Resistance Network for Verifying the Accuracy of Resistance Bridges

D. Rod White, *Member, IEEE*, and Jonathan M. Williams

***Abstract*— A novel resistance network for verifying the accuracy of resistance bridges is described. The network of four resistors generates 35 four-terminal resistance values all interrelated by the formulae for series and parallel connections of resistors, and by appropriate choice of the resistors different resistance scales can be generated. Although many resistance bridges have internal calibration procedures, an independent check of the accuracy of such bridges when measuring actual resistances is often desirable. The network provides such a check and can be used to verify the accuracy of resistance bridges on a regular basis. It has been used to assess the accuracy of a cryogenic current comparator bridge and demonstrates an accuracy approaching 1 nΩ/Ω at 100 Ω.**

***Index Terms*—Bridge, comparator, cryogenic, Hamon, linearity, ratio, resistance.**

I. INTRODUCTION

A RESISTANCE scale can be generated using a known voltage, current, or resistance ratio. A variety of resistance bridges make use of, for example, room temperature current comparators [1], cryogenic current comparators [2] and self-calibrating binary resistive dividers [3] for dc measurements, and inductive current dividers [4] and inductive voltage dividers for ac measurements [5]. In each case, the accuracy of the system is ensured by appropriate attention to possible sources of error. However, an independent test of accuracy is desirable to confirm that no contributions have been overlooked. Also such a test, if convenient, can be used to check the accuracy of a system as a whole on a regular basis.

This paper describes the use of a novel resistance network to verify the performance of the National Physical Laboratory (NPL) cryogenic current comparator (CCC) resistance bridge. The network was developed at the Measurement Standards Laboratory of the New Zealand Institute for Industrial Research and Development with a view to assessing the accuracy of resistance thermometry bridges [6], [7]. The aim of the work here was to demonstrate the use of the network to verify the overall accuracy of the NPL CCC bridge at the 1 nΩ/Ω. Although a binary build-up technique is used to check the accuracy of the cryogenic current comparator itself [8], it is important to test the whole measuring system under conditions of actual use.

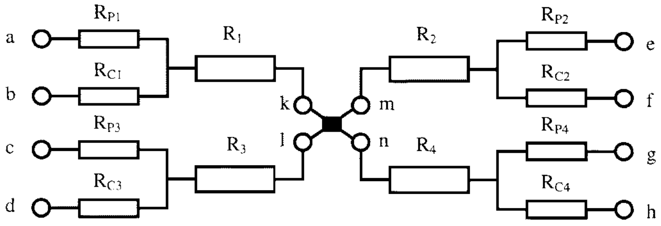

Fig. 1. Diagram of the network showing the four-terminal junction in the center, the connection points labeled a-n, the base resistors R_1 to R_4, the potential sharing resistors R_{P1} to R_{P4}, and the current sharing resistors R_{C1} to R_{C4}.

II. RESISTANCE NETWORK

The network is closely related to Hamon build-up resistors [9]. It consists of four main resistors R_1 to R_4permanently connected to a common four-terminal junction and a set of switches that allows a total of 35 different combinations of the resistors R_1 to $R_4$to be selected. The combinations consist of: i) any single resistor, ii) any two resistors in series, iii) any two resistors in parallel, iv) any two resistors in parallel and in series with a third, v) any three resistors in parallel and in series with the fourth and vi) any two pairs in parallel connected in series. The four-terminal junction is carefully engineered so as to have a four terminal resistance that is less than 10 nΩ, and the resistors are connected in parallel via auxiliary resistors which form combining networks [10], as shown in Fig. 1. External connections (k-n) are included so that all the combinations can be obtained. For example, resistor R_1 may be measured by making connections to terminals a, b, m, and n.

The range of resistances generated by the network can be varied by appropriate choice of the four main resistors. If the network is fitted with four equal resistors then a resistance range of 4 : 1 is achieved with some combinations giving the same resistance value. However, if required, resistors can be chosen so that 35 distinct resistance values can be realized.

Resistance bridges usually measure the ratio of an unknown resistance to a known standard resistance. This means that any error, caused by, for example leakage resistance or lead resistance which is associated with the standard side of the bridge, will contribute equally to all the measurements made with the network. To check for this it is necessary to include the complementary measurements which are obtained by exchanging the standard and unknown connections between the bridge and the network.

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TABLE I
SUMMARY OF THE COMBINATIONS AND NUMBER OF NOMINALLY EQUAL RESISTANCES REALIZED BY A NETWORK WITH FOUR NOMINALLY EQUAL BASE RESISTORS

Combination	Resistance	Number
(i) : R	R	4
(ii) : R + R	2R	6
(iii) : R//R	R/2	6
(iv) : R + R//R	3R/2	12
(v) : R + R//R//R	4R/3	4
(vi) : R//R+R//R	R	3
Total		**35**

TABLE II
A SUMMARY OF THE MOST SIGNIFICANT FACTORS CONTRIBUTING TO UNCERTAINTY

Source of uncertainty	Uncertainty (nΩ, 1-sigma)
junction cross resistance	5
combining network	5
self heating	40
temporal drift	30
temperature coefficients	100
bridge resolution	100
Total	**150**

The inclusion of the complementary ratios means that there are up to 70 interrelated measurements available to assess the accuracy of the bridge/network system. The results are analyzed by least-squares fitting values for the four base ratios and the coefficients in a simple algebraic model of the bridge. Since the network is completely characterized by the four base resistances there are up to 66 degrees of freedom in the least-squares analysis.

III. CRYOGENIC CURRENT COMPARATOR BRIDGE

The NPL automated cryogenic current comparator bridge was developed for routine measurement of room temperature resistors [8]. It has been designed to measure integer ratios of resistance in the range 1 : 1 to 1 : 25 for resistance values in the range 1 Ω to 10 kΩ. In use the bridge measures deviations from the nominal resistance ratio of up to 1 part in 10^4.

For a given ratio of the CCC bridge, the deviation of the resistance ratio from the bridge ratio is measured using an analog to digital converter. To check the linearity of this scale, a selection of resistors with deviations which are within one part in 10^4 is required.

The degeneracy of the network obtained when all the resistors are nominally equal provides a convenient method for generating such a restricted range of values. If the resistors are all identical and have a nominal value R, then the combinations listed in Table I are obtained.

It can be seen from Table I that the largest degeneracy of 12 occurs for combination (iv). Accordingly a network with four 200/3 Ω resistors has been made so that combination (iv) gives resistances of nominally 100 Ω. The range of the deviations of the resistor values from nominal is such that the 12 combinations give a spread of about ±10 parts in 10^6 from a 100 Ω value.

When the bridge and the network are used with a 1 : 1 bridge ratio and the complementary measurements are included, a total of 24 measured points on the resistance scale are available. The bridge-network system is then sufficiently over-determined to enable estimates of the four base ratios that characterize the network and the error in the bridge ratio to be made.

IV. UNCERTAINTY ANALYSIS

Table II summarizes the most important factors contributing to the uncertainty in the measurements. Where possible the estimates of the uncertainty have been derived from data generated according to accepted models of the error and analyzed using the same software used to analyze the results. Six factors are considered:

A. Junction Cross Resistance

Measurements of four-terminal junctions of the same design and manufacture as that used in the network indicate that the cross resistances are less than 10 nΩ. An analysis using Riley's model of the junction [11] shows that the resulting uncertainty for the 12 degenerate combinations of the network is not more than 5 nΩ.

B. Combining network

For two equal valued resistors connected in parallel the error in the parallel resistance is given by [11]

$$\Delta R = \frac{\Delta R_{\mathrm{P}} \cdot \Delta R_{\mathrm{C}}}{8R_{\mathrm{P}}} \tag{1}$$

where R_{P} is the mean value of the potential sharing resistances and ΔR_{P} and ΔR_{C} are the differences between the potential and current sharing resistances associated with each of the two resistors. With a combining network having 10 Ω potential sharing resistors and with mismatches in the network typically below 1 mΩ, the error is less than 12 nΩ and has an rms value near 5 nΩ. Note that errors due to the junction cross resistance and the combining network are correlated. The correlation coefficient may be positive or negative depending on the exact values of the resistances; a value of zero is assumed.

C. Power Coefficients

When resistors are connected in parallel the sensing current is divided so that the self heating of the resistors is reduced. For equal valued resistors in parallel the resulting difference between the measured and calculated values for a parallel combination due to the self heating in one resistor can be shown to be

$$\Delta R_{\mathrm{sh}} = \tfrac{3}{16} R^2 h I_0^2 \qquad (2)$$

where h is the power coefficient, R is the resistance of each of the two resistors, and I_0 is the sensing current. For the resistors used in the network h is less than 10^{-8}/mW, hence for a 3 mA sensing current the errors are less than 75 nΩ. The net effect of the errors will be an uncertainty in the values of the base resistors of about 40 nΩ.

D. Resistor Drift

At the level of accuracy hoped for the temporal stability of the resistors will contribute some uncertainty. The typical drift in similar resistors is of the order of 1 $\mu\Omega/\Omega$ per year. Thus over the few hours when the measurements are made the drift in resistance will be less than 100 nΩ and gives rise to an uncertainty in the measurements of less than 30 nΩ.

E. Temperature Coefficients

The main source of uncertainty when using the network arises from the temperature control of the resistors. The resistors all have temperature coefficients of 10^{-7}/K and the temperature is controlled to 0.01 K rms. Thus the uncertainty in the network resistances is about 100 nΩ.

F. Bridge Resolution

Throughout the measurements the bridge was operated with a 1-sigma resolution of 100 nΩ. The resolution is dominated by the thermal noise in the resistors with small contributions from varying thermoelectric voltages and bridge detector noise.

The total uncertainty is estimated to be 150 nΩ at the 1-sigma level or equivalently 1.5 nΩ/Ω at 100 Ω.

V. RESULTS

All 35 combinations of the network comprising $6 \times 33.3\,\Omega$, $7 \times 66.6\,\Omega$, $4 \times 88.8\,\Omega$, $12 \times 100\,\Omega$, and $6 \times 133.3\,\Omega$ were measured, some normally and some as the complementary ratios. The twelve 100 Ω combinations measured both normally and as the complementary ratios make a total of 47 measurements. The analysis of the experimental results was based on two models of the bridge. The results are summarized in Fig. 2 and Table III.

The first model assumes that the bridge does not have any ratio errors. Accordingly, the least-squares fit determines the best values for the four base ratios only. Fig. 2 plots the residuals of the fit against bridge ratio. The standard deviation of the residuals (error of fit) is 2.9 nΩ/Ω, which is about twice that expected according to Table II. Note that the residuals at all bridge ratios have a small positive bias indicating that the bridge may in fact have a small ratio error.

Therefore, the second model assumes that the bridge exhibits an error proportional to the ratio and accordingly the least-squares fit determines a correction to the main ratio in addition to values for the four base ratios. The fitted correction corresponds to a ratio error of 1.6 nΩ/Ω with an uncertainty of 0.3 nΩ/Ω (1 sigma). The distribution of residuals around the dotted line in Fig. 2 describes the results of the fit with the second model. With the correction applied to the data, the standard deviation of the residuals is reduced to 2.2 nΩ/Ω, which is more compatible with the estimate of the uncertainty given in Table II. Higher order fits gave no further improvement in the error of fit.

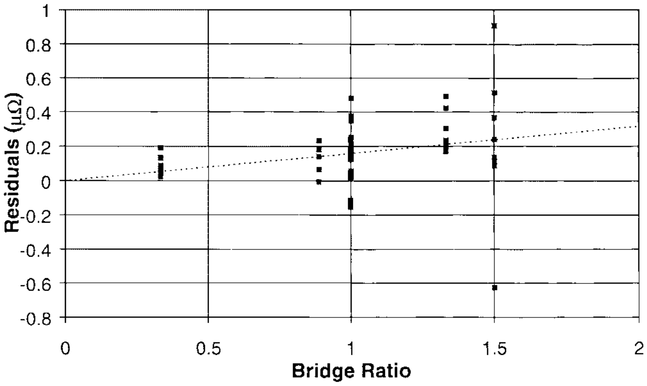

Fig. 2. The results of evaluating the NPL CCC bridge with the network. The residuals are the difference between the measured ratios and the ratios calculated from the fitted values of the network ratios assuming that the bridge is free of error (model 1). The dotted line is the fitted value of the error assuming that the bridge has a constant ratio error (model 2).

TABLE III
SUMMARY OF THE RESULTS OF THE LEAST-SQUARES ANALYSIS SHOWING THE VALUES FOR THE FITTED PARAMETERS, R_1–R_4 ARE THE NETWORK BASE RESISTANCES AND R_s THE STANDARD RESISTANCE. THE DIGITS IN PARENTHESES ARE THE STANDARD UNCERTAINTIES OF THE LAST DIGIT GIVEN

Parameter	**Model 1**	**Model 2**	**Difference**
R_1/R_S (Ω/Ω)	0.666 662 898 9(9)	0.666 662 899 2(7)	3×10^{-10}
R_2/R_S (Ω/Ω)	0.666 659 479 9(9)	0.666 659 480 0(7)	1×10^{-10}
R_3/R_S (Ω/Ω)	0.666 678 877 4(9)	0.666 678 878 2(7)	8×10^{-10}
R_4/R_S (Ω/Ω)	0.666 670 306 0(9)	0.666 670 306 4(7)	4×10^{-10}
ratio error (nΩ/Ω)		1.6(3)	
error of fit (nΩ/Ω)	2.9	2.2	

VI. CONCLUSION

The results show that the network is capable of accuracies approaching 1 nΩ/Ω. The main factor limiting its performance is the temperature control of the air bath used to thermostat the network. Secondary factors contributing to uncertainty include temporal drift of the resistors and self-heating of the resistors.

The network is a useful tool for evaluating the performance of complete dc resistance bridges at the 1 nΩ/Ω level. It provides a convenient method of checking a variety of ratios together with a linearity check for small deviations from one ratio.

The results also demonstrate the overall accuracy of the NPL CCC bridge which appears to have a small error in the 1 : 1 ratio of about 1.6 nΩ/Ω. This is comparable to the 1-sigma resolution of the bridge and so can only be detected with a large number of detailed measurements such as have been carried out here. It confirms the limited tests of the overall accuracy of the bridge which had been made previously by interchanging resistors using a $1:1$ ratio and by comparing the combination of a $2:1$ and a $5:1$ measurement with a $10:1$ measurement. This error is sufficiently small that it does not compromise the day-to-day use of the system.

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