CALCULATING PRECISION VOLTAGE DIVIDERS

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In recent years precision dc voltage dividers (R313–accuracy class 0.001; R35 and R356–accuracy class 0.005) have been developed and manufactured commercially. These voltage dividers are equipped with switching and adjustment systems which provide fast checking and adjustments without the use of measurement resistance coils, and with reduction of the division-factor errors to a minimum regardless of whether the errors are caused by time instability of the resistances, the effect of the external temperature, or by other factors.

The basis of the design of the voltage-divider circuits is the idea developed in [1] whose essence reduces to the fact that in the presence of a group of n resistors $R_1, R_2, \ldots, R_n$ which have identical nominal values R_n and are close in absolute magnitude, the ratio between the equivalent values for series (R_s) and parallel (R_p) connections is equal to

$$R_s/R_p = n^2(1-\delta) \tag{1}$$

with a high degree of accuracy, where $\delta = \left(\frac{1}{n}\sum_{i=1}^{i=n}\gamma_i\right)^2 - \frac{1}{n}\sum_{i=1}^{i=n}\gamma_i^2$ is the relative error of the ratio R_s/R_p; γ is the relative deviation of the real value R_i of the resistance of the i-th resistor from R_n.

From the expression for δ it follows that both terms of the formula are positive; if $\gamma_1 = \gamma_2 = \ldots = \gamma_n$, then $\delta = 0$, for fairly large n the appearance of $\gamma_i > 0$ and $\gamma_i < 0$ is equiprobable, as a result of which $\sum_{i=1}^{i=n}\gamma_i = 0$ and then the maximum value of the conversion error is equal to

$$\delta_{\max} = -\frac{1}{n}\sum_{i=1}^{i=n}\gamma_i^2.$$

For example, if $\gamma_1 = \gamma_2 = \ldots = \gamma_n = 5 \cdot 10^{-4}$ (0.05%), then $\delta_{\max} = -25 \cdot 10^{-8}$ ($-25 \cdot 10^{-6}$%).

Consequently, $R_1, R_2, \ldots, R_n$ can be adjusted in such a way that $\delta_{\max}$ is a negligibly small quantity. In this case

$$R_s/R_p = n^2. \tag{2}$$

The transformation of the circuits from a row of resistors connected in series to a row of resistors connected in parallel is carried out by means of switches $S_1, S_2, \ldots, S_n$, which form jumpers between resistors.

Equation (1) was obtained on the assumption that with the switches $S_1, S_2, \ldots, S_n$ closed, the resistance r_j of the jumpers is equal to zero, while with the switches open the insulation resistance r_{ins} between the contacts of each switch is equal to infinity. Under practical conditions r_j and r_{ins} have finite values which distort the value of R_s/R_p and lead to the appearance of additional errors δ_j and δ_{ins} of the division factors of the voltage divider.

This paper represents an attempt to find the expressions for determining the allowable values of r_j and r_{ins} as a function of stipulated values r of the section resistances of the transformed part of the voltage divider, and the allowable error δ_k of the division factor.

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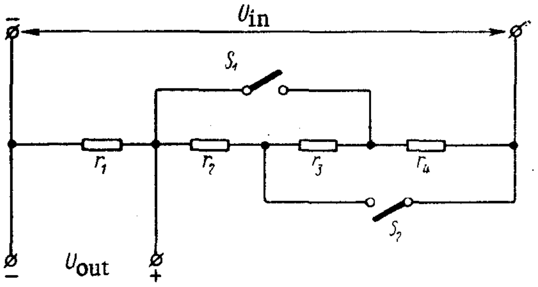

Fig. 1

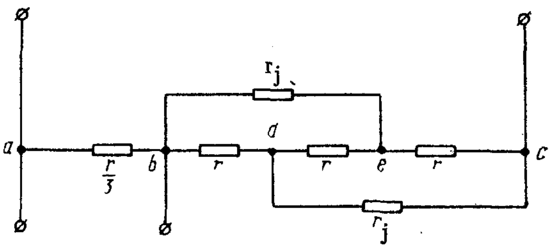

Fig. 2

The effect of the resistance of the jumpers shall be considered using a voltage divider circuit having a nominal division factor $K_n = 10:1$; the circuit is illustrated in Fig. 1, where $3r_1 = r_2 = r_3 = r_4 = r$; V_{in} is the input voltage; V_{out} is the output voltage; S_1 and S_2 are switches.

In checking the voltage divider the switches S_1 and S_2 are closed, and the equivalent resistance r_p of the sector bc is compared with the resistance r_1 taken as the reference. We assume that the resistance of the jumpers are equal to r_j with the switches closed. Under these conditions the circuit of the voltage divider takes the form shown in Fig. 2.

The resistance r_p of the sector bc (see Fig. 2) is equal to

$$r_p = \frac{rr_j(3r + r_j) + 2r(r + r_j)^2}{(2r + r_j)(3r + r_j)}. \tag{3}$$

A comparison of the real value of r_p with its nominal value $r_{p,n} = r/3$ (for $r_j = 0$) shows that $r_p > r_{p,n}$ by the amount

$$\delta_p = \frac{8}{1 + 3\frac{r}{r_j}}. \tag{4}$$

If the voltage divider is adjustable [3] and just one of the resistances of the resistors r_2, r_3 or r_4 (see Fig. 1) is variable, then by means of the adjustment one can achieve a situation in which $r_p = r_1$. Under these conditions it is necessary to reduce the adjustable resistor by $3\delta_p$ in order to reduce r_p by δ_p. As a result we have the following expression after S_1 and S_2 have opened:

$$r_s' = 9r_1(1 - \delta_p). \tag{5}$$

With allowance for (5) the real value of the division factor of the voltage divider, as determined from $K = 1 + r_s/r_1$, will be

$$K' = 10(1 - 0.9\delta_p).$$

From this we find that, making allowance for (4), the error δ_j of the division factor caused by the resistance r_j of the jumpers is equal to

$$\delta_j = \frac{7.2}{1 + 3\frac{r}{r_j}}.$$

For $r \gg 1$ we have

$$\delta_j = 2.4\frac{r_j}{r}. \tag{6}$$

From (6) it follows that for r_j = const the value of δ_j increases with decreasing r.

Assuming $\delta_j = \delta_f/m$ where $m > 1$, we use (6) to find the largest allowable value of the jumper resistance $r_{j\,max}$:

$$r_{j\,max} = \frac{\delta f}{2,4m} r. \tag{7}$$

Example. In a voltage divider of the R35 type (accuracy class 0.005) with an error ($\delta_f = 5 \cdot 10^{-5}$) the smallest resistance of a section is $r_{min} = 3 \cdot 10^4\,\Omega$. Having assumed n = 10, we find $r_{j\,max} = 0.06\,\Omega$ from (7).

The effect of the insulation resistance between the contacts of the switches S_1 and S_2 when they are open is considered using the circuit shown in Fig. 2, where the resistors r_{ins} are connected in place of the resistors r_j.

In this case the resistance r_s'' of the sector bc can be determined from (2), where r_j has been replaced by r_{ins}. From the difference between r_s'' and its nominal value $r_{s,n}'' = 3r$ (for $r_{ins} = \infty$) we find

$$r_s'' = 3r - \frac{8r}{3r + r_{ins}}. \tag{8}$$

With allowance for (8) the real value of the division factor will be

$$K'' = 10\,(1 - \delta_{ins}),$$

where $\delta_{ins} = 2.4r/(3r + r_{ins})$ is the relative error of the division factor caused by the resistances r_{ins}. For $r_{ins} \gg r$, we have

$$\delta_{ins} = 2.4\,\frac{r}{r_{ins}}. \tag{9}$$

From (9) it follows that for r_{ins} = const the value δ_{ins} increases with increasing r. Assuming $\delta_{ins} = \delta_f/l$, where $l > 1$, we find the least allowable value $r_{ins\,min}$ of the insulation resistance between switch contacts from (9):

$$r_{ins.\,min} = \frac{2.4l}{\delta_f}\,r. \tag{10}$$

Example. In the R35 voltage divider the largest section resistance is $r_{max} = 3 \cdot 10^6\,\Omega$. Having assumed $l = 10$, we find $r_{ins\,min} = 1.44 \cdot 10^{12}\,\Omega$ from (10).

CONCLUSIONS

In adjustable voltage dividers the resistance of the jumpers and the insulation resistance between the contacts of the switches in the jumper circuits cause a positive systematic error of the division factor.

The effect of the resistance of the jumpers increases as the resistance of the sections of the transformed portion of the voltage divider decreases, which is one of the causes which limit the reduction of the voltage-divider resistance. The effect of the insulation resistance between switch contacts in the jumper circuits increases as the resistance of the sections of the transformed part of the voltage divider increases. This latter factor is one of the causes limiting the increase in the resistance of the voltage divider.

These conclusions apply equally to measurement of a ratio of resistances under conditions in which the method of series-parallel transformation is used.

LITERATURE CITED

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