IIR Filter Design Examples

IIR - infinite impulse response

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

Filter Design - find a_k, b_k to satisfy specifications

1) Choose prototype continuous-time lowpass filter $H_{LP}(s)$

2) Prewarp critical frequencies $\Omega = 2\tan\left(\frac{\omega}{2}\right)$

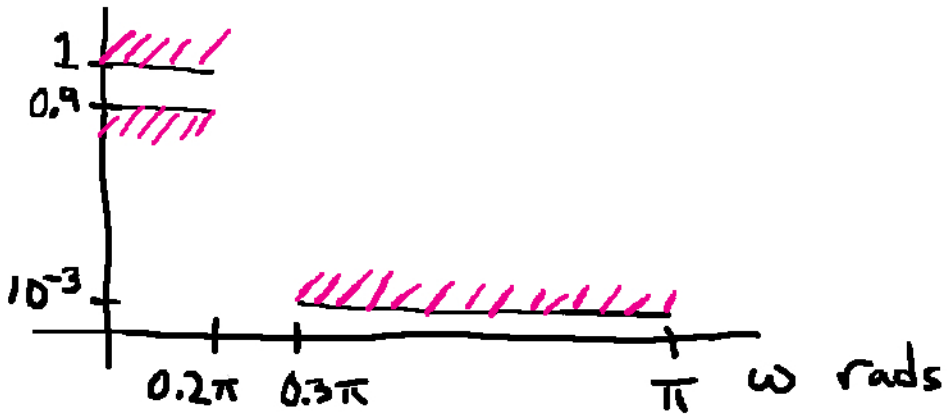
3) Apply frequency transformation

$$H(\tilde{s}) = H_{LP}(s)\Big|_{s=f(\tilde{s})}$$

4) Apply bilinear transformation

$$H(z) = H(\tilde{s})\Big|_{\tilde{s}=2\frac{(1-z^{-1})}{(1+z^{-1})}}$$

Example 1- Low pass

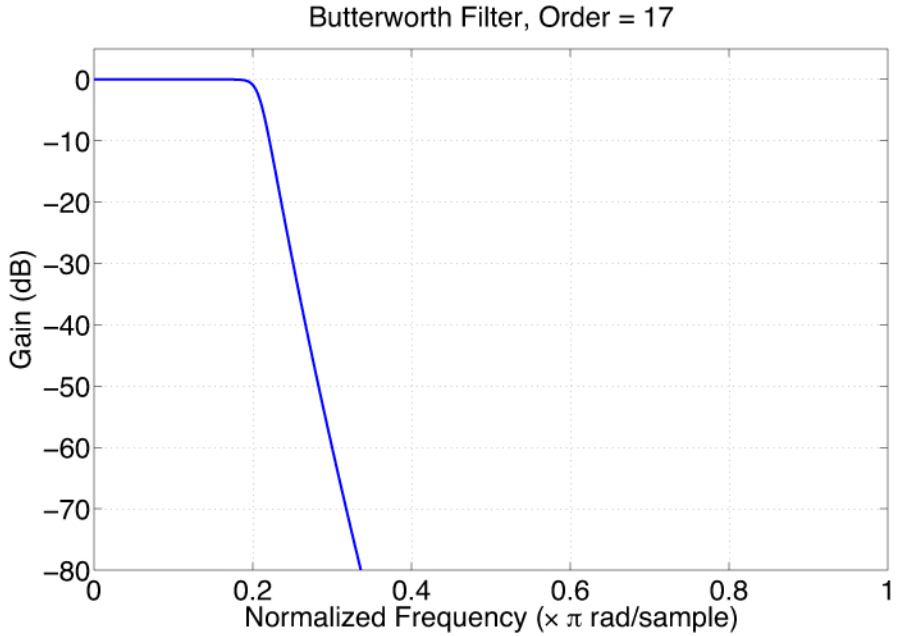


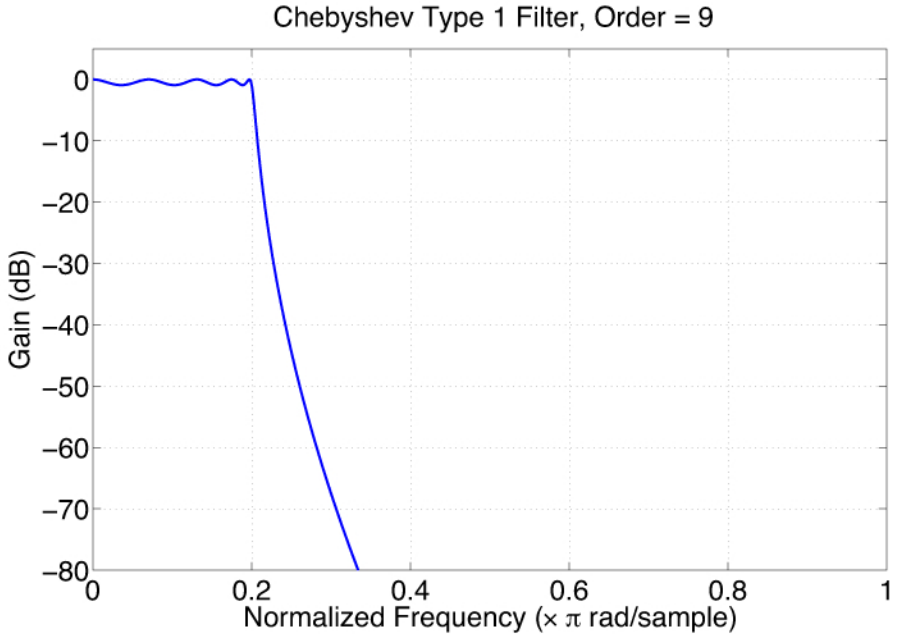
Compare Butterworth, Chebyshev 1 + 2, and elliptic designs

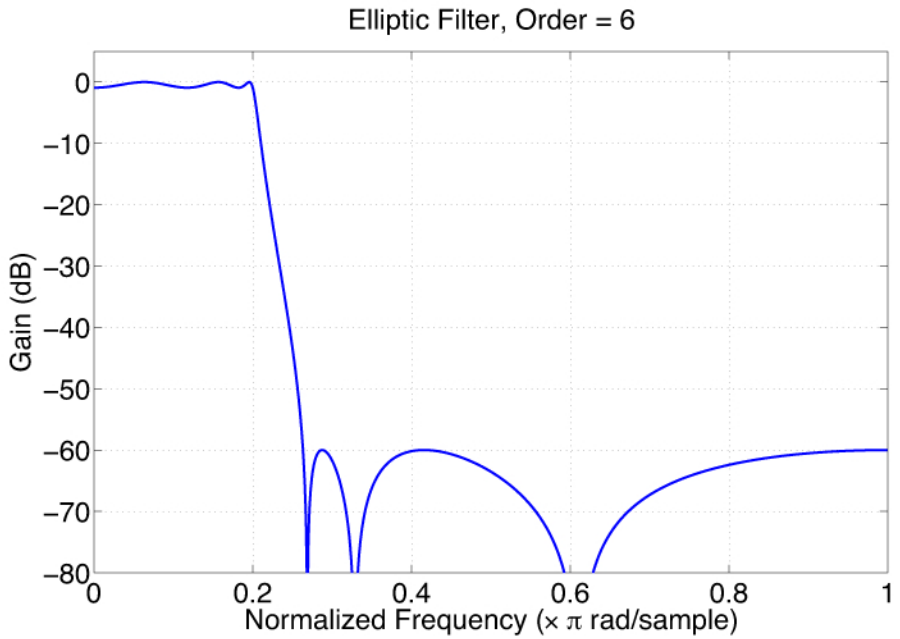
MATLAB: normalized frequency ω/π [0 1]
ripple in dB

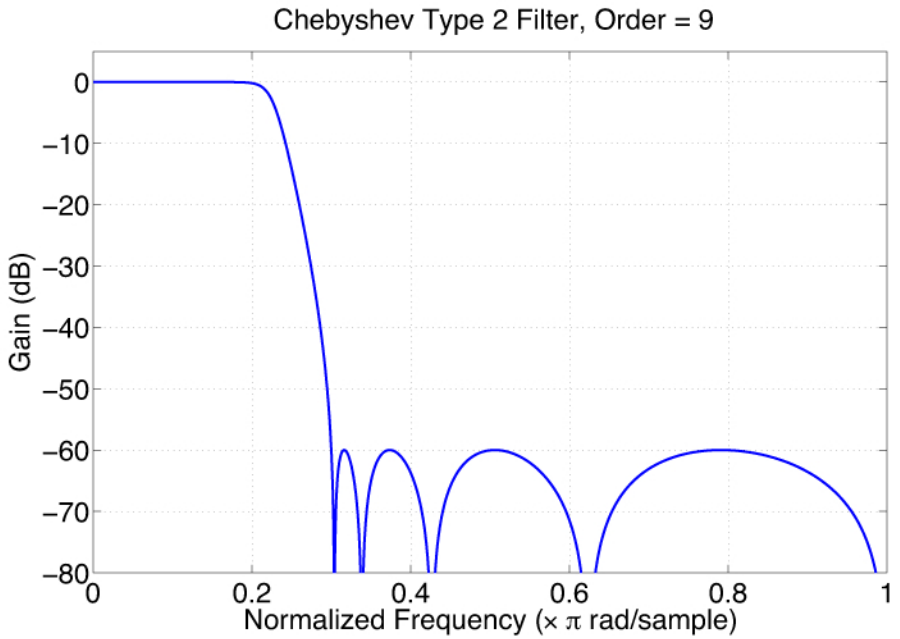
```
passband = 0.2*pi/pi;
stopband = 0.3*pi/pi;
passrip = -20*log10(0.9);
stopatten = -20*log10(0.001);

[Nc1, Wnc1] = cheb1ord(passband, stopband, passrip, stopatten);
[Bc1,Ac1] = cheby1(Nc1,passrip,Wnc1);
```

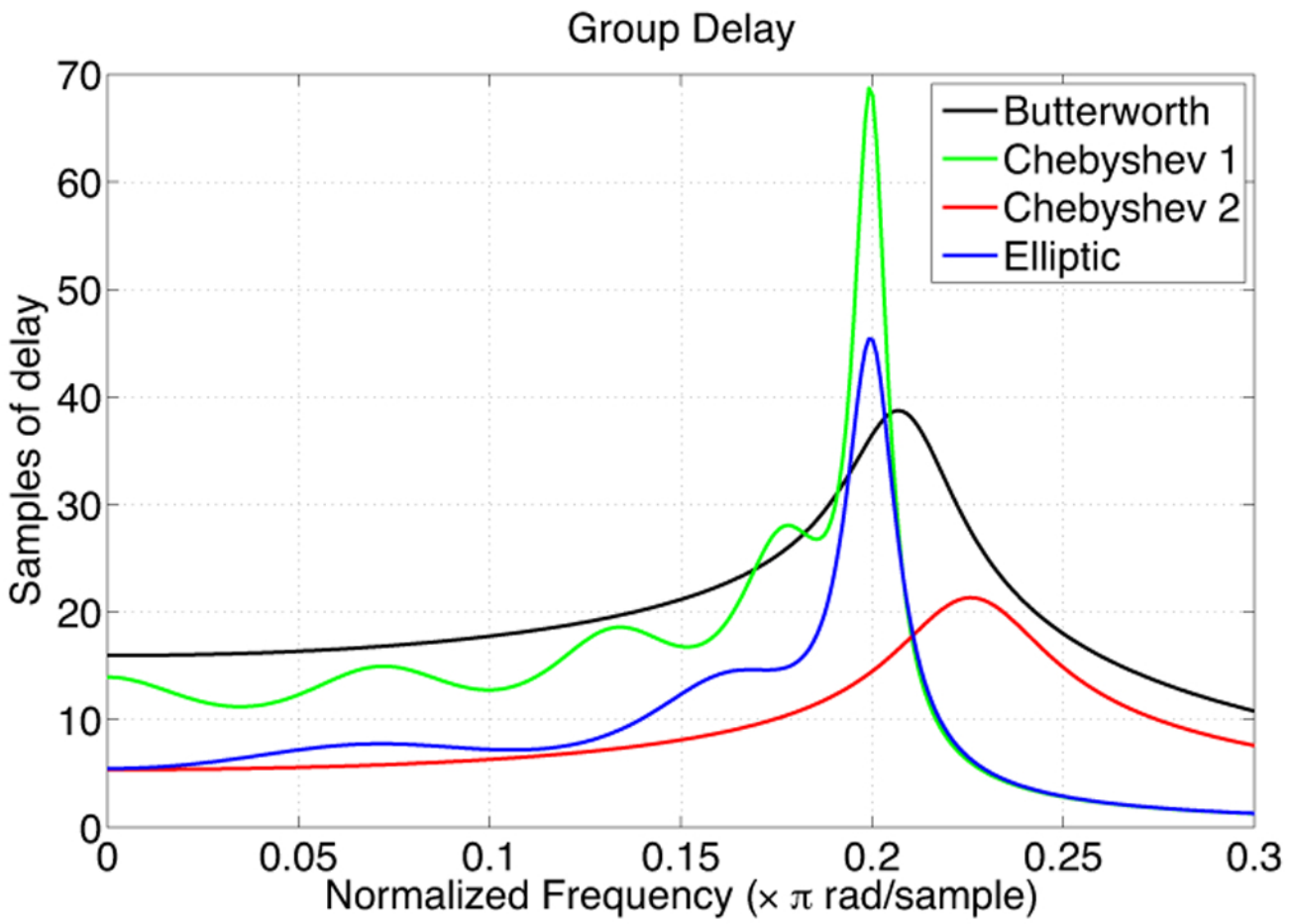
filter order
filter coefficients


Butterworth Filter, Order = 17
Gain (dB)
0
−10
−20
−30
−40
−50
−60
−70
−80
0
0.2
0.4
0.6
0.8
1
Normalized Frequency (× π rad/sample)


Chebyshev Type 1 Filter, Order = 9
Gain (dB)
0
−10
−20
−30
−40
−50
−60
−70
−80
0
0.2
0.4
0.6
0.8
1
Normalized Frequency (× π rad/sample)

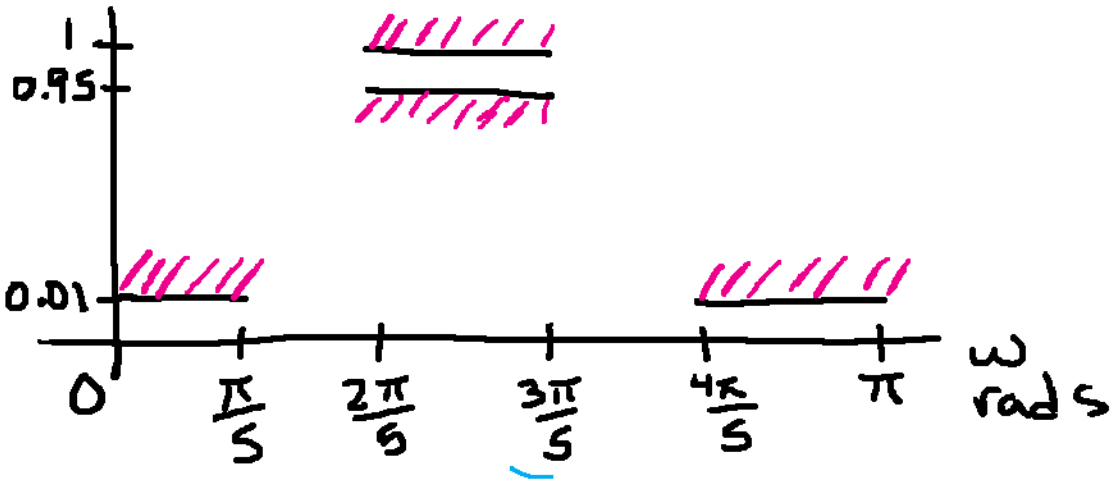

Elliptic Filter, Order = 6
Gain (dB)
0
−10
−20
−30
−40
−50
−60
−70
−80
0
0.2
0.4
0.6
0.8
1
Normalized Frequency (× π rad/sample)


Chebyshev Type 2 Filter, Order = 9
Gain (dB)
0
−10
−20
−30
−40
−50
−60
−70
−80
0
0.2
0.4
0.6
0.8
1
Normalized Frequency (× π rad/sample)



Filter	Order
Butter	17
Cheby1	9
Cheby2	9
Elliptic	6

Example 2: Bandpass

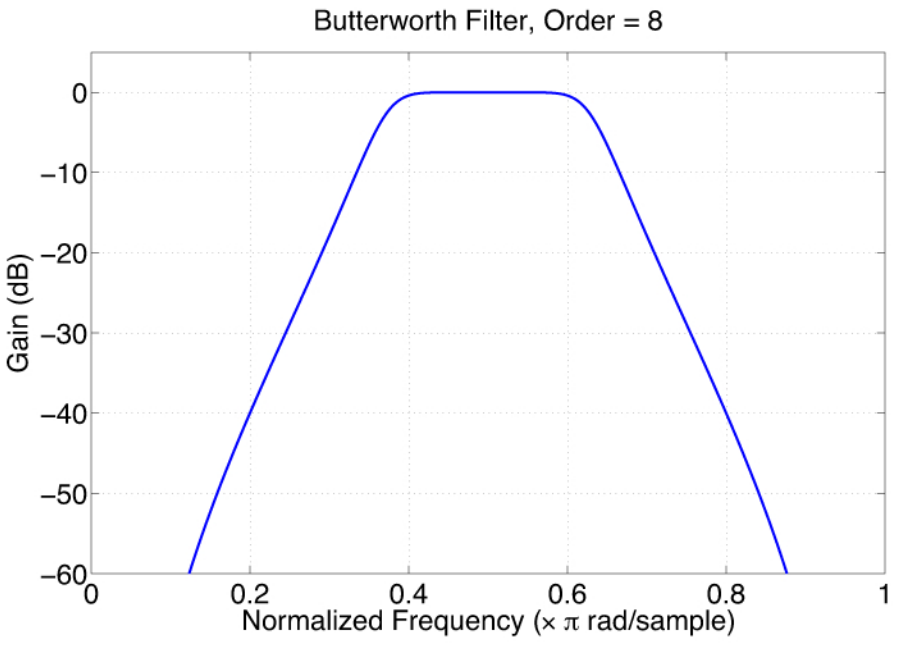


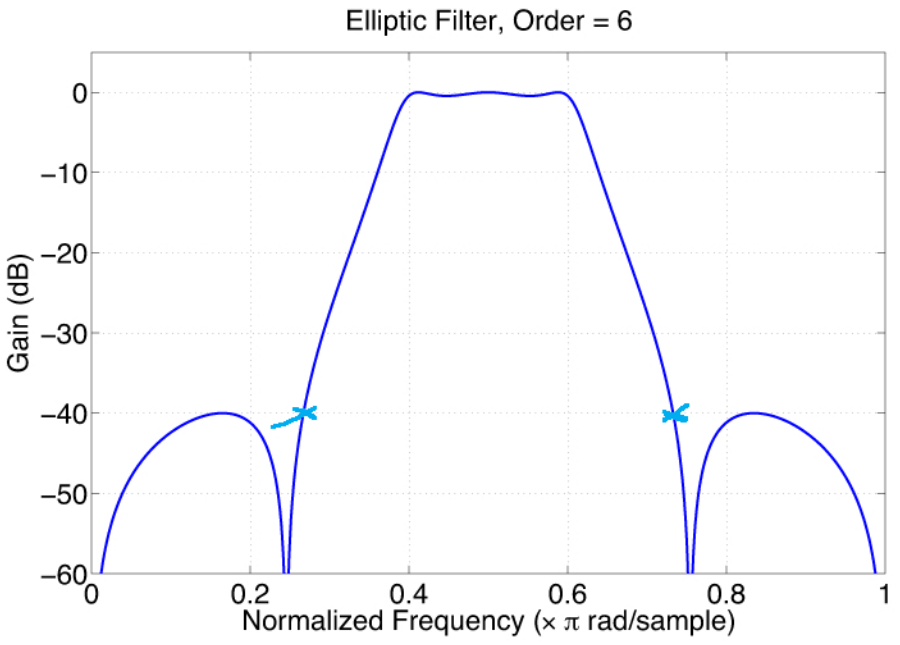
Compare Butterworth and elliptic designs

```
passband = [ 0.4*pi 0.6*pi]/pi;
stopband = [ 0.2*pi 0.8*pi]/pi;
passrip = -20*log10(0.95);
stopatten = -20*log10(0.01);

[Ne,Wne] = ellipord(passband,stopband,passrip,stopatten);
[Be,Ae] = ellip(Ne,passrip,stopatten,Wne);
```

normalize frequency
pass/stopband ripple in dB

filter order
filter coefficients


Butterworth Filter, Order = 8
Gain (dB)
0
−10
−20
−30
−40
−50
−60
0
0.2
0.4
0.6
0.8
1
Normalized Frequency (× π rad/sample)


Elliptic Filter, Order = 6
Gain (dB)
0
−10
−20
−30
−40
−50
−60
0
0.2
0.4
0.6
0.8
1
Normalized Frequency (× π rad/sample)