

Review of Introduction to Probability and Statistics

Chris Mack, <http://www.lithoguru.com/scientist/statistics/review.html>

Homework #1

- Define the sample space (that is, list all of the elements) for each example below:
 - the set of integers between 1 and 60 evenly divisible by 7
 - the set $\Omega = \{x \mid x^2 + 4x - 5 = 0\}$
 - the set of outcomes when a coin is tossed until a tail or three heads appear
 - all points in the first quadrant inside a circle of radius 3 with center at the origin
- An engineer tests three products and pronounces the results for each as pass (P) or fail (F).
 - List the elements of a sample space Ω
 - List the elements corresponding to the event E that at least two products pass
 - Describe the event that has as its elements {FFF, PFF, FFP, PFP}
- In a battery life study, each of 3 laptops is tested using 5 different brands of batteries running 7 different software suites. If 2 testers are used in the study, and test runs are made once under each distinct set of conditions for each tester, how many test runs are needed?
- A recent study concluded that by following seven basic health rules a man's life can be extended by 11 years on the average and a woman's life by 7 years. These seven rules are: don't smoke, exercise regularly, use alcohol moderately, get 7 to 8 hours of sleep each night, maintain proper weight, eat breakfast every day, and do not eat between meals. In how many ways can a person adopt five of these rules:
 - If the person presently violates all 7 rules?
 - If the person never drinks and always eats breakfast?
- How many permutations can be made from the letters of the word *healthy*? How many of these permutations start with the letter y?
- How many permutations can be made from the letters of the word *infinity*?
- Given the digits 0, 1, 2, 3, 4, 5, and 6,
 - How many three-digit numbers can be formed from if each digit can be used only once?
 - How many of these are odd numbers?
 - How many of these are greater than 330?
 - How many three-digit numbers can be formed from if each digit can be used more than once?
- In how many ways can 5 people be seated around a circular table?
- In wedding photo, how many ways can I line up the bride, groom, and 5 other guests making sure that the bride and groom stand next to each other?

10. Genes can be thought of as paragraphs comprised of three letter words, where the letters are A, T, G or C. Each word or “codon” calls out an amino acid of the protein the gene codes for. For example, GGG is a valid codon for the amino acid glycine. How many different codon words can be written with the four letters if the letters can be repeated?
11. Consider a discrete sample space $\Omega = \{1, 2, 3, 4\}$. Determine if the following probability laws are valid.
- $\mathbb{P}(1) = 0.26, \mathbb{P}(2) = 0.25, \mathbb{P}(3) = 0.26, \mathbb{P}(4) = 0.25$.
 - $\mathbb{P}(1) = 0.15, \mathbb{P}(2) = 0.28, \mathbb{P}(3) = 0.33, \mathbb{P}(4) = 0.24$.
 - $\mathbb{P}(1) = 0.26, \mathbb{P}(2) = 0.35, \mathbb{P}(3) = -0.04, \mathbb{P}(4) = 0.43$.
12. One-third of Americans are overweight. Selecting two Americans at random, what is the chance that both are overweight?
13. A batch of one hundred widgets is inspected by testing four widgets selected at random. The batch will be rejected if one or more widgets from the sample is found to be defective. What is the probability that the batch will be accepted if the batch has 5 defective widgets?
14. Consider an experiment where two fair 6-sided dice (marked with numbers 1 through 6) are thrown and all 36 possible outcomes are equally likely. Are the events $A = \{\text{first roll is a 1}\}$ and $B = \{\text{sum of the two rolls is 7}\}$ independent?
15. You are playing in a single-elimination scrabble tournament (one loss and you lose the tournament). The odds makers say the probability of you beating half the players (call them type 1 players) is 0.3, the probability of beating a quarter of the players (type 2 players) is 0.4, and the probability of beating the remaining quarter (type 3 players) is 0.5. What is the probability of winning the tournament?
16. Referring to the tournament of problem 15, suppose that you won the first game. What is the probability that the first game was played against a type 1 player?