IIR Filter Design Procedure

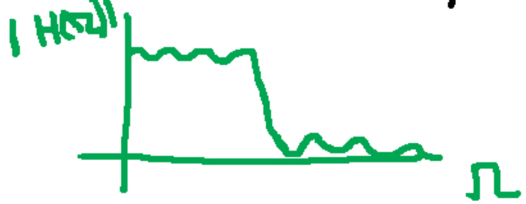
IIR discrete-time filters are designed from analog (continuous-time) prototypes

Butterworth – monotonic in both pass/stop bands

Chebyshev Type 1 – passband equiripple, stopband monotonic

Chebyshev Type 2 – passband monotonic, stopband equiripple

Elliptic – equiripple in both pass/stop bands



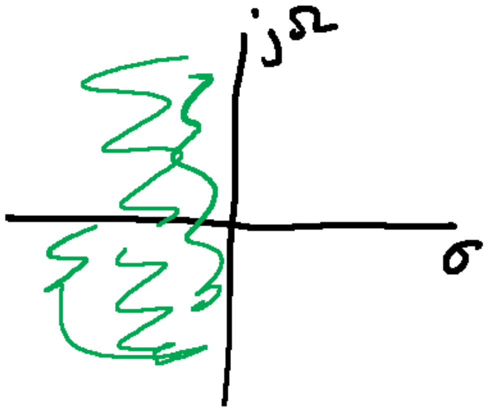
Continuous-time filters

Frequency response $H(\Omega)$ (Fourier transform)

Transfer function $H(s)$ (Laplace transform)

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt \qquad s = \sigma + j\Omega \quad \text{complex}$$

s-plane



$$H(\Omega) = H(s)\Big|_{\sigma=0}$$

Differential Equations

$$H(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} = \frac{b_M}{a_N} \frac{\prod_{k=1}^{M} (s - c_k)}{\prod_{k=1}^{N} (s - d_k)}$$

$$h(t) = \sum_{k=1}^{N} A_k e^{d_k t} u(t)$$

Stable/causal Systems

$Re\{d_k\} < 0 \Rightarrow$ poles in left half of s-plane

1) Prototype low pass filter $H_{LP}(s)$

2) Translate critical ω_k to Ω_k (prewarping)

3) Apply "frequency transformation" $s = f(\tilde{s})$

$$H(\tilde{s}) = H_{LP}(s)\Big|_{s=f(\tilde{s})} \Rightarrow H(\tilde{\Omega}) = H_{LP}(\Omega)\Big|_{\Omega=f(\tilde{\Omega})}$$

so critical frequencies of $H(\tilde{\Omega})$ are Ω_k

4) Use bilinear transform $\tilde{s} = 2\,\dfrac{1-z^{-1}}{1+z^{-1}}$

$$H(z) = H(\tilde{s})\Big|_{\tilde{s}=2\frac{1-z^{-1}}{1+z^{-1}}}$$

5) Verify that $H(z)$ satisfies the specifications