Homework 5 solutions

5.1

Consider

$$x[n] = \begin{cases} e^{j\omega_0 n} & 0 \le n \le N-1 \\ 0 & \text{else} \end{cases}$$

a/

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{n=0}^{N-1} e^{j\omega_0 n} e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} e^{jn(\omega_0-\omega)}$$

$$= \sum_{n=0}^{N-1} \left[e^{j(\omega_0-\omega)}\right]^n$$

Geometric series

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$X(e^{j\omega}) = \begin{cases} \dfrac{1-e^{j(\omega_0-\omega)N}}{1-e^{j(\omega_0-\omega)}} & \omega \ne \omega_0 \\ N & \omega = \omega_0 \end{cases}$$

b/

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn}$$

$$= \sum_{n=0}^{N-1} e^{j\omega_0 n} e^{-j(2\pi/N)kn}$$

$$= \sum_{n=0}^{N-1} e^{jn\left(\omega_0 - \frac{2\pi k}{N}\right)}$$

$$= \sum_{n=0}^{N-1} \left[e^{j(\omega_0 - 2\pi k/N)}\right]^n$$

$$X[k] = \begin{cases} \dfrac{1 - e^{jN(\omega_0 - 2\pi k/N)}}{1 - e^{j(\omega_0 - 2\pi k/N)}} & k \neq \dfrac{\omega_0 N}{2\pi} \\ 0 & k = \omega_0 N/2\pi \end{cases}$$

$X[k]$ just $X(e^{j\omega})$ evaluated at $\omega = 2\pi k/N$

$$X[k] = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}}$$

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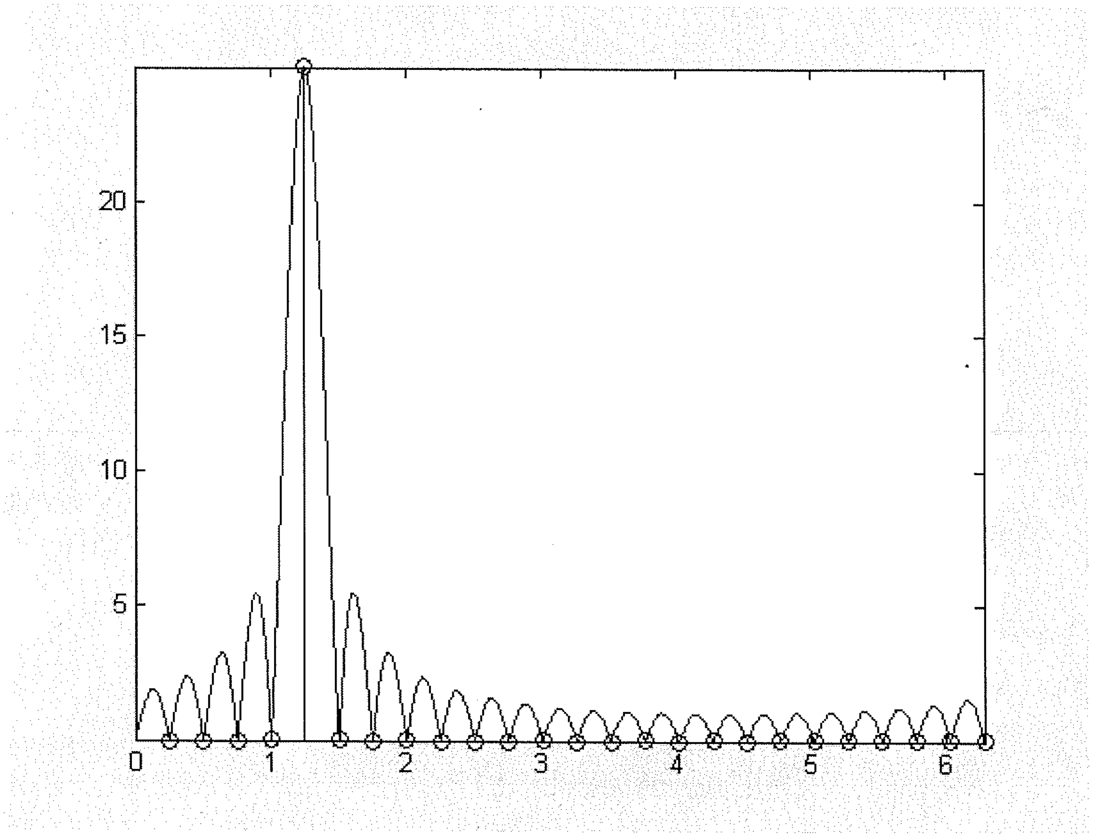
Homework 5 Solution

```
clc
clear
close all
```

Problem 5.2a)

```
w = [0:.001:2*pi]; %we could let w run from -pi to pi, but this makes plotting easier
N =25

w0 = 2*pi/5
X_DTFT = (1-exp(j*N*(w0 - w)))./(1-exp(j*(w0 - w)));

k = [1:1:N];
X_DFT = (1-exp(j*N*(w0 - 2*pi*k/N)))./(1-exp(j*(w0 - 2*pi*k/N)));

%Check to see if w0*N/(2*pi) is an integer - then X[w0*N/(2*pi)] should be
%set to N
if w0*N/(2*pi) == round(w0*N/(2*pi))
    X_DFT(w0*N/(2*pi)) = N;
end

plot(w,abs(X_DTFT))
axis tight
hold on
stem(k*2*pi/N,abs(X_DFT))
```

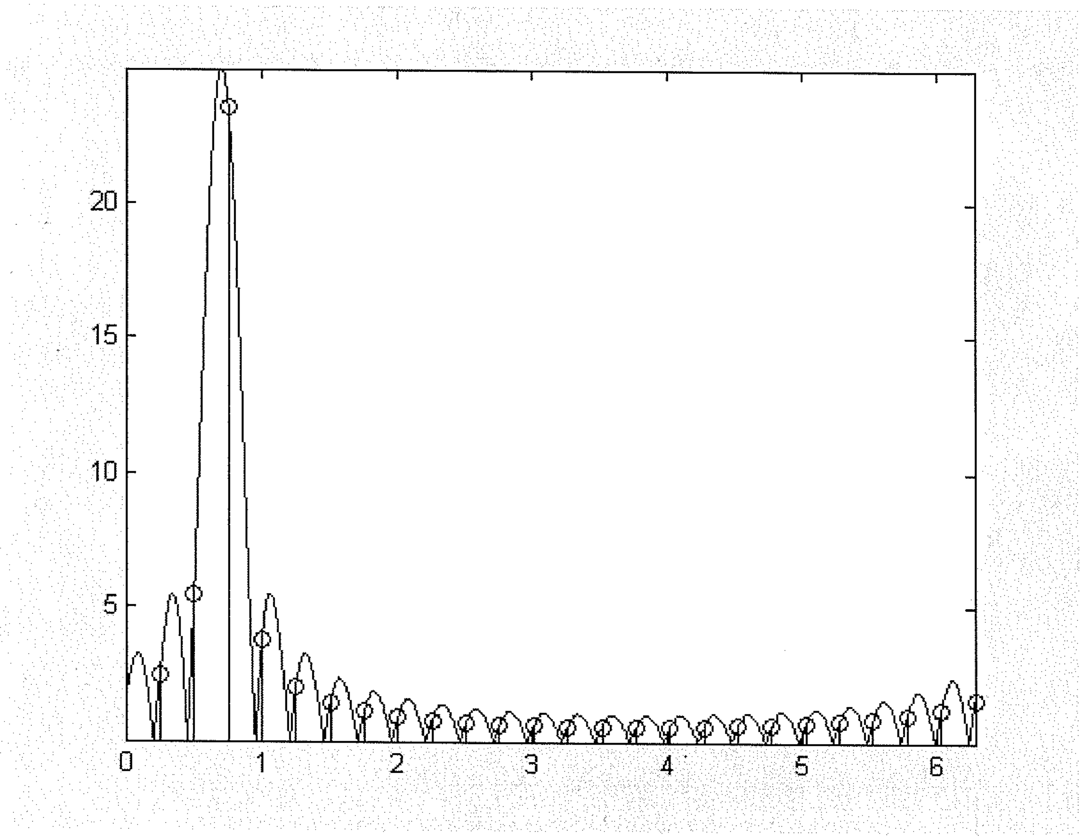
```
N =

    25


w0 =

    1.2566
```



Problem 5.2b)

```
clear

w = [0:.001:2*pi];
N =25

w0 = 9*pi/40
X_DTFT = (1-exp(j*N*(w0 - w)))./(1-exp(j*(w0 - w)));

k = [1:1:N];
X_DFT = (1-exp(j*N*(w0 - 2*pi*k/N)))./(1-exp(j*(w0 - 2*pi*k/N)));

%Check to see if w0*N/(2*pi) is an integer - then X[w0*N/(2*pi)] should be
%set to N
if w0*N/(2*pi) == round(w0*N/(2*pi))
    X_DFT(w0*N/(2*pi)) = N;
end

figure
plot(w,abs(X_DTFT))
hold on
axis tight
stem(k*2*pi/N,abs(X_DFT))
```

```
N =

    25


w0 =

    0.7069
```



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5.2c

We expect the DFT to look like a sampled sinc function → In (b), the stem plot look like a sinc. In (a), we see we are sampling the sinc function when its abs is zero or N. This may or may not be expected.

d/ When $w_0 = k 2\pi/N$ we have

$$X[k] = \frac{1 - e^{jN(2\pi k/N - 2\pi k/N)}}{1 - e^{j(2\pi k/N - 2\pi k/N}}$$

This can be evaluated as a limit where

$$\lim_{w_0 - 2\pi N/k \to 0} X[k],$$ or going back to

(5.1b) we have

$$X[k] = \sum_{n=0}^{N-1} \left[e^{j(w_0 - 2\pi k/N)}\right]^n$$

$$= \sum_{n=0}^{N-1} 1 = N$$

5.3

N samples of $x(t)$

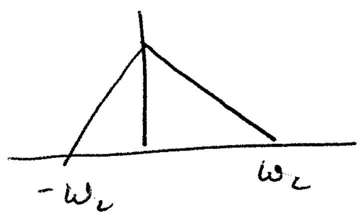
$x[n] = x(nT)$

and

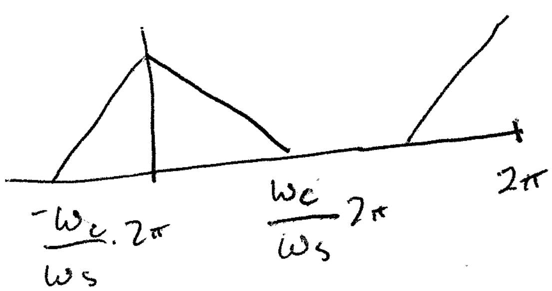
$X[k] \overset{DFT}{\longleftrightarrow} x[n]$

a/ Drawing pic to help

$X(j\Omega)$



$X(e^{j\omega})$



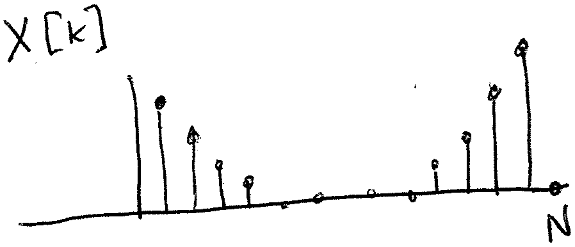
$X(e^{j\omega}) = X_s(j\frac{\omega}{T})$

$\omega = 2\pi$

Since $X[k] = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}}$

$\Rightarrow X[k] = X_s\left(j\frac{2\pi k}{NT}\right)$

$X[k]$



So

$$\boxed{X[k] = X(j\Omega)\Big|_{\Omega = \frac{\Omega_s}{N} \cdot k}}$$

$\Omega_s = \frac{2\pi}{T}$

for $k = 0 \ldots N-1$

b/

$x(t)$ bandlimited to 20 kHz

$N = 2500$ at $T = \frac{1}{5} \times 10^{-4}$ s

$$\Omega = \frac{2\pi k}{NT} = 40\pi \cdot k \quad \text{for } k = 0 \ldots 2499$$

5.4

a/ Duration = $T \cdot \text{length}(y)$ = 23.78 s

b/ From 3a, just scale axis as

$$\Omega = \frac{\Omega_s}{N} \cdot k = \frac{2\pi}{NT} \cdot k \rightarrow$$

look at plot.

Spectrum has nearly zero energy beyond

$\sim$ 15 KHz. So we could

Sample at 30 KHz.

c/

$$\Omega = 2\pi \cdot 500\,\text{Hz} = \frac{2\pi}{NT} \cdot k$$

$k \simeq 11889$

k that corresponds to 500 Hz.

d/

We would need to 'load' whole song before using ifft → this would introduce a delay of $\boxed{NT}$ = 23.78 s

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Home 5 Problem 4

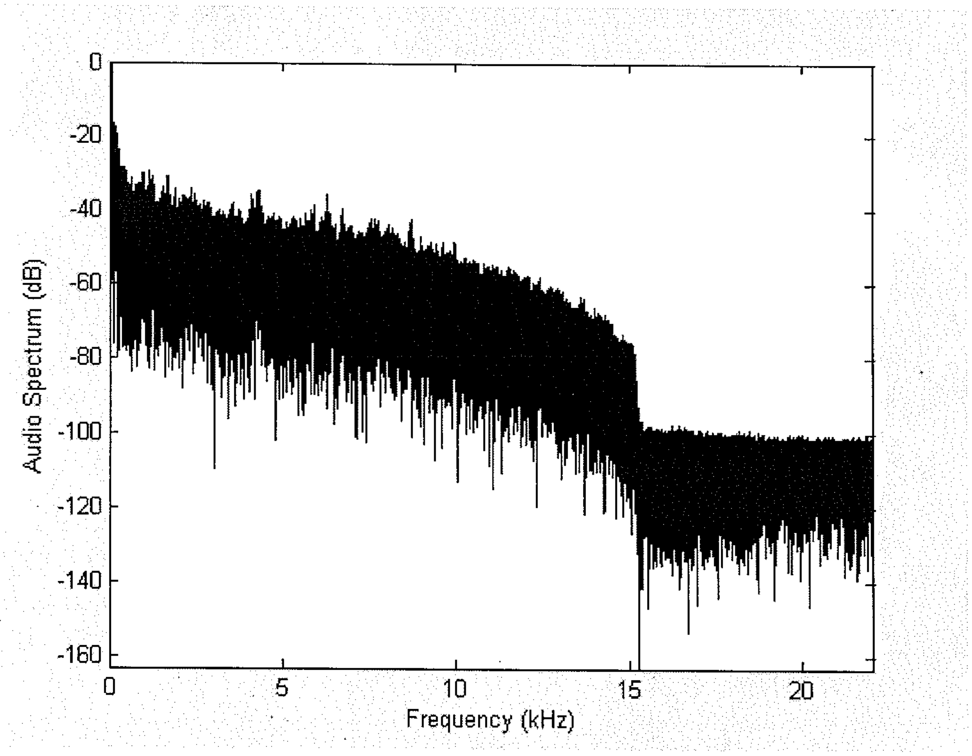
```
clear
close all
clc

load rap.mat
%sound(y,Fs)
```

Problem 4a)

```
N = length(y);
T = 1/Fs;
duration = T*N   %duraiton of audio in seconds


duration =

   23.7772
```

Problem 4b)

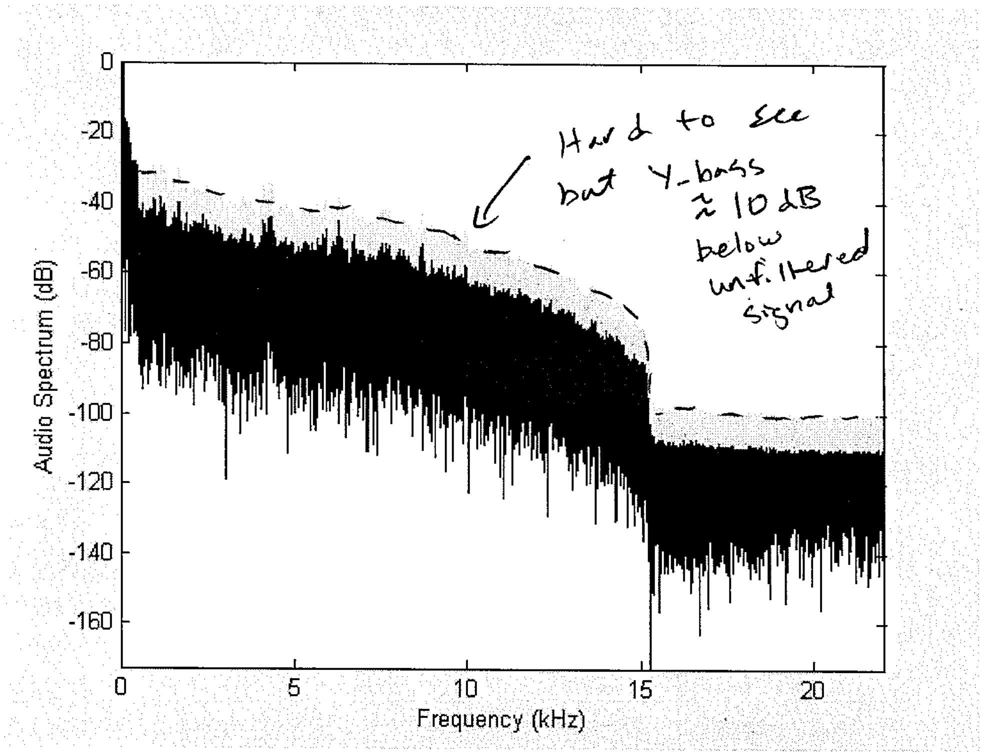
```
Y = fft(y);
k = [0:1:N-1];

%Scale axis using equation from 5.3 -
Omega = 2*pi*k/(N*T);
f = Omega/(2*pi);
Y_dB = 20*log10(abs(Y)/max(abs(Y)));

%We only need to plot first half of spectrum (Real signal, so even FFT)
plot(f(1:length(f)/2)/1e3,Y_dB(1:length(f)/2))
axis tight
ylabel('Audio Spectrum (dB)')
xlabel('Frequency (kHz)')
```



Problem 4b)

```
k_cutoff = round(500*N*T)
Y_bass = [Y(1:k_cutoff) Y(k_cutoff+1:length(Y)-k_cutoff-1)/3 Y(length(Y)-k_cutoff:length(Y))];
Y_bass_dB = 20*log10(abs(Y_bass)/max(abs(Y_bass)));
y_bass = ifft(Y_bass);
%sound(real(y_bass),Fs)

figure
plot(f(1:length(f)/2)/1e3,Y_dB(1:length(f)/2),'yellow')
hold on
plot(f(1:length(f)/2)/1e3,Y_bass_dB(1:length(f)/2),'black')
axis tight
ylabel('Audio Spectrum (dB)')
xlabel('Frequency (kHz)')
```

```
k_cutoff =

       11889
```



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