# NICKEL-CHROMIUM-ALUMINIUM-COPPER RESISTANCE WIRE

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#### SUMMARY

A review is given of the principal materials used for the construction of resistance standards. The difficulty of producing manganin and constantan commercially with the requisite small value of temperature coefficient at room temperature makes attractive the newer alloys, whose temperature coefficients can be controlled by simple heat treatment. One of these alloys, having the additional advantage of a resistivity three times that of manganin, has been studied at the National Physical Laboratory. It is composed of nickel, chromium, aluminium and copper and is known commercially as Evanohm. When the temperature coefficient, at a given temperature, has been reduced to zero by heat treatment, the curvature of the resistance/temperature characteristic is only one-tenth that of manganin. The stability of resistors constructed of this material has been investigated and has been generally found to be of the order of a few parts in 105 per year. The investigations are continuing and it is hoped that better figures may be obtained for well-aged standards. The stability is not adversely affected-and may be improved-by operation at temperatures up to 120°C. Above 140°C there is usually an increase of resistance, but even at 400°C this increase is not rapid. Operation above 400°C is not recommended even for low-accuracy resistors.

Further advantages of the material are a low thermal e.m.f. to copper, high mechanical strength and high ductility. The disadvantages are the necessity of hard soldering and the susceptibility of the resistance to change due to cold working, including vibration.

The resistance standards under investigation for long-term stability are five 1-ohm standards, and one standard of each of the following values: 1 000 ohms, 100 000 ohms, 1 megohm and 10 megohms. An additional 100 000-ohm standard of an alternative design has recently been constructed.

#### (1) INTRODUCTION

The qualities required of a resistance alloy have been well stated in a recent paper,<sup>1</sup> which incidentally contains a useful list of earlier references. These qualities are:

(a) It should have a low temperature-coefficient of electrical resistance over a wide temperature range.

(b) It should have a low thermal e.m.f. against copper.

(c) Its resistance should be stable over long periods of time.

(d) It should be workable by conventional wire and rollingmill practices.

(e) It should be capable of being easily soldered or welded.

To these qualities may be added the following for an alloy

required for resistors of high ohmic value: (f) It should be possible to draw it down to wire of 0.001 in or less in diameter.

(g) It should have a high resistivity.

### (1.1) Resistance/Temperature Characteristic of Resistance Alloys

The resistance of conductors, whether metals or alloys, as distinct from semi-conductors, is lower at the absolute zero of temperature than at the melting temperature. The average temperature-coefficient over this range is therefore positive. The ideal of a resistance alloy with zero temperature-coefficient over the whole range from absolute zero to melting temperature does not seem likely to be attainable, and investigators have concentrated their attention on developing alloys with a change of curvature in their resistance/temperature characteristics, so that over a restricted temperature range the temperature coefficient may be very small. Fig. 1 shows the ideal type of curve, while

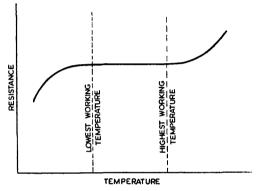


Fig. 1.--Ideal resistance/temperature curve of an alloy.

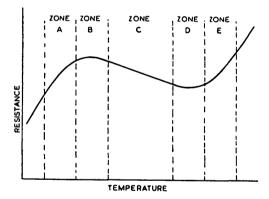


Fig. 2.—Practical approximation to ideal curve of Fig. 1.

Fig. 2 shows the sort of curve which is more likely to be attained in practice.

In Fig. 2 the part of the resistance/temperature curve which is of interest has been divided into five zones: zone A of positive temperature-coefficient at temperatures below that at which the resistance reaches a maximum, zone B containing the maximum value, zone D containing a minimum value, zone C intermediate between zone B and zone D and having a negative temperaturecoefficient and zone E of positive temperature-coefficient at temperatures above that at which there is a minimum resistance value. If an alloy can be produced in which the maximum and minimum resistances are very nearly equal, resistors of this alloy may be operated over the whole temperature range of zones B, C and D. If, however, the maximum and minimum resistances differ appreciably, the operating temperature range should be restricted to zone B or zone D. The width of zone C shown in Fig. 2 may vary from a few degrees to hundreds of

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440

degrees, according to the alloy. The resistance/temperature relation may also be more complicated than that shown.

### (1.2) Early Resistance Alloys

The earliest resistance alloy used was probably the coppernickel-zinc alloy known as german silver. This alloy had a positive temperature-coefficient of about one-tenth that of copper and the zinc content was a source of instability. A better alloy was the binary alloy of  $\frac{2}{3}$  silver and  $\frac{1}{3}$  platinum which was used in the middle of the last century for the construction of the British Association resistance standards. It was not realized at that time that a resistance/temperature curve of the shape of Fig. 2 was attainable, and it was not until about 1888 that the first alloy with a negative temperature-coefficient at room temperature was discovered. The discovery seems to have been made by Edward Weston, an Englishman working in the United States, but there was much parallel work in Germany at the Physikalisch-Technischen-Reichsanstalt (P-T-R) and at the Isabellen Metal Works (now the Isabellen-Hutte Heuser K.G.). The alloy was the binary alloy of 60% copper and 40% nickel described by the Germans as Konstantin and more commonly known in England as constantan. Since that date the composition of constantan and other similar alloys has varied somewhat, so that room temperature has occurred variously in zone A, zone B or zone C. It has never proved commercially practicable to produce constantan of such a kind that room temperature was consistently in zone B, and the best wire has only been obtained by selection. Constantan has been extensively used for a.c. resistors required to operate over considerable temperature ranges, but it is less satisfactory for d.c. resistors on account of its high thermal e.m.f. to copper, about  $40 \,\mu V$  per deg C. For use with direct current it has been almost entirely superseded by another alloy discovered a few years later. This was the ternary alloy of copper, manganese and nickel which was given the name of manganin by the Germans. This name is now almost universally used, although other proprietary names have been introduced by various manufacturers. It is possible that this alloy was also discovered by Edward Weston, but this is not certain. The composition varied appreciably at first; the modern composition of approximately 84% copper, 12% manganese and 4% nickel probably dates from about 1895.

Manganin has a resistivity of about 40 microhm-cm and a thermal e.m.f. to copper of  $2\mu V$  per deg C. Room tempera-ture may occur in zone A, zone B or zone C, but it has proved somewhat easier to obtain material such that room temperature lies in zone B than it was with constantan. In two respects manganin is inferior to constantan. First, it is more difficult to solder. Soft soldered joints are not only difficult to make but are often a source of instability of resistance. Secondly, the curvature of the resistance/temperature curve is greater than that of constantan, so that even though the mean operating temperature of a resistance standard is exactly that at which the resistance is a maximum, there is a second-order resistance change with temperature which is greater for manganin than for constantan. However, the advantage of a low thermal e.m.f. to copper outweighs these disadvantages. In respect of stability of resistance it is quite possible that constantan is superior to manganin since it is less subject to surface oxidation. It must, however, be remembered that the stability of a resistance standard is due only in part to the qualities of the resistance alloy and is dependent to a great extent on the design and construction of the standard. The instant popularity of manganin for resistance standards must be largely ascribed to the fine workmanship of Otto Wolff, who specialized in the manufacture of the Reichsanstalt resistors in universal use as national standards until

well after the First World War. Since 1914 a great deal of work has been carried out both here and in the United States on improving the method of construction of resistance standards, and both countries now possess a number of 1-ohm manganin standards which change their values by less than one part in a million each year. The development work necessary for the production of these standards has occupied a period of over fifty years, and an equal or greater period may well elapse before the stability of resistance standards of newer alloys is established with sufficient certainty to enable them to compete with manganin. Although manganin is well established for national standards of resistance, it has not been possible to achieve commercial production of manganin to the same consistent high standard. Some manganin is unstable in resistance, and large quantities are produced of a kind in which room temperature is in zone A or zone C. Manganin joints have also been a source of weakness. Thus there may well be room in the commercial production of resistors for an alternative alloy, even though manganin is now too well established for national standards to be in danger of displacement for many years.

Before this aspect is considered, however, the efforts which have been made to develop alternative materials for national standards will be briefly reviewed.

### (1.3) Alternative Materials for National Resistance Standards

Resistors have been constructed at the National Physical Laboratory with platinum as the resistance material in the belief that it is less subject to physical or chemical change. To obtain the same accuracy with these standards as was obtained with the manganin standards it was necessary to maintain the temperature of the coils constant to one four-thousandth of a degree centigrade or better. The difficulties involved in such close temperature control were largely overcome, but the experiment was ultimately suspended when the results showed that the superiority of platinum over manganin with respect to stability could not be established with any certainty.

At the National Bureau of Standards in the United States, J. L. Thomas investigated the properties of gold-chromium alloys, and was successful in producing samples in which the width of zones B, C and D together was only about  $10^{\circ}$  C and the difference between the maximum resistance in zone B and the minimum resistance in zone D was less than one part in a million. Moreover, these zones could be brought to the neighbourhood of room temperature by heat treatment at the low temperature of  $150^{\circ}$  C. The stability of resistors constructed with this material, however, appears to be inferior to that of manganin resistors and the thermal e.m.f. to copper is three or four times as great.

Thomas also investigated the properties of an alloy which had been developed commercially as early as 1910. This alloy, known as Therlo, differs in composition from manganin mainly in the replacement of the nickel content by aluminium. Thomas made a series of alloys of slightly different composition and found the best to be 85% copper, 9.5% manganese, 5.5% aluminium and a very small amount of iron. The resistivity of this alloy was similar to that of manganin, while the thermo-electric e.m.f. to copper was only about one-tenth. The curvature of the resistance/temperature curve at the maximum resistance value in zone B was about one-half that of manganin. The most remarkable property of this alloy, however, was that the temperature for maximum resistance in zone B could be altered by heat treatment at the low temperature of 140°C. Provided that suitable insulation was used it was therefore possible to adjust the temperature coefficient of a completed resistance standard to zero at the mean working temperature. Early results showed

that resistance standards of Therlo had a stability comparable to those of manganin, and if these results should be confirmed over the years it would appear that Therlo should be a powerful competitor to manganin. Alfred Schulze, who carried out work along the same lines in Germany during 1933–41, was unable to produce stable resistors of Therlo but succeeded with alloys having similar qualities but slightly different compositions. Two of these alloys are known by the commercial names of Isabellin and Novokonstant.

The present position, therefore, is that the three alloys, Therlo, Isabellin and Novokonstant, are superior to manganin in respect of their thermal e.m.f. to copper and may have a slight superiority in respect of the curvature of the resistance/temperature curve in zone B. They have the great advantage over manganin that the mid-temperature of zone B can be controlled by heat treatment at a temperature considerably lower than the annealing temperature, and lower than the maximum operating temperature of some insulating materials. Their stability over long periods is not yet established and they are likely to be unsuitable for operations involving large temperature rises.

#### (1.4) High-Resistivity Alloys

None of these alloys meets the demand for a material of high resistivity, and until recent years the only alloy available was the binary alloy of 80% nickel and 20% chromium which has a resistivity at room temperature about three times that of manganin and a positive temperature-coefficient of about 60 parts in  $10^6$  per deg C. The maximum resistance in zone B occurs at about  $500^\circ$  C. The temperature coefficient is rather high for the better classes of resistors and the stability of resistance is not good. The curvature of the resistance/temperature relation in zone A is very small.

The stability of the nickel-chromium alloy has been greatly improved, and the valuable property of control of temperaturecoefficient by heat treatment at moderate temperatures has been achieved by the addition of two more metals. Two alloys of this type are of particular interest. The one, commercially known as Evanohm, has an approximate composition of 73% nickel, 21% chromium, 2% aluminium, 2% copper and the balance of other metals. The other, commercially known as Karma, has a similar composition except that the copper is replaced by iron. Both these alloys have a resistivity three times that of manganin, a thermal e.m.f. to copper generally rather less than that of manganin, high tensile strength (making possible the drawing of fine wires) and a temperature-coefficient which may be controlled by heat treatment at moderate temperatures. Moreover, when the temperature-coefficient is adjusted so that the mean working temperature comes in the middle of zone B, the curvature of the resistance/temperature relation is about one-tenth that of manganin and less than one-half that of constantan. The alloys must be hard-soldered. Apart from this one disadvantage, they appear admirably suited for resistors of high value or required to operate over large temperature ranges. Investigations of their stability have been started both at the National Bureau of Standards and at the National Physical Laboratory.

The results obtained at the National Physical Laboratory so far have shown that the alloy containing copper is superior in stability to the one containing iron. Results of tests on the copper alloy only will be given in the paper. The samples tested were obtained commercially, and it does not necessarily follow that all samples of the copper alloy will be superior to samples of the iron alloy, especially in view of the conflicting results which have been obtained by other workers on other alloys. However, the purpose of the investigation was to determine the suitability of one of these alloys for resistance standards, and the results obtained so far have been encouraging.

### (2) THE TEMPERATURE-COEFFICIENT OF ELECTRICAL RESISTANCE OF EVANOHM

Experiments were carried out at the National Physical Laboratory to confirm and amplify the information given in an earlier paper<sup>2</sup> on the resistance/temperature characteristic.

The annealing temperature of Evanohm is in the neighbourhood, of 1000°C, and in wire form the alloy may be annealed either by heating in an oven at this temperature or by what is sometimes a simpler process-that of passing sufficient current through the wire to raise it to the required temperature. In either case the cooling must be sufficiently rapid to prevent changes in resistance and temperature-coefficient occurring at lower temperatures. It was found that when annealing was carried out in air the wire was slightly tarnished by oxidation. Some samples were annealed in hydrogen and in a partial vacuum in the hope of avoiding oxidation, but the experiment was abandoned when it was found that the temperature coefficient was about 50% higher than when annealing was carried out in air, and that it could not be reduced by subsequent heat treatment at a lower temperature. When properly annealed the wire has a temperature-coefficient of electrical resistance similar to that of the nickel-chromium alloy, namely approximately 60 parts in  $10^6$  per deg C with negligible variation over a temperature range from room temperature or lower to about 400°C. It is not possible to determine a true value of the temperature-coefficient above 400° C, since part of the resistance change is permanent above this temperature. This permanent resistance change is at first an increase and is accompanied by a decrease of temperature coefficient at room temperature. Both the amount and rate of change are dependent on the temperature. With heat treatment at 550°C the resistance first increases approximately 15% and then decreases. The temperature coefficient first decreases through zero to a negative value of about 20 parts in 10<sup>6</sup> per deg C and then increases to a positive value approaching that for the annealed condition. However, the resistance/temperature curve loses its linearity after prolonged heat treatment, and it is not desirable to continue the treatment beyond the point at which the temperature-coefficient first becomes zero.

There is some difficulty in achieving the correct amount of heat treatment, since in the neighbourhood of zero temperaturecoefficient the wire may change its characteristics very rapidly. Commercial heat treatment is controlled to give a temperaturecoefficient within the limits of  $\pm 20$  parts in 10<sup>6</sup> per deg C, but commercial supplies within the limits of  $\pm 5$  parts in 10<sup>6</sup> can be obtained by selection. The temperature coefficient may be affected to some extent by manufacturing processes carried out after heat treatment, such as insulating the wire. When, therefore, it is desired to construct a resistance standard with zero temperature-coefficient at a particular temperature, the final heat treatment must be carried out after the completion of all processes involving cold working of the wire.

The linearity of the resistance/temperature relation of the annealed wire is not maintained as the temperature-coefficient is reduced by heat treatment. The curvature introduced is very small and appears to be dependent to some extent on the temperature at which heat treatment is given. When the temperature-coefficient is zero the curvature is about one-tenth that of manganin so that the working temperature range of a resistor may be three times as great for the same change of resistance.

# (3) THE CAUSES OF INSTABILITY OF RESISTANCE

Resistance changes in the alloy itself may be due to physical or chemical changes occurring either spontaneously or as a result of treatment of the wire.

Chemical changes occur mainly on the surface of the wire and

may be accelerated by heat, humidity and other causes or retarded by protective coatings or by immersion in a dry atmosphere of an inert gas. Insulating liquids have also been used to protect the surface of wires, but there is more difficulty in obtaining an inert liquid than an inert gas.

Physical changes occur mainly in the body of the wire and may be accelerated by heat treatment or by cold working, including vibration.

The importance of surface changes becomes greater as the ratio of surface area to volume is increased, and it is therefore an advantage to use a wire as large in diameter as practicable. In this respect Evanohm has an advantage over manganin for resistances of all values on account of its higher resistivity. It seems probable also that the surface is less liable to contamination than that of manganin, and the encouraging results of stability tests on resistors constructed with wire of 0.001 in diameter lend support to this view. These results are given in Sections 6–10.

In respect of physical changes it is not so likely that Evanohm will show up advantageously. The presence of aluminium in its composition is believed to be favourable to enlargement of grain size with time, but it is also believed that the presence of copper will counteract this tendency. However this may be, it is certain that both the resistance and the temperature-coefficient of this alloy are peculiarly susceptible to cold working in any form, including vibration. Further, the very advantage of being able to control the temperature-coefficient by heat treatment at moderate temperatures makes the alloy suspect if it is normally operated over a wide temperature range.

It is unlikely that Evanohm will prove a formidable competitor to manganin for resistance standards of low value and of the highest class, since, on the one hand, the advantage of a vanishingly small temperature-coefficient is of minor importance for resistance standards which are operated under closely controlled temperature conditions and with small self-heating, and, on the other hand, manganin standard resistances of proved stability are already in existence, whereas Evanohm is unproved. The field for Evanohm is more likely to be in working resistance standards which are operated over a considerable temperature range on account of self-heating. The exact temperature of a resistor heated by its own current is always difficult to determine, so that it is a great advantage if it is constructed of wire of small temperature coefficient, while a moderate amount of secular instability may be relatively unimportant for a working standard. Working resistance standards of manganin are quite commonly operated over a temperature rise of 30° C, so that for the same performance it should be possible to operate an Evanohm standard over a range of 100°C. This would make possible the construction of a standard of smaller physical dimensions at lower cost. For a.c. work the smaller dimensions would generally result in a reduction of errors from capacitance. It is therefore of importance to establish the stability of Evanohm when subjected to cyclic temperature changes of at least 100°C, and it may be observed that, although a slow drift in resistance might not matter in a working standard, a corresponding drift in the temperature-coefficient would be objectionable unless it were extremely slow.

The stability of manganin resistance standards of high value— 1000 ohms and upwards—is not so good as those of lower value, and Evanohm may prove competitive, even for standards of the highest class, in this field.

The experimental work carried out to determine the stability of Evanohm is described in Sections 5–10. Much of it required measurements of the highest precision on properly constructed standards, but in order to accelerate the work many measurements were made to a lower order of accuracy on resistors of simple construction. When the preliminary results proved encouraging, a number of properly made standards were constructed for long-term measurements of the highest accuracy.

### (4) EFFECT OF HEAT TREATMENT

The simplest method of applying heat treatment to short lengths of Evanohm wire is to pass current through them. It was found experimentally that a current of 1.2 amp through a wire of 0.0048 in diameter supported horizontally in air was sufficient to raise it to the annealing temperature of  $1000^{\circ}$  C. Maintenance of this current for five to ten minutes restored the wire to the annealed condition, whatever the previous heat treatment at lower temperatures. The annealed wire has a temperature-coefficient of +60-65 parts in  $10^{6}$  per deg C at  $20^{\circ}$  C.

A number of samples of annealed wire were subjected to currents between 0.85 amp and 1.2 amp for varying periods from a few seconds to five minutes, and the temperature-coefficient at  $20^{\circ}$ C was determined after the heat treatment. The values obtained were all within the range of +55-70 parts in  $10^{6}$ , and it was concluded that the condition of the wires did not differ significantly from the annealed condition. When the current was reduced below 0.7 amp its duration had a marked effect on the final temperature-coefficient, and Fig. 3 shows the results

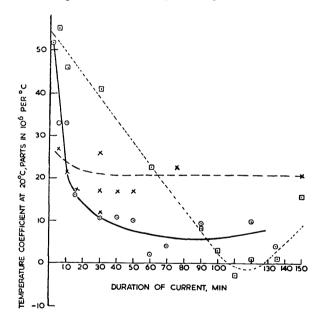


Fig. 3.—Effect on temperature-coefficient of passing current through Evanohm wire supported horizontally in air.

| Wire diameter | 0.0048in. |
|---------------|-----------|
|               | 0.60 amp. |
| ;;            | 0.65 amp. |
| — — × — —     | 0.68 amp. |

obtained with currents of 0.68, 0.65 and 0.60 amp. There are inconsistencies in many of the observed points but the general trend is clear: the temperature-coefficient is reduced by the passage of the current. The rate of reduction falls as the current is reduced although the maximum change of coefficient increases and is eventually sufficient to reverse its sign. The temperature of the wire with a current of 0.6 amp passing through it is probably between 500 and  $600^{\circ}$  C.

An alternative method of heat treatment is to place the sample in a heated oven. It is easier by this method to maintain the maximum temperature of the sample at a constant and known value, but as the oven takes some hours to warm up, and even longer to cool down, part of the heat treatment of the wire occurs at a temperature below the maximum value. A sample of

annealed 0.0048 in-diameter wire was subjected to prolonged heat treatment in an oven according to the following procedure.

The oven was warmed up slowly from room temperature, and the resistance/temperature relation of the wire was determined up to 300°C. A final temperature of 450°C was reached in 24 h and the oven was then allowed to cool off to room temperature. On succeeding days the maximum temperature was raised by 25° C until a maximum of 550°C was reached. On all succeeding days the maximum was 550°C, but on some days it was maintained for a period before the oven was allowed to cool. It was found that over the whole temperature range from 20°C to 300°C the resistance/temperature relation could be represented very closely by the parabolic law given in eqn. (1):

$$R_t = R_{20} [1 + \alpha(t - 20) + \beta(t - 20)^2] \quad . \quad (1)$$

where  $R_t$  = Resistance at a temperature of  $t \deg C$ .  $R_{20}$  = Resistance at a temperature of 20°C.

 $\alpha$  and  $\beta$  are constant coefficients.

Table 1 shows the experimentally determined values of  $\alpha$  and  $\beta$ . For any given experimental resistance/temperature curve it is possible to vary  $\alpha$ , provided that  $\beta$  is also suitably varied, over

The variation of the coefficient  $\beta$  with heat treatment follows a somewhat irregular course until the maximum negative value of  $\alpha$  is reached. Thereafter  $\beta$  increases fairly consistently in the negative direction. The desirable objective is, of course, to heattreat the wire so that both  $\alpha$  and  $\beta$  are zero simultaneously. It is apparent from Table 1 that this objective is not likely to be achieved when  $\alpha$  passes through zero from a negative to a positive value, and in all later work heat treatment was designed to bring the coefficient  $\alpha$  to the first zero, changing from a positive to a negative value. The irregular variations of the coefficient  $\beta$ in the early stages of heat treatment suggest that the exact nature of the heat treatment might affect the value of  $\beta$  when  $\alpha$  is zero. The correct heat treatment to make  $\alpha$  and  $\beta$  simultaneously zero has not yet been found, although a few experiments with different heat treatments were tried.

The third point of interest emerging from Table 1 is that the change of  $\alpha$  from its value for the annealed wire is approximately proportional to the change of resistance from its value for the annealed wire. This relation may be expressed by eqn. (2) thus

$$\frac{R_2}{R_1} = 1 + 2000(\alpha_1 - \alpha_2) \quad . \quad . \quad . \quad (2)$$

Table 1

EFFECTS OF PROLONGED HEAT TREATMENT OF 0.0048IN-DIAMETER WIRE

| Day   | Maximum<br>oven temperature   | Period at<br>maximum<br>temperature  | α   | β  | Increase in<br>resistance<br>(Y)   | Remarks  |
|---|---|--|---|--|--|--|
|   | deg C   | hours  | $+58 \times 10^{-6}$  | $+12 \times 10^{-9}$   | %<br>0`•0  | Annealed wire<br>Average temperature coefficient over  |
| 1<br>2<br>3<br>4<br>5<br>6<br>7<br>8<br>9<br>10<br>11<br>12<br>13<br>14<br>15<br>16<br>17<br>18 | 450<br>475<br>500<br>525<br>550<br>550<br>550<br>550<br>550<br>550<br>550<br>55 | Momentary<br>Momentary<br>Momentary<br>Momentary<br>Momentary<br>0 $\cdot$ 5<br>1<br>Momentary<br>2<br>2<br>2<br>2<br>4<br>0 $\cdot$ 5<br>4<br>4 | $\begin{array}{c} +49 \times 10^{-6} \\ +43 \times 10^{-6} \\ +36 \times 10^{-6} \\ +12 \times 10^{-6} \\ -11 \times 10^{-6} \\ -22 \times 10^{-6} \\ -21 \times 10^{-6} \\ -21 \times 10^{-6} \\ -21 \times 10^{-6} \\ -11 \times 10^{-6} \\ -11 \times 10^{-6} \\ -5 \times 10^{-6} \\ 0 \\ +1 \times 10^{-6} \\ +4 \times 10^{-6} \\ +7 \times 10^{-6} \\ \end{array}$ | $\begin{array}{c} -10 \times 10^{-9} \\ -23 \times 10^{-9} \\ -32 \times 10^{-9} \\ -32 \times 10^{-9} \\ -3 \times 10^{-9} \\ -3 \times 10^{-9} \\ -4 \times 10^{-9} \\ -12 \times 10^{-9} \\ -10 \times 10^{-9} \\ -11 \times 10^{-9} \\ -29 \times 10^{-9} \\ -35 \times 10^{-9} \\ -40 \times 10^{-9} \\ -44 \times 10^{-9} \\ -51 \times 10^{-9} \\ -56 \times 10^{-9} \\ -60 \times 10^{-9} \\ -60 \times 10^{-9} \end{array}$ | $2 \cdot 0$<br>$3 \cdot 8$<br>$5 \cdot 4$<br>$8 \cdot 5$<br>$11 \cdot 3$<br>$12 \cdot 4$<br>$13 \cdot 0$<br>$13 \cdot 5$<br>$14 \cdot 1$ | range 20–300°C = + 61 × 10 <sup>-6</sup><br>Voltage leads of copper burnt away.<br>Evanohm leads substituted |

a small range with very little loss of fit. The values given in Table 1 are those which give the best fit to each individual curve and not those which give the most consistent results for the complete set of curves. The Table shows that the value of  $\alpha$  falls from  $+58\times10^{-6}$  for the annealed wire to a negative maximum of  $-23 \times 10^{-6}$ . Thereafter, further heat treatment causes it to change in a positive direction. In another experiment, in which the oven temperature was eventually raised to 700°C in order to accelerate the heat treatment, the average temperature-coefficient over the range 20-100° C finally reached a value of  $+84 \times 10^{-6}$ , but over the range 20–300° C the average coefficient was only  $+54 \times 10^{-6}$ . The resistance/temperature relation, although approximately similar to that for the annealed condition, had a much greater curvature and there was no indication that heat treatment at this temperature would ever bring the wire back to the annealed condition.

where  $R_2$  is the resistance corresponding to the coefficient  $\alpha_2$  and  $R_1$  is the resistance corresponding to the coefficient  $\alpha_1$ . This equation is sometimes convenient when heat-treating a wire to obtain zero temperature coefficient. It is also of value in judging whether a secular resistance change of given amount is likely to be accompanied by a significant change of temperature-coefficient.

It is apparent from the results given in Fig. 3 and Table 1 that the temperature coefficient of the wire may be changed by heat treatment at various temperatures. The rate of change becomes less as the temperature is lowered, but it is important to discover whether significant changes occur at moderate temperatures, since they might limit the possible working temperature of a resistor. It was found possible to bring the temperature coefficient of annealed wire to zero by heat treatment at a temperature of 400° C. The time required for this was 265 h. It would not, therefore, be practicable to operate a precision resistor up to

400°C. However, if the accuracy required were only 1% it would be possible to adjust the temperature coefficient of the wire to +25 parts in 10<sup>6</sup> per deg C initially, and the coefficient would gradually decrease with time of operation at 400°C, eventually reaching a negative value of about 25 parts in 10<sup>6</sup> per deg C. The coefficient would then begin to change in a positive direction. It is likely that the time of operation at 400° C would exceed 1 000 h before the coefficient returned to its initial value of +25 parts in 10<sup>6</sup> per deg C. It would then be necessary to anneal the resistance material and readjust the temperature coefficient by heat treatment. There would be a resistance change between 20 and 400° C on account of temperature-coefficient of 1% or less. There would also be a resistance change of about 10% accompanying the change of temperaturecoefficient. It would, therefore, be necessary to adjust the resistance at intervals in order to maintain it near to its nominal value.

For precision work, changes of this magnitude would not be tolerable, and experiments were carried out to determine what was the maximum possible operating temperature for accurate work. It is not possible to assign a rate of change of resistance and temperature-coefficient to each maximum operating temperature, since the rates of change vary with time of operation. The resistance changes occurring in one resistor with various periods of operation at various temperatures have, however, been ascertained, and are shown in Table 2. The temperature-

#### Table 2

Change of Resistance with Time at Various Temperatures of a Resistor of 0.0048 in-diameter Wire

| Temperature       | Change of<br>resistance<br>at 20° C                 | Total change<br>of resistance<br>at 20° C   |
|-------------------|---|---|
| °C<br>100         | Parts in 10 <sup>6</sup><br>+4<br>-36               | Parts in 10 <sup>6</sup><br>+4<br>-32   |
| 200<br>200        | +46<br>+137   | +14<br>+151   |
| 300<br>100        | +950<br>+12   | +320<br>+1270<br>+1282  |
| 150<br>150<br>150 | +10 + 8 - 6   | +1 292<br>+1 300<br>+1 294  |
|                   | °C<br>100<br>100<br>200<br>250<br>300<br>100<br>150 | $\begin{array}{c c} \hline Temperature \\ \hline \\ $ |

coefficient at the start of the tests was -4 parts in  $10^6$  per deg C, and it changed by less than one part in 10<sup>6</sup> per deg C during the tests. It is apparent from this Table that the resistance change at 300°C is not tolerable, and that even at 200°C the changes are larger than are desirable. At 150 and 100°C the changes are small and irregular in direction, and the effect of prolonged operation at a temperature of about 150°C was therefore studied in more detail. A 10000-ohm coil of 0.001 indiameter wire was immersed in silicone oil. On five days each week the temperature of the oil was raised to 143°C in a period of 3h and held at that temperature for 5h. This treatment was continued for 12 months. During the first three months the resistance increased by 29 parts in 10<sup>6</sup> and after this time it increased at an average rate of 20 parts in 10<sup>6</sup> per year. Thus, for resistors intended to be accurate within one part in 10<sup>4</sup> it would only be necessary after the first three months to adjust the value once every ten years, on account of operation at 143° C. It is, therefore, quite practicable to operate up to 140° C, but because of secular change the maximum working temperature should probably not exceed 120°C on account of the curvature of the resistance/temperature characteristic. From tests on a number of samples it appears that  $\beta$  usually lies between  $-20 \times 10^{-9}$  and  $-40 \times 10^{-9}$  for small values of  $\alpha$ , and if the temperature-coefficient is adjusted so that the maximum resistance occurs at 70°C, the resistance at 20°C and at 120°C is 0.005% less than at 70°C for the lower limits of  $\beta$  and 0.01% less for the upper limit of  $\beta$ .

# (5) LONG-TERM STABILITY

Whilst the experiments already described were proceeding the long-term stability of a number of resistors was investigated. It was generally found that the stability, after an initial settling-down period, was of the order of a few parts in 10<sup>5</sup> per year, and was quite adequate for resistors which were to be operated with a high temperature-rise and which could be readjusted every few years to their nominal value. There was little evidence of a definite drift in one direction, except during periods of hightemperature operation, and the resistance variations which occurred might well be due to imperfections in the construction of the resistors rather than defects in the material itself.

For resistors of the highest class, operated over a small temperature range, say from 15 to  $25^{\circ}$  C, a higher stability was desired and a few well-made resistors were selected for study in this respect. It was considered that Evanohm was likely to be used for high-resistance standards rather than for low-resistance ones and the early coils manufactured were of high value. Standards of lower value have been constructed more recently, but there is little data on stability yet available for these.

The method of construction of standards may affect their stability, and a description of this for each standard is given in Sections 6-10, together with the resistance measurements.

The method of soldering the copper lead-in wires to the resistance wire was the same for all standards. It is somewhat similar to a method already described;<sup>3</sup> joints have been found easy to make and have not subsequently given trouble. The section of a stainless-steel rod,  $\frac{1}{4}$  in in diameter and 5 in long, was reduced to about  $\frac{1}{40}$  in<sup>2</sup> over the middle 2 in lengths, by grinding two flats. A small bead of a special coppersilver solder was located in a small depression drilled in the middle of one flat. Pure borax was used as flux and the minimum heat necessary was provided by passing an appropriate current, 100 to 150 amp, through the rod. The surface of the Evanohm wire was cleaned by scraping with a sharp-edged tool before soldering. After soldering, the borax on the joint was cracked and the joint immersed and vibrated for a few minutes, first in dilute sulphuric acid, then successively in distilled water, an ammonia solution and finally in distilled water again.

### (6) 100 000-OHM STANDARD

The wire used for this standard was 0.001 in in diameter; it was enamelled and had a resistance of 760 ohms/ft. The preparatory heat treatment was applied by the manufacturer before enamelling and no further heat treatment was carried out. The average temperature-coefficient over the range 20-50°C was +5 parts in 10<sup>6</sup> per deg C, the variation between various samples, in general, not exceeding one part in 10<sup>6</sup>.

The wire was wound in a single layer on a silk-covered cylindrical metal former and mounted in a case of a type described in an earlier paper,<sup>4</sup> making what is usually known as a Class S standard. The insulation resistance of the coil mounting was  $10^{12}$  ohms. After construction, the coil was allowed to age at room temperature for six months. It was then placed in a desiccator for two years, after which it was finally sealed.

The resistance was measured to 1 part in 10<sup>5</sup> during the period in which it was in the desiccator and to 1 part in 10<sup>5</sup> after sealing.

| Date       | Temperature | Resistance   | Remarks   |
|------------|-------------|--------------|---|
|            | °C          | ohms         |   |
| 11.7.1952  | 20          | 100 009      | After one week in desiccator  |
| 8.12.1952  | 20          | 100 007      | Insulation resistance $5 \times 10^{10}$ ohms<br>In desiccator<br>Insulation resistance $1.6 \times 10^{11}$ ohms   |
| 22.12.1953 | 20          | 100 007      | In desiccator   |
| 6.5.1954   | 20          | 100 008      | Insulation resistance $0.8 \times 10^{12}$ ohms<br>In desiccator<br>Insulation resistance $1.3 \times 10^{12}$ ohms |
| 7.7.1954   | 20          | 100 01 1     | After removal from desiccator and sealing in dry air<br>Insulation resistance $0.9 \times 10^{12}$ ohms             |
| 7.7.1954   | 20          | 100012.5     | Measured on precision bridge  |
| 23.7.1954  | 15          | 100 007 • 0] |   |
| 23.7.1954  | 20          | 100012.6     | Average temperature-coefficient $+0.000011$   |
| 23.7.1954  | 25          | 100018.2     |   |
| 22.9.1954  | 20          | 100 012 · 8  |   |
| 29.4.1955  | 20          | 100 014 · 6  |   |
| 24.10.1955 | 20<br>20    | 100015.1     |   |
| 7.2.1956   | 20          | 100 015 · 3  |   |

 Table 3

 RESISTANCE OF 100 000-OHM S-COIL

Table 3 shows the measurements made up to the present time. The resistance fell slightly during the first 5 months in the desiccator and thereafter it increased about 4 parts in  $10^5$  during the remaining 19 months in the desiccator. Since the coil was sealed the resistance has increased a further  $2\frac{1}{2}$  parts in  $10^5$  in 15 months. The resistance has thus increased at an approximately constant rate of 2 parts in  $10^5$  per year for the last 3 years. It may also be observed that the temperature-coefficient of the standard is about twice that of the samples of wire tested before it was constructed. This difference might be due to differences in the temperature-coefficient of the later evidence showing a steady secular resistance increase, it appears more likely that both the resistance increase and the higher temperature-coefficient arise from strains imposed on the wire by the former on which it is wound.

The 1-megohm and 10-megohm standards, described in Sections 7 and 8, were constructed on a different principle, the wire being wound on mica cards. The temperature-coefficient of these standards was found to be substantially the same as that of samples of the wire used for them; it is therefore probable that the strain involved in this method of winding is small. A new 100000-ohm standard has been constructed in which the wire is wound on a mica card. This standard is mounted in a rectangular metal box with S-type terminals. The temperaturecoefficient of the completed standard is +4 parts in 10<sup>6</sup> per deg C over the range 15–25° C, substantially in agreement with the value obtained for samples of the wire used. The resistance was 99969.5 ohms on 24th October, 1955 (immediately after sealing), and 99969.8 ohms on 7th February, 1956.

## (7) SUBDIVIDED 1-MEGOHM RESISTANCE STANDARD

This resistor was constructed with 0.001 in-diameter enamelled Evanohm wire similar to that used for the 100000-ohm S-coil. The wire was wound in a single layer on thin mica cards, each card carrying a length measuring 100000 ohms. The cards were mounted in an unsealed Perspex box. The ends of the wire and tappings for each 100000 ohms were brought to terminals on the top of the box. Table 4 shows the resistance values obtained.

The temperature-coefficient of the completed standard was +4 parts in 10<sup>6</sup> per deg C over the range 15-25° C. This value is substantially the same as that of the wire before winding.

Table 4

SUBDIVIDED 1-MEGOHM RESISTANCE STANDARD

| Sections | Resistance at 20°C at dates given |           |           |           |           |  |
|----------|-----------------------------------|-----------|-----------|-----------|-----------|--|
| bections | 15.3.54                           | 10.8.54   | 4.4.55    | 4.8.55    | 2.2.56    |  |
|          | ohms                              | ohms      | ohms      | sohms     | ohms      |  |
| 1        | 100 003                           | 100 006   | 100 004   | 100 006   | 100 006   |  |
| 1 and 2  | 200 005                           | 200 012   | 200 008   | 200 01 1  | 200 0 1 1 |  |
| 1–3      | 299 997                           | 300 008   | 300 001   | 300 006   | 300 006   |  |
| 1-4      | 400 008                           | 400 023   | 400 01 3  | 400 019   | 400 020   |  |
| 1–5      | 500 01 5                          | 500 033   | 500 021   | 500 028   | 500 029   |  |
| 1–6      | 599 998                           | 600 020   | 600 006   | 600 014   | 600 01 5  |  |
| 1–7      | 699 965                           | 699 991   | 699 975   | 699 984   | 699 985   |  |
| 1-8      | 800 007                           | 800 036   | 800 017   | 800 027   | 800 028   |  |
| 1–9      | 900 048                           | 900 080   | 900 058   | 900 069   | 900 070   |  |
| 1-10     | 1 000 003                         | 1 000 039 | 1 000 014 | 1 000 027 | 1 000 028 |  |
|          |                                   |           |           |           |           |  |

### (8) SUBDIVIDED 10-MEGOHM RESISTANCE STANDARD

The construction of this standard was generally similar to that of the subdivided 1-megohm standard, except that each card contained wire measuring 500 000 ohms and the whole resistor was subdivided into ten sections of 1 megohm each. Table 5 shows the resistance values obtained.

### Table 5

SUBDIVIDED 10-MEGOHM RESISTANCE STANDARD

| Sections | Resistance at 20°C at dates given |          |          |          |  |
|----------|-----------------------------------|----------|----------|----------|--|
| Sections | 1.12.54                           | 5.4.55   | 13.8.55  | 2.2.56   |  |
|          | megohms                           | megohms  | megohms  | megohms  |  |
| 1        | 1.000.02                          | 1.000 02 | 1.000.03 | 0.99993  |  |
| 1 and 2  | 2.000.06                          | 2.00007  | 2.00010  | 1.99998  |  |
| 1–3      | 3.000.08                          | 3.000.09 | 3.00013  | 3.000.01 |  |
| 1-4      | 4.00007                           | 4.000.09 | 4.00014  | 4.000 02 |  |
| 1–5      | 5.000.06                          | 5.000.08 | 5.00015  | 5.00003  |  |
| 1–6      | 6.000.09                          | 6.00012  | 6.00020  | 6.000.08 |  |
| 1–7      | 7.00012                           | 7.00015  | 7.00024  | 7.00012  |  |
| 1–8      | 8.000.09                          | 8.00019  | 8.00030  | 8.00018  |  |
| 1–9      | 9.00025                           | 9.00035  | 9.00047  | 9.00034  |  |
| 1–10     | 10.00019                          | 10.00028 | 10.00041 | 10.00028 |  |
|          | 1                                 | l        |          |          |  |

The temperature-coefficient of samples of the wire used was found to be within the limits of  $\pm 1$  part in 10<sup>6</sup> per deg C, while that of the completed standard was -2 parts in 10<sup>6</sup> per deg C over the range 15–25° C.

### (9) 1000-OHM STANDARDS

Three 1 000-ohm lengths of wire wound on mica cards were subjected to various kinds of heat treatment, to ascertain whether the stability was affected. The wire used was 0.0048 in in diameter and it had been enamelled by the manufacturers after adjustment of the temperature-coefficient to a low value.

The first coil was maintained at air temperature after construction. Its resistance increased 6 parts in  $10^6$  during the first 3 months and then decreased 8 parts in  $10^6$  during the next 4 months; thereafter, it decreased at a constant rate of 13 parts in  $10^6$  per year for 1 year.

The second coil was maintained at 200° C for 25 h and then allowed to rest for one week before measurements were made. Its resistance has remained constant to within  $\pm 3$  parts in 10<sup>6</sup> for 20 months without showing any steady drift.

The third coil was maintained at  $200^{\circ}$  C for 35 h, during which time its resistance increased by 227 parts in 10<sup>6</sup>. This was followed by a period of 30 h at 180° C entailing an increase of resistance of 17 parts in 10<sup>6</sup>. A further period of 155 h at 160° C caused an increase of 34 parts in 10<sup>6</sup>.

The temperature was then reduced to and held at 140°C. Measurements at long intervals gave the following results:

After the first 500 h at  $140^{\circ}$  C resistance increased 10 parts in  $10^{6}$ .

After the second.500 h at 140° C resistance decreased 25 parts in 106.

After the third 500 h at 140°C resistance unchanged.

After the fourth 500 h at  $140^{\circ}$  C resistance increased 2 parts in  $10^{6}$ .

The coil was then maintained at air temperature for 3 months, during which time its resistance increased by 4 parts in  $10^{6}$ .

These figures, although not conclusive, suggest that some heat treatment at moderate temperatures is beneficial. Some strain is always put on the wire during the construction of a resistor, and the heat treatment probably removes it. The final stability figures of a few parts in  $10^6$  for the second and third coils were very good, and justified the construction of an S-type 1 000-ohm coil for further investigation. The coiled-coil method of construction<sup>5</sup> was adopted and the wire was annealed after coiling to remove strains.

The wire used was 0.0048 in in diameter and was coated with synthetic enamel. Two processes for removing the enamel were tried. In the first the wire was immersed in pure phenol at  $50^{\circ}$ C for one hour and then washed in warm water. The enamel could then be removed easily with a cloth. An alternative and simpler treatment was to burn off the enamel by passing a current through the wire. After removal of the enamel the wire was coiled on a  $\frac{1}{16}$  in-diameter mandrel and afterwards annealed for two minutes at red heat. Heat treatment at 425°C followed, to bring the temperature-coefficient to the region of zero. The coil was then mounted on to a cylindrical Perspex former and fitted into its case, and after a period in a desiccator it was sealed. No heat treatment was applied after the coil was mounted on its Perspex former since the strains imposed during this operation were considered to be negligible.

The resistance and temperature coefficient of the coil were then measured. The average temperature-coefficient over the range  $15-25^{\circ}$  C was found to be less than one part in  $10^{6}$  per deg C.

The initial resistance was 999.990 ohms on 17th June, 1955.

The value has increased by 2 parts in  $10^6$  during the two months to 16th August, 1955, and by a further 1 part in  $10^6$  in the following six months to 2nd February, 1956.

### (10) 1-OHM STANDARDS

The manganin standards which have proved to have the highest stability have been 1-ohm standards, and a number of 1-ohm standards of Evanohm were constructed for purposes of comparison, although it was realized that this material was not likely to be competitive with manganin for coils of this value. For these standards, wire of 0.048 in diameter was used. The requisite length of wire, less than 100 cm, was bent at its mid-point to form a loop and the double wire coiled round the former. The wire was sufficiently stiff to keep its shape and the former served merely to prevent vibration. It consisted of four grooved Perspex rods secured to a metal cylinder, each rod being provided with a grooved cover of Perspex. The wire was held lightly in the grooves with very little strain. The covers could be taken off and the coil removed for heat treatment without changing the coil shape or applying significant strain. Heat treatment was applied to the shaped coil at 400°C to bring the temperaturecoefficient to zero. Five of these standards have been constructed. Their initial values after sealing and their temperaturecoefficients are given in Table 6. Heat treatment at 80° C for a

Table 6

| Temperature-<br>ard coefficient<br>over range |  |  | tes given   |
|---|--|--|---|
| over range<br>15–25° Č                        | 1.8.55   | 20.9.55  | 20.2.56   |
| parts in<br>10° per deg C<br>+1.0             | ohms<br>0 • 999 931  | ohms<br>0.999933   | ohms<br>0 · 999 933<br>1 · 000 039  |
| +0.3 +0.6                                     | -  | 1.000067<br>1.000019   | $1 \cdot 000039$<br>$1 \cdot 000067$<br>$1 \cdot 000023$<br>$0 \cdot 999954$  |
| -   | $\begin{array}{c} \begin{array}{c} \text{coefficient} \\ \text{over range} \\ 15-25^{\circ}\text{C} \end{array}$ $\begin{array}{c} \\ \begin{array}{c} \text{parts in} \\ 10^{\circ} \text{ per degC} \\ +1 \cdot 0 \\ -0 \cdot 8 \\ +0 \cdot 3 \end{array}$ | $\begin{array}{c} \begin{array}{c} coefficient \\ over range \\ 15-25^{\circ}C \end{array} & 1.8.55 \end{array}$ | $\begin{array}{c c} \hline coefficient \\ over range \\ 15-25^{\circ}C \\ \hline 1.8.55 \\ \hline 20.9.55 \\ \hline \\ parts in \\ 10^{\circ} per deg C \\ +1 \cdot 0 \\ -0 \cdot 8 \\ -0 \cdot 8 \\ -0 \cdot 3 \\ +0 \cdot 3 \\ -0 \cdot 6 \\ \hline \\ +0 \cdot 6 \\ \hline \\ \hline \\ \end{array} \begin{array}{c c} ohms \\ ohms \\ 0 \cdot 999 \ 931 \\ 0 \cdot 999 \ 933 \\ 0 \cdot 999 \ 933 \\ -0 \cdot 999 \ 931 \\ -0 \cdot 999 \ 933 \\ -0 \cdot 999 \ 931 \\ -0 \cdot 999 \ 933 \\ -0 \cdot 99$ |

few hours was applied to the coils when finally mounted in order to ease any slight strain imposed during this operation.

### (11) OTHER STANDARDS

A number of other resistance standards have been constructed of this material and are in use for general measurements. One of these was described in an earlier paper.<sup>6</sup>

The stability of these resistors has proved satisfactory so far, and there seems little doubt that, provided the wire is not subjected to strain or vibration, the resistance will not change more than a few parts in  $10^5$  per year. The figures obtained on specially constructed standards indicate the possibility of better stability, but more time will be required before this can be established with certainty.

### (12) EFFECT OF COLD WORKING

During the course of the experiments already described, evidence accumulated of the susceptibility of Evanohm to resistance and temperature-coefficient changes caused by strains. A direct experiment was made to determine the effect of coiling wire of 0.0048 in diameter on to a  $\frac{1}{10}$  in-diameter mandrel. It was found that the temperature-coefficient, if near zero, was made more positive by from 4 to 8 parts in 10<sup>6</sup> per deg C. The temperature-coefficient could be brought back to its initial value by heat treatment extending over several days at 200° C, but showed no tendency to do so when a coil was left untouched at room temperature for three months. It may be observed that the effect of cold working is to move the temperature-coefficient in the opposite direction to that resulting from heat treatment. When, therefore, heat treatment applied to a wire to bring the temperature-coefficient to zero has proved excessive it is tempting to apply cold working or vibration to correct the overshoot. It is likely, however, that the stability of the coil would be adversely affected by such treatment and the alternative method of annealing the wire and repeating the heat treatment is preferable.

In view of the effects of cold working it is undesirable to use Evanohm for resistors which are subject to vibration or rough handling.

### (13) CONCLUSIONS

The experiments described show that Evanohm has many desirable qualities as a resistance material. Its stability of resistance, when free from vibration, is good, and for high resistances of fine wire may prove to be superior to manganin. For resistors of moderate value, using wire of substantial section, the stability is also good, but the best figures obtained up to the present are not equal to the best figures for manganin. Better figures may possibly be obtained when the resistors under observation have had a longer settling-down period.

The material is of greatest value for resistors required to operate over a considerable temperature range on account of self-heating, since it is possible to adjust the temperaturecoefficient to zero by a simple heat treatment. The change of resistance over the working temperature range resulting from the curvature of the resistance/temperature characteristic is smaller than for constantan and much smaller than for manganin.

The high resistivity, high tensile strength and ductility of the alloy make it very suitable for resistances of high value. For lower values the high resistivity is a disadvantage from the point of view of heat dissipation, but is still an advantage from the point of view of resistance stability since the ratio of surface area to volume is less than that of materials of lower resistivity.

## (14) ACKNOWLEDGMENTS

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# DISCUSSION ON

# AN EXTENDED ANALYSIS OF ECHO DISTORTION IN THE F.M. TRANSMISSION OF FREQUENCY DIVISION MULTIPLEX'\*

where

**Mr. L. Lewin** (*communicated*): Messrs. Medhurst and Small state that an echo of amplitude equal to the signal cannot give rise to intermodulation noise, a brief demonstration being given in their Appendix 10.2. This result seems rather surprising, since for small echo amplitudes the distortion certainly increases with echo strength. It also runs counter to one's intuition—admittedly not an infallible guide—of the effects of selective fading on the signal. Since, for equal echo and signal the resulting amplitude modulation is 100%, the conventional limiter cannot deal with the signal in this case. It is therefore interesting to consider in detail the more practical case in which the echo is just a little different in amplitude from the signal.

Accordingly, we take for the combined signal and echo the expression

$$S = \cos(\omega_c t + \phi) + r \cos(\omega_c t + \psi) \quad . \quad . \quad (A)$$

where  $\phi = \phi(t)$  is the phase angle representing the frequencymodulated signal and  $\psi = \phi(t - \tau) - \omega_c \tau$  is the phase angle of the delayed echo. Eqn. (A) can be put in the form

$$S = A \cos(\omega_c t + \theta)$$
 . . . (B)

$$A^2 = 1 + 2r \cos(\phi - \psi) + r^2$$

and  $\tan \theta = (\sin \phi + r \sin \psi)/(\cos \phi + r \cos \psi)$ 

The recovered signal is the instantaneous frequency and is given by  $s = d\theta/dt$ . From (B),

$$s = \frac{\dot{\phi} + r^2 \dot{\psi} + r(\dot{\phi} + \psi) \cos{(\phi - \psi)}}{1 + 2r \cos{(\phi - \psi)} + r^2} \quad . \quad . \quad (C)$$

If we define two angles  $\xi = \frac{1}{2}(\phi + \psi)$  and  $\eta = \frac{1}{2}(\phi - \psi)$  then (C) can be put in the form

$$s = \xi + \frac{(1-r^2)\dot{\eta}}{(1-r)^2 + 4r\cos^2\eta} = \xi + \Delta$$
, say . (D)

On letting  $r \to 1$  we appear to get  $s = \xi$ , which is the authors' result. However, on closer inspection, we see that the additional term  $\Delta$  becomes large when  $\eta$  is close to  $(n + \frac{1}{2})\pi = \eta_n$ , say. In the neighbourhood of  $\eta_n$ ,  $\Delta$  can be written

$$\Delta \simeq \frac{(1 - r^2 \dot{\eta})}{(1 - r)^2 + 4r(\eta - \eta_n)^2} \quad . \quad . \quad . \quad (E)$$

<sup>\*</sup> MEDHURST, R. G., and SMALL, G. F.: Paper No. 2006 R, March, 1956 (see 103 B, p. 190).