Continuous Time Chebyshev and Elliptic Filters

Butterworth - monotonic in pass/stop bands

Chebyshev Type I - ripple in pass band,
monotonic in stop band

Chebyshev Type II - monotonic in passband,
ripple in stop band

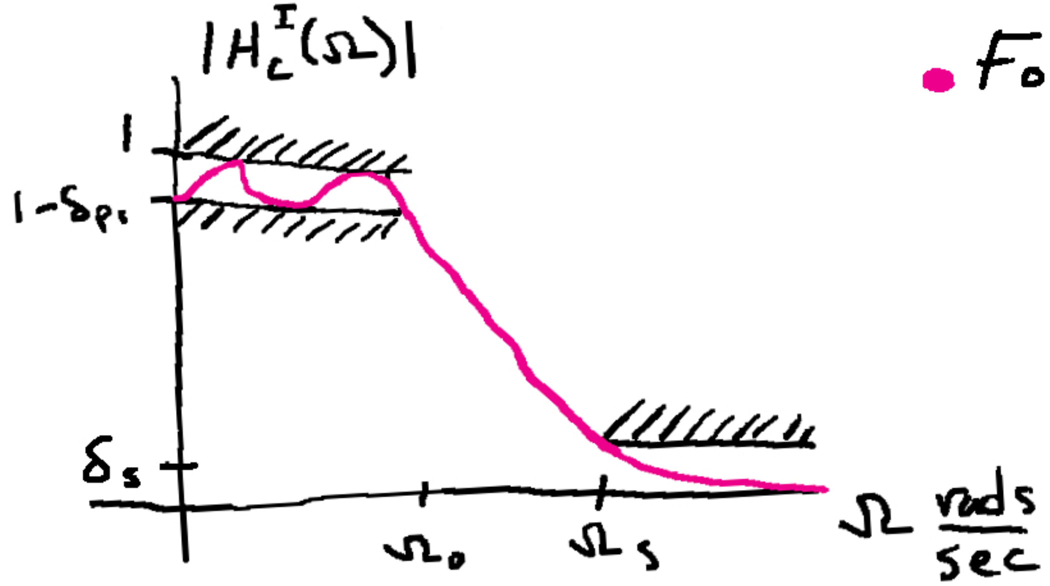
- Elliptic - ripple in both pass/stop bands

Chebyshev Type I

$$|H_c^I(\Omega)|^2 = \frac{1}{1+\varepsilon^2 T_N^2\left(\frac{\Omega}{\Omega_0}\right)}; \qquad T_N(x) = \begin{cases} x, & N=1 \\ 2x^2-1, & N=2 \\ \vdots & \end{cases}$$

- For $\left|\frac{\Omega}{\Omega_0}\right| < 1$, $T_N\left(\frac{\Omega}{\Omega_0}\right)$ oscillates between ± 1, $T_N(1) = 1$

  $\Rightarrow |H_c^I(\Omega)|^2$ oscillates between $\frac{1}{1+\varepsilon^2}$ and 1



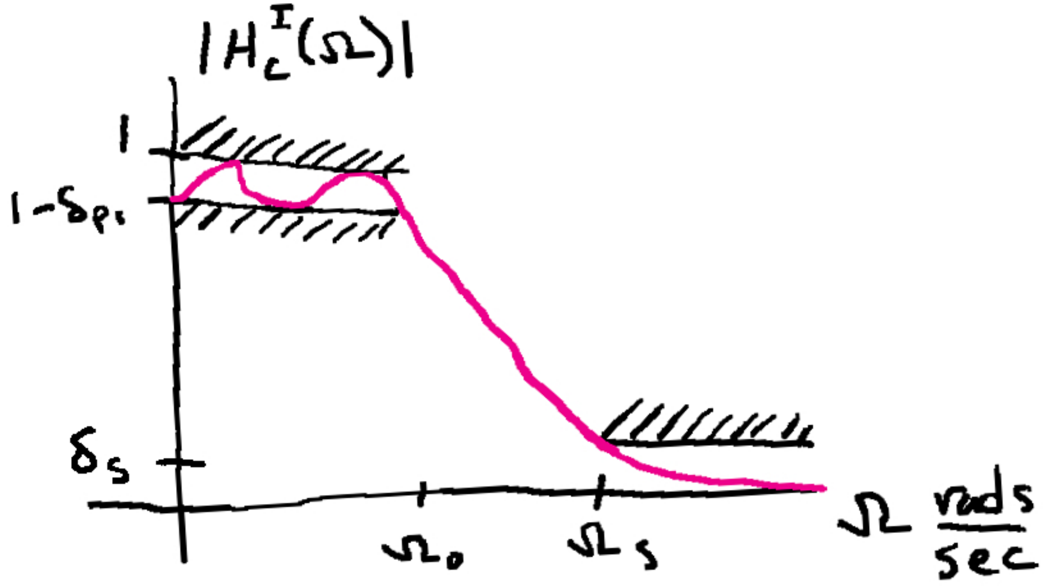
- For $\left|\frac{\Omega}{\Omega_0}\right| > 1$, $T_N\left(\frac{\Omega}{\Omega_0}\right)$ monotone increasing

$$|H_c^I(0)|^2 = \begin{cases} 1, & N \text{ odd} \\ \frac{1}{1+\varepsilon^2}, & N \text{ even} \end{cases}$$

Three design parameters: $N, \varepsilon^2, \Omega_0$

- Set Ω_0 = passband edge
- Choose ε^2 so $1 - \delta_{p_1} = \left(\frac{1}{1+\varepsilon^2}\right)^{1/2}$
- Choose N to satisfy stopband constraint

$$\delta_s^2 \geq \frac{1}{1 + \varepsilon^2 T_N^2\left(\frac{\Omega_s}{\Omega_0}\right)}$$



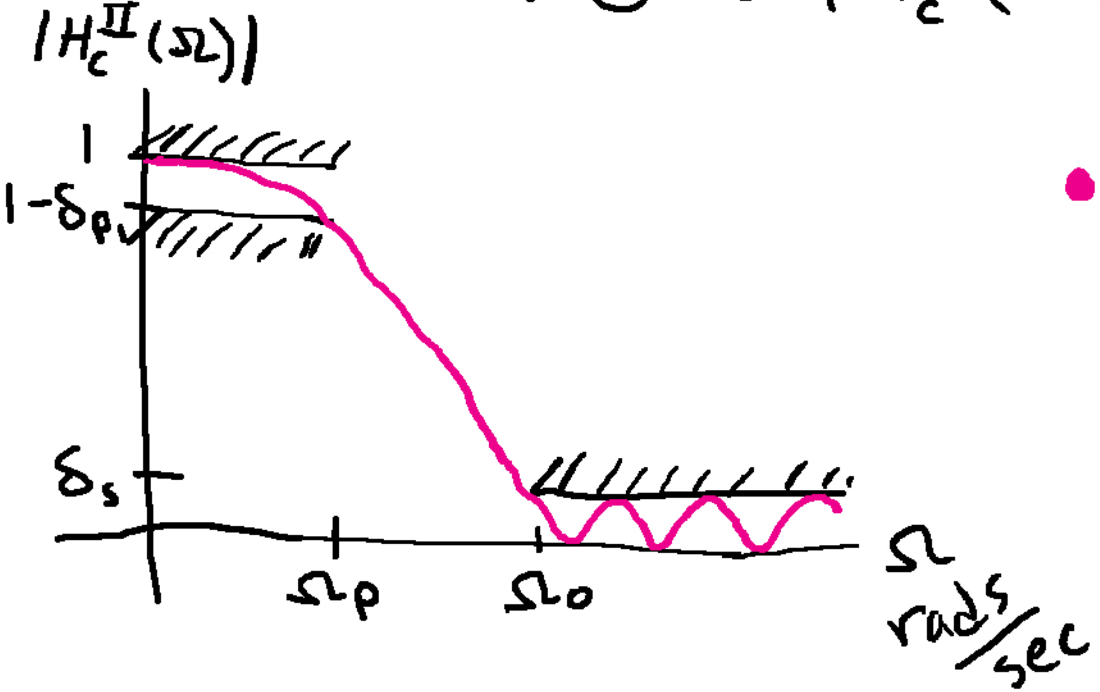
- Filter poles located on an ellipse in the s plane

Chebyshev Type II

$$|H_c^{II}(\Omega)|^2 = \frac{1}{1+\left[\varepsilon^2 T_N^2\left(\frac{\Omega_0}{\Omega}\right)\right]^{-1}} = \frac{\varepsilon^2 T_N^2\left(\frac{\Omega_0}{\Omega}\right)}{1+\varepsilon^2 T_N^2\left(\frac{\Omega_0}{\Omega}\right)}$$

- For $\left|\frac{\Omega_0}{\Omega}\right| < 1$ $(|\Omega| > \Omega_0)$, $T_N\left(\frac{\Omega_0}{\Omega}\right)$ oscillates between ± 1

$$\Rightarrow 0 \leq |H_c^{II}(\Omega)|^2 \leq \frac{\varepsilon^2}{\varepsilon^2+1}, \quad |\Omega| > \Omega_0$$



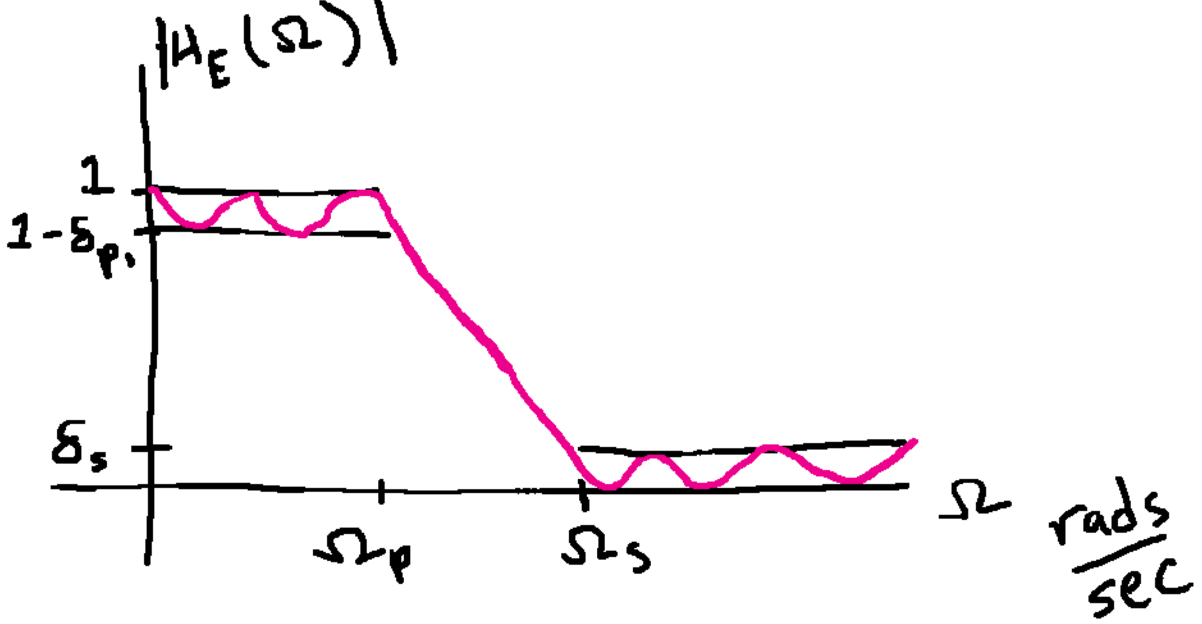
- For $|\Omega| < \Omega_0$, $T_N\left(\frac{\Omega_0}{\Omega}\right)$ increases as $\Omega \rightarrow 0$

$$|H_c^{II}(0)| = 1$$

Elliptic Filters

$$|H_E(\Omega)|^2 = \frac{1}{1 + \varepsilon^2 U_N^2(\Omega)}$$; $U_N(\Omega)$ Jacobian elliptic function

equiripple error in both pass/stop bands



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