Linear Phase FIR Filters

FIR Filter

$$H(e^{j\omega}) = \sum_{k=0}^{M} b_k e^{-jk\omega} = \sum_{k=0}^{M} h[k] e^{-jk\omega}$$

Generalized linear phase

$$H(e^{j\omega}) = A(e^{j\omega}) e^{-j\alpha\omega + j\beta}$$

$\Rightarrow$ constant group delay

Four types of FIR filters w. generalized linear phase

M even or M odd, even or odd symmetry

Type I: M even, even symmetry

$$h[n] = h[M-n], \quad 0 \le n \le M$$

$$H(e^{j\omega}) = \sum_{k=0}^{M} h[k] e^{-jk\omega}$$

$$= e^{-jM/2\,\omega}\Big(h[0]e^{j\frac{M}{2}\omega} + h[M]e^{-j\frac{M}{2}\omega} + h[1]e^{j(\frac{M}{2}-1)\omega} + h[M-1]e^{-j(\frac{M}{2}-1)\omega}$$

$$+ \dots + h[M/2]e^{j0\omega}\Big)$$

$$= e^{-j\frac{M}{2}\omega}\Big(2h[0]\cos(\tfrac{M}{2}\omega) + 2h[1]\cos((\tfrac{M}{2}-1)\omega) + \dots + h[\tfrac{M}{2}]\Big)$$

$$= e^{-j\frac{M}{2}\omega}\sum_{k=0}^{M/2} a[k]\cos(\omega k) \; ; \qquad a[0] = h[M/2]$$

$$a[\ell] = 2h[\tfrac{M}{2}-\ell], \quad \ell = 1,\dots,\tfrac{M}{2}$$

$$H(e^{j\omega}) = e^{-j\frac{M}{2}\omega} \underbrace{\sum_{k=0}^{M/2} a[k]\cos(\omega k)}_{\text{real}}$$

Generalized Linear Phase

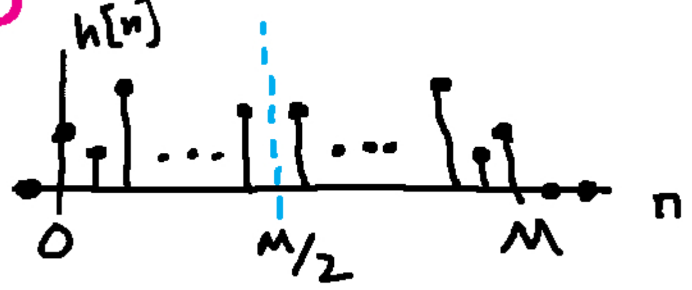
$$H(e^{j\omega}) = A(e^{j\omega})\, e^{-j\alpha\omega + j\beta}$$

$$\Rightarrow A(e^{j\omega}) = \sum_{k=0}^{M/2} a[k]\cos(\omega k)$$

$$\alpha = M/2$$

Group delay $M/2$

Type II : M odd, even symmetry



$$h[n] = h[M-n], \quad 0 \le n \le M$$

$$\Downarrow$$

$$H(e^{j\omega}) = e^{-j\omega(M/2)} \sum_{k=1}^{(M+1)/2} b[k] \cos[\omega(k - 1/2)]$$

$$b[k] = 2h[(M+1)/2 - k], \quad k = 1, 2, \ldots, (M+1)/2$$

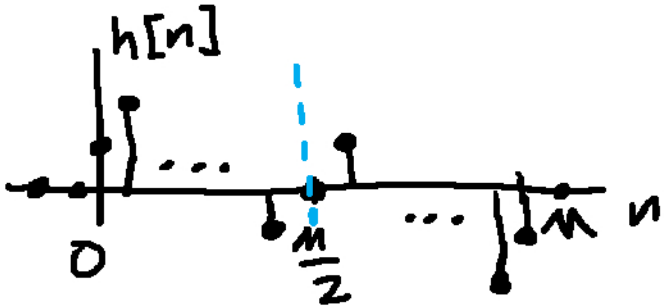
$$\cos[\pi(k - 1/2)] = 0 \quad \text{for all integer } k$$

$\Rightarrow H(e^{j\pi}) = 0$ (zero at $z = -1$)

no highpass filters

Type III: M even, odd symmetry

$$h[n] = -h[M-n]$$



$$H(e^{j\omega}) = j e^{-j\omega M/2} \sum_{k=1}^{M/2} c[k] \sin \omega k$$

$$c[k] = 2h[M/2 - k]$$

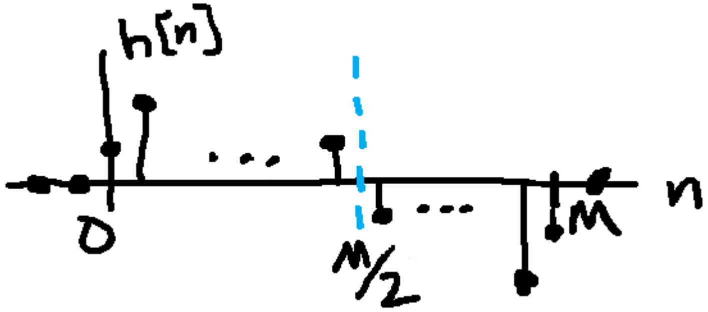
$\sin(0k) = 0$, $\sin(\pi k) = 0$ for all integer k

$\Rightarrow$ $H(e^{j0}) = 0$, $H(e^{j\pi}) = 0$ (zeros at $z = \pm 1$)

no lowpass or highpass filters

Type IV: M odd, odd symmetry

$$h[n] = -h[M-n]$$



$$H(e^{j\omega}) = j e^{-j\omega M/2} \sum_{k=1}^{(M+1)/2} d[k] \sin[\omega(k-1/2)]$$

$$d[k] = 2h[(M+1)/2 - k], \quad k = 1, 2, \cdots (M+1)/2$$

$\sin[0(k-1/2)] = 0$ for all k

$\Rightarrow H(e^{j0}) = 0$ (zero at $z=1$)

no lowpass filters

Type I + II (even symmetry)

$$H(z) = z^{-M} H(z^{-1})$$

Type III + IV (odd symmetry)

$$H(z) = -z^{-M} H(z^{-1})$$

zeros not on $|z|=1$ are in conjugate reciprocal pairs