ECE431 Homework 6
DFT and fMRI

Due 3pm Friday, October 12 to 431 lock box in WisCEL.

6.1. Consider the signals $x_1[n]$ and $x_2[n]$ defined as

$$x_1[n] = \begin{cases} 1, & 0 \leq n \leq 99 \\ 0, & \text{otherwise} \end{cases}$$

$$x_2[n] = \begin{cases} 1, & 0 \leq n \leq 9 \\ 0, & \text{otherwise} \end{cases}$$

Note that the N point FFT of a vector x is computed in MATLAB with fft(x,N). If you omit N, MATLAB defaults to computing an FFT of length equal to the length of x.

(a) Use conv in MATLAB to compute the linear convolution of $x_1[n]$ and $x_2[n]$. Plot your result.

(b) Use fft and ifft to compute the 100 point circular convolution of $x_1[n]$ and $x_2[n]$. Plot your result.

(c) Use fft and ifft to compute the 110 point circular convolution of $x_1[n]$ and $x_2[n]$. Plot your result.

(d) Explain the differences observed in (a)-(c).

6.2. In this problem we explore functional magnetic resonance imaging (fMRI) of a subject performing repetitive finger tapping. The site http://en.wikipedia.org/wiki/Fmri gives an overview of fMRI. The MATLAB file fmrisig.mat contains the fMRI signal (as a function of time) recorded from a region in the motor cortex. The signal reflects the variation in the blood oxygenation level in the region as a function of time. The dominant sinusoidal component (excluding DC) is correlated with the repetitive finger tapping. We will identify the dominant component of this template waveform and use this knowledge to obtain an image of activation as a function of location in a slice of the brain.

(a) Plot the magnitude of the DFT for the signal in fmrisig.mat.

(b) Identify the dominant component (excluding DC).

(c) If the sampling rate is 4 samples/sec, what is the frequency (in Hz) of the dominant sinusoidal component?

(d) Using the magnitude and phase of the DFT coefficients, approximate the signal using a DC term and the dominant sinusoidal component. Plot and compare your fit with the raw data.

6.3. We now use the results of the previous problem to produce an image of neural activity in the brain. The fMRI data consists of a series of MRI brain images collected as a function of time. Oxygenated and deoxygenated hemoglobin have different magnetic characteristics, so the variations in the MRI intensity are related to the blood flow, and thus to the neural activity that is nourished by the blood flow. The data are noisy and the variations in intensity due to neural activation are very subtle. Normally statistical signal detection methods are used to derive an activation map - a binary image of active and non active pixels in the brain.

This problem develops a DFT-based procedure for producing an activation map. A data file fmri.mat contains 122 2-dimensional MRI images of 64 by 64 pixels collected at 4 images/sec. That is, each pixel contains a 122 sample time signal obtained at a sampling frequency of 4 Hz. The basic idea is to perform a DFT of the time signal associated with each pixel and check if it has a "peak" at the frequency corresponding to the finger tapping activation signal derived in the previous problem. You may determine whether there is a peak at the frequency of interest by comparing the magnitude squared at that frequency to the average magnitude squared at the other frequencies, since these other frequencies tell us the size of the noise. That is, if the pixel is inactive, the magnitude squared of the DFT coefficient corresponds to "noise", while if it is active, then the magnitude squared of the DFT coefficient is noise **plus** a signal term. If the ratio is much greater than one, then you decide that pixel is "active", while if it is close to one, then the pixel is "inactive". This approach assumes the noise has equal power at all frequencies - a reasonable assumption if we ignore DFT coefficients corresponding to DC and π radians.

The M-file fmri.m is a simple program to help get you started manipulating and displaying the data. Due to the way the data was acquired, some of the pixels outside the head have all zeros for their time series, e.g., the (1,1) pixel. Also, since each pixel involves a length 122 time series, there are 122 DFT coefficients. We are discarding the one corresponding to DC, are only using the coefficients corresponding to positive frequencies (due to conjugate symmetry), and will discard the coefficient corresponding to frequency π radians. Hence the available coefficients have MATLAB indices ranging from 2 through 61. One of these is associated with the frequency of interest, so we have $M = 59$ DFT coefficients available for estimating the noise power.

(a) Determine an appropriate threshold for deciding whether a pixel is active. Discuss how and why you selected this particular threshold.

(b) Produce a color map of activated pixel locations superimposed on one of the MRI images in fmri.mat. You can do this by applying the image command to the sum of the activation map and the MRI image - provided you scale the amplitudes of the MRI image so they occupy the lower portion of the available color range and scale the activation map so it occupies the upper part of the color range. Try small, medium, and large thresholds relative to the threshold identified in (a). Can you identify the part of the brain active in finger tapping (motor cortex)?

(c) Why is it important to normalize by the noise? That is, why not look solely at the magnitude squared at frequency of interest? Hint: display the map of estimated noise level as a function of location.