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Online Review Course of
Undergraduate Probability and Statistics

# Review Lecture 12
# Hypothesis Testing

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## What is Hypothesis Testing?

- We wish to make inferences about the population based on data from a sample
  - Does a certain treatment change the glass transition temperature of a certain polymer?
  - Does the reliability of this new device meet its specification?
  - Does this new drug perform better than a placebo?
  - Is the mean time to perform a certain task less than 2 hours?
- Hypothesis testing is a formal method for answering these types of questions

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## Hypothesis Testing Example

- Nationwide, 26% of college students smoke
  - We'll assume that this came from survey data on a very large sample, so that we can ignore the uncertainty in this proportion
- My survey of 100 randomly chosen UT students found that 21% smoke
  - We'll ignore measurement error, and consider only sampling error
- Does the data from this sample give me enough confidence to say that UT students smoke at a different rate than all college students?
- To answer this, we must set up a formal hypothesis test

(I made up these numbers!)

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## Hypothesis Testing Steps

- Step 1 – define the null hypothesis
  - Null Hypothesis = ordinary, expected state of affairs (nothing has changed, one group is the same as another group); it is the default position that there is no relationship between two measured phenomena
  - Form: $H_o$: $\theta = \theta_o$ (parameter equals a number)
  - Example: $H_o$: $p_{UTsmokers} = 0.26$ (the proportion of UT smokers equals the national proportion)
- We assume that the null hypothesis is true unless there is convincing evidence otherwise

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## Hypothesis Testing Steps

- Step 2 – define the alternative hypothesis
  - One of three possible alternative to the null hypothesis
  - Form: $H_a$: $\theta \neq \theta_o$ (two-sided or two-tailed test)
    $H_a$: $\theta > \theta_o$ (one-sided or one-tailed test)
    $H_a$: $\theta < \theta_o$ (one-sided or one-tailed test)
  - Example: $H_a$: $p_{UTsmokers} \neq 0.26$ (the proportion of UT smokers does not equal the national proportion)

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## Hypothesis Testing Steps

- Step 3 – define $\alpha$, the significance level of the test
  - This defines the minimum confidence level ($1 - \alpha$) for our conclusions
  - Example: $\alpha = 0.05$ (we'll be able to make our conclusions with at least a 95% confidence level)
- Step 4 – calculate the p-value for our data
  - p-value: the probability that data at least this unusual could have come about by chance, given that the null hypothesis is true

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## P-Value Calculation Example

- From our sample, $\hat{p} = 0.21$
- Assuming the null hypothesis is true,
  - *np* > 10, *nq* > 10, use normal approximation

$$SE(p) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.26(0.74)}{100}} = 0.0439$$

  - $\hat{p} \sim N(0.26, 0.0439^2)$
- Calculate z: $z = \frac{|\hat{p} - p|}{SE(p)} = \frac{|0.21 - 0.26|}{0.0439} = 1.14$

 

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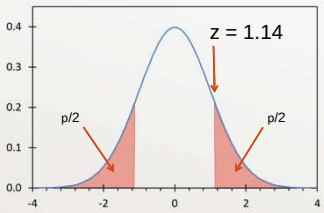
## P-Value Calculation Example

For a two-tail test




$z \sim N(0,1)$

- From Normal Distribution calculator, ℙ(z < -1.14) = 0.127
- Thus, p-value = 0.254

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## Hypothesis Testing Steps

- Step 5 – compare the p-value to the significance level
  - If p-value < α, we reject $H_o$ in favor of $H_a$
  - If p-value > α, we fail to reject $H_o$
- Note that the null hypothesis is privileged: we only reject it if there is sufficient reason to do so
- For our Example: p-value > α
  - We cannot conclude from this data that UT students smoke at a different rate than the national rate for college students

 

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## Ways to Game the Hypothesis Test

- Pick α after calculating the p-value
  - Think carefully about why one significance level is preferred over another
- Switch between one-sided and two-sided tests to try to get the desired result
  - This is equivalent to changing the alternative hypothesis
- Pick the null hypothesis with care – it is the privileged hypothesis
- The sample size is extremely important
  - If the sample is too small, it becomes almost impossible to reject the null hypothesis

 

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## Hypothesis Testing Errors

- Type I error (also called an α-error)
  - The null hypothesis is true, but we reject it
  - The probability of making a type I error ≤ α
- Type II error (also called a β-error)
  - The null hypothesis is false, but we fail to reject it
  - The probability of making a type II error depends on the effect size and the sample size
- Making α smaller diminishes the probability of making a type I error, but increases the probability of making a type II error

 

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## Review #12: What have we learned?

- What are the five steps of an hypothesis test?
- Explain the different roles of the null hypothesis and the alternative hypothesis
- Define significance level
- Define p-value
- Explain the meaning of type I and type II errors

 