HW 3 solutions

3.1

Digital Differentiator

$$x(t) \longrightarrow \underbrace{\boxed{\text{Differentiator}}}_{H(\Omega)} \longrightarrow \frac{dx(t)}{dt}$$

a/ Find $H(\Omega)$ by comparing FT of $x(t)$ and $\frac{dx(t)}{dt}$.

→ First from def. of FT

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)\, j\Omega\, e^{j\Omega t}\, d\Omega$$

So

$$\frac{d\,x(t)}{dt} \overset{FT}{\longleftrightarrow} j\Omega X(j\Omega)$$

So, in frequency domain, our system is

$$X(\Omega) \longrightarrow \boxed{H(\Omega)} \longrightarrow j\Omega X(j\Omega)$$

so $\boxed{H(\Omega) = j\Omega}$

b/

approximate $\frac{dx}{dt}$ with filter $g[n] = \frac{\delta[n] - \delta[n-1]}{T}$

$x[n] = x_c(nT)$

$y[n] = g[n] * x[n]$

$$y[n] = \frac{\delta[n] - \delta[n-1]}{T} * x[n]$$

$$= \frac{x[n] - x[n-1]}{T}$$

$$G(e^{j\omega}) = \sum_{k=-\infty}^{\infty} g[k] e^{-j\omega k} = \boxed{\frac{1}{T} - \frac{1}{T} e^{-j\omega}}$$

see published results - on next page

c/ See plots next page.

d/ $f[n] = \frac{\delta[n+1] - \delta[n-1]}{2T}$

$$F(e^{j\omega}) = \frac{1}{2T} e^{j\omega} - \frac{1}{2T} e^{-j\omega} = \frac{j \sin \omega}{T}$$

$\longrightarrow$ The phase matches exactly.

e/ Not one answer — from graph can see what is reasonable

Contents

ECE 431 HW 3 Solution

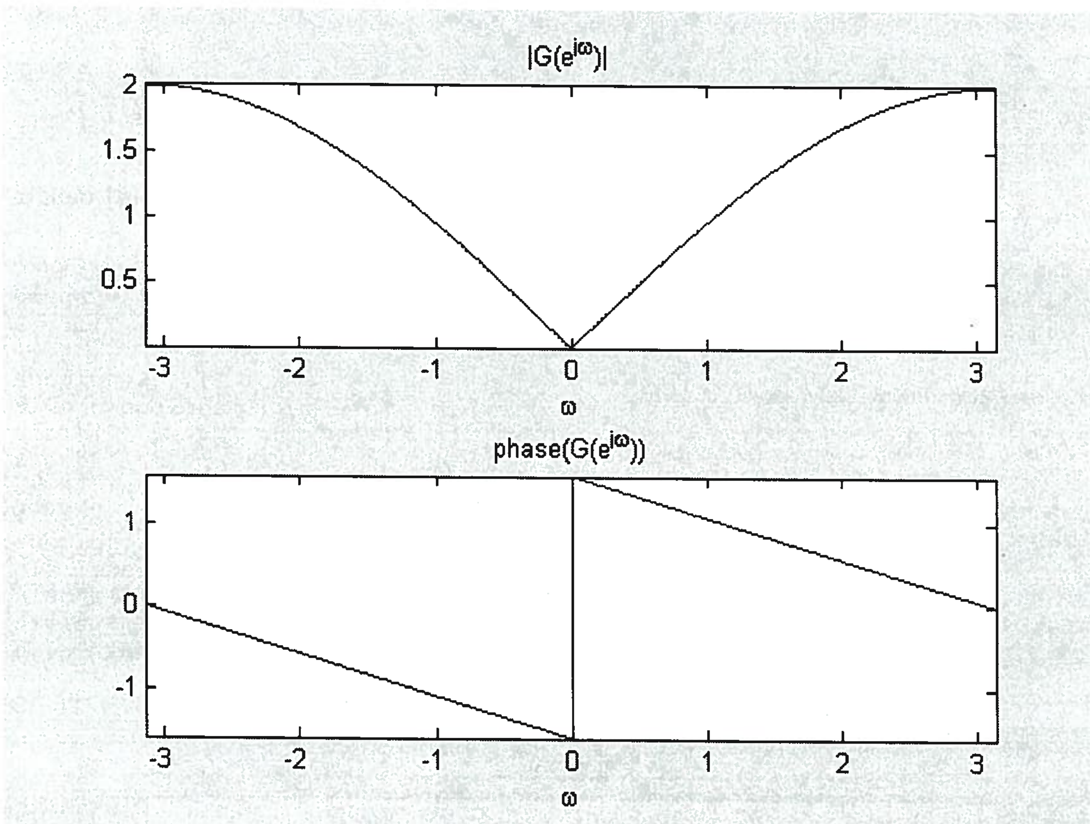
```
clear
close all
clc
```

Problem 1b)

```
T =1;
omega = [-pi:.01:pi];
G = 1/T - 1/T*exp(-j*omega);

subplot(2,1,1)
plot(omega,abs(G))
title('|G(e^{j\omega})|')
xlabel('\omega')
axis tight

subplot(2,1,2)
plot(omega,phase(G))
xlabel('\omega')
title('phase(G(e^{j\omega}))')
axis tight
```

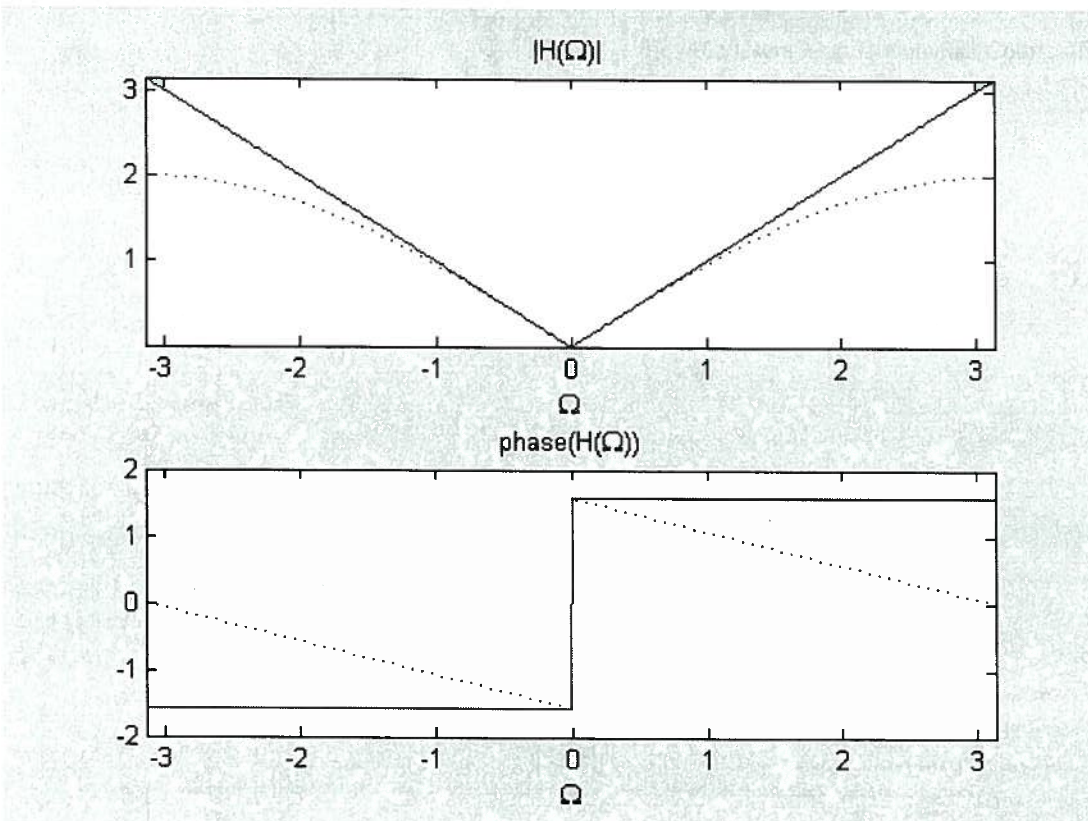


Problem 1c)

```
Omega = [-pi:.01:pi];
H = j*Omega;

figure
subplot(2,1,1)
plot(Omega,abs(H))
hold on
plot(omega,abs(G),':')
title('|H(\Omega)|')
xlabel('\Omega')
axis tight

subplot(2,1,2)
plot(omega,phase(H))
hold on
plot(omega,phase(G),':')
xlabel('\Omega')
title('phase(H(\Omega))')
axis([-pi pi -2 2])
```



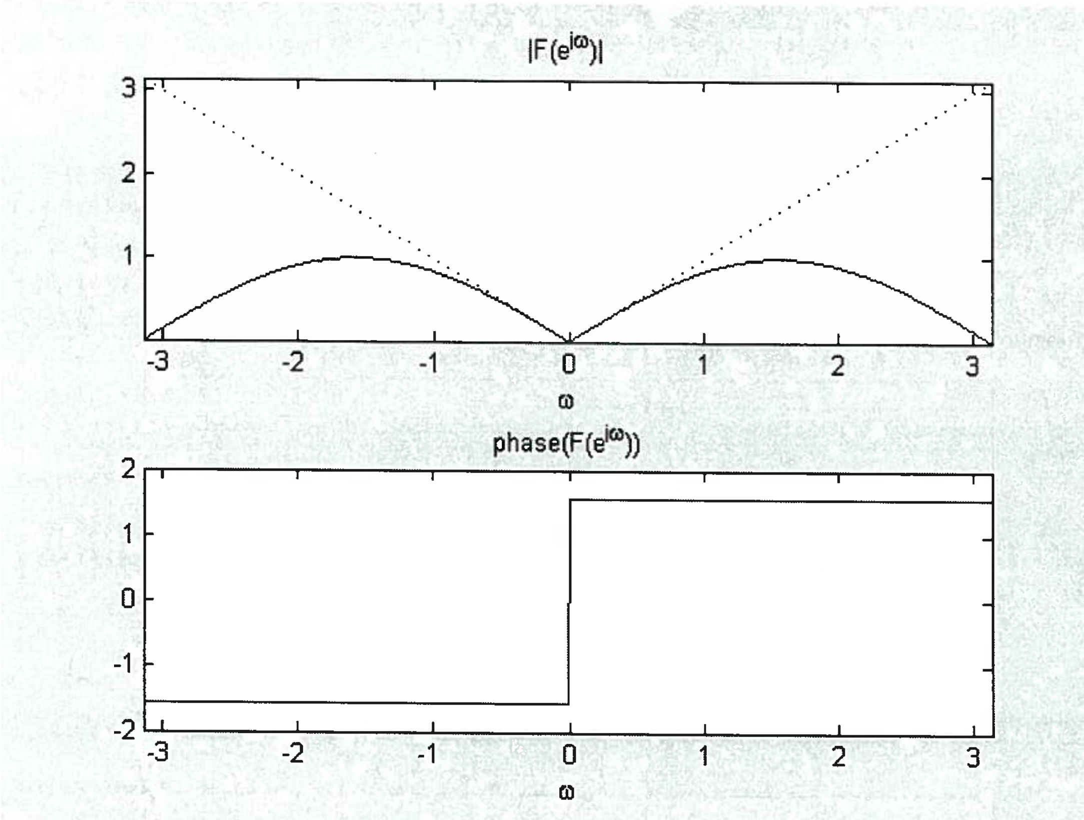
Problem 1d)

```
clear Omega
omega = [-pi:.01:pi];
F = j*sin(omega)/T;

figure
subplot(2,1,1)
plot(omega,abs(F))
hold on
plot(omega,abs(H),':')
title('|F(e^{j\omega})|')
xlabel('\omega')
```

```
axis tight

subplot(2,1,2)
plot(omega,phase(F))
hold on
plot(omega,phase(H),':')
xlabel('\omega')
title('phase(F(e^{j\omega}))')
axis([-pi pi -2 2])
```



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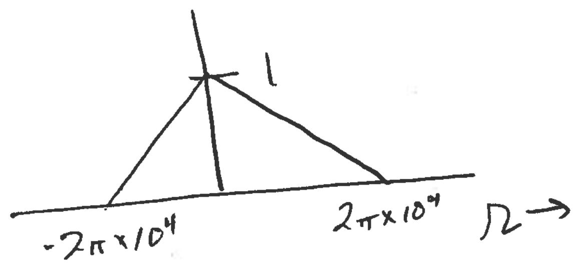
3.2

OSB 4.31

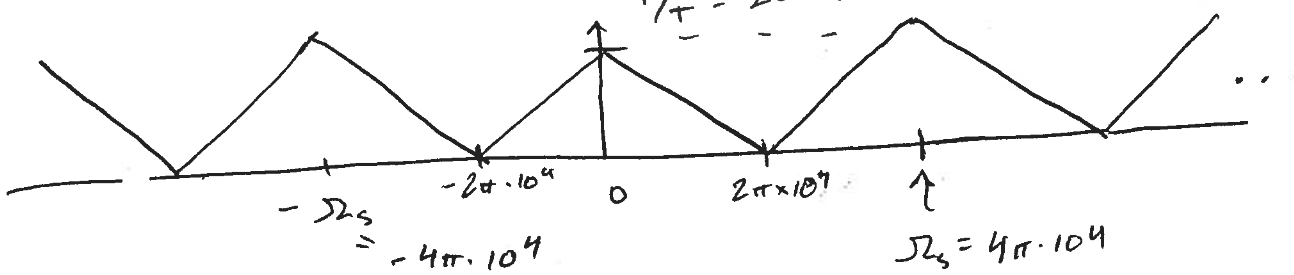
Sketch $X_s(j\Omega)$ and $X(e^{j\omega})$ for $\frac{1}{T} = 20\text{kHz}$

$X_c(j\Omega)$

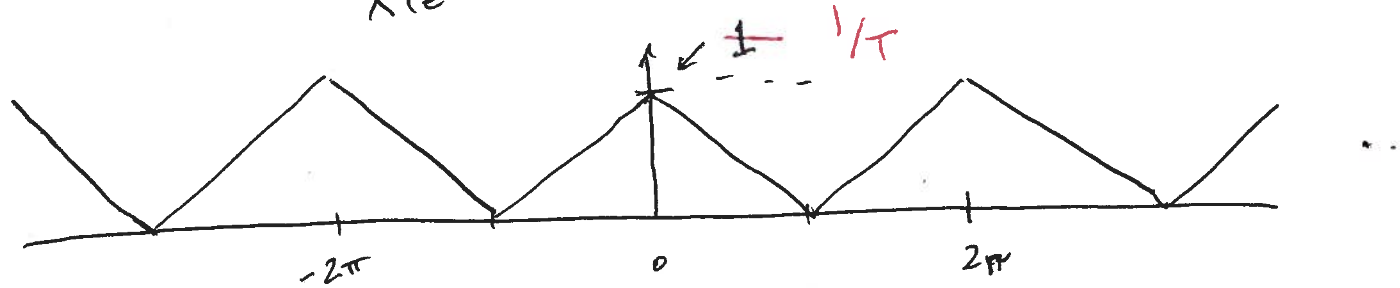


a/ So, $\frac{1}{T} = 20\text{ kHz} \Rightarrow \Omega_s = 2\cdot\pi\cdot 20\text{kHz} = 4\pi\cdot10^4$

$X_s(j\Omega)$

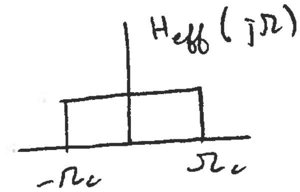


$X(e^{j\omega})$

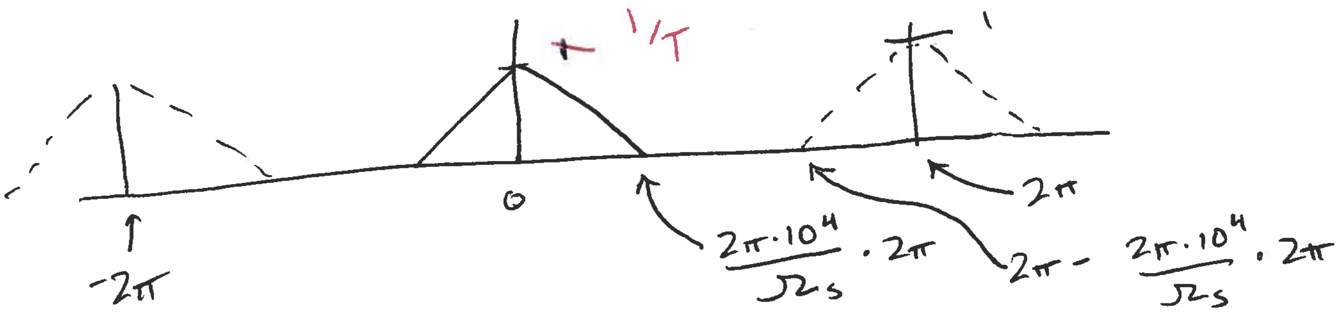


b/ Determine range of T for which systems are equivalent.

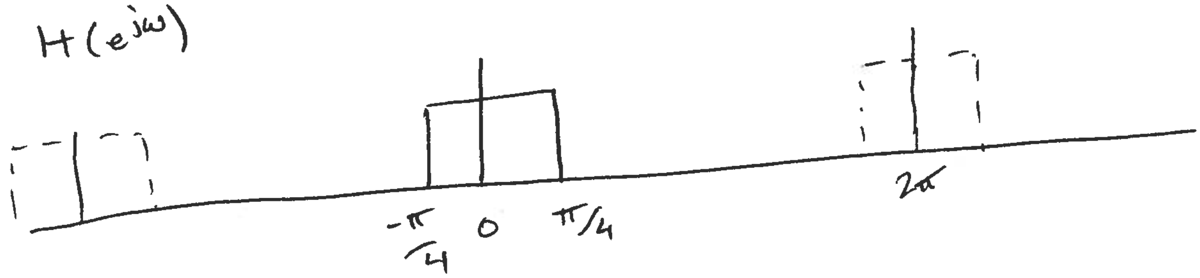
So, for an arbitrary T, we have



$X(e^{j\omega})$



$H(e^{j\omega})$



So, we can allow some aliasing, since part of the signal is lost after digital filter.

From 1st graph, as long as

$$2\pi - \frac{2\pi \cdot 10^4}{\Omega_s} \cdot 2\pi > \pi/4$$

$$\Rightarrow \Omega_s \geq \frac{16\pi \cdot 10^4}{7}$$

$$\boxed{T \leq \frac{7}{8} \cdot 10^{-4} \text{ seconds}}$$

Also, since problem asks for a range, we assume some signal is filtered off.

i.e.

$$\frac{2\pi \cdot 10^4}{\Omega_s} \cdot 2\pi \geq \pi/4$$

$$\Omega_s \leq 16\pi \cdot 10^4$$

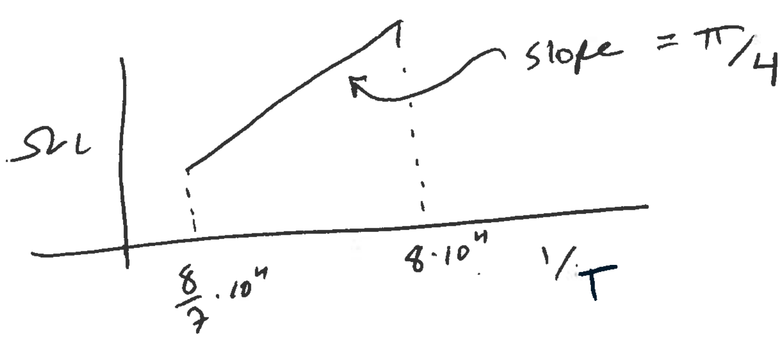
$$\boxed{T \geq \frac{1}{8} \cdot 10^{-4}}$$ seconds

Combining

$$\boxed{\frac{1}{8} \cdot 10^{-4} \leq T \leq \frac{7}{8} \cdot 10^{-4} \text{ sec}}$$

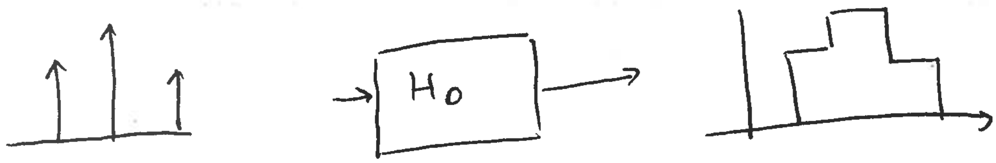
i/

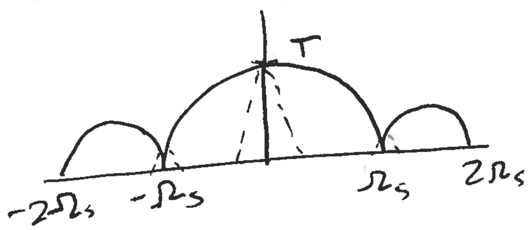
$$\Omega_c = \frac{1}{8}\Omega_s \Rightarrow \Omega_c = \frac{1}{4}\frac{\pi}{T}$$



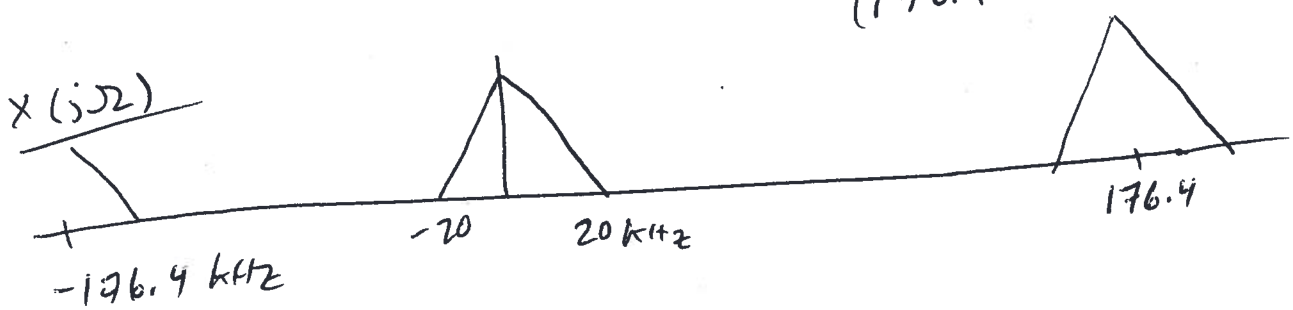
3.3

Zero order hold - 4x over sampling





So, 4x over sampling $\Rightarrow$ 44.1 kHz $\cdot$ 4 (176.4 KHz)



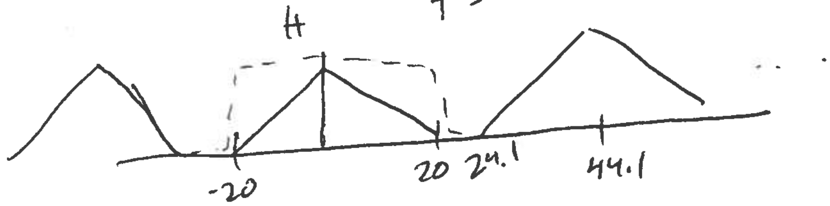
Design Constraint - 4x

$$\boxed{\begin{array}{ll} 0.99 < |H_0(f)| \cdot |H_c(f)| < 1.01 & -20\text{ kHz} \le f \le 20\text{ kHz} \\ |H_0(f)|\,|H_c(f)| \le 10^{-3} & 176.4 \cdot m\text{ kHz} - 20\text{K} \le f \le 176.4 \cdot m\text{ kHz} + 20\text{K} \\ & m \ne 0,\ m \text{ integer.} \end{array}}$$

where $$|H_0(f)| = \left| \frac{2e^{-j2\pi f T/2} \sin(2\pi f T/2)}{2\pi f} \right|$$

3.3 a continued

Anti - imaging filter



8x compared to

1x

$.99 < |H| < 1.01$ $\qquad -20\text{ kHz} \le f \le 20\text{ kHz}$

$|H_a| < 10^{-4}$ $\qquad 44.1\text{ kHz}\cdot m - 20\text{ kHz} \le f \le 44.1\text{ kHz}\cdot m + 20\text{ kHz}$

$m \ne 0$, m integer

8x

$f = 352.8$

$.99 < |H_a| < 1.01$ $\qquad -20\text{ kHz} \le f \le 20\text{ kHz}$

$|H_a| < 10^{-4}$ $\qquad 352.8\text{ kHz}\cdot m - 20\text{ kHz} \le f \le 352.8\text{ kHz}\cdot m + 20\text{ kHz}$

$m \ne 0$, m integer

Contents

ECE 431 HW 3 Solution

```
clear
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```
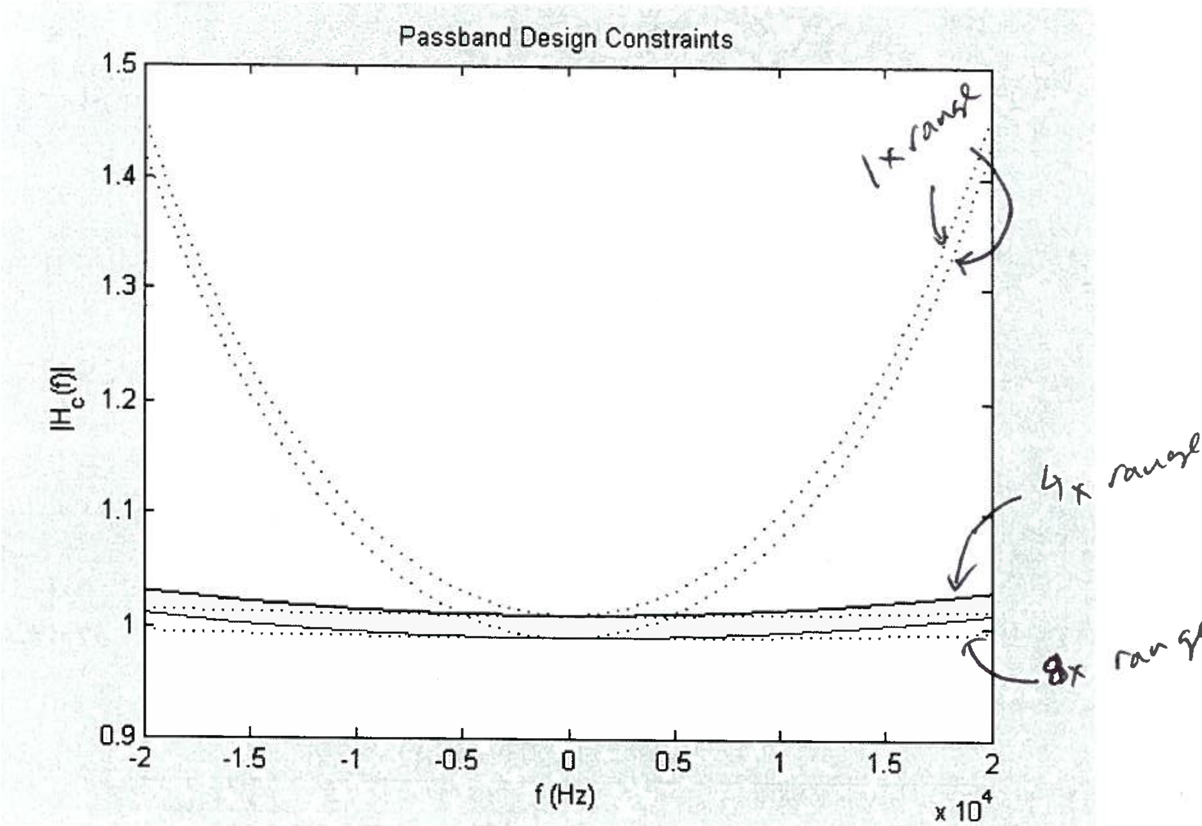

Problem 3a) - Passband constraint

```
%4x
T = 1/(4*44.1e3);
f1 = [-20e3:10:20e3];
H0 = 1/T*2*exp(-j*2*pi*f1*T/2).*sin(2*pi*f1*T/2)./(2*pi*f1);
mag_Hc_min = .99./abs(H0);
mag_Hc_max = 1.01./abs(H0);
plot(f1,mag_Hc_min)
hold on
plot(f1,mag_Hc_max)

%8x
clear T
T = 1/(8*44.1e3);
f1 = [-20e3:10:20e3];
H0 = 1/T*2*exp(-j*2*pi*f1*T/2).*sin(2*pi*f1*T/2)./(2*pi*f1);
mag_Hc_min = .99./abs(H0);
mag_Hc_max = 1.01./abs(H0);
plot(f1,mag_Hc_min,':')
hold on
plot(f1,mag_Hc_max,':')

%1x
clear T
T = 1/(1*44.1e3)
f1 = [-20e3:10:20e3];
H0 = 1/T*2*exp(-j*2*pi*f1*T/2).*sin(2*pi*f1*T/2)./(2*pi*f1);
mag_Hc_min = .99./abs(H0);
mag_Hc_max = 1.01./abs(H0);
plot(f1,mag_Hc_min,':')
hold on
plot(f1,mag_Hc_max,':')

xlabel('f (Hz)')
ylabel('|H_c(f)|')
title('Passband Design Constraints')
```

```
T =

  2.2676e-005
```



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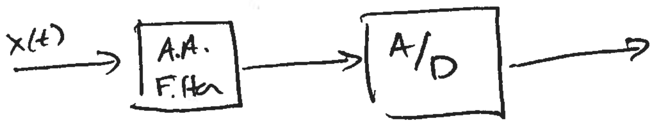
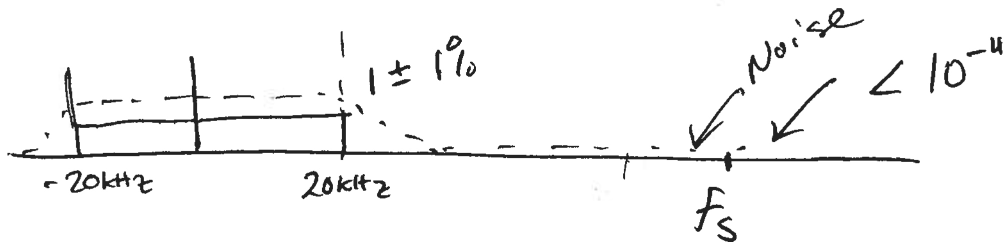
3.3 b



Want...



8x

$.99 < |H_{aa}| < 1$ $\quad -20k < f < 20kHz$

$|H_{aa}| < 10^{-4}$ $\quad f > f_s - 20kHz$

$f_s = 8 \cdot 44.1kHz$

1x

$.99 < |H_{aa}| < 1$ $\quad -20k < f < 20kHz$

$|H_{aa}| < 10^{-4}$ $\quad f > 24.1kHz$