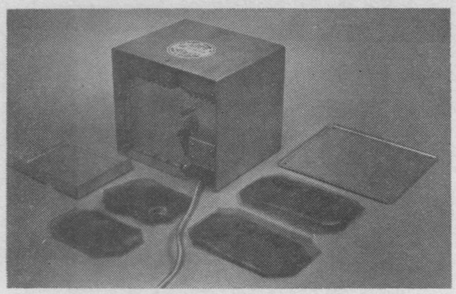

Fig. 7. Inner bath showing gallium alloy pouches.

to-months experienced for cells shipped without temperature control. Application of this bath has been instrumental in updating the voltage-calibration service offered by the Sandia Primary Standards Laboratory. The redundant methods of determining cell temperature have also had application in evaluating the resistance-measuring capability of customer laboratories. Typical agreement in measured temperature has been within the same thousandth of a degree celsius—much better than had been anticipated.

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Resistive Voltage-Ratio Standard and Measuring Circuit

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***Abstract*—This paper describes a highly stable, guarded dc voltage-ratio standard and the measuring network and techniques used to establish the values of its ratios to an accuracy of 0.2 ppm. The entire system is housed within a dry-air enclosure whose temperature is maintained at 23 ± 0.05°C. Discrete ratios from 1:1 to 1000:1 are provided, with maximum rated voltage set at 1000 volts. The design of the standard was chosen so that a redundancy of measurement could be incorporated in the system. Thus each successive ratio is measured by a substitution or "bootstrap" method and by satisfying the conditions of the series-parallel principle, the 10:1, 100:1, and 1000:1 ratios are measured by a second independent method. The design also admits additional checks on the validity of the measurements. An analysis of measurement errors and a discussion of their possible origin are included. Since the intent also was to design the ratio standard for low-frequency operation some preliminary data are included on its ac performance.**

Introduction

RECENT years have brought challenges to the scientific and technological communities that were barely discernible a few decades ago. Advances made in meeting these have been possible only through the more precise and accurate measurement of physical quantities that often are interweaved within vast complex measuring systems and often cover a range of magnitudes of several decades. Measurement of dc voltage from the 1-volt level of the standard cell to 1 or 2 kV is a case in point—a necessary and important link in the system. Extension from the approximate 1-volt level is realized through ratio networks that most generally are built up from resistive elements. Those that can be more aptly classed as ratio standards contain a limited but adequate number of discrete ratios and are most often used in comparison circuits to assign values to working standards of somewhat lower accuracy class.

The ratio standard that for many years performed this function at NBS was a Silsbee design [1]. It featured a separate guarding network for the more critical sections, exhibited only small changes with self-heating, and admitted to a self- or step-up calibration process. The stability and the accuracy to which its ratios were known were of the order of 5 ppm of ratio. Some degradation in accuracy followed when using it in a comparison network to assign values to other types of ratio networks.

The need for an improved standard, with a more accurate assignment to its ratios has become evident. Voltage supplies of constant known output and digital voltmeters for monitoring voltage are approaching claimed accuracies of 10 or 20 ppm. These must be evaluated through ratio networks, leading ultimately

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to an assignment in terms of the national standard of EMF. Thus, ratios must be known with increasing accuracy at each higher echelon of measurement thereby placing stringent requirements on those ratio measurements at the national level.

To help provide the increased accuracies required in such a chain of measurements, the intent at NBS was to design a more versatile standard of improved stability and with ratios more closely equal to their nominal values, together with a measuring network and technique for assigning those values of ratios to an accuracy better than 0.5 ppm of the output (5×10^{-10} of the 1000-volt input).

During this study and prior to the final design described in this paper, several ratio networks were considered. One, which looked promising, was developed and examined as an experimental model DD-2 before it was rejected for practical reasons. Theoretically, the design admits errors only of second order but the problems encountered in its construction, operation, and in the assignment of its errors outweighed the advantage predicted by theory. However, the basic design, although not new, might be useful in some applications and provide high accuracy where a model of limited range or one with few ratios is desired. For this reason, a brief discussion is included in Appendix I.

Rejection of this design led to the one described that is simpler in construction, more versatile, and includes a redundancy of measurement feature. Study of an experimental model (DD-3) built in 1967 proved its feasibility and furnished the guidelines for its design for both dc and low-frequency ac operation. Further advantage was gained by providing the ratio standard with its own measuring network so that the combination would serve as a measurement system. This paper describes the measurement system, identified as model DD-4, and includes an analysis of the measured ratios that led to an estimate of 0.2 ppm for the uncertainty.

It is interesting to note that a similar basic design for a ratio standard was being explored simultaneously and independently by Ohlon [2].

Basic Circuits

The measurement system as regarded in this application includes two networks housed within the same enclosure. One is the ratio standard, the second is the main bridge array used to measure its ratios. Each is electrically separate from the other and either is available for use with external circuitry. The voltage supply, detector, and connecting leads are the only external accessories required. The system is shown schematically in Fig. 1 and its physical arrangement appears in Fig. 2.

Ratio Standard

The "working" branch of the ratio standard designed for 1000 Ω/V is indicated in the schematic circuit of Fig. 3. Carefully selected resistance elements are arranged singly or in combination to form 28 sections. Taking the first section as the reference with a nominal value of $R = 1$ kΩ, the remaining sections divide into three groups A, B, and C, each containing nine sections. Those of group A have a nominal value of 1 kΩ while those of groups B and C have nominal values of 10 kΩ and 100 kΩ, respectively.

In the series mode as shown, 27 ratios (referred to the first section) are available, extending over three decades to a maximum of 1000:1. The resistance of each section is nominally equal to either the resistance of the preceding section or to the summation of all preceding sections. As stated in [1], this satisfies the criterion by which such a network is judged to be self-calibrating. Thus, the bootstrap technique described in [1] is applicable and requires only the measurement and appropriate summations of small differences between nearly equal quantities.

In addition to the series mode, the nine sections comprising a given group are geometrically arranged for easy transfer to a parallel configuration [3]. Links, or their equivalent (indicated by dotted lines), from a to b and from c to d reduce group A to a parallel resistance of 1 kΩ, nominal; similarly, those bridging groups B and C reduce these groups to nominal values of 10 and 100 kΩ, respectively. Inclusion of the paralleling feature increases the number of available ratios with some convenient duplications, but more important, it makes possible an intended redundancy of measurement so that more accurately known ratios are assured.

The ratio standard also carries a "guard" chain of 200 Ω/V arranged and mode operated in similar fashion to that of the working branch. Operating in parallel with the working branch, its function is twofold: 1) it maintains the proper potentials on metal shields that surround the critical parts of the working branch and prevents the loss of current through the insulation, 2) it completes the shielding network of the measuring circuit and, through its accessible terminals, maintains the branches of this network at the proper potentials, so that leakage paths through insulation are diverted from the measuring circuit.

Measuring Circuit

The measuring network, although accessible for external use, is designed as an integral part of the system for the ready calibration of the ratio standard. With suitable connecting leads, it accomodates the ratio standard in either its parallel or series mode and can be used entirely or partially depending on the calibration method employed. The circuit as it would be used for either mode of the standard is indicated schematically in Fig. 4. Ratio arms A and B operate on the principle of the direct-reading ratio set (DRRS) [4]. Resistor B and resistor A, when at midscale setting, have nominal values of 1 kΩ, with A adjustable by 50 ppm on either side to a least count of 0.1 ppm. (Interpolation extends the resolution to 0.01 ppm.) Three resistance elements having nominal values of 1, 10, and 100 kΩ are separately

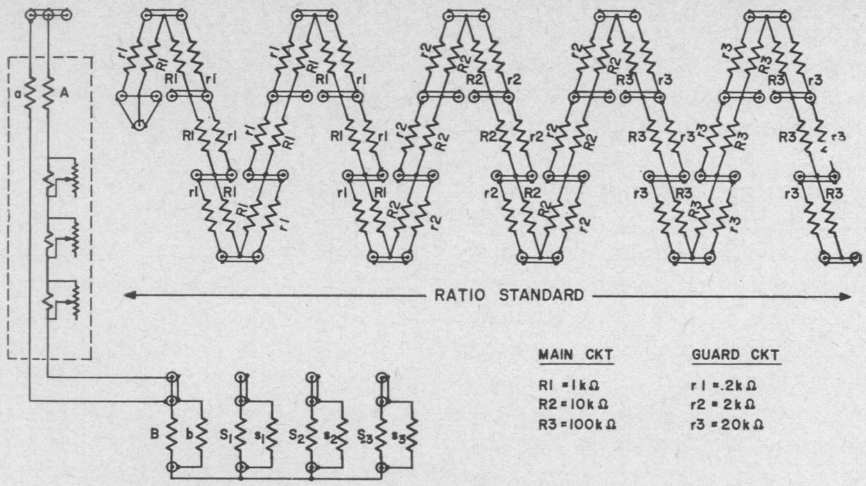

Fig. 1. Model DD-4 voltage ratio standard and measuring circuit.

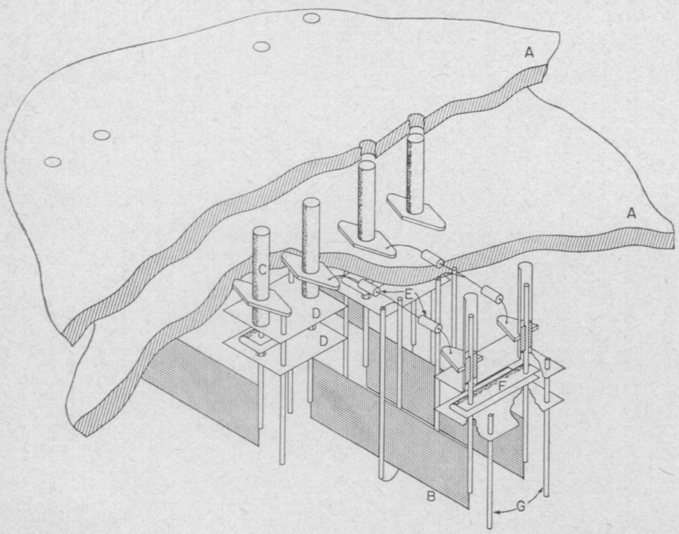

Fig. 2. Physical arrangement of DD-4. *A*—Plexiglass top pieces. *B*—Card-type resistor. *C*—Coaxial terminal. *D*—Brass guarding plates. *E*—Guard resistors. *F*—Folded Cu 4-terminal junction. *G*—Brass guarding rods.

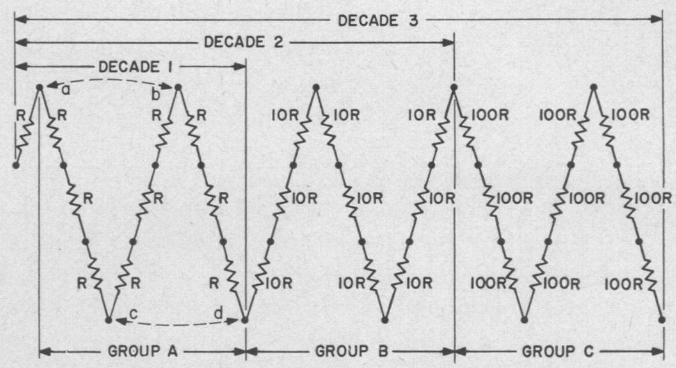

Fig. 3. "Working" branch of DD-4.

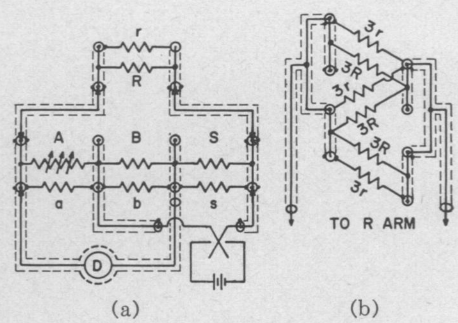

Fig. 4 (a) Simplified measuring circuit for a single section (b) for a series–parallel array.

available as S. The R arm is occupied in sequence by the sections of the standard whose differences are being determined. Since the bridge measures only small differences between nearly equal quantities, only moderate accuracy of measurement is required.

The guarding network is essentially a mirror image of the main bridge. With the shields maintained at proper potentials, leakage currents are directed from shield to shield and confined to the guarding network.

Design Parameters

Ratio Standard

The performance of this type of ratio standard is governed in great measure by the behavior and structure of the resistance elements that comprise the working branch. These were carefully selected to reduce the largest sources of error to manageable size.

Special effort was made to acquire elements with the lowest possible temperature coefficient. Measurements and proper selection assured a temperature coefficient less than 0.5 ppm/°C for the 1 and 10 kΩ elements. Thus, each section of groups A and B (Fig. 3) comprises a single element. A similar selection provided 100-kΩ elements with temperature coefficients not exceeding 1 ppm/°C. Allotting four elements to each section of group C and arranging them in a series-parallel network with some matching of positive and negative coefficients reduced the temperature coefficient per section to less than 0.5 ppm/°C.

A related and equally troublesome source of instability and uncertainty arises from temperature changes produced in the elements when under load. This can be alleviated by designing for the lowest power consumption except that an upper limit is imposed on the value of resistance that can be chosen. Leakage currents and their control become paramount problems so that some compromise is required. The design for 1000 Ω/V offers a low operating current without placing too stringent demands on the guarding system, the sensitivity of detection, and the stability of the resistors. Further reduction of uncertainties associated with heat flow was assured by selecting single-layer card-type elements of Evanohm with each having a radiating area of 16 cm^2 per surface. Each element is suspended between its respective terminals with its two surfaces exposed to the coolant and removed sufficiently from neighboring elements to alleviate problems from proximity heating.

The nine elements comprising a given group were closely matched (adjusted if necessary) to satisfy the requirement that the proportional correction to the resistance of a group when in the parallel mode be identical to that when in the series mode within the accuracy sought.

The high accuracy and stability prescribed for this system dictated that complete guarding along the working resistance chain be provided. The departure of each guard section from its nominal value of resistance does not exceed 0.01 percent so that the guarding sleeves and the working terminals they enclose are essentially at the same potential. Thus, with guarding and high insulation resistance, errors caused by leakage currents are less by several orders than the accuracy prescribed for the measurement process. To prevent heat generated in the guard from influencing the working circuit, the guard chain is mounted in an upper channel external to the enclosure and open to the outside.

Measuring Circuit

The DRRS, although available to external circuits, was essentially tailored to the application at hand. The adjustable arm is a small and compact commercial unit with Waidner–Wolff circuitry and card-type resistance elements. The resistor that forms the B arm and those that occupy the S arm (Fig. 1) in sequence are also the card type having been selected by the same criteria and mounted in the same fashion as those of the ratio standard.

Additional Considerations

Terminal Junctions

Proper selection and arrangement of elements that satisfy design criteria neither guarantee that the values assigned to the ratios are conclusive nor that they are valid when the standard is used to calibrate other ratio networks. Unless other precautions are taken, values obtained by two or more independent methods of equal precision could differ by significant amounts with no assurance, which, if any, represents the value of the ratios as used in calibration. Thus, special attention was given to the design of the terminal junctions and provision was made so that effects from their interconnections and their connections to both internal and external measuring circuits could be predicted and verified experimentally.

An ideal junction is four terminals, designed and adjusted so that all four-terminal resistances are zero [5]. This is the preferred junction, since, if four-terminal techniques are used to measure a chain of nominally equal resistance elements, it defines each resistance precisely. Furthermore, through the use of compensating fans or their equivalent, it permits the parallel arrangement of these elements without introducing spurious resistances through the paralleling process. This subject has been covered quite thoroughly in the literature [6], particularly the use and analysis of compensating fans as proposed by Page [7]. If the ideal junction and compensating fans were applied to the circuitry discussed in this paper as indicated in Fig. 5, no discrepancies would exist between the identity of the measured ratios and those in use.

However, a study of the preliminary experimental model (DD-3) indicated that strict adherence to the geometry of the ideal junction (three terminals symmetrical to a fourth) was not necessary even though the measuring technique selected would not be strictly four terminal. For these and practical reasons, each junction is a small thin copper plate with its terminals located at the four corners of a square (see Fig. 2). The plug-type coaxial terminals are silver-plated brass with polytetrafluoroethylene (PTFE) insulation. The plug-socket resistance of a connection is approximately 450 μΩ, and the contact resistance is repeatable to within ±10 μΩ.

The four-terminal resistances of each junction measured 5.5 ± 0.5 μΩ for both the direct and cross resistances. The maximum uncertainty from junction effects and from possible variations in lead and connector resistances was estimated at less than 0.05 ppm. Appendix II indicates how this estimate was approached. The results and their analysis in Appendix III indicate the experimental verification.

Detection System

The detection system consists of a photoelectric galvanometer amplifier (PGA) whose unbalance signal is further amplified and displayed by an electronic detector. To provide proper negative feedback for the PGA, a 500-ohm resistor is connected across its output terminals. The system is operated at a sensitivity of 30 nV/mm. The noise level is less than 40 nV peak-to-peak during the time period for a single measurement. By visually integrating the signal voltage over a time period of approximately 3 seconds, it is possible to detect 0.01-ppm changes in the bridge circuit at the 1-mA level.

The metal cases of the amplifier and electronic detector as well as the shields of all leads in the detection system are driven at the appropriate guard potential. Thus, the integrity of the insulation is maintained and effects

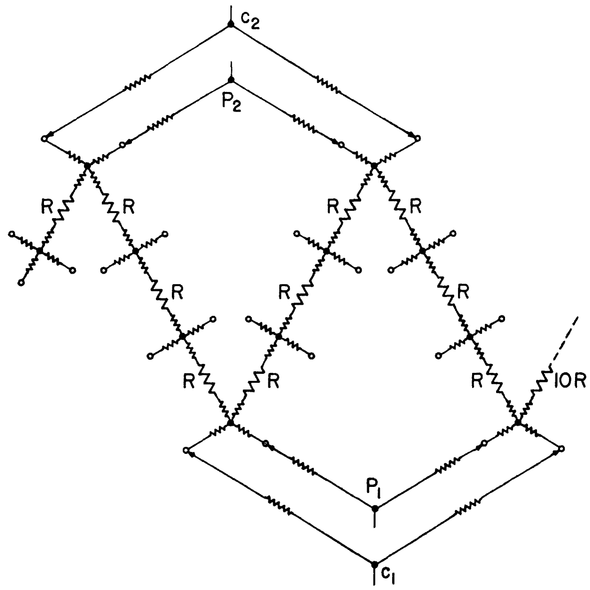

Fig. 5. The parallel mode of the ratio standard, emphasizing ideal junctions and their interconnections. The unlabeled resistance elements are lead and junction resistances.

caused by leakage are reduced to a negligible level. In addition, electrostatic interference is suppressed. The detection system is isolated from the 60-Hz power circuits to reduce noise problems.

During a measurement operation, the DRRS is adjusted while the bridge current is periodically reversed until a null condition results.

Controlled Enclosure

The ratio standard and the measuring network are immersed in a dry-air bath maintained at a temperature of 23 ± 0.05°C. Heat transfer occurs by forced convection using two fans that provide laminar flow across the resistance elements. The thermally lagged bath of approximately 0.1 meter^3 consists of two aluminum boxes separated by 2.5 cm of foam polystyrene and two plexiglass top pieces separated by a 2.5-cm air space.

The temperature control system consists of 1) a thermoelectric heat-pump assembly that extracts a constant amount of heat and 2) a proportional control circuit that regulates the power input of two (0–15 watts) heater elements. Twelve copper-constantan thermocouples strategically located within the bath monitor the temperature to within ±0.05°C.

A recently installed regenerative drying system holds the humidity constant at 4 percent relative humidity. A small amount of filtered and desiccant-dried air under pressure is continually forced into the bath while two small ports located at the opposite end of the bath serve as the exhaust.

Methods of Measurement

Four essentially independent methods were used during the period of evaluation and several lead assemblies were introduced for each method. Two of these were considered as the principal methods while the others served as occasional checks. The one identified as the bootstrap method is regarded as primary since values are obtained for each consecutive ratio. The other identified as series–parallel serves as the redundant method, providing additional values for the 10:1, 100:1, and 1000:1 ratios. Although both techniques utilize the built-in measuring circuit they are sufficiently different to be regarded as independent.

Bootstrap

The circuit of Fig. 4(a) shows the exact arrangement of leads and shielding as used in the bootstrap method with one section of the standard (e.g., the reference section), in the R arm. The 1-kΩ sections, beginning with the first or reference section, are introduced in the R arm in rapid succession and the bridge is balanced for a null on the detector. The balance equation for each k section is of the form

$$R_{1,k} = S_1[1 + D_{1,k} - D_0] \tag{1}$$

where S_1 has the same nominal value as $R_{1,k}$. The reading on the DRRS stated in proportional parts is $D_{1,k}$ while D_0 corresponds to the reading that would exist for an *exact* 1:1 ratio of $A:B$. (The derivation of (1) appears in [1].) The correction to each successive ratio through 10:1 is obtained by taking differences between each equation and the reference equation and averaging their successive sums. The summation $\sum_{k=1}^{k=10} R_{1,k}$ is the resistance of the first decade that is nominally equal to the resistance of each section in group B (Fig. 3). A comparable set of 10 balances is then made beginning with the first decade and continuing through the B group with S_1 changed to $S_2 = 10$ kΩ. A third set of similar balances carries the measurements through the 1000:1 ratio. Under rated conditions each measurement is made with 1 mA through each section. The general equation from which any ratio (referred to the first section) can be computed is

$$\frac{S_{m,k}}{R_{1,1}} = k10^{m-1}\left[1 + \frac{\left(\sum_{k=1}^{k=10} d_{1,k} \cdots + \sum_{k=1}^{k=10} d_{m-1,k}\right)}{10} + \sum_{k'=1}^{k'=k} \frac{d_{m,k'}}{k'}\right] \tag{2}$$

where $S_{m,k}$ is the sum of all resistances up to the kth tap of decade m, and $R_{1,1}$ is the reference section. Each term in the parenthesis is the sum of all the measured differences throughout a decade while the last term accounts for the measured differences up to the k section of interest. (The derivation of (2) is similar to the one appearing in [1].) Since $k10^{m-1}$ is the nominal ratio, (2) is of the form

$$N = N_n[1 + c]$$

where c is the correction to a ratio in proportional parts.

It is important to emphasize that a systematic error that might exist in each measurement is not cumulative within a decade since it, like the differences, partakes of the averaging process. Any accumulation occurs only when passing from one decade to another.

Series–Parallel

This method provides corrections to ratios 10:1, 100:1, and 1000:1 and requires six measurements in contrast to the 30 of the previous method. In each successive set of two measurements, each group (A, B, and C) is converted, in turn, into its parallel mode and compared against *all* the series-connected sections preceding it. Thus in the first set of two balances, the reference section occupies the R arm and is followed by group A connected in its parallel mode, as shown in Fig. 4(b). In the second set, with group A returned to its series connection, the first decade followed by group B in its parallel mode occupies the R arm. A similar sequence repeated once again carries the measurements through the 1000:1 ratio. For each successive set, S takes on the values 1, 10, and 100 kΩ. Three sets of differences and the average of their successive sums with the series–parallel principle applied yield values for the three ratios. The general equation for any ratio (referred to the first section) is

$$N_m = 10^m\left[1 + \frac{9}{10}\sum_{p=1}^{p=m} d_p\right]$$

where m is the number of the decade and

$$\sum_{p=1}^{p=m} d_p.$$

is the sum of the measured differences.

Two principal lead assemblies were used. In the one case the paralleling of groups was effected by using both current and potential fans. In the other only one pair of compensating fans was used. No difference in the results was noted so that the simpler arrangement was preferred. This is further indicated in Fig. 9 of Appendix II where the connections for both the reference section and the parallel array are shown.

Additional Methods

The two "check" methods alluded to earlier are identified as the "interchange" and "Kelvin bridge" methods. In the former, only arms A and B of the internal bridge are used. The other two arms of the bridge are occupied in turn by two adjacent sections with each pair advancing by one section. Since decade (m) forms the first section of decade ($m + 1$) the process can be carried forward through the remaining sections. In this kind of measurement, D_0 must be determined by the usual interchange technique. The measuring process is similar to that of the bootstrap method in that a value is obtained for each ratio.

The second method, as the name implies, is a four-terminal measurement. The step-by-step procedure of the bootstrap method is followed but the sections are measured as four-terminal resistances using a Kelvin bridge. This method was used only for the 1-kΩ decade where effects from terminal junctions and lead arrangements would be greatest.

Results

An analysis of all dc data accumulated from December 1968 to November 1969 is contained in Appendix III. Data taken since the completion of the analysis and too late for inclusion continue to substantiate the conclusions as stated.

The dc data obtained by the two principal methods covered 50–60 *complete* sets of measurements for each of two applied voltages. (Additional complete or partial sets obtained on occasion from the two supplementary methods totalled 14). The initial study covering about six months was made at 50 percent rated voltage. When evidence indicated that the measuring system was sufficiently stable and that measurements probably could be made to better than the 0.5 ppm anticipated, the system was moved to a location where ambient influences could be better controlled. The concentrated study continued for 4 months with measurements made at the reduced voltage over a 2-month period followed by those at rated voltage.

Analysis of the data at rated voltage shows that for a 99 percent confidence interval no difference greater than 0.1 ppm exists between the two methods. This excellent agreement between two independent methods makes it very unlikely that a systematic error greater than 0.1 ppm has gone undetected.

Additional information is available from Fig. 11 where the results at rated voltage are plotted against elapsed time for the 10:1, 100:1, and 1000:1 ratios. Fig. 11(c) is interesting since it appears to indicate a difference approaching 0.3 ppm between the two methods. However, as shown in the appendix, about 0.2 ppm arises because the power level at which each section operated was appreciably different for the two methods. Hence, this portion of the observed difference simply reflects a difference in test conditions and can not be regarded as a systematic error that lies hidden in one method or the other. Furthermore, the load conditions on the sections at rated voltage for the bootstrap method are essentially the same as those that would exist when in use, so that this difference could be regarded as a systematic error only if the series–parallel method were used alone to assign values at so-called rated voltage. Adjustment of the values to account for the known change with load led to the assignment of 0.1 ppm as the systematic error through the 1000:1 ratio.

As justified in Appendix III, greater weight was accorded the values obtained at rated voltage. The stability of the system improved after its transfer to the inner room and the subsequent analysis indicated that the standard deviations at 50 percent rated voltage were reduced, the change being appreciable on the 1000:1

ratio, and were more consistent with those at rated voltage. Also toward the end of the measurement series at reduced voltage, when the values were scrutinized more closely, it became evident that humidity effects, though small, could no longer be considered negligible. These are discussed more fully in the appendix; however, they are evident in Fig. 11 where the changes in the drying agent are noted along the abscissa. Evidence indicates that much of the random behavior arises from this cause. Data accumulated after an improved humidity control was installed have produced no evidence to the contrary. There are two known mechanisms that could produce the humidity effects observed: 1) surface leakage across the resistance elements and 2) forces on the fine wire caused by dimensional instability of the wire insulation or mica former. Leakage across the insulation between the guard and working circuit appears negligible. An estimate of σ for the first decade where effects from humidity are least is 0.05 ppm. This includes not only the random error of the measurement process but also any random changes in the ratio standard over a period of about 2 months. Taking this value as applying also to the other decades (under dry conditions), an uncertainty is assigned at 0.2 ppm, based on a 3σ confidence limit for random errors with a 0.1-ppm estimate for the residual systematic error.

AC Characteristics

Several features were incorporated in the design of the ratio standard for operation at 100 Hz or below. These include:

1) the use of single-layer card-type resistors having small residual inductances and shunt capacitances;

2) the use of air as the heat-exchange fluid for the bath instead of oil or a similar fluid having a larger dielectric constant;

3) the design of junctions that could be guarded easily, along with the use of guarded connectors as the terminations of these junctions;

4) a low guard resistance (200 Ω/V) to minimize the effect of external shunt capacitances on the guard ratio and thus further reduce their effect on the main ratio [8];

5) the use of brass rods, connected to appropriate guard-circuit taps, at various points to suppress errors caused by capacitive currents from the resistors to their surroundings.

The ratio standard was calibrated at 100 Hz by the substitution method using an inductive voltage divider. The output of an auxiliary inductive voltage divider connected to the common point of the guard-circuit resistors in the bridge provided for the phase adjustment. The calibration procedure is similar to that described previously in the bootstrap method. Results of these tests compared to the dc values agree to 0.1 ppm for ratios 1:1–10:1, 0.2 ppm for ratios 10:1–100:1, and 2.5 ppm for ratios 100:1–1000:1. The good agreement of the lower ratios further substantiates the accuracy of the dc values. The larger disagreement of the 100-kΩ sections is due probably to the uncompensated residual shunt capacitances and to significant dielectric absorption of the mica winding form of the resistors. The above ac data are only preliminary and further study is planned.

Summary

A stable voltage-ratio standard with extremely small errors and the networks for measuring its dc ratios to an accuracy of 0.2 ppm were described. Preliminary study of the standard at 100 Hz indicates a comparable performance can be expected at low ac frequencies. Investigation of the ac performance will continue. This development can lead to a more accurate assignment of ratios and relatively large dc voltages throughout the entire chain of measurements.

Appendix I

In the experimental model DD-2, a "base" element, which provides a reference ratio, contains two sets of n nominally equal resistors connected in series [9]. Each set is arranged so that its n resistors can be connected either in series or parallel. Two ratios are possible. A ratio $N_1 \approx 1 + n^2(1 + \alpha_s - \beta_p)$ is obtained by connecting the α set in its series mode and the β set in its parallel mode. A second ratio $N_2 \approx 1 + n^2(1 + \beta_s - \alpha_p)$ is formed when each set is reconnected for its alternate mode and the two are interchanged in position. (The α and β terms are small corrections to the resistance of the respective sets as they occupy the series or parallel position.) If the series–parallel principle is satisfied $\alpha_s = \alpha_p$ and $\beta_p = \beta_s$ and the mean ratio $N \approx 1 + n^2$. Thus a reference ratio is provided that, theoretically and to first order, is a function only of the number of elements. Extension of the principle to higher ratios is effected by adding sections whose resistances could be adjusted to the mean value of the base element.

Appendix II

This section outlines the procedure that was used to predict from circuit considerations the uncertainty introduced in a measurement of the combined effects from 1) the finite resistance of the four-terminal junction, 2) the changes in connections when transferring from a series to a parallel mode, and 3) the variations in lead and contact resistance as each section is introduced into the bridge. Since the uncertainties would be greatest for the lowest valued sections, only the first decade was considered.

Three circuits must be examined. Two of these duplicate the networks associated with the two principal methods of measurement while the other identifies the standard as its ratios appear in use. It is necessary that the ratios measured be identical to those in use or that a correction be applied to compensate for the discrepancy.

A four-terminal junction that departs from the ideal configuration has the equivalent circuit of Fig. 6 [6]. As applied to the junctions of the DD-4, the six resistances are calculable from the plug resistance of the connectors,

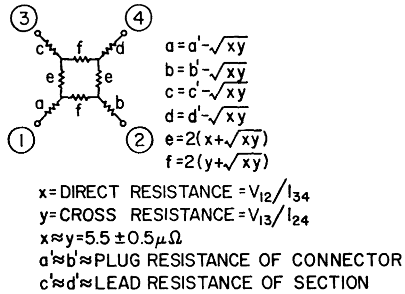

Fig. 6. Equivalent circuit of a four-terminal junction.

the lead resistance of the sections and the readily measureable direct and cross resistances X and $Y (X \approx Y = 5.5 \pm 0.5\ \mu\Omega)$. Fig. 7(a) identifies the circuit of DD-4 when it is *used* in the NBS ratio comparator to measure an unknown ratio. (The 10:1 ratio is indicated in the figure.)

Making the appropriate Δ-Y transformations reduces the circuit to that of Fig. 7(b). The ratio in use

$$N_u = \frac{\sum_{k=1}^{k=n} R_k}{R_1} = N_u[1 + c_n]$$

where $R_k = R_k' + Y_k + d_k' + c_{k+1}'$.

The problem now reduces to a similar examination of the two measuring networks to determine the extent to which the measured ratios differ from those in use.

The measuring network for the bootstrap method is indicated in Fig. 8(a) and simplified in (b), where g_k(or h_{k+1}) is the sum of the socket and contact resistances and l_k(or m_{k+1}) is the lead resistance.

Applying the latter network to each successive measurement through the decade gives a defining equation of the form

$$\frac{\left[\sum_{k=1}^{k=n} R_k\right]_{\text{bootstrap}}}{R_1} = n\left[1 + \sum_{k=1}^{k=n} d_k/n - \sum_{k=1}^{k=n} \frac{(\epsilon_{1k} + \epsilon_{2k} + \epsilon_{3k})}{n}\right]$$

where

$$\epsilon_{1k} = [(X_k - X_1) + (X_{k+1} - X_2)]1/R_1$$

$$\epsilon_{2k} = [(a_k' - a_1') + (a_2' - a_{k+1}') + (g_k - g_1) + (h_2 - h_{k+1})]1/R_1$$

$$\epsilon_{3k} = [(l_k - l_1) + (m_2 - m_{k+1})]1/R_1$$

and

$$\frac{\sum_{k=1}^{k=n} d_k}{n} = c_n.$$

From an experimental study of the junctions, connectors, and connecting leads, it is unlikely that the sum of the ϵ terms would exceed 0.03 ppm. It appears that the ratio measured and the ratio when used are identical to well within the accuracy assigned.

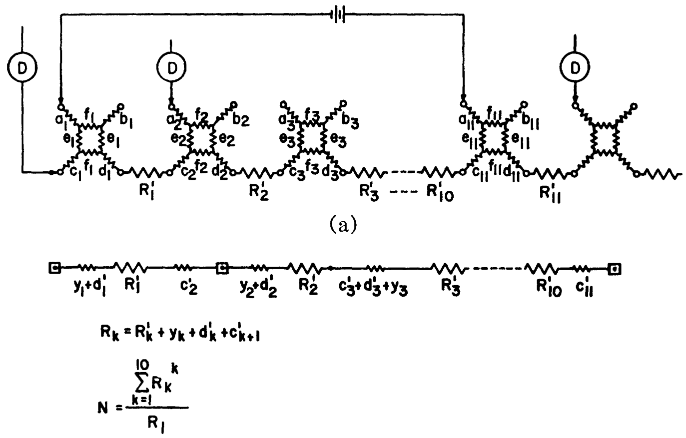

Fig. 7. Circuit of DD-4 (a) and its equivalent (b) when in *use*.

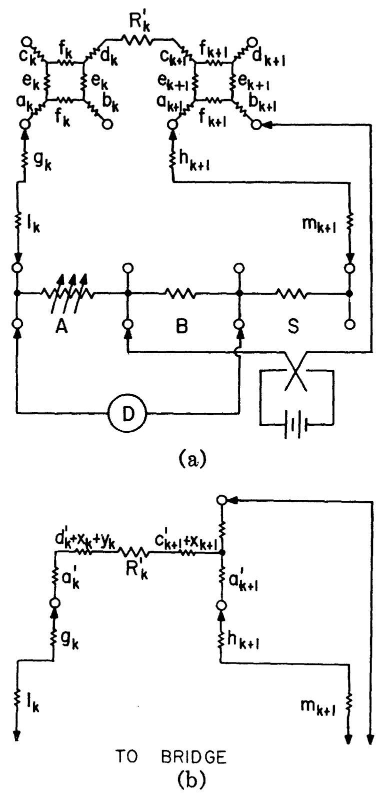

Fig. 8. Circuit and its equivalent for DD-4 when measured by bootstrap method.

The circuits for the series–parallel method are shown in Fig. 9 and refer to the connections when the reference section and the parallel array of the first decade occupy the R arm.

All resistances are identified as before except for two pairs of compensating resistors (u_1—u_2) and (v_1—v_2). Resistors u_1 and v_1 are adjusted to bring points p_1 and p_2 to the same potential. Resistors u_2 and v_2 are adjusted to bring points p_3 and p_4 to the same potential. Since the u leads, in contrast to the v leads, carry current to or from *two* branches, their resistances are half those

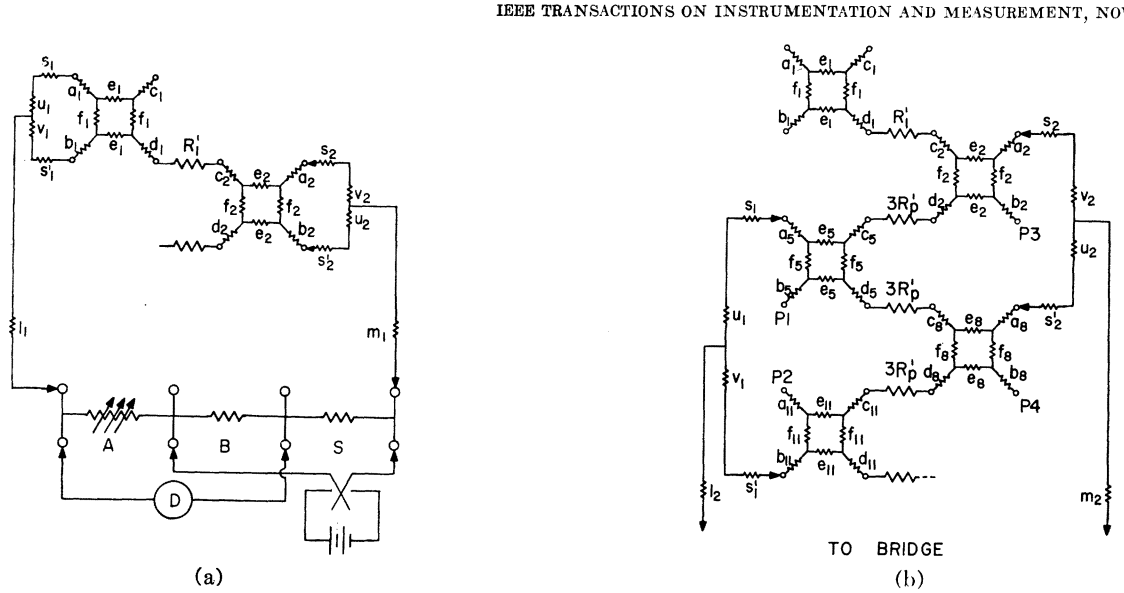

Fig. 9. Circuits for DD-4 when measured by series–parallel method.

of the v leads. The corresponding simplified circuits are shown in Fig. 10.

The balance equation when the first section occupies the R arm is

$$S[1 + D_1 - D_0] = R_1\left[1 + \frac{1}{R_1}\left(l_2 + m_2 + X_2 + \frac{\alpha_1\beta_1}{\alpha_1 + \beta_1} + \frac{\alpha_2\beta_2}{\alpha_2 + \beta_2}\right)\right]$$

where R_1 is that portion of the section that is identical to the section when in use and the $\alpha\beta$ terms are the resistances of the parallel arrays at the two ends. The balance equation when the parallel group occupies the R arm is

$$S[1 + D_p - D_0] = R_p\left[1 + \frac{1}{R_p}\left(l_2' + m_2' + \frac{X_{11}}{9} + \frac{\alpha_1'\beta_1'}{\alpha_1' + \beta_1'} + \frac{\alpha_2'\beta_2'}{\alpha_2' + \beta_2'}\right)\right].$$

From these two equations and making the parallel-to-series conversion, the defining equation for the 10:1 ratio is

$$\frac{\left[\sum_{k=1}^{k=10} R_k\right]_{S-P}}{R_1} = 10\left[1 + \frac{9}{10}d_p + \frac{9}{10}(\epsilon_1' + \epsilon_2' + \epsilon_3')\right]$$

where

$$\epsilon_1' = (X_{11} - 9X_2)1/9R_1$$

$$\epsilon_2' = [(l_2' - l_2) + (m_2' - m_2)]1/R_1$$

$$\epsilon_3' = \left[\left(\frac{\alpha_1'\beta_1'}{\alpha_1' + \beta_1'} - \frac{\alpha_1\beta_1}{\alpha_1 + \beta_1}\right) + \left(\frac{\alpha_2'\beta_2'}{\alpha_2' + \beta_2'} - \frac{\alpha_2\beta_2}{\alpha_2 + \beta_2}\right)\right]1/R_1$$

and

$$\frac{9}{10}d_p = c_{10}.$$

The sum of the ϵ' terms is calculated to be about 0.02 ppm. From this value and that given for the ϵ term of the bootstrap method a difference no greater than 0.05 ppm would be expected between the two methods. This is in good agreement with the value of 0.02 ppm deduced from experiment for the 10:1 ratio.

Appendix III

The *principal* argument of the analysis deals with the results obtained for the 10:1, 100:1, and 1000:1 ratios by the bootstrap and series–parallel methods, since these were more apt to disclose junction and connection uncertainties as well as uncertainties arising from environmental influences. Results for the intermediate ratios that are available only from the bootstrap method provide additional information. The supplementary methods further substantiate the argument.

Data from the first series of measurements obtained at 50 percent rated voltage for each of the three ratios are analyzed separately from those of similar sets obtained subsequently at rated voltage. The principal argument covers data obtained after the system was relocated in a room environment that was more easily controlled. This required the deletion of only three pairs of measurements for each ratio.

All results are stated as *corrections* to the nominal ratios in ppm.

Range

To first provide a feel for the data and the magnitudes involved, *all* data, irrespective of elapsed time, method, or innovation were plotted as histograms (not included) for each of the ratios at the two voltages. The values at *rated voltage* for each ratio approximate a normal distribution and the same is true for the values at reduced voltage when a few of the initial and relatively extreme outliers are deleted. Under this condition, the average

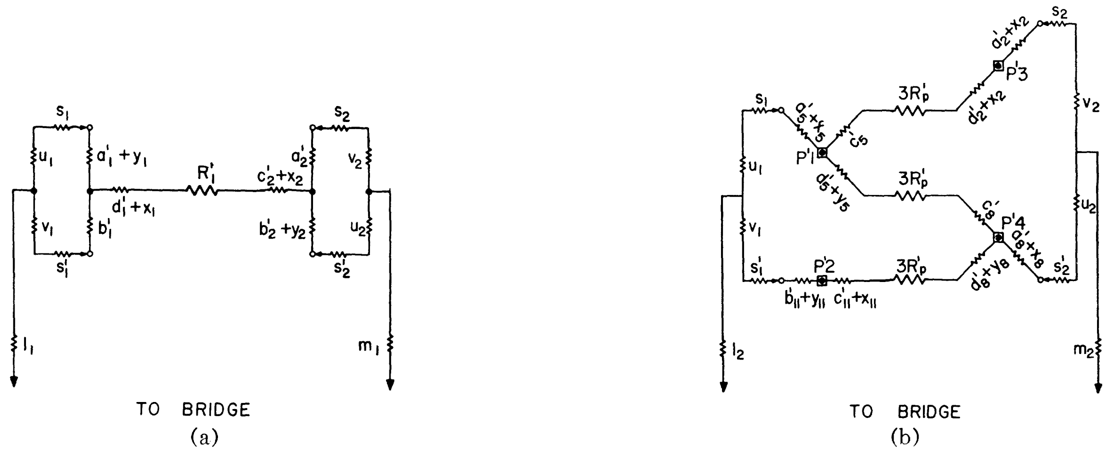

Fig. 10. Equivalent circuits for DD-4 when measured by series–parallel method. In text,

$$\alpha_1 = u_1 + s_1 + a_1' + Y_1 \qquad \beta_1 = v_1 + s_1' + b_1'$$
$$\alpha_2 = v_2 + s_2 + a_2' \qquad \beta_2 = u_2 + s_2' + b_2' + Y_2$$
$$\alpha_1' = u_1 + s_1 + a_5' + X_5 \qquad \beta_1' = v_1 + s_1' + b_{11}' + Y_{11}$$
$$\alpha_2' = v_2 + s_2 + a_2' + Y_2 \qquad \beta_2' = u_2 + s_2' + a_8' + X_8.$$

TABLE I
Ranges for all Measured Values and Their Averages (ppm)

Ratio	Percent Rated Voltage	n	Correction Average (ppm)	Range (ppm) Maximum	Range (ppm) Minimum
10:1	50	57	−0.072	+0.13	−0.30
	100	59	+0.199	+0.28	+0.08
100:1	50	46	+0.528	+0.96	0.00
	100	59	+1.012	+1.26	+0.56
1000:1	50	43	+1.432	+1.89	+0.84
	100	53	+1.456	+1.73	+1.14

TABLE II*

Ratio	Percent Rated Voltage	n	$\bar{d}$	s_d	Confidence Interval Estimates
10:1	50	13	+0.042	0.089	−0.033 → +0.117
	100	28	−0.008	0.033	−0.025 → +0.009
100:1	50	13	+0.101	0.082	+0.031 → +0.171
	100	28	−0.047	0.036	−0.066 → −0.028
1000:1	50	13	+0.056	0.211	−0.123 → +0.235
	100	28	−0.250	0.044	−0.273 → −0.227

* The 99 percent confidence interval estimates of the average difference between the two principal methods with the averages and the standard deviations of the differences included.

and range have the values given in Table 1, where n specifies the number of measurements. Inclusion of the outliers increases each range at reduced voltage by about 1 ppm. The corresponding effect on the averages is no greater than 0.2 ppm.

t Test

This portion of the analysis is an examination for any difference between the two principal methods. A sufficient number (n) of measurement pairs extending over the 4-month period were available for which a value by the one method was obtained within 1 hour of the other. The 99 percent confidence interval estimates of the average difference between the methods are given in Table II along with the averages $\bar{d}$ and the standard deviations s_d of the differences. The difference was taken as the bootstrap value less the series–parallel value.

The following facts and *apparent* inconsistencies are to be noted.

1) There is strong evidence at *rated* voltage of a negative difference between the methods that is essentially negligible for the 10:1 ratio but which increases to almost 0.3 ppm for the 1000:1 ratio, i.e., for the 100:1 and 1000:1 ratios, the limits $[\bar{d} - (ts_d/\sqrt{n})] < 0 < [\bar{d} + (ts_d/\sqrt{n})]$ are not satisfied.

2) A consistent set of limits is lacking at 50 percent rated voltage, and the intervals are wider.

3) The standard deviations at rated voltage are essentially equal and small in magnitude but exhibit perhaps a slight upward trend with increase in ratio.

4) The corresponding standard deviations at reduced voltage are inhomogeneous in magnitude with s_d at the 1000:1 ratio being unduly large. The set lacks the self-consistency feature exhibited by that at rated voltage.

The data and associated test conditions warranted further examination.

Data at Rated Voltage

Further examination of the data and the graphs of Fig. 11 indicates that $\bar{d}$ and the confidence interval for the 1000:1 ratio at rated voltage are more negative than would be expected. Several known factors could account for the apparent difference between methods, but these are equally applicable at the lower ratios and at reduced

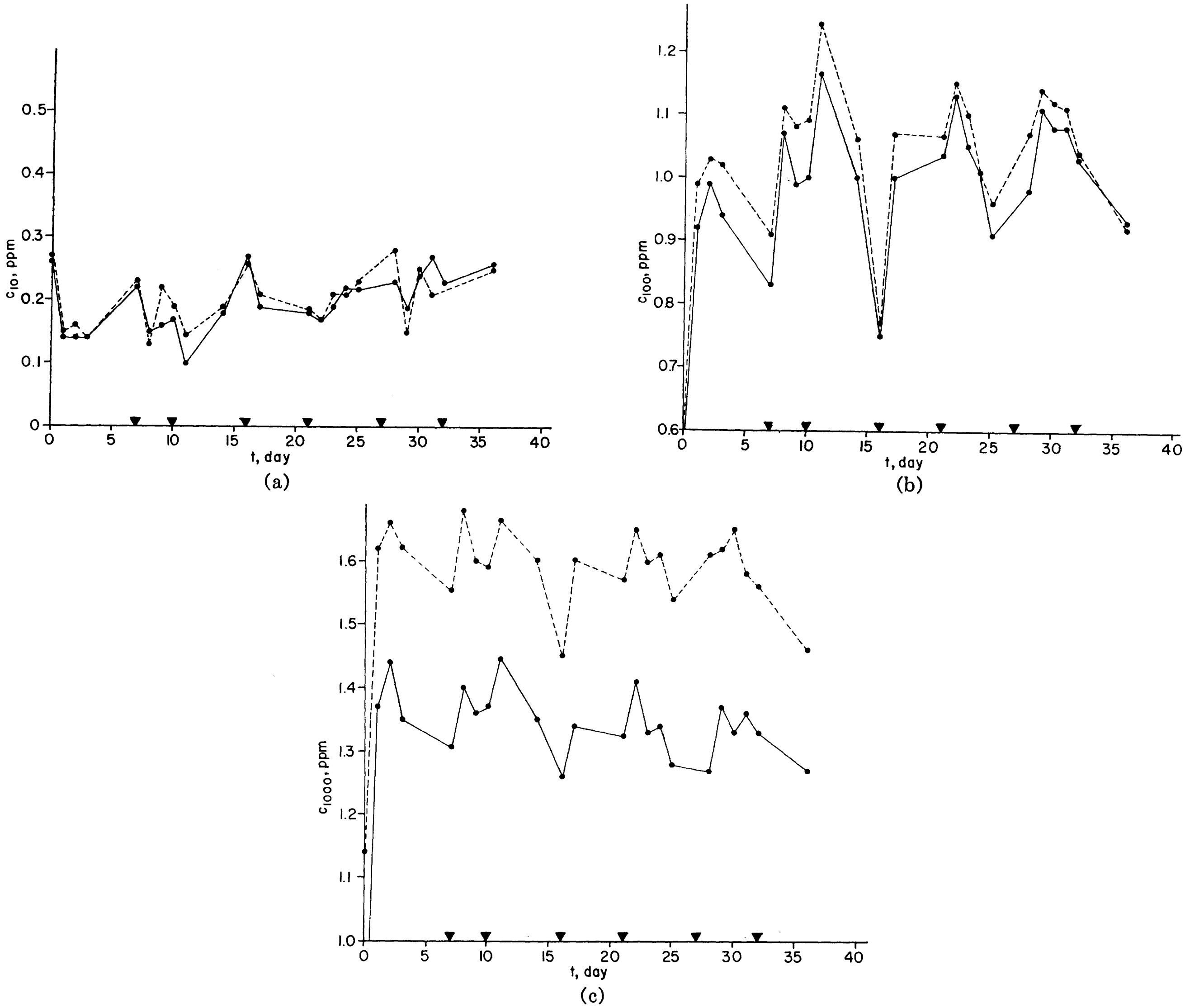

Fig. 11. Corrections to the ratios at rated voltage in ppm versus elapsed time. Solid line indicates bootstrap method; dashed line indicates series–parallel; arrows indicate desiccant was changed after the given measurement. (a) 10:1 ratio. (b) 100:1 ratio. (c) 1000:1 ratio.

voltage, with one notable exception. The power dissipated in the resistance elements in the bootstrap technique is nine times that in the series–parallel, and any contribution to the difference from this source would be magnified at the higher voltage.

The temperature coefficient of ratio is negative for all ratios and attains, with the bootstrap technique, a maximum of -0.6 ppm/°C for the 1000:1 ratio. Consideration of the power per centimeter squared of radiating surface with the resistive elements in contact with the circulating air leads to an estimated temperature rise at the surface of about 0.3°C at rated voltage. This would account for an equivalent change in the 1000:1 ratio of about -0.17 ppm. It is reasonable to assume a negligible heating effect for the parallel mode. With this assumption, it would be expected that values by the bootstrap method for the 1000:1 ratio would be more negative by the above amount than those by the series–parallel method.

If the bootstrap values are adjusted upwards (or series–parallel downwards) to eliminate the difference arising from heating effects, the confidence interval becomes $(-0.103 \rightarrow -0.057)$ in contrast to $(-0.273 \rightarrow -0.227)$ given in the table. A similar treatment of the data for the 100:1 ratio reduces the limiting values given in the table by about 0.010. Thus, the evidence supplied by the data at *rated* voltage would indicate a real difference approaching 0.1 ppm between the two methods.

Data at Reduced Voltage

An equivalent concinnity among the data at reduced voltage would be expected unless spurious influences, not present to the same extent at rated voltage, were affecting the measurement. Two obvious factors must be considered—an inherent instability in the ratio standard and effects from humidity. Since the series of measurements at reduced voltage preceded those at rated voltage, either or both might exist. However, since the two series were immediately consecutive, an inherent instability in the standard appears a less likely cause.

Furthermore, with self-heating greatly reduced, changes in the resistance of the elements would be reflected in both methods unless the instability were of an unusual nature, e.g., a steady but rapid drift. No evidence of this behavior existed. It is more logical to suppose that a humidity not under careful control would be the more likely cause for the discrepancies in Table II, since its effect on the response of resistive networks is difficult to predict. This conclusion is reached in the next section.

Table III completes the present argument and introduces the section that follows. It includes the standard deviations s_b and s_p for the bootstrap and series-parallel methods, respectively, and the ratios of their squares F. The values at rated voltage are included for comparison.

There is no evidence from the F values that the precision differs for the two methods at either applied voltage. As indicated later, it is probable that the poorer agreement between the standard deviation pairs s_b and s_p at reduced voltage arises from insidious humidity effects.

TABLE III
INDIVIDUAL STANDARD DEVIATIONS FOR THE TWO PRINCIPAL METHODS AND THE RATIO OF THE VARIANCES SQUARED

	50 Percent Rated Voltage				Rated Voltage			
Ratio	n	s_b	s_p	F	n	s_b	s_p	F
10:1	13	0.093	0.062	2.25	28	0.048	0.045	1.15
100:1	13	0.151	0.123	1.51	28	0.134	0.134	1.00
1000:1	13	0.100	0.165	0.37	28	0.110	0.107	1.05

Humidity Influence

No attempt was made to control the humidity within the enclosure during the measurement series at reduced voltage. The small effects from this parameter were given attention during the measurement series at rated voltage when it appeared that an accuracy of 0.2 ppm or better was achievable.

During the latter period, changes were effected in the humidity condition by replenishing a drying agent at suitable intervals. These changes are entered in Fig. 11 where the arrows along the abscissa indicate that the drying agent was replaced after the corresponding reading on the ordinate. The humidity effect is evident in all cases, and no trend with elapsed time of any significance is apparent. Changes in ratio following replacement of the desiccant are in the same direction for the two methods and agree to 0.1 ppm or better.

An examination of values obtained for *all* nominal ratios by the bootstrap method disclose with one exception the following behavior.

1) On the $X1$ range, corrections for all ratios go more negative with a maximum at 0.2 ppm.

2) On the $X10$ and $X100$ ranges, corrections for all ratios go more positive with maximum at 0.3 and 0.2 ppm, respectively.

3) As to the exception and for no apparent reason, the signs of the changes are reversed to those given in 1) and 2).

In contrast to Fig. 11, similar curves for the data at reduced voltage (omitted from the paper) exhibit downward trends for the two highest ratios. The maximum apparent drift amounts to about 0.3 ppm for the series–parallel method on the 1000:1 ratio and about 0.2 ppm for all others. Furthermore, the effects from the first recorded change in the drying agent, made near the end of the series, are appreciable. The changes in values for the respective methods differ by 0.2 ppm but have a maxima of 0.6 and 0.4 ppm for the 100:1 and 1000:1 ratios.

Omission of the data at reduced voltage in assigning an uncertainty of measurement is based on the facts and argument presented. These indicate that the measurement process was not under statistical control during operation at reduced voltage.

On the basis that the relative humidity is now closely controlled, the standard deviation σ is taken as the average of s_p and s_b as given for the 10:1 ratio. The uncertainty of measurement is then estimated at 0.2 ppm, based on the 3σ confidence limit for random errors, and 0.1 ppm for the systematic error. Occasional values obtained by the two "check" methods substantiate the above conclusion.

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