A Simple Resistance Network for Calibrating Resistance Bridges

D. Rod White, *Member, IEEE*, Keith Jones, *Member, IEEE*, Jonathan M. Williams, and Ian E. Ramsey

Abstract— **This paper describes a simple, passive, low-cost resistance network, closely related to Hamon build-up resistors, that enables the calibration of dc and low-frequency ac resistance and conductance bridges. The network is configured so that the four component resistors can be connected to realize 35 distinct four-terminal resistances, all interrelated by the usual formulas for the series and parallel connections of resistors. Theoretical analysis and experimental results show that with due care in the design, the network can be readily constructed to achieve an accuracy of better than 1 $\mu\Omega$ for resistances of the order of 100 Ω ($1:10^8$) for angular frequencies from dc to 10^4 rad/s.**

Index Terms—**Bridge circuits, calibration, resistance measurement, resistive circuits, switched resistor circuits, temperature measurement.**

I. Introduction

Accurate resistance thermometry requires the measurement of the ratio of four-terminal resistances with accuracies better than one part in 10^6 and approaching one part in 10^7. While there are many ac and dc resistance bridges available for this purpose, there remains an almost universal problem: how to calibrate the bridges and hence demonstrate the traceability of the resistance and temperature measurements.

The difficulty of calibration is a particular problem for ac bridges which are designed to operate at low frequencies, typically 25 Hz to 90 Hz. DC standards and techniques generally do not extrapolate well to ac, and ac standards are generally optimized for performance near and above 10^4 rad/s (1.6 kHz) and are not suited for use at low frequencies.

The most accurate techniques for calibrating ac bridges employ an auxiliary transformer to inter-compare and determine the errors in the ratios of all the taps on the inductive voltage divider (IVD) [1]–[10]. With attention to the circuit topology and electrostatic screens, accuracies of a few parts in 10^{10} are achievable. Avramov-Zamuric *et al.* [11] recently developed a calibration technique based on the comparison of binary and decimal IVD's. The technique exploits the linear independence of the algebraic models of the errors for the two IVD's. Experimental results have demonstrated a 2σ accuracy of 0.05 ppm.

In general transformer build-up techniques are not applied to the calibration of commercial bridges for several reasons. First, since the techniques involve the disassembly of the bridge to gain access to the taps on the IVD's, they require a detailed working knowledge of the bridge. Second, commercial bridges are not designed to facilitate calibration, so the disassembly is not conducive to maintaining the reliability of the bridge. Third, there may be a difficulty ensuring that the loading and defining conditions for the IVD's are the same in use as in calibration. Overall, the expense in terms of both time and expertise makes the cost of these techniques prohibitive on a commercial scale. The simplest methods for checking resistance bridges are based on the inter-comparison of stable four-terminal resistors. In the most sophisticated form equal-valued resistors are connected in series using four-terminal junctions [12], much as in a 100:1 Hamon build-up resistor [13]. Each resistor is measured individually with the bridge, then series combinations are measured to expose departures from linearity. While this greatly enhances the confidence in a bridge, it fails to provide the necessary information for a calibration, namely a means to calculate corrections and the uncertainty in the corrected bridge readings.

This paper describes a simple resistor network [14], [15], related to Hamon build-up resistors, that is suitable for calibrating all dc and low frequency ac resistance and conductance bridges. It is low in cost, is accurate to one part in 10^8 at 100 Ω, provides a thorough check on all the dials on a bridge, and provides sufficient data to make statistical estimates of the corrections and uncertainty in bridge readings.

II. Principles of the Network

The equivalent circuit of the network is shown in Fig. 1. It consists of four resistors (R_1 to R_4 in Fig. 1), each connected to one terminal of a four-terminal junction, J. The terminals of the junction as well as the free terminals of the resistors can be connected to realize 35 distinct nonzero four-terminal resistances. The various connections required are summarized in Fig. 2.

The additional resistors in the potential and current leads of the four base resistors (R_{p1} to R_{p4} and R_{c1} to R_{c4}, respectively, in Fig. 1) form a combining network which is necessary for those combinations that require parallel connections of resistors.

There are two main features of the network that contribute to its utility. First, the 35 different resistances are interrelated

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D. R. White and K. Jones are with the Measurement Standards Laboratory, New Zealand Institute for Industrial Research and Development, Lower Hutt, New Zealand (e-mail: r.white@irl.cri.nz; k.jones@irl.cri.nz).

J. M. Williams is with the Electrical Science Division, National Physical Laboratory, Teddington, Middlesex TW11 0LW, U.K.

I. E. Ramsey is with Automatic Systems Laboratories, Bradville, Milton Keynes, Bucks MK13 7FH, U.K.

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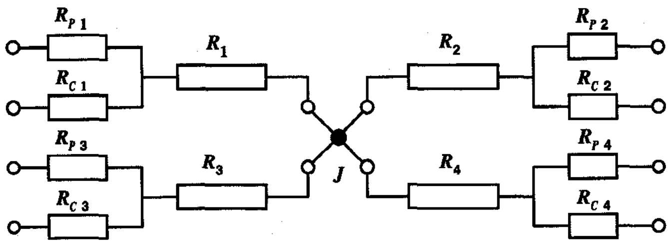

Fig. 1. An equivalent circuit for the resistance network. R_1 to R_4 and J, the four-terminal junction, are the main components. The resistors R_{p1} to R_{p4} and R_{c1} to R_{c4} are the potential and current sharing resistors in the combining network.

to the four base resistances by the formulas for the series and parallel connection of resistances. From measurements of the 35 different resistances it is possible to calculate, by least squares, values for the four base resistances and the coefficients in an equation describing the errors in the bridge.

Because all resistance bridges measure resistance by comparing the unknown resistance R_x with a reference resistor R_s, the 35 ratio measurements made of the resistance network are interrelated by a single scale factor, the reference resistance R_s. This in turn means that the 35 measurements of resistance, by themselves, will provide information only on the linearity of the bridge, and not on its absolute accuracy.

With bridges that use an external reference resistor the absolute accuracy can be determined by including one or more complement ratios in the assessment. Complement ratios are obtained by exchanging the reference and unknown connections to the bridge. The inclusion of the complement ratios also means that there are up to 70 interrelated measurements available to assess the linearity of the bridge.

The second feature contributing to the network utility is that, unlike a Hamon build-up resistor, the values of the base resistances in the network are not critical. This means that the calibration requirements on the network itself are satisfied relatively easily, and the resistance values can be chosen according to other rationale.

To provide a sound test of the accuracy of a bridge the 35 resistance values available with the network should be distinct and cover as wide a range as practical. For example, four equal base resistors will yield only five distinct resistance values. With the appropriate choice of base resistances the 35 combinations will yield 35 distinct values. A suitable figure-of-merit for assessing the quality of the distribution generated by a particular set of base resistors is

$$\beta_d = \frac{1}{P_1 - P_{35}} \left[34 \sum_{i=2}^{35} (P_i - P_{i-1})^2 \right]^{1/2} \tag{1}$$

where the P_i are the 35 resistance ratios R_i/R_s sorted into descending order. The figure-of-merit is a minimum when all of the ratios are spaced as much as possible, and equal to 1.0 when the ratios are equally spaced.

A computer search yielded a minimum figure-of-merit near 1.2 with the four base resistors

$$R_1 = 0.6254902 R_{\max} \quad R_3 = 0.2884911 R_{\max}$$
$$R_2 = 0.3745098 R_{\max} \quad R_4 = 0.2226752 R_{\max}$$

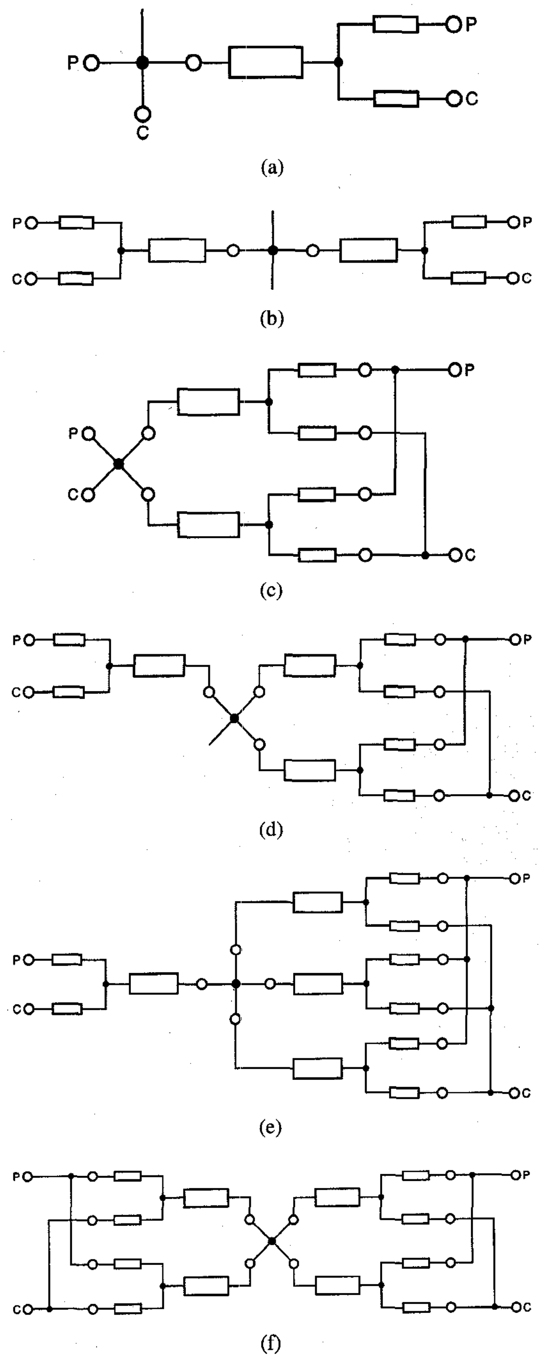

Fig. 2. The various connections required to realize all 35 four-terminal resistances. (a) A single resistor—four combinations; (b) two resistors in series—six combinations; (c) two resistors in parallel—six combinations; (d) one resistor in series with two in parallel—12 combinations; (e) one resistor in series with three in parallel—four combinations; (f) two parallel resistors in series with two parallel resistors—three combinations.

where $R_{\max}$ is the resistance of the largest combination $(R_1 + R_2)$.

A second criterion for the choice of resistance values for the four base resistors is that the 35 combinations should cause every numeral of every digit in the bridge reading to be displayed. In this way the test with the network gives confidence that all of the internal switches that connect the various taps on the IVD of an ac bridge are working correctly.

TABLE I
A SUMMARY OF THE MOST SIGNIFICANT NETWORK ERRORS

Source of Error	Equation for Error	Maximum Error at dc (μΩ)	Maximum Additional Error at ac (μΩ)		Combinations Affected
			500 rad/s	10^4 rad/s	
junction cross resistance	$R_{jx1}+R_{jx2}$	0.02	-	-	series
combining network at dc	equations (3), (4)	0.2	-	-	parallel
combining network at ac	$-\dfrac{L^2(R_1-R_2)^2\omega^2 R}{(R_1+R_2)^3 R_p}$	-	0.0025	1.0	parallel
component reactance	$\dfrac{R_1^2R_2^2\left(\dfrac{L_1}{R_1}-\dfrac{L_2}{R_2}\right)^2\omega^2}{(R_1+R_2)^3}$	-	0.0001	0.04	parallel
stray admittances	equation (8)	0.008	0.004	0.1	series
cable admittance	equation (8)	0.008	0.9	7.5	all
power coefficients	$-\dfrac{R_1^3R_2^2(R_1+2R_2)\alpha_1 h_1 I_0^2}{(R_1+R_2)^4}$	0.2	-	-	parallel

$R_{jx1}=R_{jx2}=10\ \text{n}\Omega$	$R_L=R_C=1\text{T}\ \Omega$	I_0=1 mA
$R_1=81\ \Omega$, $R_2=48\ \Omega$, $R_3=36\ \Omega$, $R_4=31\ \Omega$	$\delta_L=\delta_C=10^{-4}$	α=1 ppm/°C
$R/R_p=81$, $R_{ci}=3\ \text{m}\Omega$	$C_C=200$ pF	h=0.25°C/mW
$L_1=L_2=L=0.5\ \mu\text{H}$	$C_L=10$ pF	

A suitable figure of merit for the search is

$$\beta_e=\sum_{i=0}^{9}\sum_{j=1}^{N} f_{ij}^2\exp(-j) \tag{2}$$

where f_{ij} is the frequency of occurrence of the ith numeral (0–9) in the jth digit of an N digit bridge. The values of R_1 to R_4 given above may be used as starting values for the search. This figure-of-merit (2) preferentially weights the most significant digits and is a minimum when each numeral of each digit is exercised three or four times. The figure-of-merit is defined assuming that the IVD uses decade ratios but the principle is easily extended to binary dividers.

Because the four base resistors and the reference resistor have finite tolerances it is not possible to guarantee that the least significant digits will be fully exercised. However, if it is assumed that the numerals of those digits are distributed randomly over the 35 measurements, there is a 75% likelihood that all of the numerals of a digit will be exercised. If all 35 complement ratios are also included in a bridge assessment then the probability increases to 99.6%.

III. FACTORS AFFECTING THE ACCURACY OF THE NETWORK

A four-terminal definition of resistance is sufficient to define dc measurements of resistance to the highest accuracy, but for ac measurements there is an additional requirement to define the electromagnetic fields around the conducting elements of the resistor. Thus coaxial definitions of impedance, are necessary for the highest accuracy ac applications [1].

The ac errors, particularly those relating to failure of definition, are a major reason why Hamon build-up resistors have not previously been considered for ac applications. The mitigating factor in the application of the network to the calibration of resistance bridges is that only the real part of the impedance realized by the network is measured by a resistance bridge. This ensures that the ac imperfections of the network are reduced to second order effects.

For the purpose of estimating the errors the various components in the network are assumed to have the values as listed in Table I. These values are based on measurements made on prototypes. Table I summarizes the most significant errors [15] and lists: the source of the error; a simple algebraic approximation for the error or reference to the equation in the text; and maximum values of the error at dc, 500 rad/s and 10^4 rad/s (approximately 80 Hz and 1.6 kHz). The error is expressed as the measured resistance of the combination minus the resistance calculated from the measured resistances of the base resistors.

Since most resistance bridges measure the real part of the impedance the series-equivalent model of impedance [1] is assumed. It is also assumed that the bridge connection to the network is realized as a four-terminal coaxial connection [1] since this reduces the uncertainty in the definition of the resistances and eliminates some sources of error. This is also the definition most commonly used on commercial ac bridges.

More detailed description and derivations of the equations may be found in [15].

A. The Four-Terminal Junction

A four-terminal junction can be realized by ensuring that three of the four leads to a junction block are symmetric with respect to the fourth [16]. The junction used in the prototypes is based on Hamon's original design and is used in a number of build-up resistors made by the National Measurement Laboratory, CSIRO, Australia [17].

Riley [16] shows that the maximum errors due to the cross-resistances of the junction, which may be negative, occur for series combinations where the error is equal to the algebraic sum of the two cross resistances.

B. The Accuracy of the Combining Network

In order to realize the parallel combinations of the network, a low resistance connection must be made between each of the terminals of the resistors [Fig. 2(c)–(f)]. Because the resistance of the connections is not zero in practice, the currents through each of the resistors may not be divided, as expected, according to their resistance. If the currents are not distributed correctly then the measured resistance of parallel combinations will be in error. The solution is to introduce known resistances into the various leads so that the currents are distributed properly [1], [16]. In practice, it is usually sufficient to have sharing resistors in the potential leads and ensure that the current leads have as low a resistance as is practical.

Because the potential sharing resistors are made much larger than the current sharing resistors the expressions for the two-resistor [16] and three-resistor parallel [15] combinations respectively can be simplified to

$$R(R_1//R_2) = \frac{R_1R_2}{R_1+R_2}\left[1+\left(\frac{R_1R_2}{R_1+R_2}\right)\left(\frac{R_{p1}}{R_1}-\frac{R_{p2}}{R_2}\right) \times\left(\frac{R_{c1}}{R_1}-\frac{R_{c2}}{R_2}\right)\left(\frac{1}{R_{p1}+R_{p2}}\right)\right] \tag{3}$$

and

$$\begin{aligned}R(R_1//R_2//R_3) = {} & \frac{R_1R_2R_3}{R_1R_2+R_1R_3+R_2R_3} \\ & \times\left[1+\frac{R_1R_2R_3}{R_1R_2+R_1R_3+R_2R_3}\right. \\ & \times\left(R_{p1}\left(\frac{R_{p2}}{R_2}-\frac{R_{p3}}{R_3}\right)\left(\frac{R_{c2}}{R_2}-\frac{R_{c3}}{R_3}\right)\right. \\ & + R_{p2}\left(\frac{R_{p1}}{R_1}-\frac{R_{p3}}{R_3}\right)\left(\frac{R_{c1}}{R_1}-\frac{R_{c3}}{R_3}\right) \\ & \left.+ R_{p3}\left(\frac{R_{p1}}{R_1}-\frac{R_{p2}}{R_2}\right)\left(\frac{R_{c1}}{R_1}-\frac{R_{c2}}{R_2}\right)\right) \\ & \left.\times\frac{1}{R_{p1}R_{p2}+R_{p1}R_{p3}+R_{p2}R_{p3}}\right]\end{aligned} \tag{4}$$

where R_1, R_2, and R_3 are the three base resistors; R_{p1}, R_{p2}, and R_{p3} are the potential sharing resistors; R_{c1}, R_{c2}, and R_{c3} are the current sharing resistors; and the symbol // is used to indicate that the resistors are connected in parallel.

For ac applications the inductance of the combining network must also be considered. The effect of the inductance at high frequencies is both to increase the impedance of the combining network and to destroy the matching of the components. If it is assumed that the inductance L in each of the components of the combining network is the same, then the error increases approximately as frequency squared (see Table I).

C. Reactance of the Base Resistors

The formulas for series and parallel combinations are exact for dc resistances and complex impedances. However, ac resistance bridges measure the real part of the complex impedance. The assumption of the series representation of impedance ensures that for impedances in series the relationship

$$\mathrm{Re}(Z_1+Z_2) = \mathrm{Re}(Z_1)+\mathrm{Re}(Z_2) \tag{5}$$

is always true. However, the corresponding relationship for parallel impedances

$$\begin{aligned}&\mathrm{Re}[Z_1Z_2/(Z_1+Z_2)] \\ &\quad = [\mathrm{Re}(Z_1)\mathrm{Re}(Z_2)/(\mathrm{Re}(Z_1)+\mathrm{Re}(Z_2))]\end{aligned} \tag{6}$$

is an approximation. For impedances that have a low reactance, i.e., are predominantly resistive and have only small series inductance or parallel capacitance, and at low frequencies, the approximation is extremely good.

A similar problem arises when complement ratios are included in a bridge assessment. The implicit assumption

$$\mathrm{Re}(Z_x/Z_s) = 1/\mathrm{Re}(Z_s/Z_x) \tag{7}$$

is strictly satisfied only when the ratio Z_x/Z_s is real. For an accuracy of $1:10^8$ it is sufficient for the phase angles of the reference resistor and the network to be matched to within $1:10^4$, a condition easily satisfied by high performance film resistors for all angular frequencies up to 10^4 rad/s.

D. Coupling Within the Network

The coupling between the various parts of the network occurs through stray capacitance, mutual inductance and the imperfect insulation resistance. The most serious effects occur for the six admittances coupling the ends of series combinations. For typical insulating materials the admittances can be modeled as a leakage resistance R_L in parallel with a small capacitance C_L with a constant dissipation factor $\tan\delta_L$ [18]. The error in the measured series resistance is then approximately

$$\begin{aligned}&R(R_1+R_2)_{\text{meas}} - R(R_1+R_2)_{\text{calc}} \\ &\quad = -2R_1R_2\left(\frac{1}{R_L}+\omega C_L\tan\delta_L\right) \\ &\qquad - 3R_1R_2(R_1+R_2)\omega^2C_L^2.\end{aligned} \tag{8}$$

The leakage resistance appears to be the dominant limitation on the long-term dc accuracy of the network.

E. Limitations in the Four-Terminal Coaxial Definition

With a bridge that realizes a four-terminal coaxial connection to the network the admittances of the coaxial cables connecting the bridge to the network are indistinguishable from the admittance of the network itself. Although this effect is not strictly attributable to either the network or the bridge, a poor set of cables will affect the ability of the network to expose errors in the bridge. The effect is most pronounced for the highest-resistance combinations, and for the series combinations the error has the same form as (8) with C_L, R_L, and δ_L replaced by C_C, R_C, and δ_C, respectively.

F. The Power Coefficients of the Resistors

A simple thermal model of a resistor shows that the resistance changes in response to the sensing current according to

$$R(I_0) = R\left(1 + \alpha h R I_0^2\right) \tag{9}$$

where α is the temperature coefficient of the resistor, h is the thermal resistance between the resistor and ambient, and I_0 is the current. With most resistance bridges the excitation is provided by a constant-current source so that the sensing current for the series combinations is the same as that for the single resistor combinations, and no error occurs. However, for the parallel combinations the current is divided between the resistors and the self-heating is reduced. The effect of the self-heating in resistor R_1 propagates as

$$\begin{aligned} R(R_1//R_2)_{\text{meas}} &- R(R_1//R_2)_{\text{calc}} \\ &= -\frac{R_1^3 R_2^2 (R_1 + 2R_2)}{(R_1 + R_2)^4} \alpha h I_0^2 \end{aligned} \tag{10}$$

where I_0 is the sensing current used for all measurements.

In practice the self heating is complicated by the thermal time constants of the resistors, which for the prototype components were about 6 min. If the measurements are carried out in a short time, with the resistors allowed to return to ambient temperature between measurements, the self heating effects will be less than that implied by (10).

IV. Analysis of Results

The results of the 35 or more network measurements may be analyzed in several ways. The simplest is to use the measured values of the base ratios P_1 to P_4 (the ratios for R_1/R_s to R_4/R_s) to calculate the expected readings for the other ratios, and compare these to the measured values.

A more accurate assessment of the linearity of a bridge is gained by using a least squares fit to find the best values for P_1 to P_4 from all measurements. The least squares technique also offers a way of identifying the nonlinearities in the bridge measurements.

Suppose that the bridge error is described by a simple function such as a cubic polynomial

$$\Delta P = A + BP + CP^2 + DP^3. \tag{11}$$

The first two terms of the right-hand side characterize the zero error and scale error respectively, while the last two terms characterize any even and odd order nonlinearities. Because all the measured ratios are related by a constant scale factor through the reference resistance R_s, it is possible to determine a value for the coefficient (B) of the linear term in (11) only if complement measurements are included in the results.

The values for the coefficients in the equation and the best values for P_1 to P_4 are determined by minimizing the variance between the corrected measurements and the predicted ratios. For (11) the variance is

$$s^2 = \frac{1}{27} \sum_{i=1}^{35} \left(P_{i,m} - P_i + A + BP_{i,m} + CP_{i,m}^2 + DP_{i,m}^3\right)^2 \tag{12}$$

where $P_{i,m}$ are the measured values of the ratios, and P_i are the predicted values of the ratios calculated from the fitted values for the base ratios. Note that the number of degrees of freedom in the variance, 27, is equal to the number of measurements minus the number of fitted parameters.

In practice, a number of different fits may be used depending on the model of the error in the bridges [11], [19]. Other models might include the large-scale sawtooth errors associated with the most significant digits of decimal IVD's and the most significant bits of some analog-to-digital converters.

For the simplest least squares fit it is assumed that there is no systematic nonlinearity in the bridge readings, so only the values for the four base ratios are determined.

V. Experimental Results

A variety of experiments were carried out with four prototype networks and a large range of the commercially available bridges. One of the prototype networks is now used routinely by Automatic Systems Laboratories to test production runs for their six-, seven-, and eight-digit ac bridges. Experience with more than 35 different ac and dc bridges representative of 13 different models from nine different manufacturers has shown that a correction equation is usually necessary only on faulty bridges. With healthy bridges the four parameter fit (to the base ratios only) usually yields as low a variance as the seven or eight parameter fit (to the base ratios plus cubic equation). This suggests that for most bridges, a fit that yields statistically significant values for the coefficients in the cubic equation is indicative of a faulty bridge. Least squares fits with a sawtooth correction function proportional to the fractional part of the bridge reading revealed one bridge with a significant sawtooth error.

The standard deviation, s, is also a good indicator of the health of a bridge. For most bridges the uncertainty implied by the standard deviation was comparable with the accuracy specified by the manufacturer. Bridges with high standard deviations in their assessments were either known to be faulty or later found to be faulty. Fig. 3 shows the results of a linearity test (four parameter fit) on one of the better commercial eight-digit ac bridges.

Because none of the prototypes were temperature controlled, variations in the ambient temperature were occasionally a limiting factor in the performance of the network when testing eight-digit bridges.

In one experiment, a network with R_1 to R_4 values of 100 Ω, 100 Ω, 50 Ω, and 50 Ω, respectively, was evaluated

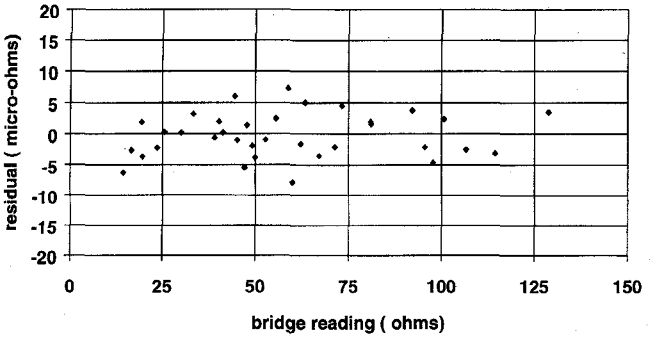

Fig. 3. The results of a linearity test of an eight-digit ac bridge. The residuals are the difference between the measured ratios and the ratios calculated from fitted values of the four base ratios. The standard deviation of 3.7 $\mu\Omega$ is equivalent to 0.03 ppm of $R_{\max}$.

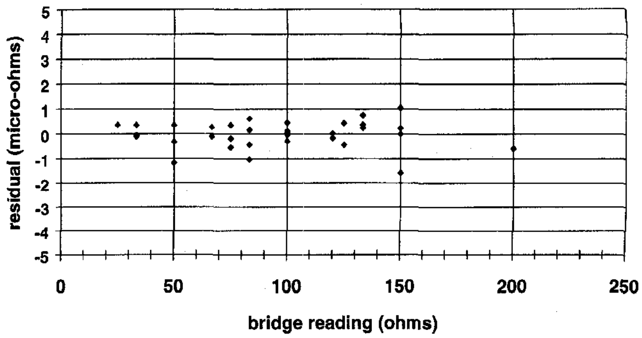

Fig. 4. The results of a dc evaluation of the network using the NPL cryogenic current comparator bridge. The residuals are the difference between the measured ratios and ratios calculated from the fitted values of the four base ratios. The standard deviation of 0.57 $\mu\Omega$ is equivalent to 0.003 ppm of $R_{\max}$.

with the cryogenic dc current comparator bridge at the National Physical Laboratory, Teddington. The bridge is known to have a one-sigma accuracy of about one part in 10^9 [20]. The network yields 12 different resistance values that are nominally simple rational multiples of 100 Ω. Fig. 4 shows the results of the evaluation.

During the measurements with the current comparator all of the network resistors showed an increase in resistance of between 3 $\mu\Omega$ and 8 $\mu\Omega$. The results were corrected for temperature assuming that the observed drift was entirely due to the concurrent increase in the temperature of the air-bath used to stabilize the network. In practice some of this drift may have been temporal: a resistor that drifts steadily at 10 ppm/year exhibits 0.8 $\mu\Omega$ change over a 7-h measurement period. In addition to the errors listed in Table I that account for approximately 0.3 $\mu\Omega$ rms, other unaccounted factors include; uncertainty in the measurements of network temperature which is believed to be the most significant factor, variable thermoelectric effects, noise, and the errors in the current comparator.

Finally, a variety of experiments were carried out to assess the fail-safe nature of the network. These included numerical experiments using data sets deliberately corrupted by known error functions, experiments with a bridge that was deliberately disabled or operated outside normal conditions, and experiments with a network with artificially high stray admittances.

It was concluded as follows.

- Bridge assessments that include complement ratios, obtained by exchanging the reference and unknown connections to the bridge, in addition to normally measured ratios will expose almost all known errors that occur in resistance bridges. Assessments without complement ratios will fail to detect errors of scale.
- The network is fail-safe in the sense that any fault in the network will be exposed by a poor standard deviation in the four-parameter fit (base ratios only).
- The presence of systematic nonlinearities in bridge readings is betrayed by either a very large standard deviation, a lower standard deviation in the cubic fit than in the four-parameter fit, and/or coefficients in the fitted correction equation with relative uncertainties of less than 40%.
- Since the least squares process effectively randomizes residual errors, in general the functional form of any error is not readily recognizable from the residual errors. Thus a fit to a correction equation is required to expose the exact form of an error.
- It is possible to construct error functions that exhibit low standard deviations in the fit, indicating incorrectly that there is little error. One example is an offset error combined with a carefully chosen two-cycle sawtooth. This gives a standard deviation about one-third of that expected from the quadrature sum of the standard deviations obtained with the two individual errors. Overall we were unable to construct an error function that would cause significant errors, be undetectable, and likely to occur in practice.

VI. Conclusion

The accuracy of the network, as indicated by both the theoretical analysis and the experimental results, substantially exceeds our initial expectations. The theoretical results in particular show that, with care in the design and use of the network, accuracies of one part in 10^8 at 100 Ω are readily achievable over the range of angular frequencies from dc to 10^4 rad/s. Since performance at higher frequencies will fall off as frequency squared, accuracies of 1 ppm should be achievable at higher audio angular frequencies (10^5 rad/s).

The network enables the calibration of all commercial low and medium frequency resistance bridges. In particular it provides a means of calibrating resistance thermometry bridges, which by virtue of their high accuracy and low operating frequency, have been prohibitively expensive to calibrate in the past.

The theoretical performance of the network at both ac and dc has yet to be demonstrated experimentally. The principle limitation in the dc performance of the prototype evaluated with the NPL cryogenic current comparator, is due to the combination of resistor temperature coefficients and a lack of temperature control of the network. A network engineered for immersion in a temperature controlled oil bath may well have a dc accuracy of 1 part in 10^9. The limiting factors in the performance at this level are likely to be the power coefficients and the short term stability of the base resistors.

The principal limitation in the ac performance of the network is due to the capacitance of the connecting cables. This is a direct consequence of the four-terminal coaxial bridge definition. The network requires only a minor modification to operate with four-terminal-pair bridges, with significantly reduced dependence on the cable capacitance. The accuracy of the network would then be limited by the inductance of the combining network to about one part in 10^8 at 10^4 rad/s.

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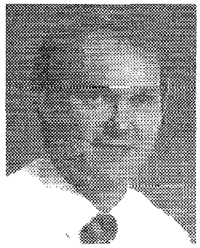
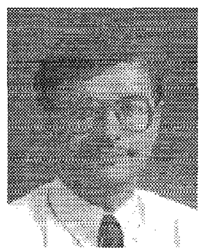


D. Rod White (M'95) was born in New Zealand in 1955. He received the M.Sc. (Hons.) degree in physics from Waikato University, Hamilton, New Zealand, in 1980.

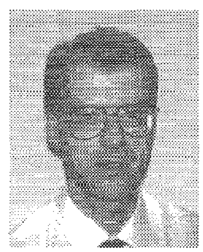
In 1979, he joined the Temperature Standards Section, Department of Scientific and Industrial Research. He is currently manager of the Temperature Standards Team of the Measurement Standards Laboratory of New Zealand, New Zealand Institute for Industrial Research and Development, Lower Hutt. His research interests include Johnson noise thermometry, radiation thermometry, low-noise electronic design, sampling, and control theory. He is co-author of the book *Traceable Temperatures: An Introduction to Temperature Measurement and Calibration* (Chichester, U.K.: Wiley, 1994).



Keith Jones (M'95) was born in Bangor, Wales, on July 19, 1956. He received the M.Sc. (Hons.) degree in physics from the University of Auckland, Auckland, New Zealand, in 1979.

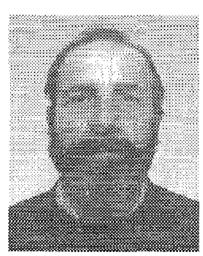
He joined the Department of Scientific and Industrial Research in 1979 and worked on the development of a calculable capacitor. He is currently involved in managing the Electrical Standards Team and developing the quantum Hall resistance standard for the Measurement Standards Laboratory of New Zealand, New Zealand Institute for Industrial Research and Development, Lower Hutt.

Mr. Jones won a DSIR Study Award to the National Physical Laboratory, U.K., to join the quantum Hall research effort in 1989.



Jonathan M. Williams was born in Hampshire, U.K., on May 25, 1962. He studied physics at Lincoln College, University of Oxford, U.K.

After graduating in 1983, he joined the Division of Electrical Science at the National Physical Laboratory (NPL), Middlesex, U.K., to develop measurement methods in optical fiber communications. Since 1988, he has been working on the application of cryogenic techniques in electrical metrology and has developed a cryogenic current comparator resistance ration bridge for routine resistance calibrations. He is currently manager of the resistance and voltage section at NPL.



Ian E. Ramsey was born in South Wales in 1941. He received the B.A. degree from Open University.

He gained his initial experience in electronics during service with the Royal Air Force. He has worked for various companies including Marconi Elliot Avionics. For the last 20 years he has worked for Automatic Systems Laboratories Ltd., Bucks, U.K., where he is now Customer Support Group Manager.