Characterizing Filter Phase Response

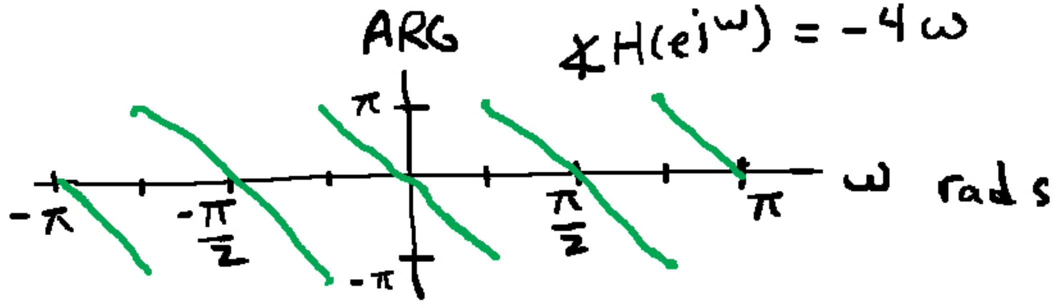
Frequency response: $H(e^{j\omega}) = |H(e^{j\omega})| e^{j\measuredangle H(e^{j\omega})}$

1) $\measuredangle H(e^{j\omega})$ is not uniquely defined

$$e^{j\measuredangle H(e^{j\omega})} = e^{j(\measuredangle H(e^{j\omega}) + \ell 2\pi)}$$

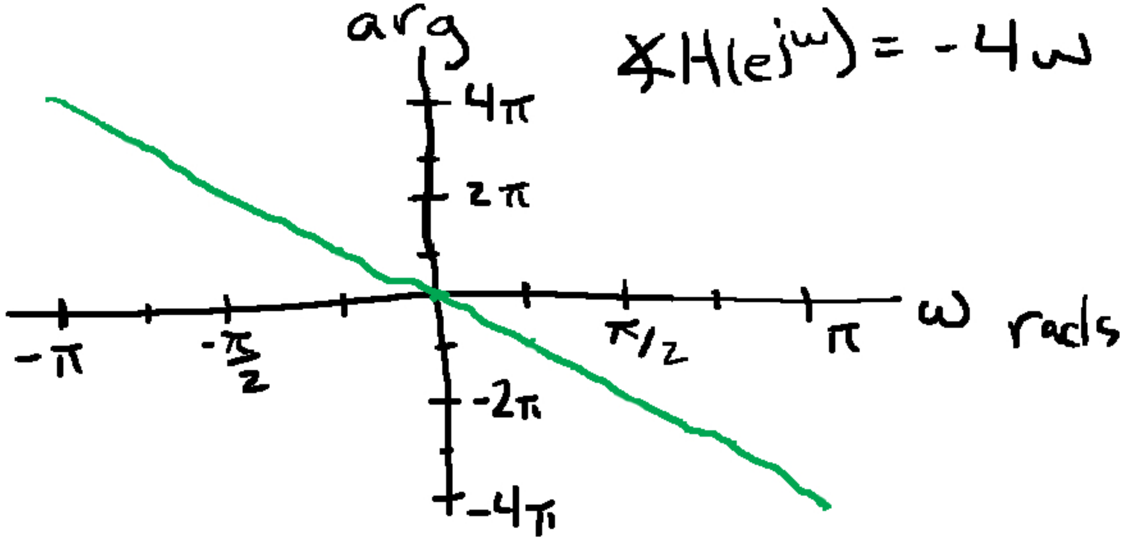
2) Principal value

$$-\pi < ARG\{H(e^{j\omega})\} < \pi$$



3) Unwrapped phase

$arg\{H(e^{j\omega})\}$



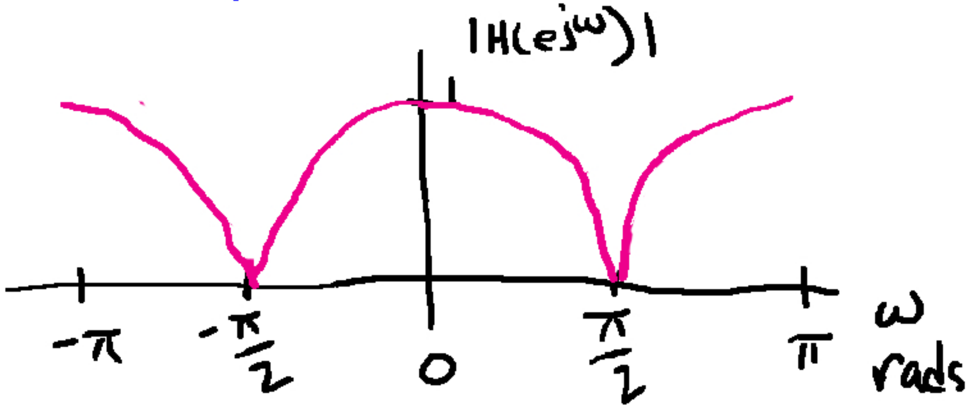
- Linear phase: $\arg\{H(e^{j\omega})\} = -\omega n_0$

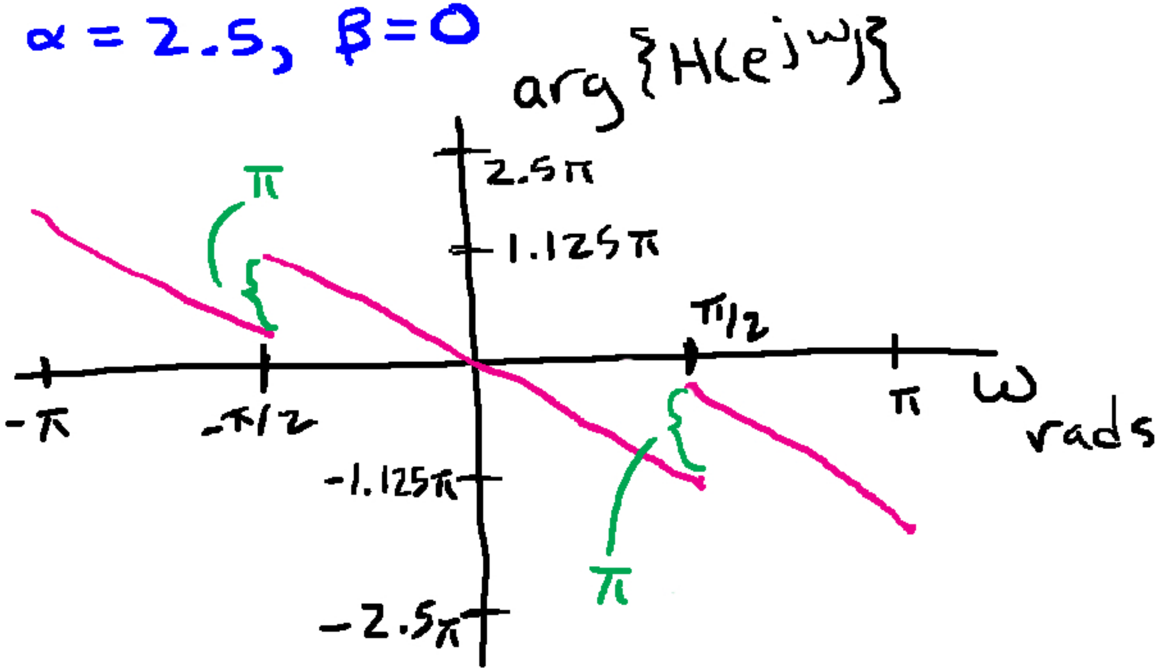
  no distortion other than a time delay

- Generalized linear phase: $H(e^{j\omega}) = A(\omega)\, e^{j\alpha\omega + j\beta}$

  $A(\omega)$ is real, but can be negative

Example: $A(\omega) = \cos(\omega)$, $\alpha = 2.5$, $\beta = 0$



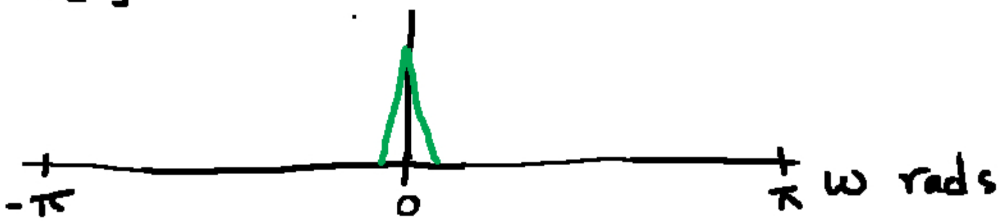


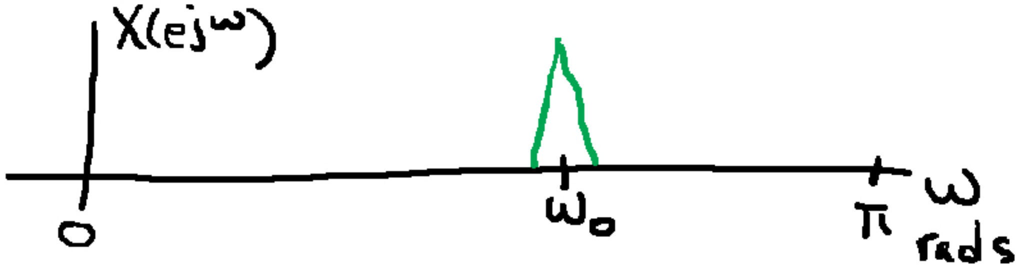
- Nonlinear phase:

Group delay – characterize nonlinear phase

$s[n] \xleftrightarrow{\text{DTFT}} S(e^{j\omega})$

$x[n] = s[n]\,e^{j\omega_0 n} \xleftrightarrow{\text{DTFT}} X(e^{j\omega}) = S(e^{j(\omega-\omega_0)})$





$|H(e^{j\omega})| = 1$

$$\boxed{\tau(\omega) = -\frac{d}{d\omega}\arg\{H(e^{j\omega})\} \quad \text{group delay}}$$

$$Y(e^{j\omega}) = X(e^{j\omega})\,H(e^{j\omega}) = S(e^{j(\omega-\omega_0)})\,e^{j[\phi_0 - \tau(\omega_0)(\omega-\omega_0)]}$$

$$= e^{j\phi_0}\,S(e^{j(\omega-\omega_0)})\,e^{-j\tau(\omega_0)(\omega-\omega_0)} \xleftrightarrow{\text{DTFT}} y[n]$$

$$y[n] = e^{j\phi_0}\,s[n-\tau(\omega_0)]\,e^{j\omega_0 n}$$

Group delay $\tau(\omega) = -\frac{d}{d\omega} \arg\{H(e^{j\omega})\}$

1) Delay experienced by narrowband envelope of signal at ω

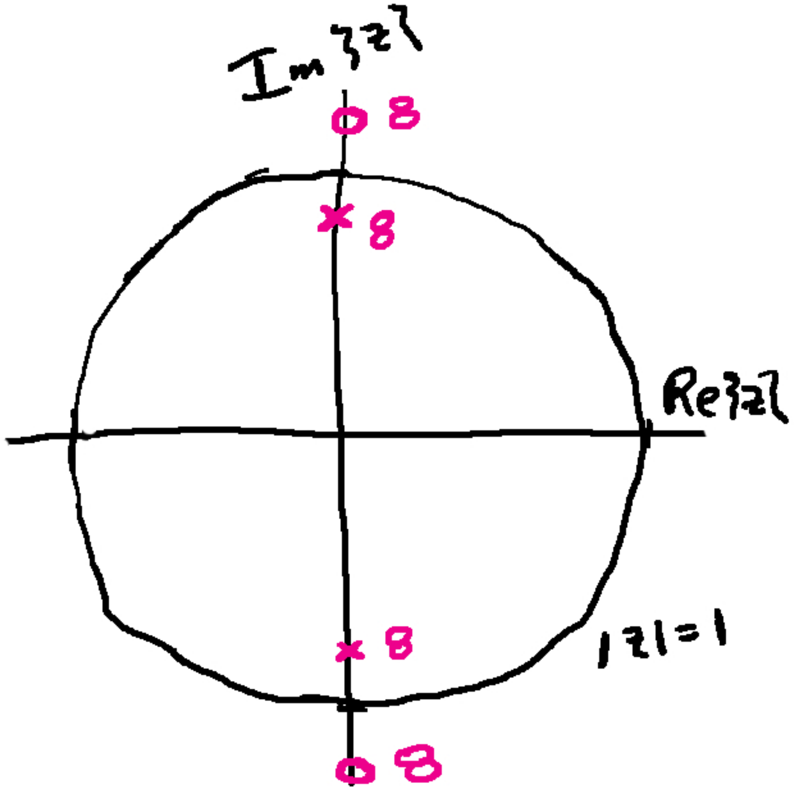
2) Linear phase $\longleftrightarrow$ constant group delay

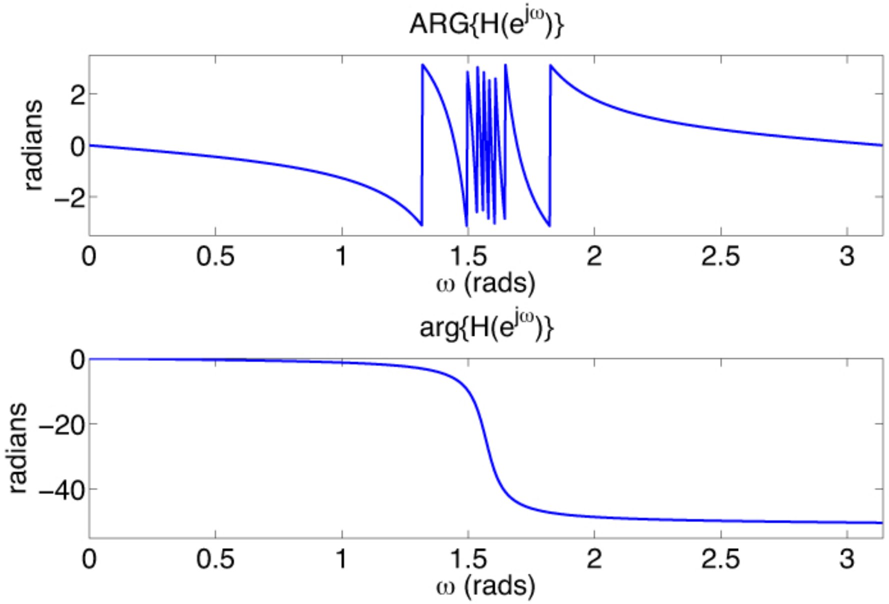
   Nonlinear phase $\longleftrightarrow$ non constant group delay

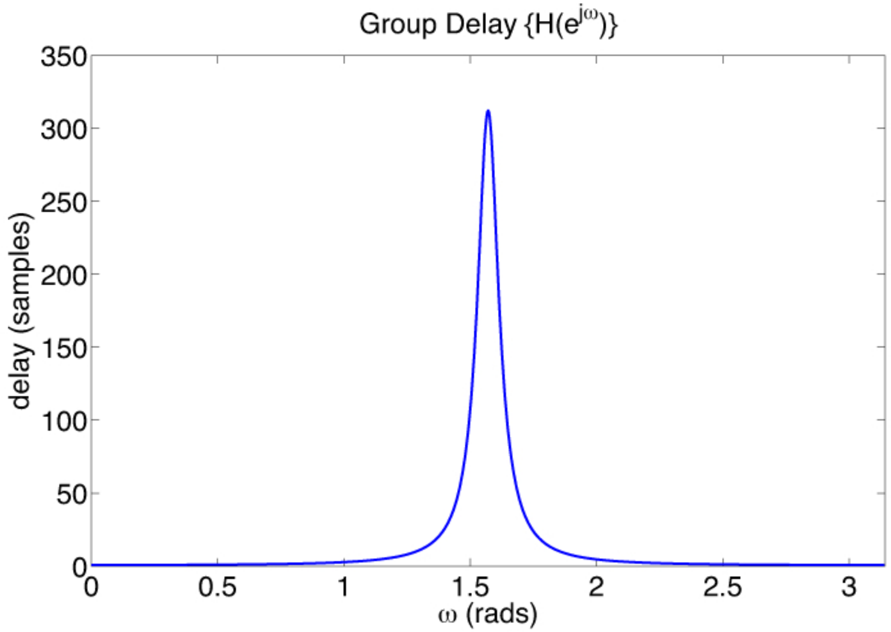
   different frequency packets experience different delays

Example: Allpass

$$H(z) = \left(\frac{z^{-2} + .95^2}{1 + .95^2 z^{-2}} \right)^8$$







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