Online Review Course of
Undergraduate Probability and Statistics

# Review Lecture 8
# Expectation Value and Variance

Chris A. Mack
*Adjunct Associate Professor*

Course Website: www.lithoguru.com/scientist/statistics/review.html





## Expectation Value

- Discrete random variables are characterized by their PMF (probability mass function)

$$p_X(x) = \mathbb{P}(X = x) \qquad \sum_{all\ x} p_X(x) = 1$$

- We define the Expectation Value (mean) of the random variable $X$ as

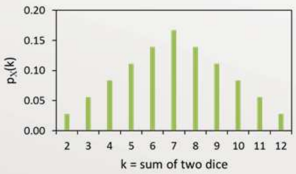
$$E[X] = \sum_{all\ x} x\, p_X(x)$$





## Expectation Value Example

- Let $X$ = sum of two six-sided, fair dice




$$p_X(2) = 1/36$$
$$p_X(3) = 2/36$$
$$p_X(4) = 3/36$$

Etc.

$$E[X] = \sum_{all\ k} k\, p_X(k) = 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + \ldots$$





## Uniform Distribution

- Let $X$ = uniformly distributed, N discrete values

$$p_X(x) = 1/N$$

$$E[X] = \sum_{all\ x} x\, p_X(x) = \sum_{i=1}^{N} \frac{x_i}{N}$$

- The expectation value for a uniformly distributed random variable is the arithmetic mean of the possible values





## Bernoulli Random Variable

- Consider a coin toss that produces a head (success) with probability *p*

$$X = \begin{cases} 1 & if\ heads\ (success) \\ 0 & if\ tails\ (failure) \end{cases}$$

$$p_X(k) = \begin{cases} p & if\ k = 1 \\ 1-p & if\ k = 0 \end{cases}$$

$$E[X] = 1(p) + 0(1-p) = p$$





## Variance

- Discrete random variables are characterized by their PMF (probability mass function)

$$p_X(x) = \mathbb{P}(X = x) \qquad \sum_{all\ x} p_X(x) = 1$$

- We define the Variance of the random variable $X$ as

$$var[X] = \sum_{all\ x} (x - E[X])^2\, p_X(x)$$

## Bernoulli Random Variable

- Consider a coin toss that produces a head (success) with probability *p*

$$X = \begin{cases} 1 \ \ if\ heads\ (success) \\ 0 \ \ if\ tails\ (failure) \end{cases} \qquad p_X(k) = \begin{cases} p & if\ k = 1 \\ 1-p & if\ k = 0 \end{cases}$$

$$E[X] = p \qquad var[X] = \sum_{all\ x} (x - E[X])^2\, p_X(x)$$

$$var[X] = (1-p)^2(p) + (0-p)^2(1-p) = p(1-p)$$



## Binomial Distribution

- Repeat a Bernoulli trial *n* times (*p* = probability of success)
- Let X = number of successes

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E[X] = n\,p$$

$$var[X] = n\,p(1-p)$$

A result of independent trials



## Geometric Distribution

- How many coin tosses are required before the first heads (success) comes up?
- Let X = number of tosses to get the first head

$$p_X(k) = (1-p)^{k-1} p$$

$$E[X] = 1/p$$

$$var[X] = (1-p)/p^2$$



## Poisson Distribution

- Consider the Binomial distribution when n >> k
- More specifically, let $n \to \infty$ while keeping $np = \lambda$ = constant

$$p_X(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$E[X] = \lambda$$

$$var[X] = \lambda$$



## Independence

- For all cases,

$$E[X+Y] = E[X] + E[Y]$$

$$E[aX+b] = aE[X] + b$$

$$var[aX+b] = a^2 var[X]$$

- If $X$ and $Y$ are independent random variables

$$E[XY] = E[X]E[Y]$$

$$var[X+Y] = var[X] + var[Y]$$



## Review #8: What have we learned?

- What is the “expectation value” and how is it calculated for a discrete RV?
- What is the “variance” and how is it calculated for a discrete RV?
- What is the mean and variance of the Bernoulli RV and a binomial distributed RV?
- What is the variance of the sum of two independent random variables?