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Online Review Course of
Undergraduate Probability and Statistics

Review Lecture 14
Inferences about a Variance

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Course Website: www.lithoguru.com/scientist/statistics/review.html

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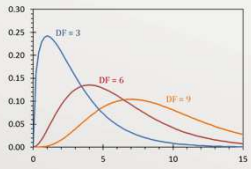
Sampling Distribution of the Variance

- Let $X_1, X_2, \ldots, X_n$ be iid random variables with $X_i \sim N(\mu, \sigma^2)$
- For a sample variance S^2, $\chi^2 = \frac{(n-1)S^2}{\sigma^2}$ is a RV with a chi square distribution and parameter DF = n -1

$$E[\chi^2] = n - 1$$
$$var[\chi^2] = 2(n-1)$$



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Sampling Distribution of the Variance

- For iid random variables with $X_i \sim N(\mu, \sigma^2)$

$$E[S^2] = \sigma^2$$

$$var[S^2] = \frac{2\sigma^4}{n-1}$$

"signal to noise" $$\frac{E[S^2]}{\sqrt{var[S^2]}} = \sqrt{\frac{n-1}{2}}$$

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Confidence Interval for the Variance

- Confidence interval:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

For α = 0.05:

DF	1	2	3	4	5	10	15	20	25	30	40	50	100
DF/$\chi^2_{0.025}$	0.199	0.271	0.321	0.359	0.390	0.488	0.546	0.585	0.615	0.639	0.674	0.700	0.772
DF/$\chi^2_{0.975}$	1018	39.5	13.90	8.26	6.02	3.08	2.40	2.09	1.91	1.79	1.64	1.55	1.35

For standard deviation confidence intervals, taking the square root of the above is a good approximation.

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Comparing Two Sample Variances

- We take samples from two normal populations (μ_1, σ_1) and (μ_2, σ_2). We wish to know if the populations have different variances.
- The F-statistic:

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$$

is a RV that follows the F-distribution with n_1-1 and n_2-1 degrees of freedom

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Caution

- Even relatively small deviations of the underlying populations from a normal distribution can make these tests and confidence intervals invalid
  - There is no equivalent of a central limit theorem to help us out
  - One must check the assumption of normality!

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Review #14: What have we learned?

- What is the sampling distribution of the variance? Under what assumptions?
- Explain the F-statistic and what it can be used for
- Explain why the statistical tests for variance could easily produce invalid results