378] Y.H. Shih, A.V. Sergienko and M.H. Rubin, Phys. Rev. A 47 (1993) 1288.
- [379] Y. Hasegawa et al., Nature 425 (2003) 45.
- [380] E. Hangley et al., Phys. Rev. Lett. 79 (1997) 1.
- [381] D.L. Moehring et al., Phys. Rev. Lett. 93 (2004) 090410.
- [382] F. Uchiyama, Phys. Lett. A 231 (1997) 295.
- [383] R.A. Bertlmann, W. Grimus and B.C. Hiesmayr, Phys. Lett. A 289 (2001) 21.
- [384] F. Benatti and R. Floreanini, Phys. Rev. D 57 (1998) R1332.
- [385] F. Benatti and R. Floreanini, Eur. Phys. Journ. C 13 (2000) 267.
- [386] M. Genovese, quant-ph 0501039.
- [387] R.A. Bertlmann and W. Grimus, Phys. Lett. B 392 (1997) 426; Phys. Rev. D 58 (1998) 034014;
- [388] G.C. Ghirardi et al., The DAΦNE Physical handbook , edited by L. Maiani, G. Pancheri, and N. Paver (INFN, Frascati, 1992) Vol. I.
- [389] J. Six, Phys. Lett. B 114 (1982) 200.
- [390] P.H. Eberhard, Nucl. Phys. B 398 (1993) 155.
- [391] A. Di Domenico, Nucl. Phys. B 450 (1995) 293.
- [392] B. Ancocha, Phys. Rev. D 60 (1999) 094008.
- [393] A. Bramon and M. Nowakowski, Phys. Rev. Lett. 83 (1999) 1.
- [394] N. Gisin and A. Go, Am.J.Phys. 69 (2001) 264.
- [395] B.C. Hiesmayr, Found.Phys.Lett. 14 (2001) 231.

- [396] R.A. Bertlman et al., Phys.Lett. A332 (2004) 355; A. Bramon et al., quant-ph0410122.
- [397] R.H. Dalitz and G. Garbarino, quant-ph 0011108.
- [398] P. Privitera and F. Selleri, Phys. Lett. B 296 (1992) 261.
- [399] F. Selleri, Phys. Rev. A 56 (1997) 3493.
- [400] A. Pompili and F. Selleri, Eur. Phys. Journ. C 14 (2000) 469;
- [401] A.Bramon and G.Garbarino, Phys. Rev. Lett. 88 (2002) 040403.
- [402] E. Leader and E. Predazzi, An introduction to gauge theoris and modern particle physics, (Cambridge Univ. press, Cambridge, 1996).
- [403] Particle Data Book, S. Eidelman et al., Phys. Lett. B 592 (2004) 1; G.V. Dass and K.V.L. Sarma, Eur. Phys. J. C5 (1998) 283.
- [404] W.H. Furry, Phys. Rev. 49 (1936) 393.
- [405] H. Albrecht, Phys. Lett. B 192 (1987) 245.
- [406] M.Genovese et al., Phys. Lett. B 513 (2001) 401.
- [407] M. Genovese et al., Foud. of Phys. 32 (2002) 589.
- [408] M. Genovese, Phys. Rev A 69 (2004) 022103.
- [409] A. Apostolakis et al., Phys. Lett. B 422 (1998) 339.
- [410] A. Go, Journ. Mod. Opt. 51 (2004) 991.
- [411] R. Foadi and F. Selleri, Phys. Rev. A 61 (2000) 012106.
- [412] A.Bramon and G.Garbarino, Phys. Rev. Lett. 89 (2002) 160401.
- [413] C.A. Sackett et al., Nature 404 (2000) 256.
- [414] G.M. D'Ariano et al., J.Opt. B 1 (1999) 576.
- [415] M.G.A. Paris et al., Phys. Lett. A 273 (2000) 153.
- [416] E.S. Fry et al., Phys. Rev. A 52 (1995) 4381.
- [417] R. Garcia-Patron Sanchez et al., Phys. Rev. Lett. 93 (2004) 130409.
- [418] G.-P. He et al., Phys. Rev. A 68 (2003) 012315.
- [419] P. Samuelsson et al., Phys. Rev. Lett. 91 (2003) 157002.
- [420] R. Ionicioiu, P. Zanardi and F. Rossi, Phys. Rev. A 63 (2001) 050101.
- [421] W. De Baere, Found. of Phys. 35 (2005) 33 and ref.s therein.
- [422] E. Nelson, Phys. Rev. 150 (1966) 1079.

- [423] L. de Broglie, in Solvay conf. 1927; Comptes rendus de l'Academie de Science 185 (1927) 380; Nuov. Cim. 1 (1955) 37.
- [424] D. Bohm, Phys. Rev. 85 (1952) 166.
- [425] P. Ebherard, in Quantum Theory and Pictures of Reality, ed. W. Schommers, (Springer, Berlin, 1989).
- [426] V. Scarani and N. Gisin, Phys. Lett. A 295 (2002) 167.
- [427] V. Scarani and N. Gisin, quant-ph 0410025.
- [428] R.D. Mattuck, Found. of Phys. 13 (1983) 481.
- [429] T. Durt, Helv. Phys. Acta 72 (1999) 356.
- [430] J.H. Tutsch, Phys. Rev. 183 (1969) 1116.
- [431] J.P. Christensen and R.D. Mattuck, Found. of Phys. 12 (1982) 4.
- [432] C. Papaliolos, Phys. Rev. Lett. 18 (1967) 622.
- [433] G. Svetlichny, Phys. Rev. D 35 (1987) 3066.
- [434] N.S. Jones et al., quant-ph 0407018.
- [435] G. Brassard et al., Phys. Rev. Lett. 83 (1999) 1874.
- [436] M. Steiner, Phys. Lett. A 270 (2000) 239.
- [437] S. Massar et al., Phys. Rev. A 63 (2001) 052305.
- [438] B.F. Toner and D. Bacon, Phys. Rev. Lett. 91 (2003) 187904.
- [439] N. Brunner, N. Gisin and V. Scarani, New Journ. of Phys. 7 (2005) 88.
- [440] P.R. Holland, The Quantum Theory of Motion, (Cambridge Univ. Press, Cambridge).
- [441] D. Duerr et al., quant-ph 0407116.
- [442] D. Duerr et al., Phys. Rev. Lett. 93 (2004) 090402.
- [443] P. Holland and C. Philippidis, quant-ph 0302076.
- [444] M.M. Ali et al., Phys. Lett. A 304 (2002) 61.
- [445] P. Ghose, Found. Phys. 26 (1991) 1441 and references therein;
- [446] N. Kemmer, Proc. Roy. Soc. A 173 (1939) 91.
- [447] J.R. Croca et al., Found. Phys. Lett. 3 (1990) 557.
- [448] A. Valentini, quant-ph 0309107.
- [449] P. Holland, Phys. Rev. A 60 (1999) 4326; quant-ph 0302076.

- [450] F. Guerra, The Foundations of Quantum Mechanics, C. Garola and A. Rossi (eds.), (Kluwer Academic Publishers, Amsterdam, 1995).
- [451] C.W. Gardiner and P. Zoller, Quantum Noise, (Springer, Berlin, 2000).
- [452] E. Nelson, Quantum Fluctuations, (Princeton Univ. Press, New Jersey, 1985).
- [453] F. Guerra and P. Ruggiero, Phys. Rev. Lett. 31 (1973) 1022.
- [454] F. Guerra and M.I. Loffredo, Lett. Nuov. Cim. 27 (1980) 41.
- [455] S.C. Lim, Lett. Math. Phys. 7 (1983) 469.
- [456] G. Parisi and Y.S. Wu, Sci. Sin. 24 (1981) 483.
- [457] P.H. Damgaard and H. Hüffel, Phys. Rep. 152 (1987) 227.
- [458] F. Guerra and P. Ruggiero, Phys. Rep. 77 (1981) 263.
- [459] D. Bohm and B.J. Hiley, Phys. Rep. 172 (1989) 93.
- [460] L.J. Wang, X.Y.Zou and L. Mandel, Phys. Rev. Lett. 66 (1990) 1111.
- [461] L.J. Wang, X.Y.Zou and L. Mandel, Phys. Rev. Lett. 68 (1992) 3814.
- [462] F. Selleri, Lett. Nuov. Cim. 1 (1969) 908.
- [463] G. Tarozzi, Lett. Nuov. Cim. 42 (1985) 438.
- [464] J.R. Croca, Found. of Phys. 17 (1987) 971.
- [465] J.R. Croca et al., Phys. Rev. Lett. 68 (1992) 3813.
- [466] X.Y.Zou, T. Grayson, L.J. Wang and L. Mandel, Phys. Rev. Lett. 68 (1992) 3667.
- [467] J. Marto and J.R. Croca, Found. of Phys. 32 (2002) 1091.
- [468] L.Felicioni et al., quant-ph 0202045;
- [469] N. Redington et al., Nuovo Cim. 109B (1994) 116
- [470] P. Ghose and M. K. Samal, Phys. Rev. E 64 (2001) 036620;
- [471] P. Ghose, Proc. of Foundations of Quantum Theory and Quantum Optics, Calcutta, ed. S.M. Roy, published in Pramana 56 (2001) 211.
- [472] P. Ghose, quant-ph/0102131, quant-ph/0208192, quant-ph/0001024.
- [473] P. Ghose, A. S. Majumdar, S. Guha and J. Sau, Phys. Lett. A 290 (2001) 205.
- [474] M. Golshani and O. Akhavan, J. Phys. A: Math. Gen. 34 (2001) 5259.
- [475] K.R. Popper, Logik der Forschung (Julius Springer Verlag, Wien, 1935).
- [476] I. Lakatos, The methodology of scientific research programmes. Philosophical papers, Vol.1 (Cambridge University Press, New York, 1980).

- [477] P. Feyerabend, *Against Method* (Verso, London, 1975); *Frontiers of Science and Philosophy*, ed. R.G. Colodny (George Allen and Unwin, Oxford, 1964) p. 189.
- [478] T. Kuhn, *The Structure of Scientific Revolutions* (Chicago, University of Chicago Press, 1970).
- [479] L. Marchildon, quant-ph/0101132(2001);
- [480] L. Marchildon, *J. Modern Optics* 50 (2003) 873.
- [481] E. Guay and L. Marchildon, quant-ph/0302085.
- [482] W.Struyve et al., *Journ. Phys. A* 34 (2003) 5259.
- [483] O. Akhavan and M. Golshani, quant-ph 0305020.
- [484] G. Introzzi, proc. of "Meeting of Italian Society of Philosophy of Science and Logic 2004" in press.
- [485] G. Brida et al., *J. Phys. B: At. Mol. Opt. Phys.* 35 (2002) 4751; *Journ. Mod. Opt.* 51 (2004) 1079.
- [486] G. Brida et al., *Phys. Rev. A* 68 (2003) 033803.
- [487] B.-G. Englert et al., *Z. Naturforsch.* 47a (1992) 1175.
- [488] Y. Aharonov, N. Erez and M.O. Scully, *Physica Scripta* 69 (2004) 81; *Phys. Lett. A* 263 (1999) 157.
- [489] D. Dürr et al., *Z. Naturforsch.* 48a (1993) 1261.
- [490] C. Dewdney et al., *Phys. Lett. A* 184 (1993) 6.
- [491] J. Barrett, *Philosophy of Science* 67 (2000) 680 and quant-ph 0002046.
- [492] M. Blasone et al., *Phys. Lett. A* 287 (2001) 205.
- [493] T.S. Biró et al., *Found. Phys. Lett.* 14 (2001) 471.
- [494] C. van De Bruck, gr-qc 0001048.
- [495] M. Blasone et al., quant-ph 0409021.
- [496] M. Blasone et al., *J. Phys. Soc. Jap. Suppl.* 72 (2003) 50.
- [497] G. 't Hooft, quant-ph 0212095; Conf. Proceedings, "Quo Vadis Quantum Mechanics", (Philadelphia, 2002).
- [498] G. 't Hooft, *Int. Journ. Th. Phys.* 42 (2003) 355.
- [499] T.S. Biró and B. Müller, hep-lat/0307028.
- [500] H.-T. Elze, *Phys. Lett. A* 310 (2003) 110.
- [501] G. Kato, *Europhys. Lett.* 68 (2004) 467.

- [502] C. Wetterlich, quant-ph 0212031.
- [503] X. Chen, quant-ph 0501034.
- [504] M.J. Hadley, Found. Phys. Lett. 10 (1997) 43.
- [505] E. Santos, quant-ph 0410193.

FIGURE CAPTIONS

Fig.1 Contour plot of the quantity CH/N (see Eq. 19; N is the total number of detections) in the plane with f (non maximally entanglement parameter, $|\psi\rangle = \frac{|H\rangle|H\rangle+f|V\rangle|V\rangle}{\sqrt{(1+f^2)}}$) as x-axis and detection efficiency η as y-axis. The leftmost region corresponds to the region where no detection loophole free test of Bell inequalities can be performed. The contour lines are at 0, 0.025, 0.06, 0.1, 0.15, 0.2.

Fig.2 Scheme of type I PDC. The two circumferences (continuous and dashed) correspond to two different wave lengths. The spots indicate the directions of emission of two entangled photons.

Fig.3 Scheme of type II PDC emission. Two circumferences, where are emitted degenerate photons (continuous line) and correlated photons of different wave lengths (dashed lines), are shown. H, V denote horizontal and vertical polarisation, respectively.

Fig.4 Franson scheme for Bell inequality test from Ref. [194]. A source emits two energy-time entangled photons that after having crossed a Mach-Zender interferometer are detected by D_1 or D'_1 and D_2 and D'_2 respectively.

Fig. 5 Set-up of Ref. [34] for a Bell inequality test. A source S emits two particles (1,2) in four beams A,B,C,D. An entangled state is realised by superposition on beam splitters.

Fig.6 Scheme of the experiment (from Ref.[40]) for a Bell inequality test on energy-time entangled photons by two remote measurement devices.

Fig. 7 Scheme for generating polarisation entangled photon states by superimposing on a beam splitter two correlated photons produced in type I PDC after having rotated polarisation of one of the two (from Ref. [206]).

Fig. 8 Typical set-up for PDC entangled photon Bell experiments. On the left one can recognise the pump laser, a titanium-sapphire mode locked laser pumped by a diode laser and with second harmonic generation. At the centre of the picture the non-linear crystal, followed by a filter for eliminating the UV pump and fibre couplers (preceded by interferential filters) collecting the photons to be addressed to the detectors (one of them is on the background).

Fig. 9 Sketch of a bright source of polarisation entangled photons realised by superimposing two type I PDC emissions (from Ref. [51]). CR1 and CR2 are two $LiIO_3$ crystals cut at the phase-matching angle of 51° . L1 and L2 are two identical plano-convex lenses with a hole of 4 mm in the centre. P is a 5 x 5 x 5 mm quartz plate for birefringence compensation and $\lambda/2$ is a first order half wave-length plate at 351 nm. U.V. identifies the pumping radiation at

351 nm. The infrared beam (I.R.) is used for system alignment (dashed line identifies the correlated emission).

Fig. 10 Outcomes of GHZ test from Ref. [264]. These data show that terms predicted by SQM (tall bars) appear in a fraction 0.85 ± 0.04 of all cases against a fraction 0.15 ± 0.02 of other terms.

Fig. 11 Experimental scheme for generating a GHZ pair from Ref. [264]. Pairs of polarisation-entangled photons are generated by a short pulse of ultraviolet light (200 fs, $\lambda = 394$ nm) pumping a BBO crystal. Observation of the desired GHZ correlations requires fourfold coincidence and therefore simultaneous emission of two pairs. The photon registered at T is always horizontally polarised (H) and thus its partner in b must be vertically polarised (V). The photon reflected at the polarizing beam-splitter (PBS) in arm a is always V, being turned into equal superposition of V and H by the $\lambda/2$ plate, and its partner in arm b must be H. Thus if all four detectors register at the same time, the two photons in D1 and D2 must either both have been VV and reflected by the last PBS or HH and transmitted. The photon at D3 was therefore H or V, respectively. Both possibilities are made indistinguishable by having equal path lengths via a and b to D1 (D2) and by using narrow bandwidth filters to stretch the coherence time to about 500 fs, substantially larger than the pulse length. Polarizers oriented at 45° and $\lambda/4$ plates in front of the detectors allow measurement of linear 45° (-45°) (circular R/L) polarisation.

Fig. 12 Experimental apparatus for tritter generation of qutrits from Ref. [330]. Different paths combination originate 5 peaks in arrival time histogram (shown on the right). Coincidences for central peak (shown on the left as a function of Alice's and Bob's phase vectors) project onto one of three orthogonal qutrit states.

Fig. 13. Bohm trajectories calculated for a particle crossing a double slit (from Ref. [473]).

Fig. 14 Outline of the experiment for testing empty waves hypothesis (from Ref. [447]). Two identical photons (produced by PDC) enter through two different beam splitters ($BS_{1,2}$) a Mach-Zender interferometer. Coincidences are measured between the two photo-detectors ($D_{1,2}$) at the exits of the second pairs of beam splitters ($BS_{3,4}$).

Fig. 15 The double slit experiment of Ref. [486]. In the upper window the set-up scheme. A pump laser at 351 nm generates type I parametric down conversion in a lithium-iodate crystal. Conjugated photons at 702 nm are sent to a double-slit (two slits of $10\mu m$ separated of $100\mu m$) by a system of two plano-convex lenses in a way that each photon of the pair crosses a well defined slit. A first photodetector is placed at 1.21 m a second one at 1.5 m from the slit. Both the single photon detectors (D) are preceded by an interferential

filter at 702 nm (IF) and a lens (L) of 6 mm diameter and 25.4 mm focal length. Signals from detectors are sent to a Time Amplitude Converter and then to the acquisition system (multi- channel analyser and counters). In the lower window the experimental coincidences data are compared with quantum mechanics predictions. On the x-axis we report the position of the first detector respect to the median symmetry axis of the double slit. The second detector is kept fixed at -0.055 m (the region without data around this point is due to the superposition of the two detectors). The x errors bars represent the width of the lens before the detector. Coincidences are clearly observed in the same semiplane at variance with [471,472,473,474] result.