5.28

$$H(z) = \frac{(1 - e^{j\pi/3} z^{-1})(1 - e^{-j\pi/3} z^{-1})(1 + 1.1765 z^{-1})}{(1 - 0.9 e^{j\pi/3} z^{-1})(1 - 0.9 e^{-j\pi/3} z^{-1})(1 + 0.85 z^{-1})}$$

expanding (by hand or Matlab / Maple)

$$H(z) = \frac{1 + 0.1765 z^{-1} - 0.1765 z^{-2} + 1.1765 z^{-3}}{1 - 0.05 z^{-1} + 0.045 z^{-2} + 0.6885 z^{-3}}$$

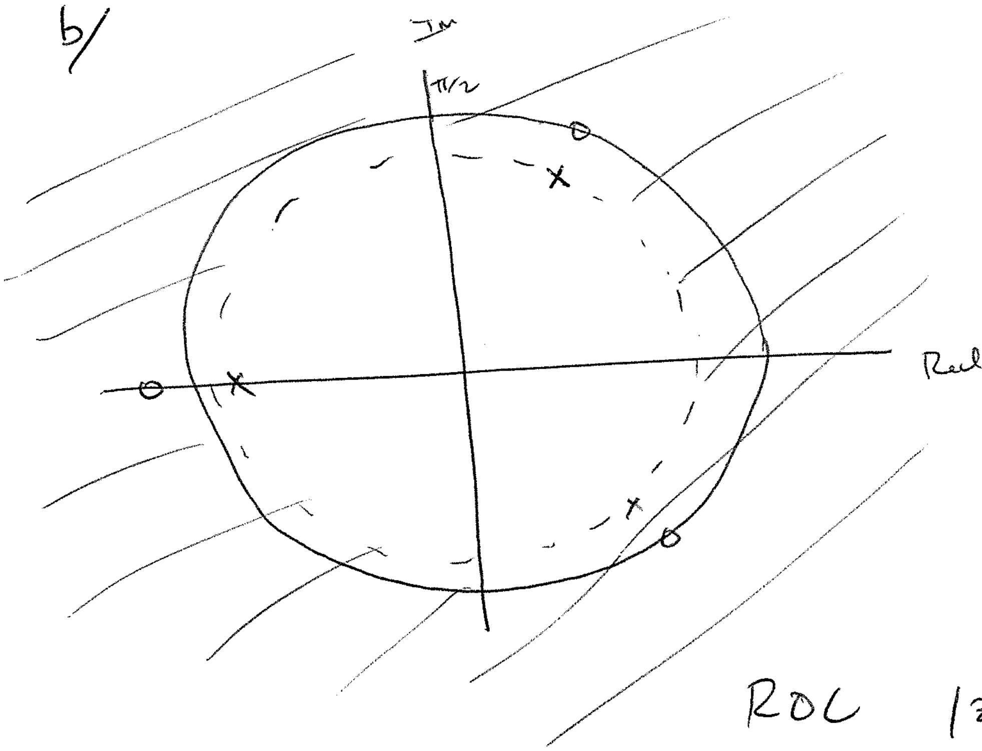
thus

a/

$$x[n] + 0.1765\, x[n-1] - 0.1765\, x[n-2] + 1.1765\, x[n-3]$$
$$= y[n] - 0.05\, y[n-1] + 0.045\, y[n-2] + 0.6885\, y[n-3]$$

See Matlab

b/



Poles

$z = 0.9 e^{j\pi/3}$

$= 0.9 e^{-j\pi/3}$

$= -0.85$

Zeros

$z = e^{j\pi/3}$

$= e^{-j\pi/3}$

$= -1.1765$

ROC $|z| > 0.9$

Contents

OS Problem 5.28

```
clc
close all
clear
```

Problem 5.28a)

```
syms x
Y = (1 - exp(j*pi/3)*x)*(1-exp(-j*pi/3)*x)*(1+ 1.1765*x)
X = (1 - 0.9 * exp(j*pi/3)*x)*(1-0.9*exp(-j*pi/3)*x)*(1+ 0.85*x)

Y = expand(Y)
X = expand(X)
```

```
Y =

(1-(1/2+1/2*i*3^(1/2))*x)*(1-(1/2-1/2*i*3^(1/2))*x)*(1+2353/2000*x)


X =

(1-(9/20+9/20*i*3^(1/2))*x)*(1-(9/20-9/20*i*3^(1/2))*x)*(1+17/20*x)


Y =

1+353/2000*x-353/2000*x^2+2353/2000*x^3


X =

1-1/20*x+9/200*x^2+1377/2000*x^3
```

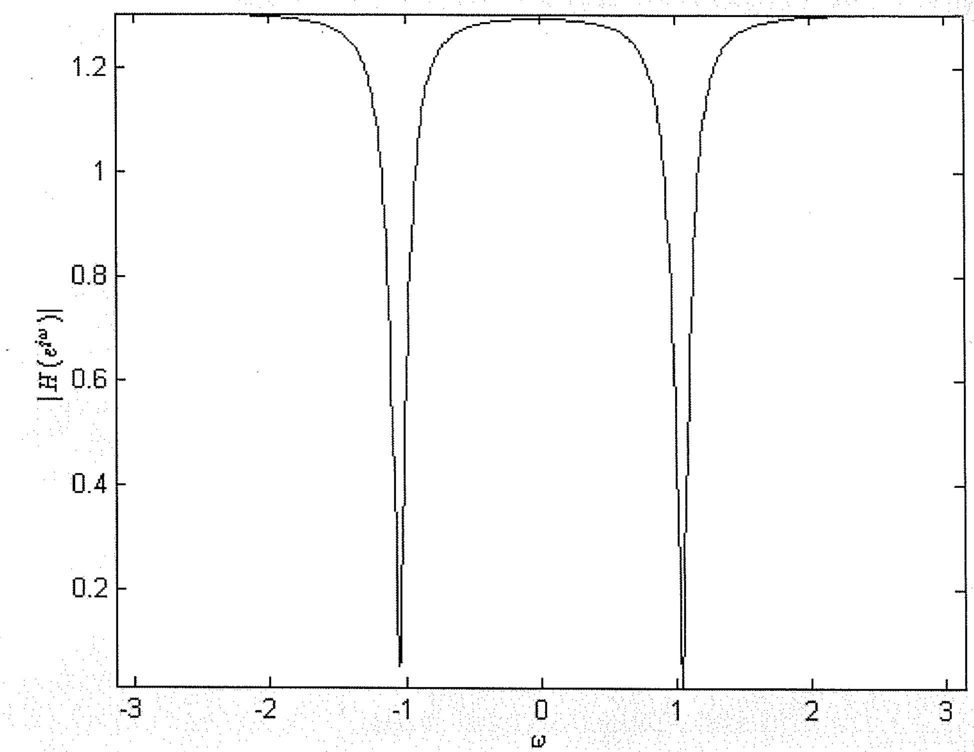
Problem 5.28b)

```
zeros = [exp(j*pi/3)  exp(-j*pi/3) -1.1765].'
poles = [0.9*exp(j*pi/3) 0.9*exp(-j*pi/3) -.85].'

figure
zplane(zeros,poles)
```

```
zeros =

   0.5000 + 0.8660i
   0.5000 - 0.8660i
```



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5.28 d

i) True - ROC includes unit circle, so must be stable.

(Since ROC includes unit circle, that implies FT exists, which implies h[n] is absolutely summable, and thus stable).

ii) False - since system is stable, h[n] must converge to zero

iii) False - There are zeros at exactly

$z = 1\angle \pm \pi/3$

iv) False - There is a zero outside the unit circle

v) False - Original system has a zero outside unit circle, so inverse has a pole outside unit circle.

Homework 10 Solutions

10.1 (OS. 5.45 a, b, c, d, f, g, i)

a/ B, C, D, E

IIR filters must have poles at $z \neq 0$.

In IIR filters, the current output, $y[n]$, is a function of previous outputs, $y[n-k]$. Relating the general form of the difference eqn to

$$H(z) = \left(\frac{b_0}{a_0}\right) \frac{\prod (1 - c_k z^{-1})}{\prod (1 - d_k z^{-1})}$$

we see that $d_K \neq 0$ for all K in an IIR Syfem.

b/ A, F

FIR filters must have a pole only at $z = 0$. by same reasoning as a).

C/

A, B, C, E, F.

ROC must include unit circle for stable system. Since systems are causal, all poles must be inside unit circle.

D/ E

Poles and zeros must be inside unit circle.

F/

C

All pass systems have form

$$\frac{z^{-1} - c^*}{1 - cz^{-1}} \leadsto \begin{array}{l} \text{zero at } z = 1/c^* \\ \text{poles at } z = c \end{array}$$

If there is a pole at $x + jy$, then zero at $\dfrac{1}{x - jy} = x + jy \cdot \dfrac{1}{x^2 + y^2}$

G/ E

Poles and zero's inside unit circle

This is high pass. Low pass is A and F.

i/ B, D

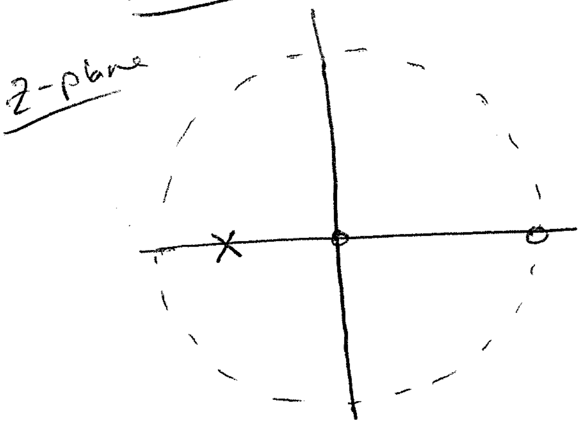
Zeros near $\omega = 0$, poles near $\omega = \pi$.

10.2 (OS 5.50)

No - Since $H(e^{j\omega}) = 0$ for some ω,

$\frac{1}{H(e^{j\omega})}$ blows up $\rightarrow$ $H(e^{j\omega})$ does not have a stable inverse.

10.3



$$|H(e^{j\pi})| = 1$$

$$H(z) = a \cdot \frac{(1 - z^{-1})}{(1 + 2/3\, z^{-1})} \cdot z$$

$$|H(e^{j\pi})| = \left| a \cdot \frac{1 - e^{-j\pi}}{1 + 2/3\, e^{-j\pi}}\, e^{j\pi} \right| = 1$$

$$\left| a \cdot \frac{1 + 1}{1 - 2/3} \cdot -1 \right| = 6|a| = 1$$

Choose $a = 1/6, -1/6$, ect.

a)

$$\boxed{H(z) = 1/6\, z\, \frac{(1 - z^{-1})}{(1 + 2/3\, z^{-1})}}$$

b/

$$Y(z) = X(z) \cdot \frac{1}{6} z \frac{(1 - z^{-1})}{(1 + 2/3 z^{-1})}$$

$$6 \cdot Y(z) \cdot (1 + 2/3 z^{-1}) = X(z)(z - 1)$$

$$6 \cdot y[n] + 4 y[n-1] = x[n+1] - x[n]$$

$$\boxed{y[n] + 2/3 y[n-1] = 1/6 x[n+1] - 1/6 x[n]}$$

c/ $H(e^{j\omega}) = H(z)\Big|_{z = e^{j\omega}}$

$$\boxed{H(e^{j\omega}) = \frac{1}{6} e^{j\omega} \frac{1 - e^{-j\omega}}{1 + 2/3 e^{-j\omega}}}$$

d/

$$|H(e^{j\omega})| = \sqrt{\frac{1}{6} e^{j\omega} \frac{1-e^{j\omega}}{1+2/3\, e^{-j\omega}} \cdot \frac{1}{6} e^{-j\omega} \frac{1-e^{+j\omega}}{1+2/3\, e^{j\omega}}}$$

$$= \frac{1}{6} \sqrt{\frac{1 - e^{j\omega} - e^{-j\omega} + 1}{1 + 2/3\, e^{-j\omega} + \frac{2}{3} e^{j\omega} + 4/9}}$$

$$= \frac{1}{6} \sqrt{\frac{4 \sin^2 \theta/2}{8/3 \cos^2 \theta/2 + 1/9}}$$

$$= \frac{1}{3} \frac{\sin \theta/2}{\sqrt{8/3 \cos^2 \theta/2 + 1/9}} =$$

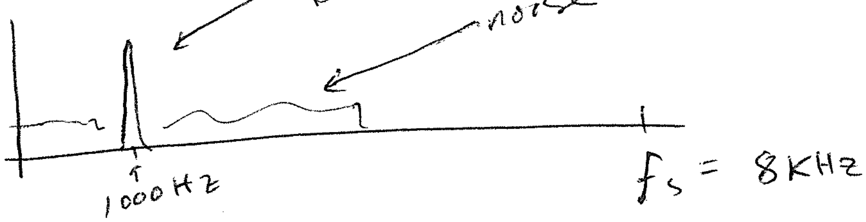
$$= \frac{1}{3} \cdot \frac{\sin \theta/2}{\sqrt{\frac{8}{3}} \cos \theta/2 + 1/3}$$

High pass filter

10.4

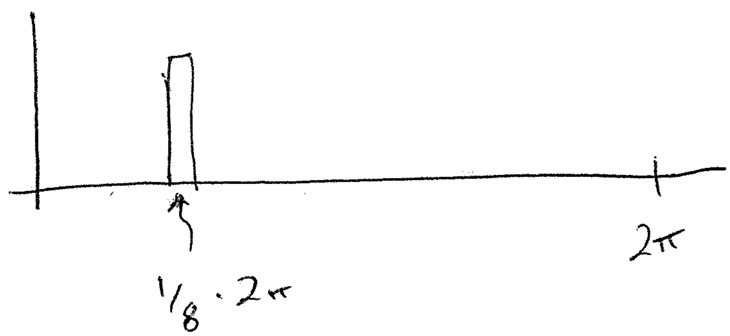
Data sampled at 8 KHz (above Nyquist)

Detect this signal



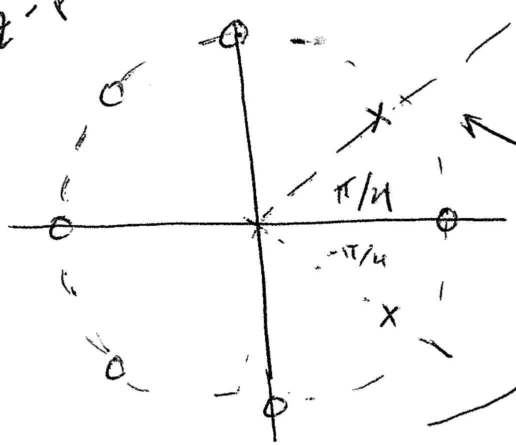
$H(e^{j\omega})$

Design something like



Example

Z-plane



place pole close to here

zeros elsewhere

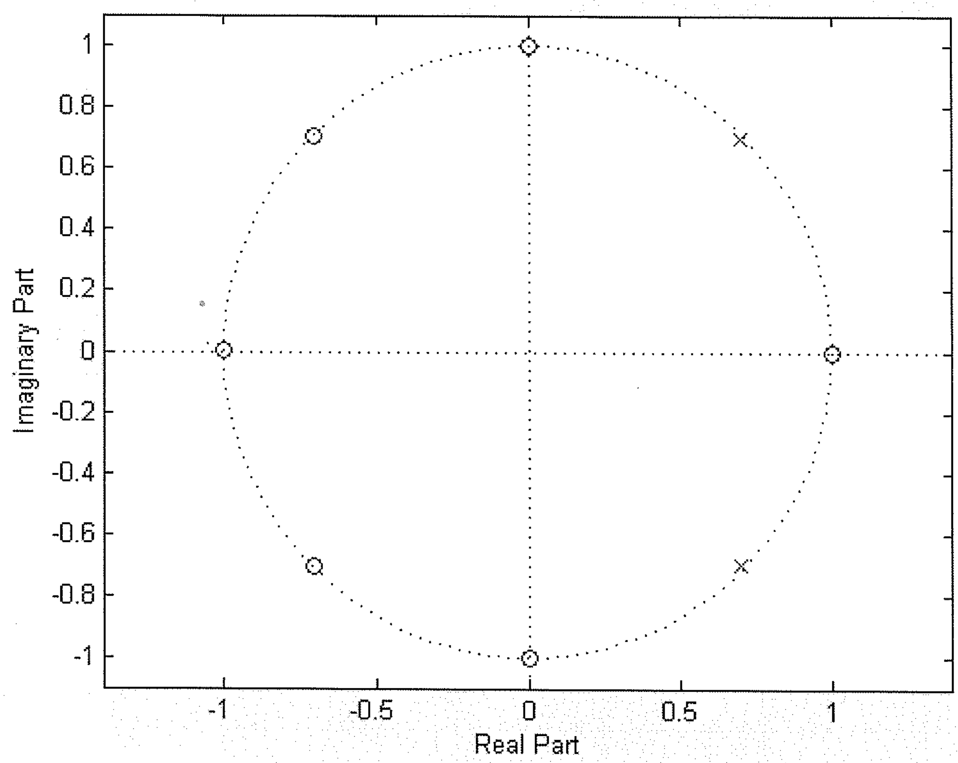
$$H(z) = \frac{\prod_{k=0,2,3,4,5,6} \left(1 - e^{jk\pi/4} z^{-1}\right)}{\left(1 - .99 e^{j\pi/4} z^{-1}\right)\left(1 - .99 e^{-j\pi/4} z^{-1}\right)}$$

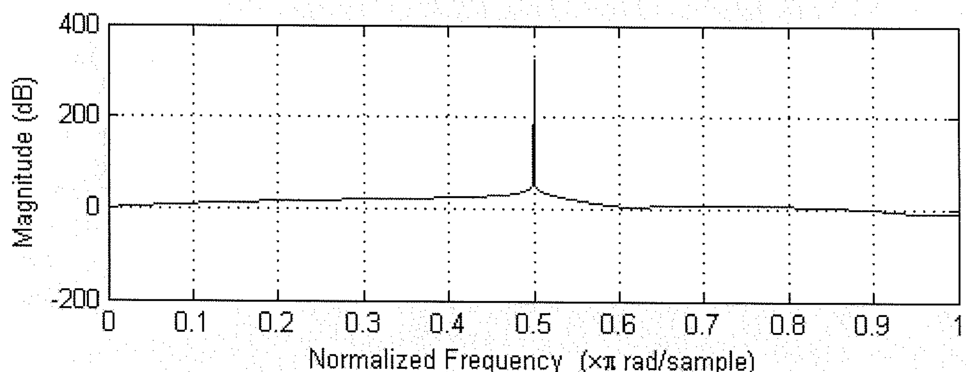
b, c/ See Matlab

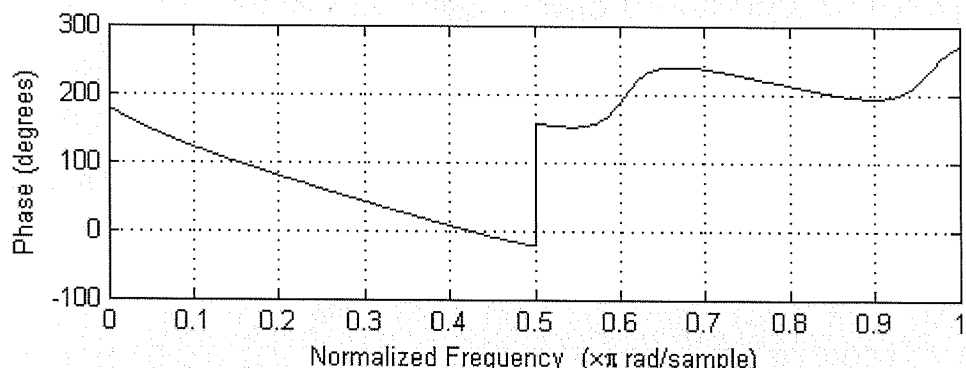
Homework 10 Problem 4

```
clear
close all
clc

a = [.99*exp(j*pi/4) .99*exp(-j*pi/4)].';

b = exp(j*pi/4.*[0 2 3 4 5 6]).';

a_num = poly(a)
b_num = poly(b)

zplane(b,a)
figure
freqz(b,a)
```

```
a_num =

    1.0000   -1.4001    0.9801


b_num =

  Columns 1 through 4

   1.0000                  1.4142 - 0.0000i   1.0000 - 0.0000i   0.0000 - 0.0000i

  Columns 5 through 7

  -1.0000 + 0.0000i  -1.4142 + 0.0000i  -1.0000 + 0.0000i
```







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