CHE384 Data to Decisions
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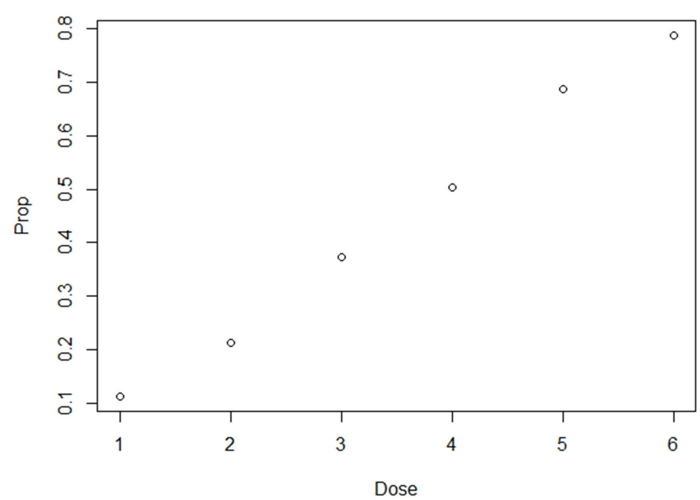
Homework #9 – Logistic Regression- Solutions

Turn in your solution with the answers to the questions below. Also, email to me the supporting spreadsheet and/or R script that you used to perform the analysis. (Please name the file using this format: HW9_yourname.R).

1. To test the toxicity of a particular substance, six groups of 250 insects each were given different doses of the toxic substance. The dose (x) is given in arbitrary units on a logarithmic scale. One day after exposure, the number of insects that were dead were counted (y).

Use R to perform a logistic regression:

a) Plot the sample proportion that died (y/n) as a function of x. Does the plot suggest that the logistic regression function is appropriate?



From the plot, it appears that a sigmoidal (S-shaped) curve will be appropriate.

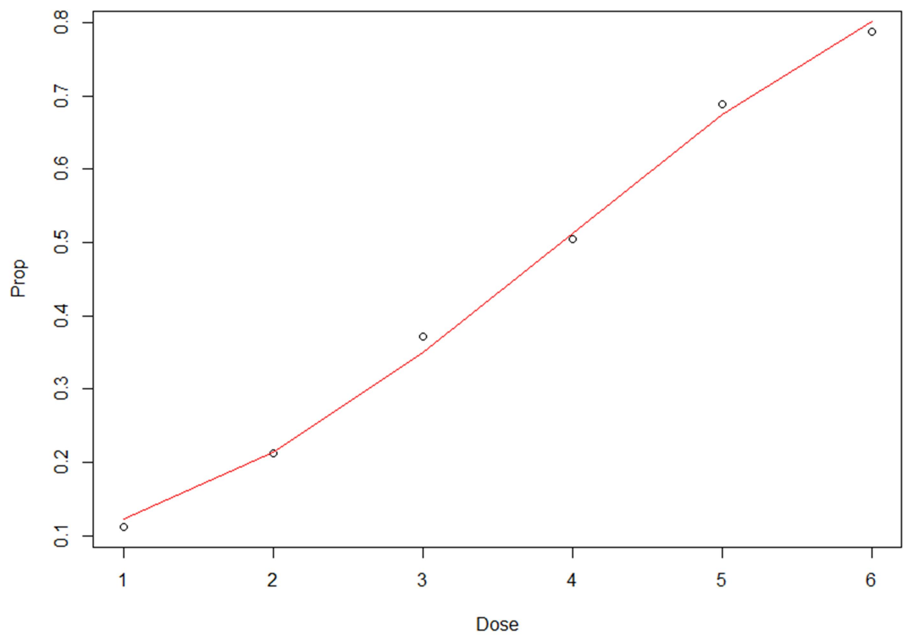
b) Find the maximum likelihood estimates for β_0 and β_1. State the fitted response function.

```
glm(formula = cbind(died, n - died) ~ Dose, family = binomial(link = "logit"), data = insects)

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.64367    0.15610  -16.93   <2e-16
Dose         0.67399    0.03911   17.23   <2e-16

                 2.5 %     97.5 %
(Intercept) -2.9554809 -2.3432165
Dose         0.5985828  0.7519688
```

Thus, using 95% confidence limits, we have $b_0 = -2.6437 \pm 1.96*0.1561$, *and* $b_1 = 0.6740 \pm 1.96*0.0391$. *The fitted response function is*

$$ln\left(\frac{\pi}{1-\pi}\right) = -2.64 + 0.674x$$

c) Plot the sample data together with the fitted response function. Does the fit appear to be a good one?



Yes, the fit appears to be good.

d) What is the estimated probability that an insect dies when the dose is x = 3.5?

log-odds:
`prediction = -0.2847003 , SE(prediction) = 0.02995912`
odds:
`Odds = 0.7522396`
Probability:
`Probability = 0.4293018`

e) What is the estimated median lethal dose (the dose where there is a 50% chance of the insect dying)?

When $\pi = 0.5$, *the logistic regression gives* $x_{0.5} = -\beta_0/\beta_1$. *Thus, the estimated median lethal dose is 2.6437 / 0.6740 = 3.922.*

2. A random sample 33 families were surveyed to determine their annual family income (x_1, in thousands of dollars) as well as the age of their oldest car (x_2, in years). One year later, a follow-up survey determined if they bought a car ($y = 1$) or did not purchase a car ($y = 0$) in the past year. Assume that a multiple logistic regression model first order in the two predictor variables is appropriate.

Use R to perform a logistic regression:

a) Find the maximum likelihood estimates for β_0, β_1 and β_2. State the fitted response function.

```
Call: glm(formula = Purchased ~ ., family = binomial(link = "logit"), data = cars)

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -4.73931    2.10195  -2.255   0.0242 *
Income       0.06773    0.02806   2.414   0.0158 *
CarAge       0.59863    0.39007   1.535   0.1249

                  2.5 %     97.5 %
(Intercept) -9.44517202 -1.0486919
Income       0.01950261  0.1317774
CarAge      -0.12216880  1.4469327
```

$$ln\left(\frac{\pi}{1-\pi}\right) = -4.74 + 0.0677 * income + 0.60 * car\_age$$

b) Obtain $\exp(b_1)$ and $\exp(b_2)$ and explain these numbers.

```
exp(coef(model))
(Intercept)      Income      CarAge
0.008744682    1.070079    1.819627
```

The term $\exp(b_1) = 1.07$ is the odds ratio for income. For each \$1,000 increase in family income, the odds of buying a new car increase by the factor 1.07. The term $\exp(b_2) = 1.82$ is the odds ratio for car age. For each year increase in the age of the oldest family car, the odds of buying a new car increase by the factor 1.82.

c) What is the estimated probability that a family with an income of \$50,000 and an oldest car that is 3 years old will buy a car in the next year?

The predicted probability is 0.609 with a standard error of 0.12.