Online Review Course of
Undergraduate Probability and Statistics

Review Lecture 7
Discrete Random Variables

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Course Website: www.lithoguru.com/scientist/statistics/review.html

TEXAS
WHAT STARTS HERE CHANGES THE WORLD

What is a Random Variable?

- Our probabilistic model assigns probabilities to events, which are collections of outcomes from our sample space
  - Events, outcomes, and sample spaces are represented mathematically by sets
  - In science, we want models that deal with numbers
- Random Variable: a real-valued function of the outcomes of the experiment
  - $\Omega \rightarrow \mathbb{R}$ (maps sample space onto the real numbers)

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What is a Random Variable?

- Formally,
  - Let $\omega \in \Omega$, $X(\omega) = x$, $x \in \mathbb{R}$
- Example
  - Ω = {all UT students}, X = height of randomly selected student
- Discrete versus Continuous random variable
  - Option 1: round height measurement to the nearest cm. Result = discrete RV
  - Option 2: measure height with infinite precision. Result = continuous RV.

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Random Variable Examples

- Examples of random variables
  - The number of heads in a sequence of 12 coin tosses
  - The sum of two rolls of a die
  - The number of coin tosses until the first head is obtained
  - The money won or lost in a particular game of chance
  - The time required for a text message to travel from sender to receiver
- Discrete and continuous RVs are handled separately
  - Similar but slightly different mathematics

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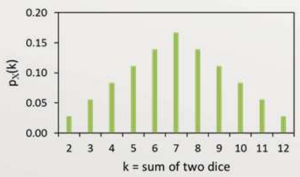
Probability Mass Function (PMF)

- For a discrete random variable

PMF of X → $p_X(x) = \mathbb{P}(X = x)$

- Ex: Let X = sum of two six-sided dice



Note:

$$\sum_{all\ k} p_X(k) = 1$$

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Bernoulli Random Variable

- Consider a coin toss that produces a head (success) with probability p

$$X = \begin{cases} 1 & if\ heads\ (success) \\ 0 & if\ tails\ (failure) \end{cases}$$

$$p_X(k) = \begin{cases} p & if\ k = 1 \\ 1 - p & if\ k = 0 \end{cases}$$

- Used to model many binary-outcome experiments
  - Bernoulli trial: p = constant, each trial is independent

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Binomial Distribution

- Repeat a Bernoulli trial n times (p = probability of success)
- Let X = number of successes

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

- Called the Binomial Distribution
  - Assumes n = constant, plus all the assumptions of a Bernoulli trial
  - Two parameters: n and p

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Geometric Distribution

- How many coin tosses are required before the first head (success) comes up?
- Let X = number of tosses to get the first head

$$p_X(k) = (1-p)^{k-1} p$$

- Called the Geometric Distribution

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Poisson Distribution

- Consider the Binomial distribution when n >> k
- More specifically, let $n \to \infty$ while keeping $np = \lambda$ = constant

$$p_X(k) = \lim_{n\to\infty} \binom{n}{k} p^k (1-p)^{n-k} = \frac{\lambda^k}{k!} e^{-\lambda}$$

- Called the Poisson Distribution
  - Ex: For a solution of concentration C, how many molecules are in a volume V.
  - Ans: Poisson with λ = CV

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Functions of RVs

- Functions of random variables are random variables
- Given X and $p_X(x)$, and y = g(x), what is $p_Y(y)$?

$$p_Y(y) = \sum_{x|g(x)=y} p_X(x)$$

(For x and y discrete RVs)

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Cumulative Distribution Function

- Discrete random variables are characterized by their PMF (probability mass function)

$$p_X(x) = \mathbb{P}(X = x) \qquad \sum_{all\ x} p_X(x) = 1$$

- We define the Cumulative Distribution Function (CDF) of the random variable X as

$$F_X(x) = \mathbb{P}(X \le x) = \sum_{all\ k \le x} p_X(k)$$

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Review #7: What have we learned?

- Define “random variable”
- Explain the difference between discrete and continuous random variables
- What is the probability mass function?
- What is a Bernoulli trial (and what assumptions apply)?
- Can you explain the binomial, geometric, and Poisson distributions?
- Define “cumulative distribution function”

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