Frequency Transformations for Continuous Time Systems

Frequency transformation –

convert prototype (usually lowpass) filter to desired filter

$$s = f(\tilde{s}) \qquad \text{transformation relating } s \text{ to } \tilde{s}$$

$$H(\tilde{s}) = H_{LP}(s)\Big|_{s=f(\tilde{s})}$$

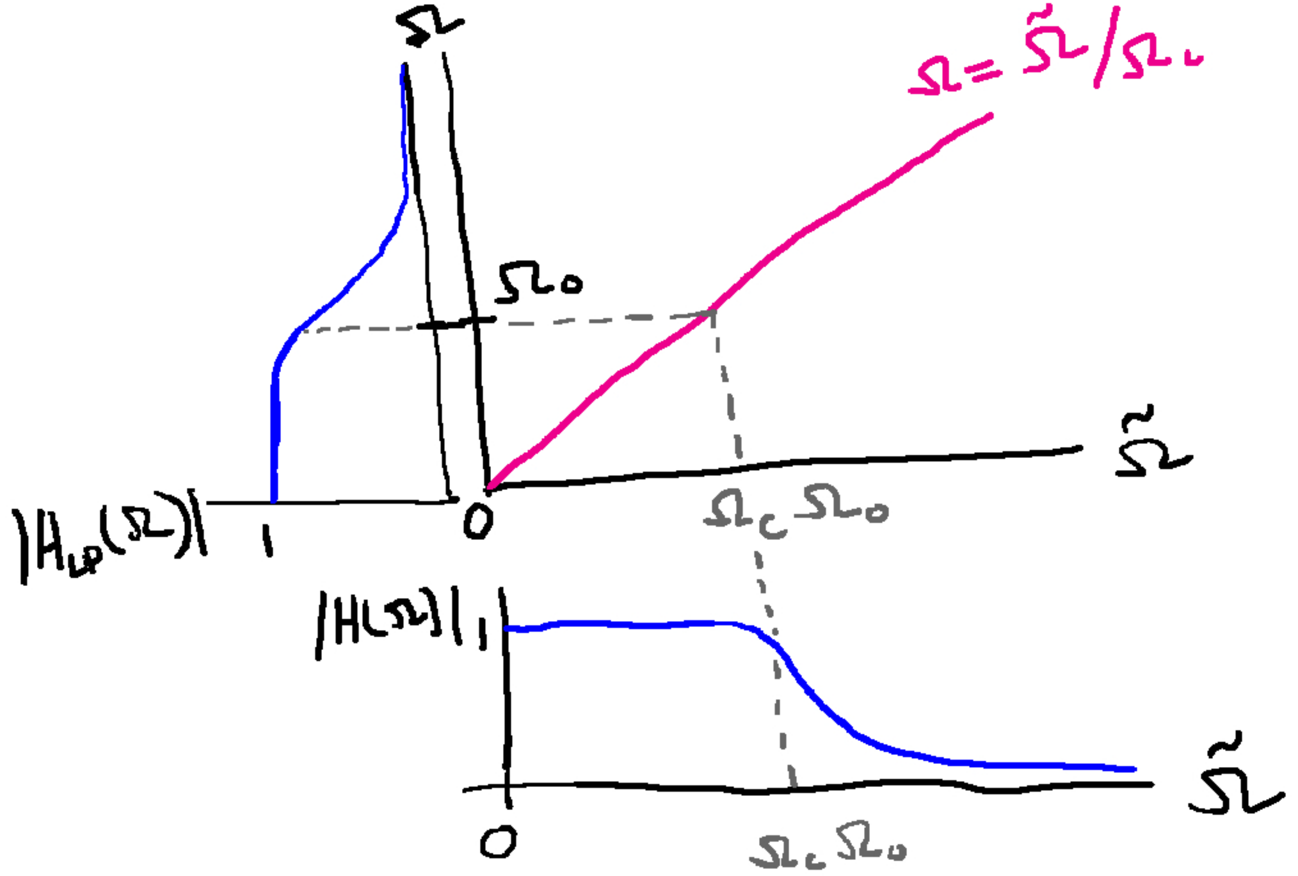
$$H(\tilde{\Omega}) = H_{LP}(\Omega)\Big|_{\Omega=f(\tilde{\Omega})} \qquad \text{"warp" frequency axis}$$

Lowpass to lowpass
Lowpass to highpass
Lowpass to bandpass

Lowpass to Lowpass

$$s = \tilde{s}/\Omega_c \underset{s=j\Omega}{\Rightarrow} \Omega = \tilde{\Omega}/\Omega_c$$

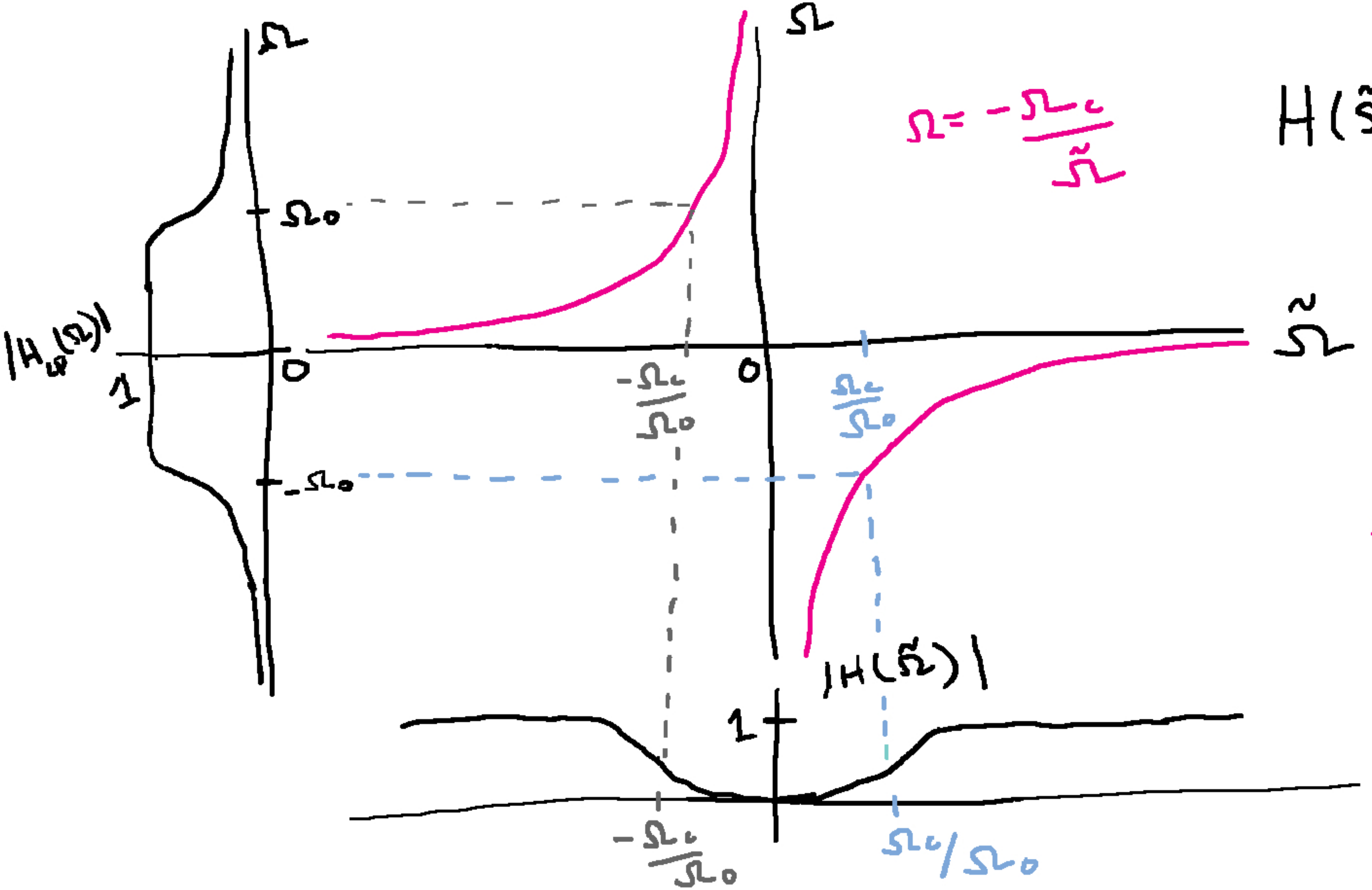
$$H(\tilde{s}) = H_{LP}(\tilde{s}/\Omega_c)$$



$\tilde{\Omega}$ axis is Ω axis stretched by Ω_c

Lowpass to Highpass: $s = \frac{\Omega_c}{\tilde{s}} \underset{s=j\Omega}{\Rightarrow} \Omega = -\frac{\Omega_c}{\tilde{\Omega}}$

$$H(\tilde{s}) = H_{LP}\left(\frac{\Omega_c}{\tilde{s}}\right)$$

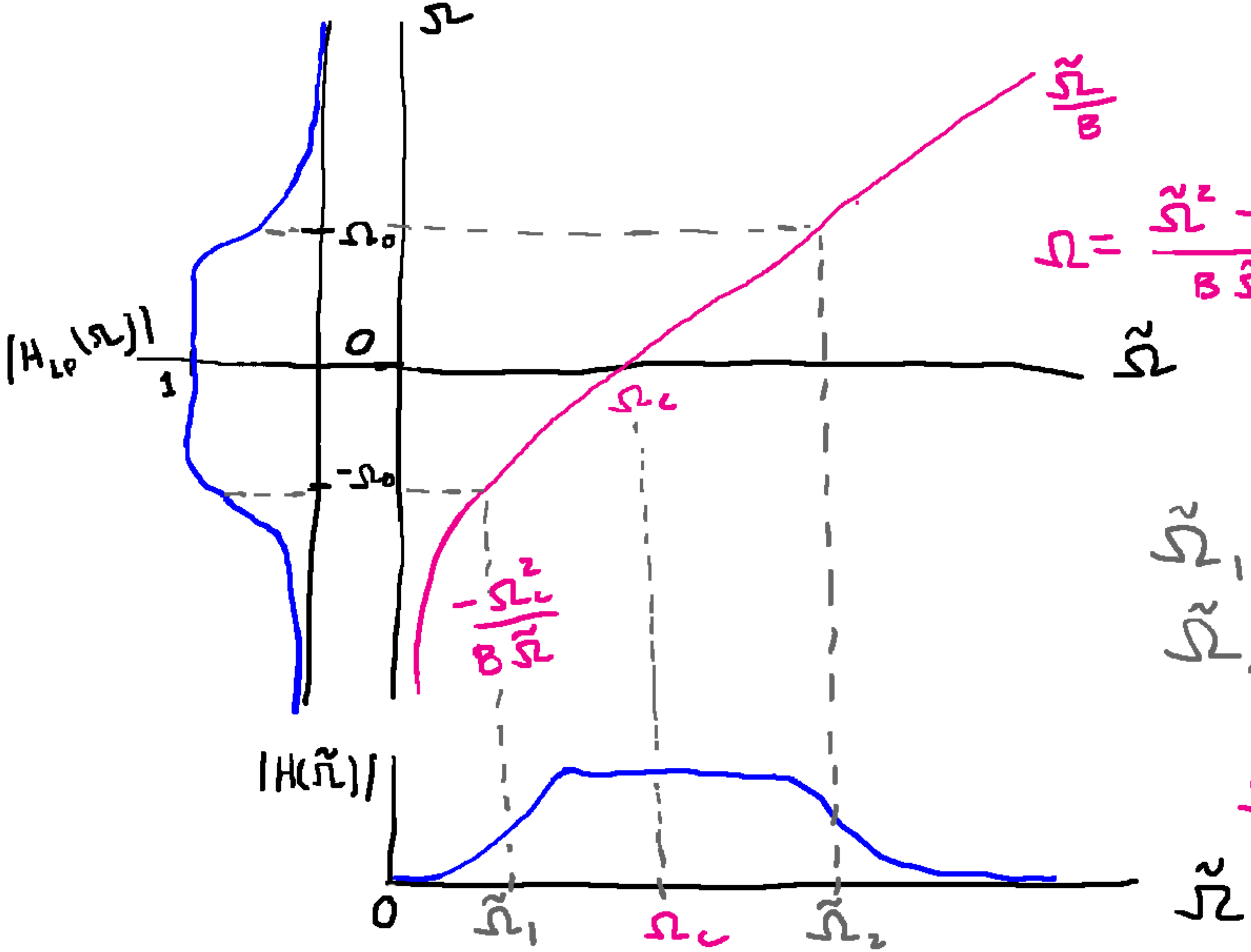


inverts the frequency axis

Lowpass to Bandpass:

$$s = \frac{\tilde{s}^2 + \Omega_c^2}{B\tilde{s}} \underset{s=j\Omega}{\Rightarrow} \Omega = \frac{\tilde{\Omega}^2 - \Omega_c^2}{B\tilde{\Omega}}$$

$$H(\tilde{s}) = H_{LP}\left(\frac{\tilde{s}^2 + \Omega_c^2}{B\tilde{s}}\right)$$



$$\Omega = \frac{\tilde{\Omega}^2 - \Omega_c^2}{B\tilde{\Omega}}$$

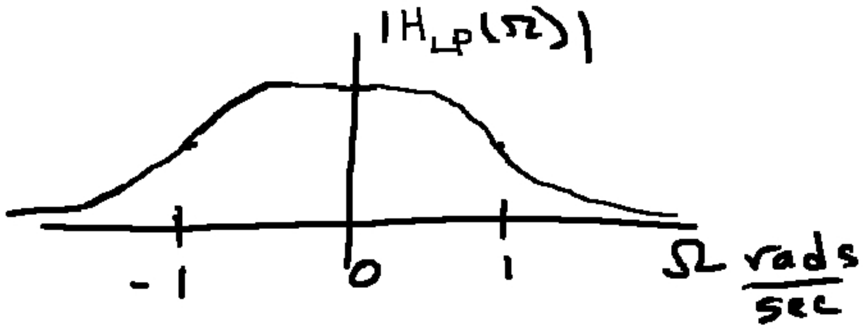
$$\tilde{\Omega}_1 = \frac{-\Omega_0 B + \sqrt{\Omega_0^2 B^2 + 4\Omega_c^2}}{2}$$

$$\tilde{\Omega}_2 = \frac{\Omega_0 B + \sqrt{\Omega_0^2 B^2 + 4\Omega_c^2}}{2}$$

$$\tilde{\Omega}_2 - \tilde{\Omega}_1 = \Omega_0 B$$

B scales width

Example: convert $H_{LP}(s) = \frac{1}{s+1}$ to bandpass $H(s)$ with passband (½ power) from $10 \le |\tilde{\Omega}| \le 14$ rads/sec



½ power bandwidth = 2 rads/sec

Desired ½ power bandwidth = 4 rads/sec $\Longrightarrow B = 4$

Desired center frequency = 12 rads/sec $\Longrightarrow \Omega_c = 12$

$$s = \frac{\tilde{s}^2 + \Omega_c^2}{B\tilde{s}} = \frac{\tilde{s}^2 + 144}{4\tilde{s}}$$

$$H(\tilde{s}) = \frac{1}{s+1}\bigg|_{s = \frac{\tilde{s}^2+144}{4\tilde{s}}} = \frac{4\tilde{s}}{\tilde{s}^2 + 4\tilde{s} + 144}$$