Advanced Problems

3.30. A causal LTI system has system function

$$H(z) = \frac{1 - z^{-1}}{1 - 0.25z^{-2}} = \frac{1 - z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})}.$$

(a) Determine the output of the system when the input is $x[n] = u[n]$.
(b) Determine the input $x[n]$ so that the corresponding output of the above system is $y[n] = \delta[n] - \delta[n-1]$.
(c) Determine the output $y[n]$ when the input is $x[n] = \cos(0.5\pi n)$ for $-\infty < n < \infty$. You may leave your answer in any convenient form.

3.31. Determine the inverse z-transform of each of the following. In parts (a)–(c), use the methods specified. (In part (d), use any method you prefer.)

(a) Long division:

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}, \qquad x[n] \text{ a right-sided sequence}$$

(b) Partial fraction:

$$X(z) = \frac{3}{z - \frac{1}{4} - \frac{1}{8}z^{-1}}, \qquad x[n] \text{ stable}$$

(c) Power series:

$$X(z) = \ln(1 - 4z), \qquad |z| < \frac{1}{4}$$

(d) $X(z) = \dfrac{1}{1 - \frac{1}{3}z^{-3}}, \qquad |z| > (3)^{-1/3}$

3.32. Using any method, determine the inverse z-transform for each of the following:

(a) $X(z) = \dfrac{1}{\left(1 + \frac{1}{2}z^{-1}\right)^2 (1 - 2z^{-1})(1 - 3z^{-1})}$,
($x[n]$ is a stable sequence)
(b) $X(z) = e^{z^{-1}}$
(c) $X(z) = \dfrac{z^3 - 2z}{z - 2}$, ($x[n]$ is a left-sided sequence)

3.33. Determine the inverse z-transform of each of the following. You should find the z-transform properties in Section 3.4 helpful.

(a) $X(z) = \dfrac{3z^{-3}}{\left(1 - \frac{1}{4}z^{-1}\right)^2}$, $x[n]$ left sided
(b) $X(z) = \sin(z)$, ROC includes $|z| = 1$
(c) $X(z) = \dfrac{z^7 - 2}{1 - z^{-7}}$, $|z| > 1$

3.34. Determine a sequence $x[n]$ whose z-transform is $X(z) = e^z + e^{1/z}$, $z \neq 0$.

5.28. A causal LTI system has the system function

$$H(z) = \frac{(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})(1 + 1.1765z^{-1})}{(1 - 0.9e^{j\pi/3}z^{-1})(1 - 0.9e^{-j\pi/3}z^{-1})(1 + 0.85z^{-1})}.$$

(a) Write the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of this system.
(b) Plot the pole–zero diagram and indicate the ROC for the system function.
(c) Make a carefully labeled sketch of $|H(e^{j\omega})|$. Use the pole–zero locations to explain why the frequency response looks as it does.
(d) State whether the following are true or false about the system:

(i) The system is stable.
(ii) The impulse response approaches a nonzero constant for large n.
(iii) Because the system function has a pole at angle $\pi/3$, the magnitude of the frequency response has a peak at approximately $\omega = \pi/3$.
(iv) The system is a minimum-phase system.
(v) The system has a causal and stable inverse.

5.29. Consider the cascade of an LTI system with its inverse system shown in Figure P5.29.

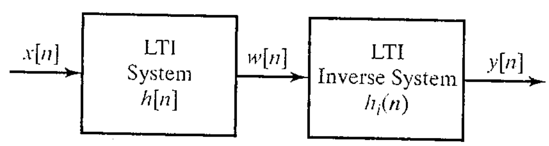

Figure P5.29

The impulse response of the first system is $h[n] = \delta[n] + 2\delta[n-1]$.

(a) Determine the impulse response $h_i[n]$ of a stable inverse system for $h[n]$. Is the inverse system causal?
(b) Now consider the more general case where $h[n] = \delta[n] + \alpha\delta[n-1]$. Under what conditions on α will there exist an inverse system that is both stable and causal?

5.30. In each of the following parts, state whether the statement is always TRUE or FALSE. Justify each of your answers.

(a) "An LTI discrete-time system consisting of the cascade connection of two minimum-phase systems is also minimum-phase."
(b) "An LTI discrete-time system consisting of the parallel connection of two minimum-phase systems is also minimum-phase."

5.31. Consider the system function

$$H(z) = \frac{rz^{-1}}{1 - (2r\cos\omega_0)z^{-1} + r^2z^{-2}}, \qquad |z| > r.$$

Assume first that $\omega_0 \neq 0$.

(a) Draw a labeled pole–zero diagram and determine $h[n]$.
(b) Repeat part (a) when $\omega_0 = 0$. This is known as a critically damped system.