CHE323/384 Chemical Processes for Micro- and Nanofabrication
Chris Mack, University of Texas at Austin

Homework #11 Solutions

1. Consider a binary pattern of lines and spaces. Which diffraction orders pass through the lens under these circumstances:

   (a) λ = 248 nm, NA = 0.8, pitch = 300 nm, on-axis coherent illumination
   (b) Same as (a), but pitch = 400 nm
   (c) Same as (b), but illumination tilted by an angle $\sin\theta' = 0.5$

   a) Spatial frequency = j/p. Cutoff frequency = NA/λ. Max $j \le pNA/\lambda = 0.96$. Thus, only the zero order makes it through the lens.
   b) Max $j \le pNA/\lambda = 1.29$. Thus, the zero and $\pm 1^{st}$ orders make it through the lens.
   c) Spatial frequency = $j/p + 0.5/\lambda$. Cutoff frequency = NA/λ. Max $j \le p(NA\text{-}0.5)/\lambda = 0.48$. Thus, the max order is 0. Min $j \ge -p(NA+0.5)/\lambda = -2.096$. Thus, the min order is -2. The 0, -1 and -2 orders go through the lens.

(Note: it is usually very helpful to draw pictures when working this type of problem.)

2. For a repeating line/space pattern and coherent illumination, derive expressions for the aerial image intensity at the center of the line and the center of the space as a function of the number of diffraction orders captured.

For lines and spaces and coherent illumination, $E(x) = a_0 + 2\sum_{j=1}^{N} a_j \cos(2\pi jx/p)$. At the center of the space, $x = 0$. Thus,

$$E(x=0) = \sum_{j=-N}^{N} a_j, \quad I(x=0) = E(x=0)E^*(x=0) = \left(\sum_{j=-N}^{N} a_j\right)\left(\sum_{j=-N}^{N} a^*_j\right)$$

. For the common case where the diffraction order amplitudes are real (true when we have a binary mask, such as chrome on glass, or an ideal phase-shift mask with only 0 and π phase shifts),

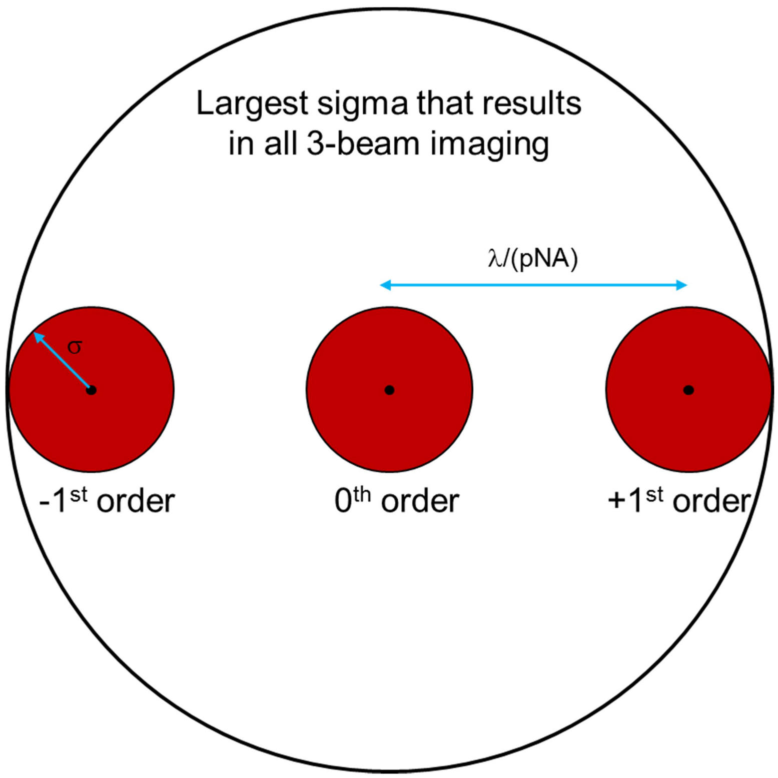
$$I(x=0) = \left(\sum_{j=-N}^{N} a_j\right)^2$$

At the center of the line, $x = p/2$. So $E(x) = a_0 + 2\sum_{j=1}^{N} a_j \cos(j\pi) = \sum_{j=-N}^{N} a_j (-1)^j$ and

$$I(x=p/2) = \left(\sum_{j=-N}^{N} (-1)^j a_j\right)^2$$

3. For $\lambda = 248$ nm, NA = 0.8, and pitch = 400 nm, below what value of σ is the image entirely made up of three-beam interference? At what σ value does one-beam imaging first appear?

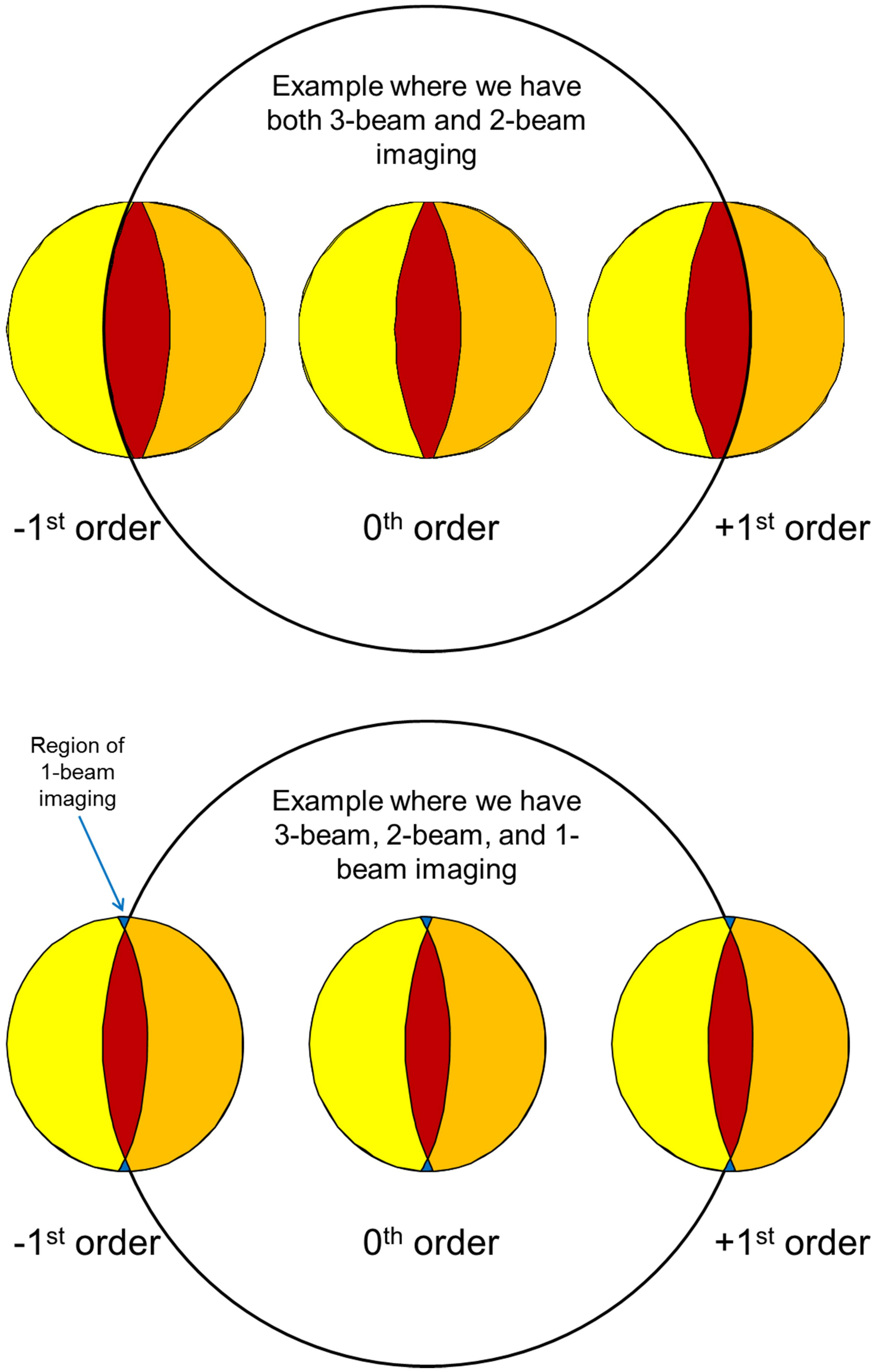
The best way to work this problem is to draw the lens top-down as a unit circle (called sigma-space). Since the radius of the lens is NA/λ in spatial frequency space, one must multiply spatial frequencies by λ/NA to convert them to sigma-space. Thus, the position of the first order, which is 1/p in frequency space, becomes λ/(pNA) in sigma-space.



From the geometry, imaging is entirely three-beam when $\frac{\lambda}{pNA} + \sigma \le 1$. Thus, $\sigma \le 1 - \frac{\lambda}{pNA}$. For this problem, that means $\sigma \le 0.225$.

As the source size increases we have both three-beam and two-beam imaging (see first figure below). Imaging begins to have a one-beam component when $\sqrt{1-\sigma^2} \le \frac{\lambda}{pNA}$. Thus,

$$\sigma \ge \sqrt{1 - \left(\frac{\lambda}{pNA}\right)^2} = 0.632.$$



Example of dense line/space imaging where only the zero and first diffraction orders are used. Red represents three-beam imaging, lighter and darker yellows show the areas of two-beam imaging, and blue represents one-beam imaging.

4. Consider the case of dense equal lines and spaces (only the 0 and $\pm 1^{st}$ orders are used) imaged with coherent illumination. Show that the peak intensity of the image in the middle of the space falls off approximately quadratically with defocus for small amounts of defocus.

$I(x) = a_0^2 + 2a_1^2 + 4a_0 a_1 \cos(\Delta\Phi)\cos(2\pi x/p) + 2a_1^2 \cos(4\pi x/p)$. Recall that $\Delta\Phi = 2\pi\delta(1-\cos\theta)/\lambda$ and that in the paraxial (small angle) limit, $\Delta\Phi = \pi\lambda\delta/p^2$, though the paraxial approximation does not need to be invoked for this derivation.

At the center of the space, $x = 0$ and $I(0) = a_0^2 + 4a_1^2 + 4a_0 a_1 \cos(\Delta\Phi)$. Using a Taylor series expansion of the cosine, for small defocus (and thus small $\Delta\Phi$),

$$\cos(\Delta\Phi) \approx \left(1 - \frac{\Delta\Phi^2}{2}\right)$$

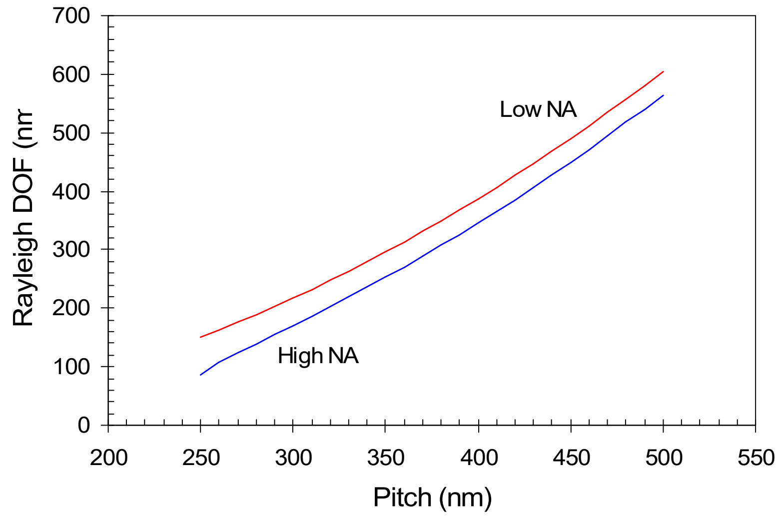
$$I(0) \approx a_0^2 + 4a_1^2 + 4a_0 a_1\left(1 - \frac{\Delta\Phi^2}{2}\right) = I_{in-focus}(0) - 2a_0 a_1 \Delta\Phi^2$$

Since $\Delta\Phi$ is directly proportional to defocus (even when we are not in the paraxial regime), one can see that the peak intensity will fall off approximately as the defocus squared. Using the equal line/space values for a_0 and a_1 gives, in the paraxial limit,

$$I(0) \approx I_{in-focus}(0) - \frac{1}{\pi}\left(\pi\lambda\delta/p^2\right)^2 = I_{in-focus}(0) - \pi\lambda^2\delta^2/p^4$$

5. Compare the depth of focus predictions of the high-NA version of the Rayleigh DOF equation to the paraxial (low-NA) version by plotting predicted DOF versus pitch (use $k_2 = 0.6$, $\lambda = 248$ nm, pitch in the range from 250 to 500 nm, and assume imaging in air).

$$DOF_{highNA} = \frac{k_2}{2}\frac{\lambda}{n(1-\cos\theta)}, \quad DOF_{lowNA} \approx k_2 \frac{\lambda}{\sin^2\theta} = k_2 \frac{p^2}{\lambda}$$



6. Consider the coherent image of a line/space pattern where only three orders are used to form the image:

$$I(x) = \left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi x / p)\right]^2$$

Calculate the image contrast, defined as

$$Image\ Contrast = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

For simplicity, assume the maximum intensity occurs in the middle of the space, and the minimum intensity occurs in the middle of the line (a common assumption).

$$I_{\max} = I(x=0) = \left[\frac{1}{2} + \frac{2}{\pi}\right]^2, \quad I_{\min} = I(x = p/2) = \left[\frac{1}{2} - \frac{2}{\pi}\right]^2, \text{ so}$$

$$Image\ Contrast = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{4/\pi}{1/4 + 8/\pi^2} = 0.9715$$