

Deming Regression

• The Deming regression estimate of the slope is the MLE estimate if both X and Y are normally distributed with independent errors  $-\varepsilon \sim N(0,\sigma_{\varepsilon}), \ \delta \sim N(0,\sigma_{\delta}), \ \text{cov}[\varepsilon,\delta] = 0$   $-\lambda = \sigma_{\varepsilon}^{2}/\sigma_{\delta}^{2} \text{ is known}$ • In general,  $\lambda$  is estimated from external knowledge -Prior measurement error analysis

Deming Regression

• Deming regression also allows an estimate for the true x-value,  $\hat{x}_i$   $\hat{x}_i = x_i + \frac{y_i - (b_0 + b_1 x_i)}{b_1 + \lambda/b_1}$ - An OLS regression of  $y_i$  vs  $\hat{x}_i$  produces the same fit as a Deming regression of  $y_i$  vs  $x_i$ • Deming (optimized) residuals:  $e_i = sign(y_i - \hat{y}_i)\sqrt{(y_i - \hat{y}_i)^2/\lambda + (x_i - \hat{x}_i)^2}$ 

Deming Regression vs. Geometric Mean

• Consider the Deming regression when  $\lambda = \frac{\sigma_{\varepsilon}^2}{\sigma_{\delta}^2} = \frac{s_y^2}{s_x^2}$ • For this case, the Deming regression slope equals the geometric mean slope =  $\sqrt{\lambda}$ – The GM approach can be justified if all of the variation in X and Y comes from randomness (i.e., we did not systematically vary X or Y)

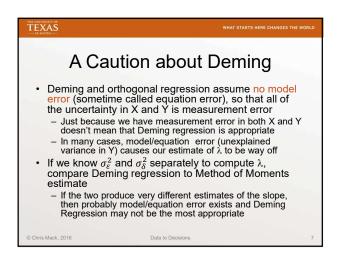
Standard Error of Coefficients

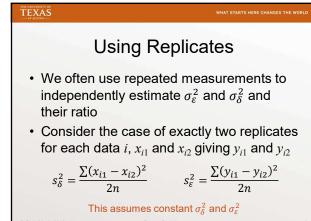
• Calculating the standard error of the coefficients for a total regression is not trivial (generally, we use a jackknife or bootstrap method)

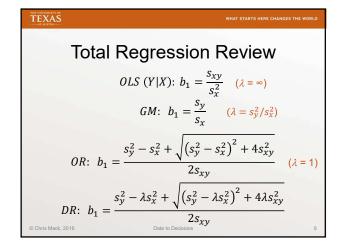
– Let our statistical software perform the calculations

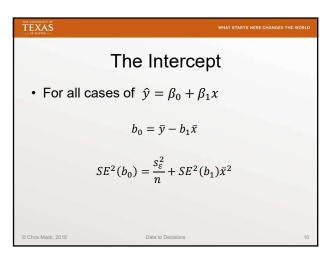
– See M. A. Creasy, "Confidence Limits for the Gradient in the Linear Functional Relationship", Journal of the Royal Statistical Society B, 18(1), 65-69 (1956).

• A Method of Moments estimator gives  $SE(b_1) = \frac{b_1}{\sqrt{n}} \sqrt{\frac{\left(s_x s_y\right)^2}{\left(s_{xy}\right)^2} - 1} = \frac{b_1}{\sqrt{n}} \sqrt{\frac{1}{b_{10L}} \frac{1}{|x|} b_{10LS(X|Y)}} - 1$ 









References on Total Regression

C.-L. Cheng and J. W. Van Ness, Statistical Regression with Measurement Error, Arnold Publishers, London (1999).

Wayne A. Fuller, Measurement Error Models, John Wiley & Sons, New York (1987 or 2006).

Albert Madansky, "The Fitting of Straight Lines When both Variables are Subject to Error", Journal of the American Statistical Association, 54(285), pp. 173-205 (Mar., 1959).

