CHE384, From Data to Decisions: Measurement, Uncertainty, Analysis, and Modeling

Lecture 45
Best Subset Regression

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Subset Regression

- Given a list of possible regressor variables, including possible interactions:
  - Define the “full” model as the model that includes every regressor and every interaction we care to consider
  - A “subset model” is a model with one or more terms missing
- Which subset model (or full model) is the best?

Testing Individual Terms

- For any individual model term, does the confidence interval for the parameter include 0?
  - The studentized parameter $b_k/SE(b_k)$ is student-t distributed with $n - p$ degrees of freedom
  - Calculate the p-value for a hypothesis test with H_0: $\beta_k = 0$
- If a term is not statistically significant, we generally leave it out of the model

Partial F-Test: Comparing Models

- Consider two models, where model 1 is a sub-set of model 2
  - Is it useful to include the extra variables of model 2 in the regression?
  - Are the added covariates informative?
- Consider this hypothesis test:
  - H_0: added predictor variables have all $\beta_j = 0$
  - H_A: at least one added predictor variable has $\beta_j \neq 0$

Partial F-Test: Comparing Models

- We want to compare the SSE (or chi square for a weighted regression) for these two models:

This is called the extra sum of squares (model 1 is a sub-set of model 2)

$$F = \frac{(SSE_1 - SSE_2)/(p_2 - p_1)}{SSE_2/(n - p_2)} = \frac{(R_2^2 - R_1^2)/(p_2 - p_1)}{(1 - R_2^2)/(n - p_2)}$$

- This statistic is F-distributed with $p_2 - p_1, n - p_2$ degrees of freedom (p = # of adjustable parameters in the model)

Partial F-Test Example

- Full Model: $\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$
- We suspect that $\beta_1 = \beta_3$
- Create a subset model with $\tilde{x}_1 = x_1 + x_3$

$$\hat{y} = \beta_0 + \beta_1 \tilde{x}_1 + \beta_2 x_2$$

- The Partial F-test of the subset model to the full model is equivalent to testing the null hypothesis that $\beta_1 = \beta_3$

Mallows C_p

- Compare a full model (with k adjustable model parameters) to a subset model (with p adjustable model parameters)

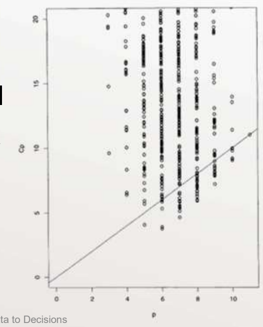
an estimate for χ^2

$$C_p = \frac{SSE_p}{\hat{\sigma}_\varepsilon^2} - n + 2p = \frac{SSE_p}{MSE_k} - n + 2p$$

- If the subset model is correct, then C_p should be about equal to p (smaller is better)
- Only useful when $n >> k$

Mallows C_p

- For moderate k, calculate C_p for every subset model
- If $C_p < p$, we have a candidate model



Colin L. Mallows, "More Comments on Cp", *Technometrics*, **37**(4), 362-372 (Nov 1995).

Likelihood Ratio

- For nested models
  - Let L_k = likelihood for full k-parameter model
  - Let L_p = likelihood for p-subset model ($L_k > L_p$)

$$G^2 = -2\ln\left(\frac{L_p}{L_k}\right)$$

- For large samples, if the subset model is correct then G^2 is approximately chi square distributed with $DF = k - p$
- If G^2 is too large, we reject the subset model

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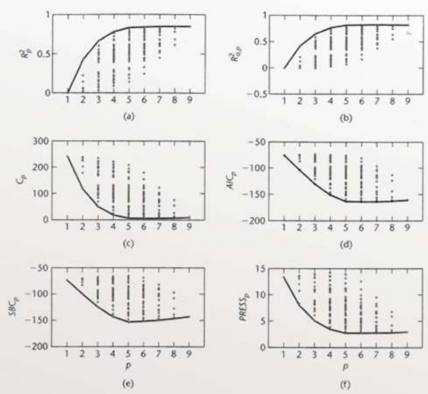
FIGURE 9.5 Plot of Variable Selection Criteria with All Eight Predictors—Surgical Unit Example.



Full Search

From Kutner, Nachtsheim, Neter, and Li, "Applied Linear Statistical Models', Fifth edition, McGraw-Hill (2005).

Lecture 45: What have we learned?

- Understand the use of the partial F-test for subset models
- How is Mallow's C_p used to evaluate subset models?
- How is the likelihood ratio used to evaluate subset models?