ECE431 Homework 5
The DFT

Due 3pm Friday, October 5. Submit to ECE431 mobile box in WisCEL.

5.1. Consider the finite-duration complex sinusoid

$$x[n] = \begin{cases} e^{j\omega_0 n} & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the DTFT, $X(e^{j\omega})$.
(b) Find the DFT, $X[k]$, using the defining equation. How is the DFT related to the DTFT evaluated in (a)?
In the next problem the DFT is evaluated numerically.

5.2. For $x[n]$ defined in the previous problem, use MATLAB to plot the DTFT magnitude, $|X(e^{j\omega})|$ and the magnitude of the DFT coefficients, $|X[k]|$ assuming $N = 25$. When you plot the DTFT, use a large number of samples in frequency - least 500 - and use the command `plot(x,y)` so that it appears as a continuous function of frequency. When you plot the DFT coefficients, use the `stem(x,y)` command so that they appear as discrete functions of frequency. Use `hold` to display both DTFT and DFT on the same graph. Evaluate the following cases.
(a) $\omega_0 = 2\pi/5$.
(b) $\omega_0 = 9\pi/40$.
(c) Comment on whether the DFT coefficients look like what you expect for a complex sinusoid.
(d) Find the expression for the DFT coefficients when $\omega_0 = k2\pi/N$ where k is integer.

5.3. MATLAB's `fft` command computes the N point DFT by sampling the DTFT at N evenly spaced points on the interval $0 \leq \omega < 2\pi$. Suppose we take N samples of a continuous-time signal $x(t)$ at a sampling interval of T so that $x[n] = x(nT)$. Let the DFT of $x[n], n = 1, 2, \ldots N$ be $X[k], k = 1, 2, \ldots N$. Note that the indexing of values here corresponds to MATLAB's convention of indexing the first element of the array with 1.
(a) How does `fft` sample the frequency Ω associated with the original continuous-time signal $x(t)$? That is, give a formula that relates k to the samples of Ω. You may assume that T is chosen so that no aliasing takes place.
(b) Suppose $x(t)$ is bandlimited to 20kHz, we collect $N = 2500$ samples at intervals $T = 1/5 * 10^{-4}$ s, and we perform an N point DFT. What continuous time frequencies Ω correspond to the DFT coefficients indices k?

5.4. Download the rap song segment from the web site, rap.mat. This segment is 1048576 samples in length and was sampled at 44.1kHz. The command sound(y,Fs) will play the song where Fs is the sampling rate.

(a) Use the sampling rate and number of samples to calculate the time duration of the song in seconds.

(b) Use the MATLAB command fft to compute the DFT of the signal and plot its spectrum in dB, that is, plot 20 times the logarithm base 10 of the magnitude of the DFT. Note that the command for the log base 10 of a signal in MATLAB is log10. Label your frequency axis in units of Hz. Is there a distinct drop in energy above a certain frequency? What is the effective bandwidth of the signal? Could a lower sampling rate have been used?

(c) Boost the bass by amplifying the low frequency components of the signal (below 500 Hz) by a factor of 3 (take the DFT, boost, then take inverse DFT). State which components of the DFT should be amplified to accomplish this. Listen to the "bass boosted" signal and plot its spectrum for comparison to the original spectrum plotted in part (b).

(d) If we construct a DSP bass boost system that processes length 1048576 "block" of full-length songs one at a time using the approach in (c), what would be the time delay introduced by our approach? I.e., how long do you have to wait to hear the song processed by your bass boost system after you give the command to play a song? You may assume that you have a fast processor so the time to compute the DFT and inverse DFT is negligible.