Homework 9

Solutions

## Contents



## Homework 9 Problem 1

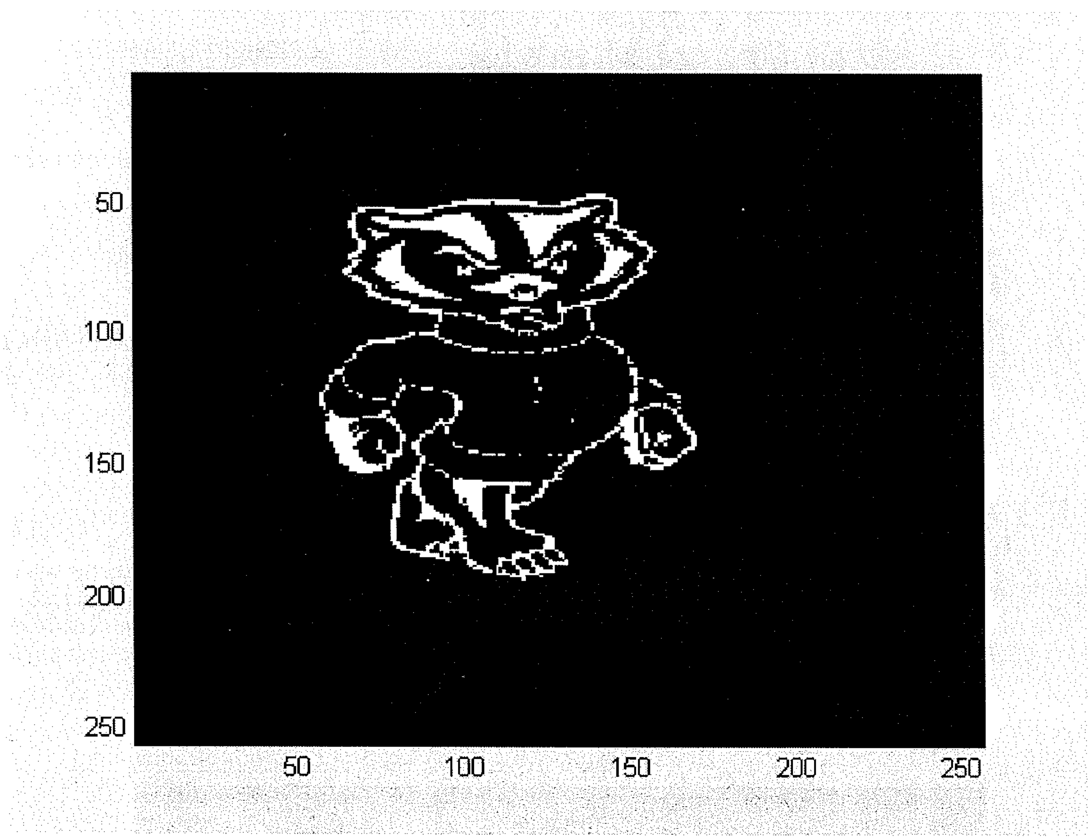
```
close all
clc
clear

load camerawm.mat
```

## Problem 1a)

```
%Convert double into a 8 bit integer
yint8 = uint8(y);

water_mark = mod(yint8,2);
colormap(gray)
imagesc(y)
imagesc(water_mark)
```

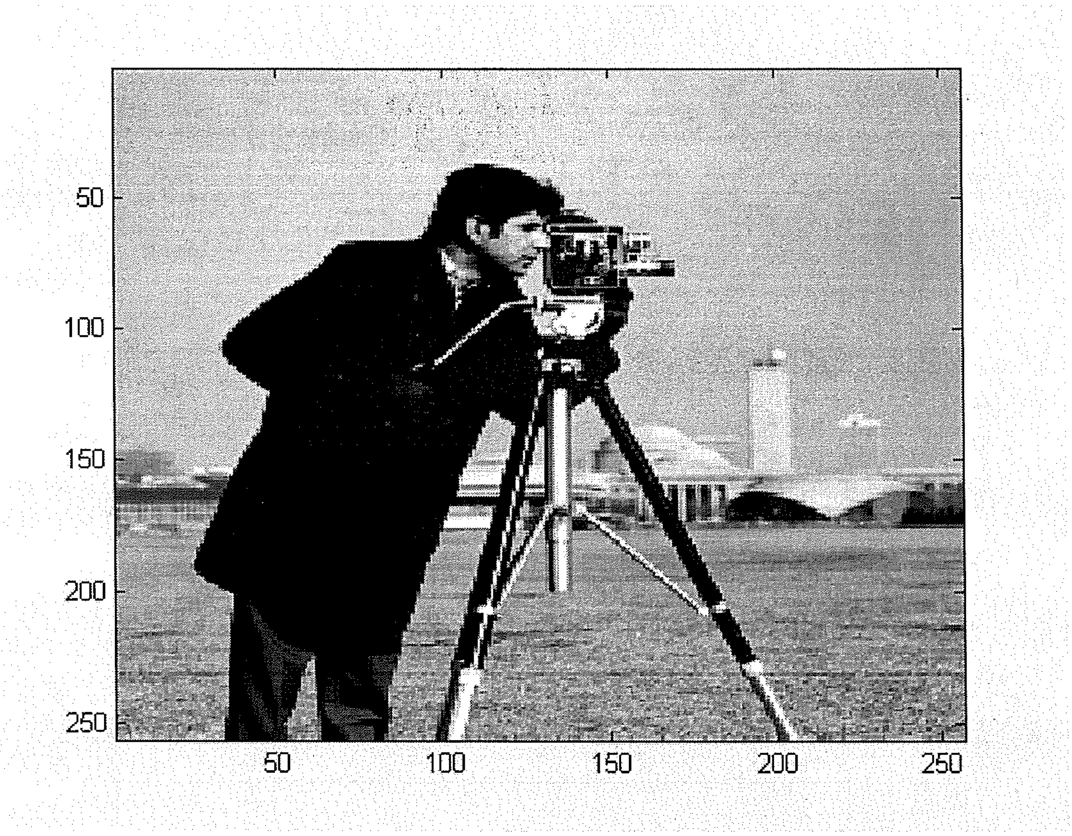


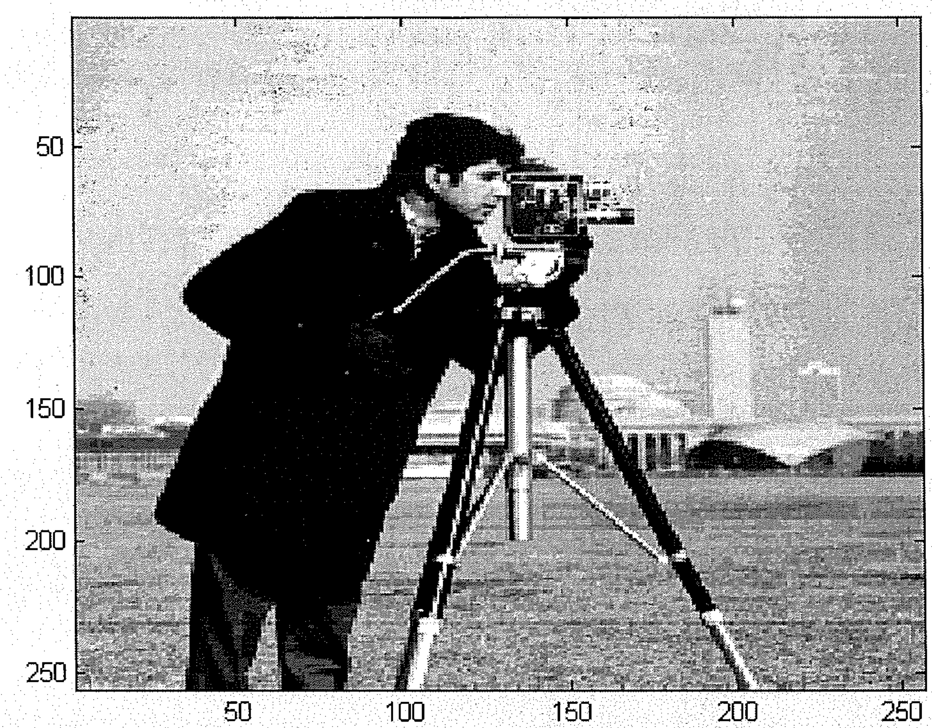

## Problem 1b)

```
%Remove bits (can be done many ways)
figure
colormap(gray)
imagesc(bitshift(yint8,-2))
```

```
figure
colormap(gray)
imagesc(bitshift(yint8,-4))

figure
colormap(gray)
imagesc(bitshift(yint8,-6))
```







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9.2

(3.30)

$$H(z) = \frac{1-z^{-1}}{1-0.25z^{-2}} = \frac{1-z^{-1}}{(1-0.5z^{-1})(1+0.5z^{-1})}$$

Causal

a) Determine the output when input is $x[n] = u[n]$

$$Y(z) = H(z)X(z)$$

$$X(z) = Z\{u[n]\}$$

$$Y(z) = \frac{\cancel{(1-z^{-1})}}{(1-0.5z^{-1})(1+0.5z^{-1})} \cdot \frac{1}{\cancel{(1-z^{-1})}} \qquad |z|>1$$

$$= \frac{A}{1-0.5z^{-1}} + \frac{B}{(1+0.5z^{-1})}$$

$$\Rightarrow \quad A + 0.5Az^{-1} + B - 0.5Bz^{-1} = 1$$

$$A + B = 1$$

$$0.5A - 0.5B = 0$$

$$A = B = 0.5$$

$$Y(z) = \frac{0.5}{1-0.5z^{-1}} + \frac{0.5}{1+0.5z^{-1}}$$

$$\boxed{Y[n] = (1/2)^{n+1}u[n] + 1/2 \cdot (-1/2)^n u[n]}$$

3.30 b

$y[n] = \delta[n] - \delta[n-1]$

$Y(z) = 1 - z^{-1}$ All z

$Y(z) = H(z)X(z)$

$$X(z) = \frac{Y(z)}{H(z)} = (1-0.5z^{-1})(1+0.5z^{-1})$$

$$= 1 - 0.25z^{-2}$$

$$\boxed{x[n] = \delta[n] - 0.25\,\delta[n-2]}$$

3.30 c

$Y[n]$ when input is $x[n] = \cos(0.5\pi n)$ $-\infty < n < \infty$

$x[n] = \ldots 1 \; 0 \; -1 \; 0 \; 1 \ldots$

use DTFT → since system is causal, from z-transform table, ROC includes unit circle.

$$H(e^{j\omega}) = H(z)\big|_{z=e^{j\omega}}$$

3.30 e — continued

$$H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{1 - 0.25\, e^{-2j\omega}}$$

$$H(e^{j\omega})\, X(e^{j\omega}) =$$

$$\frac{1-e^{-j\omega}}{1-0.25e^{-2j\omega}} \cdot \pi\, \delta(\omega - \pi/2) + \frac{1-e^{-j\omega}}{1-0.25e^{-2j\omega}}\, \pi\, \delta(\omega + \pi/2)$$

$$= \pi \frac{1 - e^{-j\pi/2}}{1 - 0.25e^{-j\pi}}\, \delta(\omega - \pi/2) + \pi \cdot \frac{1 - e^{+j\pi/2}}{1 - 0.25e^{j\pi}}\, \delta(\omega + \pi/2)$$

$$= \frac{4}{5}\pi(1+j)\, \delta(\omega - \pi/2) + \frac{4}{5}\pi(1-j)\, \delta(\omega + \pi/2)$$

$$\longrightarrow \boxed{y[n] = \frac{4}{5}\cos(\pi/2\, n) - \frac{4}{5}\sin(\tfrac{\pi}{2} n)}$$

3.31

a) Long division:

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}, \quad x[n] \text{ right sided}$$

$$\begin{array}{r|l} & -1 \\ \hline -\frac{1}{3}z^{-1} + 1 & \frac{1}{3}z^{-1} + 1 \\ & -\left(\frac{1}{3}z^{-1} - 1\right) \\ \hline & 2 \end{array}$$

$$X(z) = -1 + \frac{2}{1 + \frac{1}{3}z^{-1}}$$

Check:

$$\frac{2}{1 + \frac{1}{3}z^{-1}} - \frac{1 + \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}} = \frac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}$$

$$x[n] = -\delta[n] + 2\left(-\tfrac{1}{3}\right)^n u[n]$$

3.31 b

Partial Fraction:

$$X(z) = \frac{3}{z - 1/4 - 1/8 z^{-1}} \qquad x[n] \text{ stable}$$

$$= \frac{3z^{-1}}{1 - 1/4 z^{-1} - 1/8 z^{-2}}$$

$$= \frac{3z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + 1/4 z^{-1}\right)}$$

$$= \frac{A}{(1 - 1/2 z^{-1})} + \frac{B}{(1 + 1/4 z^{-1})}$$

$$\Rightarrow \quad A(1 + 1/4 z^{-1}) + B(1 - 1/2 z^{-1}) = 3z^{-1}$$

$$A + B + \frac{1}{4}Az^{-1} - \frac{1}{2}Bz^{-1} = 3z^{-1}$$

$$A + B = 0$$

$$1/4 A - 1/2 B = 3$$

$$A = 4$$

$$B = -4$$

$$\Rightarrow X(z) = \frac{4}{(1 - 1/2\, z^{-1})} - \frac{4}{(1 + 1/4\, z^{-1})}$$

$$x[n] = 4 \cdot \frac{1}{2}^n u[n] - 4\,(-1/4)^n\, u[n]$$

---

Since $x[n]$ is stable, must be right-sided

3.31 c

$$X(z) = \ln(1-4z) \qquad |z| < 1/4$$

Power Series

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} \qquad \text{for } |x|<1$$

So

$$X(z) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-4z)^n}{n} \qquad |z|< 1/4$$

$$\Rightarrow |-4z| < 1$$

$$\checkmark \; |z| < 1/4$$

Let $k = -n$

$$\sum_{k=-\infty}^{-1} (-1)^{-k+1} \frac{(-4)^{-k} z^{-k}}{-k}$$

$$\rightarrow \qquad x[n] = \frac{(-1)^{-n+1} (-4)^{-n}}{-n} \cdot u[-n-1]$$

3.31 d

$$X(z) = \frac{1}{1 - \frac{1}{3} z^{-3}} \qquad |z| > (3)^{-1/3}$$

Geometric Series

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \qquad |a| < 1$$

$$\frac{1}{1 - \frac{1}{3} z^{-3}} = \sum_{k=0}^{\infty} \left(\frac{1}{3} z^{-3}\right)^k \qquad \text{for} \qquad \left|\frac{1}{3} z^{-3}\right| < 1$$

$$|z^{-3}| < 3$$

$$z^3 > 3$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k z^{-3k} \qquad |z| > 3^{1/3} \checkmark$$

$$= \left(\frac{1}{3}\right)^0 z^{-3\cdot 0} + \left(\frac{1}{3}\right)^1 z^{-3} + \left(\frac{1}{3}\right)^2 z^{-6} + \dots$$

So, substitute $n = 3k$ and change index

$$= \sum_{n=0,3,6\dots}^{\infty} \left(\frac{1}{3}\right)^{n/3} z^{-n}$$

by inspection

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^{n/3} & n = 0, 3, 6, 9 \dots \\ 0 & \text{else} \end{cases}$$

9.4/
(5.29)

a/ $h_i[n]$ ?

$h[n] = \delta[n] + 2\delta[n-1]$

$H(z) = 1 + 2z^{-1}$ all $z$

$H_i(z) = \dfrac{1}{1 + 2z^{-1}}$

ROC must include unit circle

ROC $|z| < 2$

$$\boxed{h_i[n] = -(-2)^n u[-n-1]}$$

non - causal

b) $h[n] = \delta[n] + \alpha\,\delta[n-1]$

$H(z) = 1 + \alpha z^{-1}$

$H_i(z) = \dfrac{1}{1 + \alpha z^{-1}}$ for stability, ROC must include unit circle

if $\alpha > 1$ system is non - causal

need $\boxed{\alpha < 1 \text{ for inverse system to be causal.}}$