ECE431 Homework 2
Sampling

Due 3pm Friday, Sept 14. Submit in WisCEL 410B to the ECE431 lock box.

2.1. Review.

a) Show that the DTFT of $x[n] * y[n]$ is given by $X(\omega)Y(\omega)$, where $X(\omega)$ and $Y(\omega)$ are the DTFTs of $x[n]$ and $y[n]$, respectively.

b) Prove that $w(t) * (x(t) + y(t)) = w(t) * x(t) + w(t) * y(t)$.

c) Two DT LTI systems with impulse responses $g[n]$ and $h[n]$ are connected in series. Show that the input-output behavior of the series connection is independent of the order of the systems. That is, show $g[n]$ followed by $h[n]$ has the same input-output characteristic as $h[n]$ followed by $g[n]$. Hint: use the result of part a).

2.2. Consider the continuous-time signal $x(t) = \frac{\sin(\pi t)}{\pi t} \sin(9\pi t)$. Sketch the FT $X_s(\Omega)$ of the sampled signal $x_s(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$ and the DTFT $X(e^{j\omega})$ of $x[n] = x(nT)$ for the following sampling intervals

a) $T = 1/14$

b) $T = 1/7$

c) $T = 1/5$

Identify whether aliasing occurs in each case.

2.3. EKG (electrocardiogram) signals are approximately bandlimited to ± 20 Hz. EKG signals are often measured in the presence of strong 60 Hz interference, so the recorded signal consists of the EKG plus a 60 Hz noise signal. Assume you are going to sample the noisy EKG and use a DT filter to remove the 60 Hz noise, before reconstructing the CT EKG.

a) What is the minimum sampling rate required to prevent aliasing in the recorded data?

b) What is the minimum sampling rate required to reconstruct the EKG signal without 60 Hz noise? If this is less than the rate you identified in part a), justify why you can tolerate aliasing.

2.4. Pulse Sampling. Consider a more realistic model for an A/D converter in which ideal *impulse sampling* is replaced by a train of *pulses* $p(t)$ of the form

$$p(t) = \begin{cases} 0, & t < 0 \\ 1/\Delta, & 0 \le t \le \Delta \\ 0, & t > \Delta \end{cases}$$

where $0 < \Delta < T$ and T is the sampling period. That is, the sampled waveform is $x_p(t) = x(t)s_p(t)$ where $s_p(t) = \sum_{n=-\infty}^{\infty} p(t - nT)$. Characterize the impact of pulse sampling in the frequency domain by expressing $X_p(\Omega)$ in terms of $X(\Omega)$. Is it possible to reconstruct $x(t)$ from $x_p(t)$ using an ideal lowpass filter? Explain your answer.