A Room Temperature Setup to Compare the Quantized Hall Resistance with 1-Ω Standards

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***Abstract*—A method has been developed to relate the mean value of the IEN primary group of 1-Ω standard resistors to the $i = 2$ quantum level of the Hall resistance. It is based on an automated Potentiometric System, a 36 × 360 Ω Hamon network, and a commercial Kusters Current Comparator bridge. Descriptions of the equipment and of the measurement technique are given, together with a detailed uncertainty budget. The typical fractional total uncertainty of the measurement at the 1-σ level is $5 \cdot 10^{-8}$.**

I. Introduction

The Hall resistance R_H of a two-dimensional electron gas, under conditions of high magnetic field and low temperature, is quantized and assumed to be equal to h/ie^2 (h is the Planck constant, e is the electron charge, and i an integer quantum number). Due to its accuracy the use of this quantization has been recommended by the Comité International des Poids et Mesures to realize the ohm [1]. To achieve this goal the metrological laboratory needs a system to compare R_H with a group of standard resistors stable enough to maintain the unit within the time period between quantum Hall experiments.

At IEN the $i = 2$ recommended value $R_H = 12906.4035$ Ω [1] is used, while the primary group of standards is made up of ten 1-Ω Thomas-type resistors, maintained in an oil bath at (20 ± 0.001)°C. The comparison is performed using room temperature measuring apparatus, partly commercial and partly built in house. The technique is shown in Fig. 1. The value of R_H is first transferred to a nominally equal resistor R_R using an automated Potentiometric System (PS). Then R_R, in series with the auxiliary resistor R_A, is scaled down to 10 Ω using the same PS and a specially built Hamon network. The step from 10 Ω to 1 Ω is performed with a commercial Kusters Current Comparator bridge (CC) following two different paths; one is a direct comparison with the 1-Ω standard using the 10:1 ratio of the bridge, and the other is a step up to 100 Ω using the same 10:1 ratio and a step down to 1 Ω using a commercial Hamon network. Taking the mean value of the two different paths partially compensates for the 10:1 ratio error of the bridge.

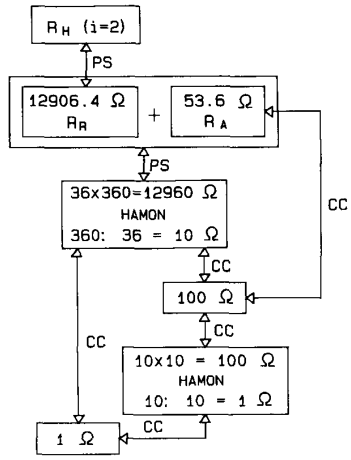

Fig. 1. Diagram of the scaling down technique.

In the following the apparatus is described and the contributions to the 1-σ uncertainty are analyzed.

II. The R_H-R_R Comparison

The reference resistor R_R (ESI SR104 type) is maintained in a thermostatic, doubly shielded, air enclosure at a nominal temperature of (27 ± 0.002)°C. A small correction resistor was added to have R_R trimmed to R_H within a few ppm (1.6 ppm, actually) to reduce the influence of nonlinearity error of the detector. The temperature coefficient of R_R was measured to be +0.2 ppm/°C, while the effect of the dissipated power is always negligible. The rate of drift of R_R, deduced from the comparisons with R_H, is −0.06 ppm/year.

The automated Potentiometric System built to compare R_H and R_R is outlined in Fig. 2, where the resistors in comparison can be connected in either the R_1 or R_2 positions. The system uses high capacity mercury batteries for the current and compensation voltage sources, a battery-powered linear amplifier as a detector, and high quality make-before-break rotary switches. Special care has been given to the thermal and electrical insulation and electri-

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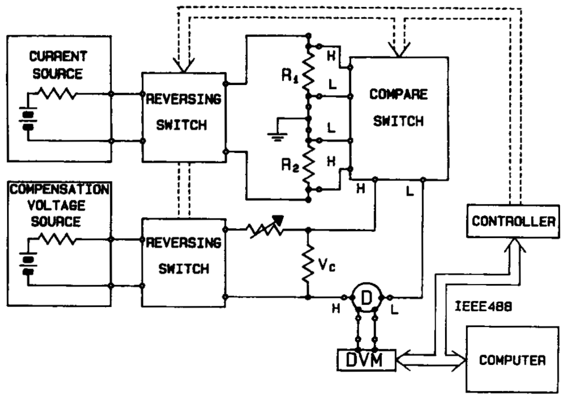

Fig. 2. Schematic diagram of the automated Potentiometric System. D is a linear amplifier and DVM is a digital voltmeter.

cal shielding of the circuit. A particular sequence of positive and negative current measurements yielding a single determination of the ratio R_H/R_R has been adopted to reduce the influence of the thermal voltages and the voltage and current drifts. More details are given in [2].

Usually six determinations, taking about 2 h, give a standard deviation of the mean (type A or random uncertainty) of 0.004 ppm for a measuring current of 25 μA. The limited insulation resistance and the difference of the measurements when R_H and R_R are interchanged in the system give the main, humidity dependent, contributions to the type B, or systematic, uncertainty. The total uncertainty is 0.009 ppm.

III. The 12960 Ω–10 Ω Step Down

The series-to-parallel transfer technique based on Hamon networks [3] is widely used to precisely compare resistors whose ratio is the square of an integer number n. If the ratio does not satisfy this condition exactly, as when 12906.4035 Ω is compared to any decadic value, one can try to add an auxiliary resistor R_A to the odd-value resistor or to the Hamon network. R_A must be a small fraction of the odd-value resistor to lessen the demand on the accuracy of its calibration. Another possibility is to remove the constraint on n using more general networks having two dual configurations [4]. In this case n is a rational number.

Starting from the nominal value of R_R, a step down to 100 Ω would require $n = 11.36$, which can be accommodated by a dual configuration network having 14 1133.33-Ω resistors, 11 in series, series connected to the remaining three in parallel and to an auxiliary $R_A = 61.96$ Ω resistor. The main difficulty in this solution comes from the treatment of the parallel connections at the internal nodes of the network [5]. On the other hand, a step down to 10 Ω requires $n = 35.93$, which can be increased to 36 by adding $R_A = 53.60$ Ω to the reference resistor R_R. This leads to a standard Hamon network made of 36 360-Ω resistors. The difficulty is postponed to the 10 Ω to 1 Ω step down, where no easy series-to-parallel transfer technique is at hand.

We chose the second solution and R_A was carefully connected, in the same thermostatic enclosure, to the trimmed standard resistor R_R. The separate calibration of R_A must be done to an accuracy of 1 ppm. The comparison of $(R_R + R_A)$ with the series Hamon network is done using the same Potentiometric System and the same technique as for the $R_H - R_R$ comparison.

A. Hamon Network Construction

A picture of the 36 × 360 Ω Hamon network during assembly is shown in Fig. 3 where the two teflon disks supporting the resistors and the parallel contacts are shown upside down. The main resistors are hermetically sealed bulk-metal-foil type with thermal self compensation (Vishay VHP101). The four-terminal copper junctions and the topology of the connections are visible in the inset. Two adjacent resistors are connected to two of three holes, 120° spaced, on the disk-shaped tail of the junction, the third hole being used for the parallel voltage lead. The stem of the junction is fixed to the thick teflon disk supporting the network and is used for the parallel current lead. The teflon cover for the parallel connection holds thick copper shorting rings for current and thinner copper rings with 20-Ω compensation resistors for voltage. Stainless steel cylinders are tightly fitted to the stems of the four-terminal junctions and form mercury-filled cups to improve the contacts. Careful mechanical work was done to assure that the 74 contact surfaces are in-plane to within 0.2 mm.

Even though a seal is provided, we preferred to keep the network in air, rather than in an oil bath. The heavy thermal insulation gives a thermal stability of 0.02°C when the room temperature is controlled to 0.2°C. The temperature coefficient of the network is +0.13 ppm/°C, and the mean drift rate is about −0.0025 ppm/day. Usually during the half-day step down to 10 Ω, the network stability is better than 0.005 ppm. The power coefficient of the network is lower than 0.3 ppm/W, giving a contribution of only 0.003 ppm to the uncertainty for 10 mW comparisons.

B. Hamon Network Verification

An analysis of the accuracy of series-to-parallel Hamon networks is done in [6], considering the deviations from nominal of the main resistors Δ_k and of their average Δ_{av}, the maximum four-terminal junction resistance M, the maximum shorting bar resistance R_f and the unbalance δ of the compensation resistors. The conclusive relation is:

$$\frac{R_P}{R_s} = \frac{1}{36^2}\left[1 \pm 4\frac{M}{R} \pm 2\frac{R_f}{R}\delta \pm \frac{1}{36}\sum_{k=1}^{36}(\Delta_k - \Delta_{av})^2\right], \quad (1)$$

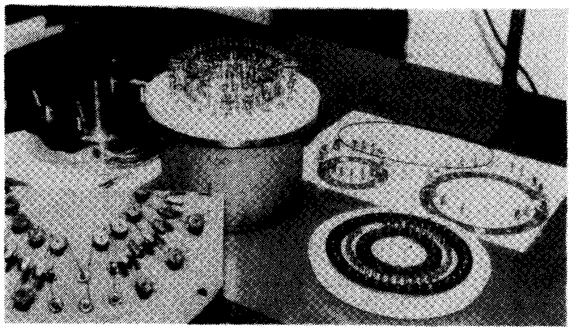

Fig. 3. Assembly process of the 36 × 360 Ω Hamon network.

where R is the nominal value of the main resistors. The contributions in (1) were measured following the techniques given in [6]. The results are:

$$M/R = 0.00075 \text{ ppm}; \quad R_f/R = 1.2 \text{ ppm}$$

$$\delta = 326 \text{ ppm}; \quad \frac{1}{36}\sum_{k=1}^{36}(\Delta_k - \Delta_{av})^2 = 0.0002 \text{ ppm}.$$

Summing up the absolute values of the uncertainty components, from (1) we get:

$$\frac{R_P}{R_s} = \frac{1}{1296}(1 \pm 4 \cdot 10^{-9}). \tag{2}$$

The 1-σ uncertainty associated with this estimated maximum deviation is 0.002 ppm. To this, the contribution arising from the nonreproducibility of the parallel contacts, evaluated to be 0.01 ppm, must be added. The total uncertainty associated with the 36 × 360 Ω Hamon network, comprehensive of that arising from the power coefficient, is 0.012 ppm. The deviation of the series network from the value of $(R_R + R_A)$, of the order of 10 ppm, which increases the effect of nonlinearity of the detector in the Potentiometric System, is another source of uncertainty. The total uncertainty of the step down to 10 Ω is typically 0.019 ppm.

IV. The 10 Ω–1 Ω Step Down

The 36 × 360 Ω Hamon network in the parallel configuration is compared with a 1-Ω standard maintained in an oil bath at (23 ± 0.002)°C. This standard is used as a transfer towards the 1-Ω primary group, but its high stability allows a first check of the reproducibility of the whole step down. In the same oil bath a 100-Ω standard and a commercial 10 × 10 Ω Hamon network (L&N mod. 4231) are also maintained.

A commercial Kusters Current Comparator bridge (Guildline mod. 9975) in both 1 : 1 and 10 : 1 ratios is used for the measurements. To compensate for systematic error, in 1 : 1 comparisons the resistors are interchanged on the two sides of the bridge, leaving the position of the first decades of the bridge unchanged. A conceptually similar approach is taken for the 10 : 1 comparison, as shown in Fig. 1; in the step down to 1 Ω (direct path) and in the step up to 100 Ω (indirect path), the same 10 : 1 ratio and possibly the same position of the first decades are used. Two main problems limit the compensation capability of the method. First, the positions of the decades in the two 10 : 1 measurements are farther apart than in 1 : 1 measurements, due to a higher deviation of the parallel 36 × 360 Ω Hamon network from nominal. Second, the systematic error of the 100 : 1 ratio of the 10 × 10 Ω Hamon network is introduced. To evaluate this systematic error measurements were carried out following the same techniques as for the 36 × 360 Ω network. Using (1) we obtain:

$$\frac{R_p}{R_s} = \frac{1}{100}(1 \pm 5.5 \cdot 10^{-8}). \tag{3}$$

The 1-σ uncertainty associated with this maximum deviation is 0.032 ppm, to which the nonreproducibility of the parallel contacts, of the order of 0.01 ppm, must be added.

The plots in Fig. 4 show the behavior in time of the interchange error for the 1 : 1 comparisons at 1-Ω and 100-Ω levels, required by the indirect path in Fig. 1, and also the relative difference in the 10 Ω to 1 Ω ratios following the two paths, which is a kind of interchange error for the 10 : 1 ratio. The global uncertainty of the 10 Ω–1 Ω step down is made up of two main contributions. The first (type A and B) is the combination of the uncertainties of the direct and the indirect paths which are considered as independent measurements; the detail is reported in ppm in Table I. The second (type B) is derived from the

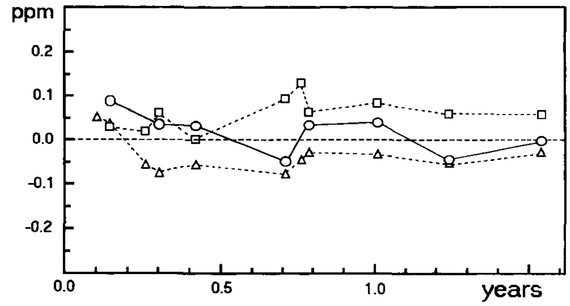

Fig. 4. Interchange errors for 1 Ω to 1 Ω (triangles) and 100 Ω to 100 Ω (squares) comparisons and differences between 10 Ω to 1 Ω ratios obtained from the direct and the indirect paths (circles).

TABLE I
THE COMBINATION OF THE UNCERTAINTIES IN ppm OF THE DIRECT AND THE INDIRECT PATHS IN THE 10 Ω–1 Ω STEP DOWN

Direct:	A - 10Ω-1Ω		0.009
Indirect:	A - 10Ω-100Ω	0.006	
	A - 1Ω-1Ω	0.010	
	A - 100Ω-100Ω	0.006	
	B - 1Ω-1Ω (interchange)	0.020	
	B - 100Ω-100Ω (interchange)	0.020	
	B - Hamon (100:1)	0.034	
	Total indirect	(RSS)	0.046
	Total	(RSS/2)	0.023

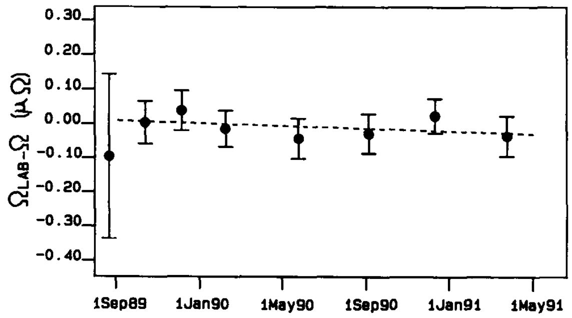

Fig. 5. Monitoring the representation of the ohm Ω_{LAB}, maintained by the IEN primary group of standards, by comparison with the quantized Hall resistance. The dashed line is a linear interpolation of all but the first measurements.

difference between the two measurements and is of the order of 0.015 ppm; this uncertainty, as that associated with the 1 : 1 interchange error, is evaluated as the standard deviation of a uniform distribution.

V. The Transfer to the Primary Group

To relate the mean value of the ten 1-Ω Thomas type standard resistors of the primary group to R_H, the 1-Ω transfer standard is compared with three of these resistors, and a complete measurement cycle on the primary group is then performed. In this cycle the ten resistors are organized in a closed circle, and each standard is compared with the two adjacent ones. The typical uncertainty of this transfer is 0.01 ppm, to which temperature and pressure effects must be added.

The pressure coefficient of the ten 1-Ω resistors is at present not known, but is of the order of 0.002 ppm/hPa. This lack of knowledge gives the main contribution to the total uncertainty of 0.035 ppm of this 1 Ω–1 Ω step.

The primary group was used until January 1, 1990, to maintain the ohm at IEN. The value of its mean resistance has been monitored in time by means of the international comparisons of resistance standards carried out by the Bureau International des Poids et Mesures (BIPM). From these comparisons and from the absolute measurements of the ohm made by CSIRO, the Australian primary metrological laboratory, a very low drift rate of about −0.009 ppm/year has been calculated [7]. Due to this high stability of the primary group, the mean resistance of the group is not reassigned every time the R_H–1 Ω step down is made. After the change in value of +0.17 μΩ on January 1, 1990 [7], to comply with the new representation of the unit, the value will be maintained until a significant drift is observed. To monitor this drift the measurement of the mean resistance of the primary group using the quantum Hall effect is made every three months. The resulting behaviour is reported in terms of the representation of the ohm maintained by the primary group, Ω_{LAB}, in Fig. 5. By linear interpolation of all but the first

TABLE II
Typical 1-σ Uncertainty Budget in ppm for the R_H–1 Ω Comparison

R_H-R_R	A - random	0.004	
	B - interchange	0.004	
	B - detector linearity	0.002	
	B - leakage	0.007	
	B - Hall quantization	0.002	
			RSS 0.009
(R_R+R_A)-10Ω	A - random	0.004	
	B - interchange	0.004	
	B - detector linearity	0.012	
	B - leakage	0.007	
	B - Hamon (1296:1)	0.012	
			RSS 0.019
10Ω-1Ω	A&B - Direct and indirect	0.023	
	B - 10:1 correction unbal.	0.015	
	B - temperature stability	0.005	
	B - pressure stability	0.002	
			RSS 0.028
R_A	A&B - calibration		0.010
1Ω-1Ω	A&B - transfer to prim. Group	0.010	
	B - temperature	0.015	
	B - pressure	0.030	
			RSS 0.035
		Total	RSS 0.051

measurements, we obtain

$$\Omega_{LAB} - \Omega = a + b(t + t_0), \tag{4}$$

where $t_0 = -0.433$ years, $a = (-0.011 \pm 0.012)\ \mu\Omega$, $b = (-0.023 \pm 0.012)\ \mu\Omega$/year and t, in years, starts on January 1, 1990.

VI. Uncertainty Summing-Up

A typical 1-σ uncertainty budget in ppm for the R_H–1 Ω comparison is summarized in Table II. The main contributions to the uncertainty arise from the incomplete correction of the 10 : 1 ratio error of the bridge, from the 10 × 10 Ω commercial Hamon network (see also Table I), and from the temperature and pressure effects on the 1-Ω primary group of standards.

Furthermore, from the results of the international comparisons of resistance standards, a residual unknown systematic error of the order of a few tenths of a ppm can not be ruled out [7]. A more conclusive check about the existence of this error will be possible when the results of the current international comparison will be available.

VII. Conclusions

A method, based on conventional room temperature techniques, has been applied to the comparison of the IEN primary group of standards with the quantized Hall resistance. Improvements, especially related to the monitoring and correction of the pressure effects on the primary group, are achievable, but the uncertainty due to the 10 : 1 ratio of the bridge can hardly be reduced. The construction of a different system based on a cryogenic current comparator has been undertaken. However, the use of a more traditional technique allowed us to set up a quantum Hall effect measuring system adequate to the current requirements of the secondary laboratories.

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REFERENCES

[1] Recommendation 2 (CI-1988) of the Comité International des Poids et Mesures, in *Procès-verbaux de la 77e session 1988.* Also in T. J. Quinn, "News from the BIPM" *Metrologia*, vol. 26, pp. 69–74, 1989.
[2] G. Marullo Reedtz and M. E. Cage, "An automated potentiometric system for precision measurement of the quantized Hall resistance," *J. Res. NBS*, vol. 92, pp. 303–310, 1987.
[3] B. V. Hamon, "A 1-100 ohm build-up resistor for the calibration of standard resistors," *J. Sci. Instrum.*, vol. 31, pp. 450–453, 1954.
[4] A. Melville Thompson, "Self-checking resistive ratios," *IEEE Trans. Instrum. Meas.*, vol. IM-27, pp. 423–425, 1978.
[5] G. W. Small, "Comparison of quantized Hall resistance with a 1-Ω standard," *IEEE Trans. Instrum. Meas.*, vol. IM-32, pp. 446–447, 1983.
[6] J. C. Riley, "The accuracy of series and parallel connections of four-terminal resistors," *IEEE Trans. Instrum. Meas.*, vol. IM-16, pp. 258–268, 1967.
[7] G. Boella and G. Marullo Reedtz, "The representation of the ohm using the quantum Hall effect at IEN," *IEEE Trans. Instrum. Meas.*, vol. 40, pp. 245–248, 1991.