FIR and IIR Filters

Two categories of discrete-time filters

Finite Impulse Response
FIR

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

$$= b_0 \prod_{k=1}^{M} (1 - c_k z^{-1})$$

$$h[n] = \begin{cases} b_n, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Infinite Impulse Response
IIR

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}, \quad a_0 = 1$$

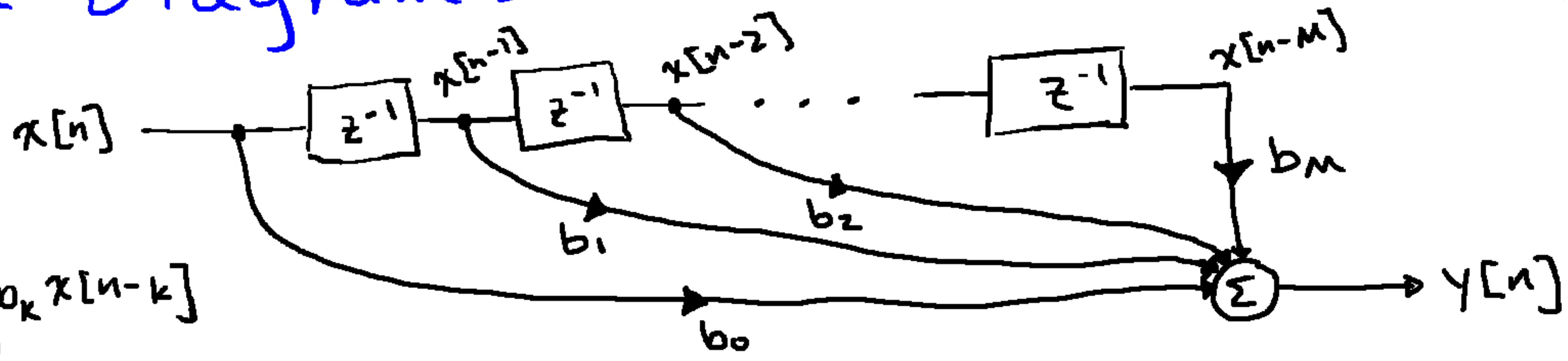
$$= \frac{b_0 \prod_{k=1}^{M} (1 - c_k z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})}$$

$$h[n] = \sum_{k=1}^{N} A_k (d_k)^n u[n]$$

Block Diagrams

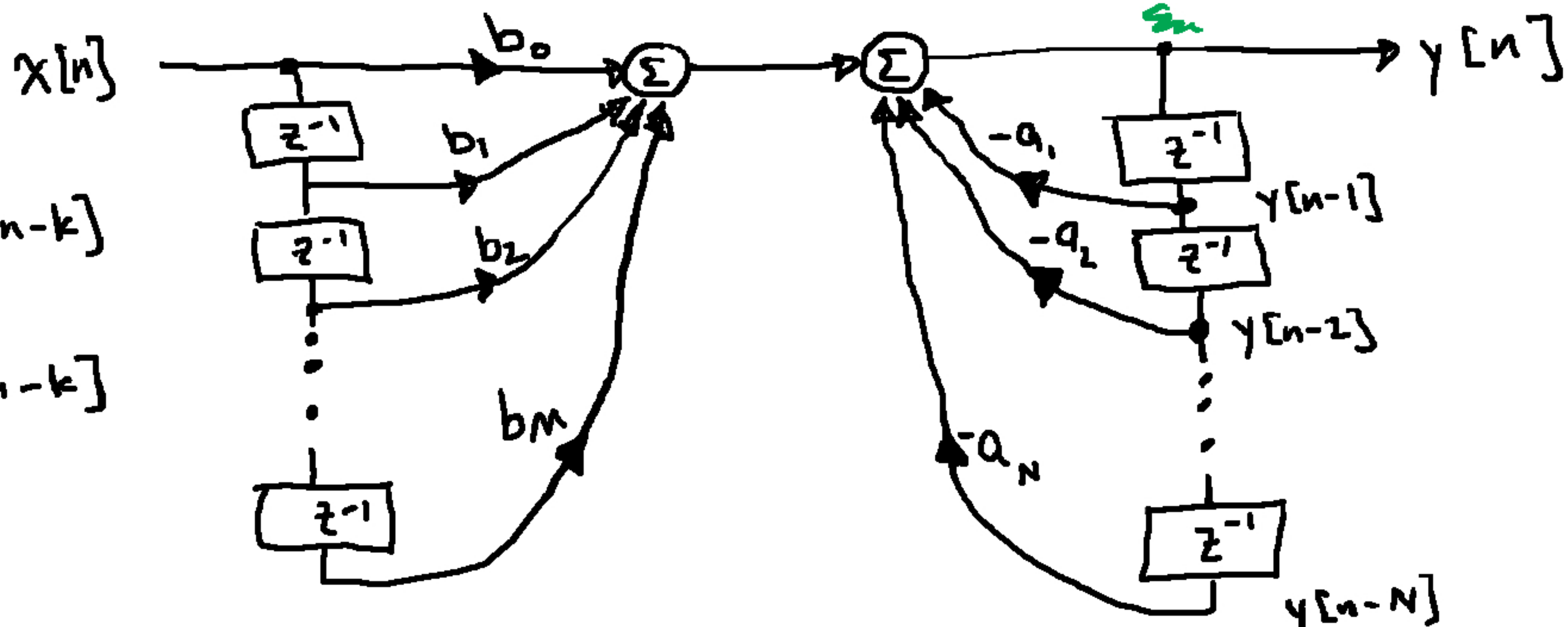
FIR

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$



IIR

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$



Filter Designs

Desired frequency response - $H_d(e^{jw})$

FIR	IIR
$H(e^{jw}) = \sum_{k=0}^{M} b_k e^{-jkw}$	$H(e^{jw}) = \dfrac{\sum_{k=0}^{M} b_k e^{-jkw}}{\sum_{k=0}^{N} a_k e^{-jkw}}$, $a_0 = 1$
- Optimization based design	- Transform analog filter designs
- Arbitrary magnitude/phase response	- Frequency selective gain
- Can obtain linear phase	- Nonlinear phase, no control
- Can require large M	- Fewer coefficients than FIR