Frequency Response Magnitude and Poles and Zeros

Influence of poles/zeros on $|H(e^{j\omega})|$

Stable $H(z) \Rightarrow$ ROC includes $|z|=1$

$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} = \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{\sum_{k=0}^{N} a_k e^{-j\omega k}}$$

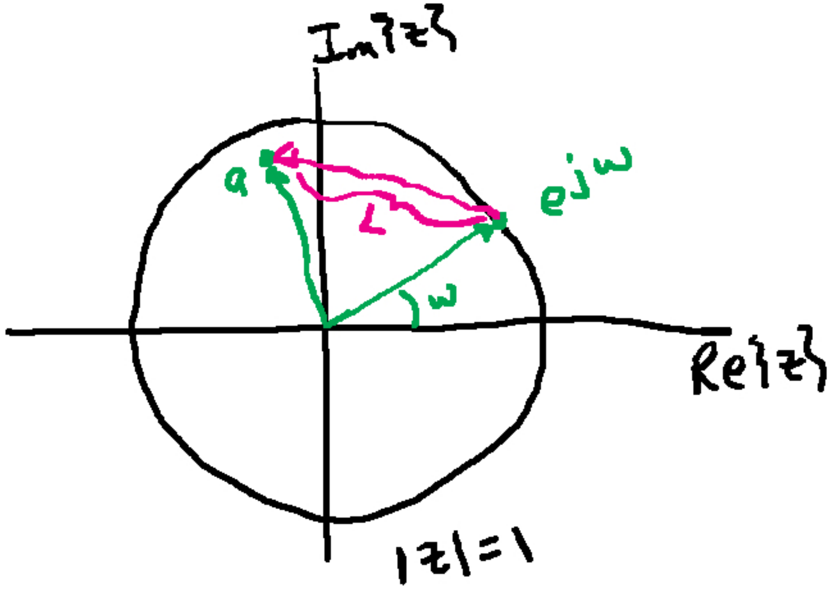
In pole/zero form

$$H(e^{j\omega}) = \frac{b_0}{a_0} \frac{\prod_{k=0}^{M}(1-c_k e^{-j\omega})}{\prod_{k=0}^{N}(1-d_k e^{-j\omega})}$$

$$|H(e^{j\omega})| = \frac{|b_0|}{|a_0|} \frac{\prod_{k=0}^{M}|1-c_k e^{-j\omega}|}{\prod_{k=0}^{N}|1-d_k e^{-j\omega}|} = \underline{\frac{|b_0|}{|a_0|} \frac{\prod_{k=0}^{M}|e^{j\omega}-c_k|}{\prod_{k=0}^{N}|e^{j\omega}-d_k|}}$$

since $\dfrac{|e^{-j\omega M}|}{|e^{-j\omega N}|} = 1$

$|H(e^{j\omega})|$ depends on terms $|e^{j\omega} - a|$



$e^{j\omega} - a$ is a vector connecting $e^{j\omega}$ to a

$L = |e^{j\omega} - a|$ is the length of the vector

$$|H(e^{j\omega})| = \frac{|b_0|}{|a_0|} \frac{\prod_{k=0}^{M} |e^{j\omega} - c_k|}{\prod_{k=0}^{N} |e^{j\omega} - d_k|}$$

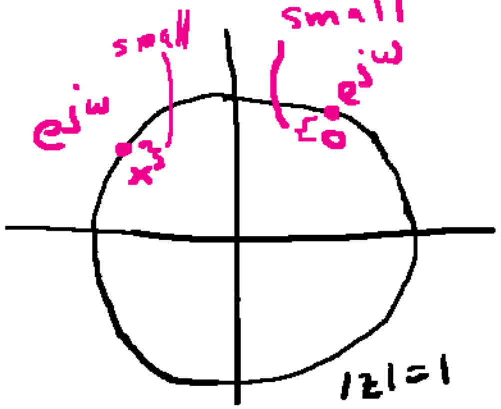
$|e^{j\omega} - c_k| =$ distance from $e^{j\omega}$ to c_k

$|e^{j\omega} - d_k| =$ distance from $e^{j\omega}$ to d_k

$$|H(e^{j\omega})| = \frac{|b_0|}{|a_0|} \frac{\prod_{k=0}^{M} \text{"distance from } e^{j\omega} \text{ to zeros"}}{\prod_{k=0}^{N} \text{"distance from } e^{j\omega} \text{ to poles"}}$$

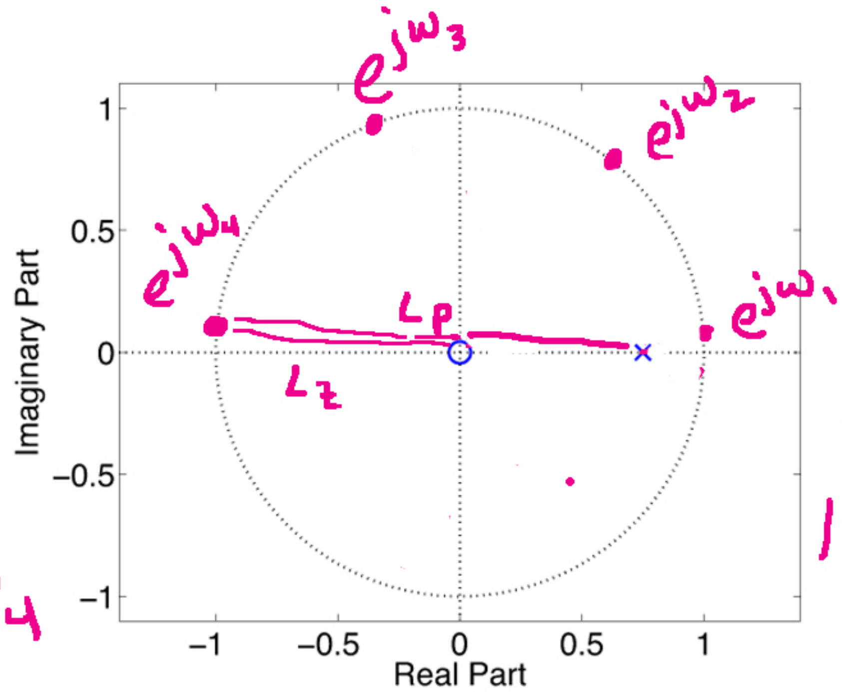
When $e^{j\omega}$ is close to a zero, $|H(e^{j\omega})|$ is "small"

When $e^{j\omega}$ is close to a pole, $|H(e^{j\omega})|$ is "large"



Example: $H(z) = \dfrac{1}{1 - \frac{3}{4}z^{-1}}$

zero: $z = 0$
pole: $z = 3/4$



$L_z = 1$
$L_p = 7/4$

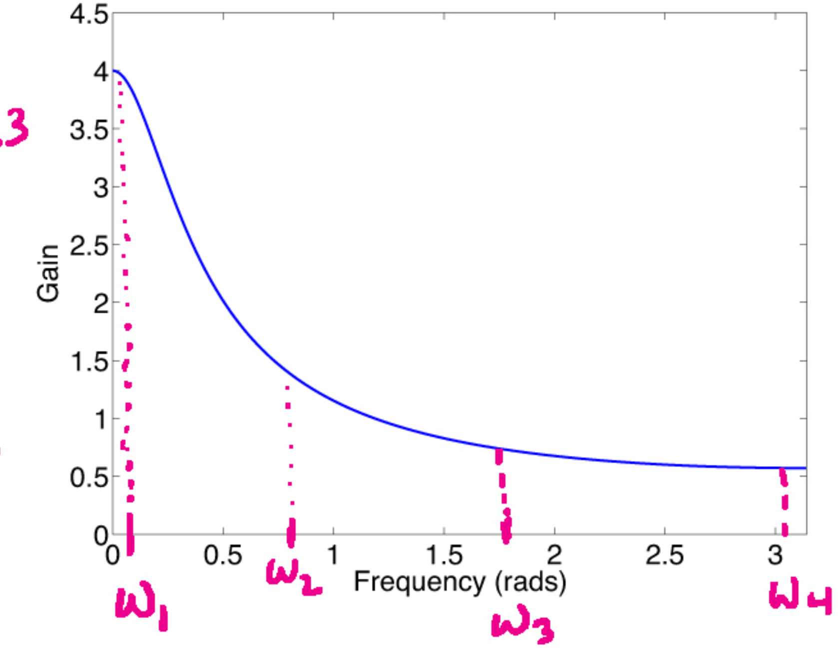
$|H(e^{j\omega_1})| \approx \dfrac{1}{1/4}$

$|H(e^{j\omega_2})| \approx \dfrac{1}{.77} \approx 1.3$

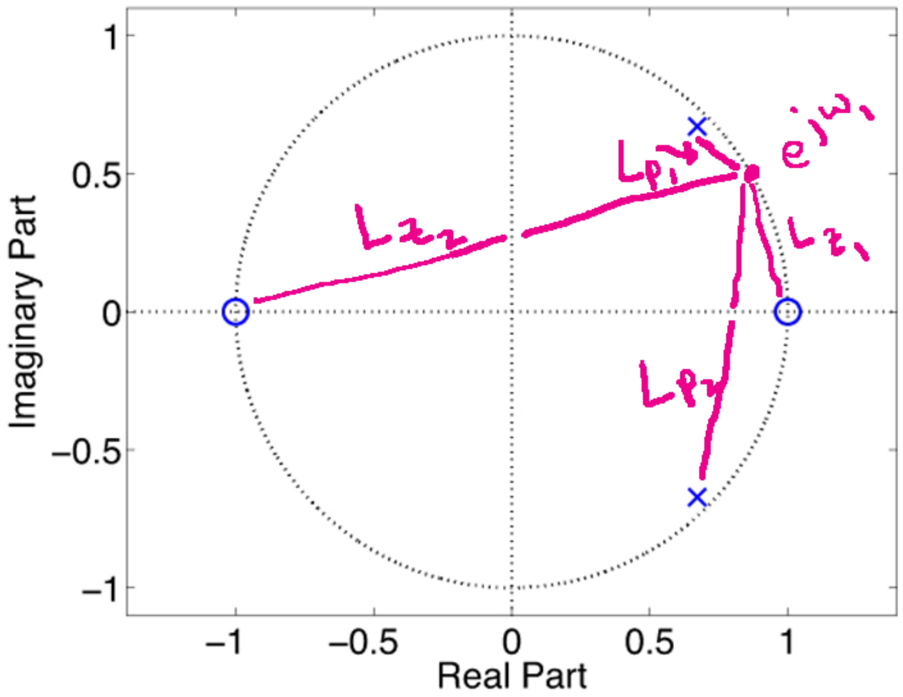
$|H(e^{j\omega_3})| \approx \dfrac{1}{1.25} = 0.8$

$|H(e^{j\omega_4})| \approx \dfrac{1}{7/4} = 4/7$

$|H(e^{j\omega})|$



Example: $H(z) = \dfrac{1 - z^{-2}}{(1-.95e^{j\pi/4}z^{-1})(1-.95e^{-j\pi/4}z^{-1})}$

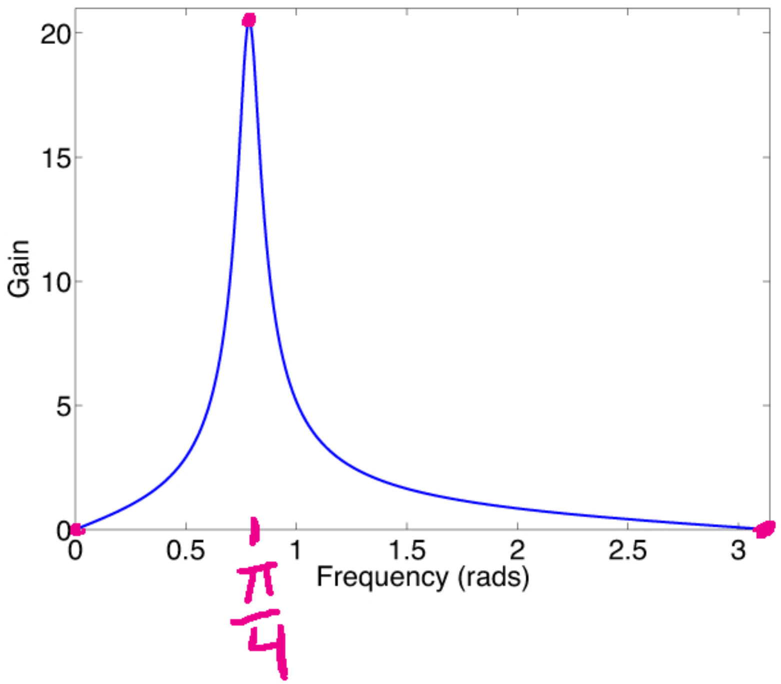


$$|H(e^{j\omega_1})| = \frac{L_{z_1} L_{z_2}}{L_{p_1} L_{p_2}}$$

$\omega_1 = 0 : L_{z_1} = 0$

$\omega_1 = \frac{\pi}{4} : L_{p_1} = 0.05$

$\omega_1 = \pi : L_{z_2} = 0$

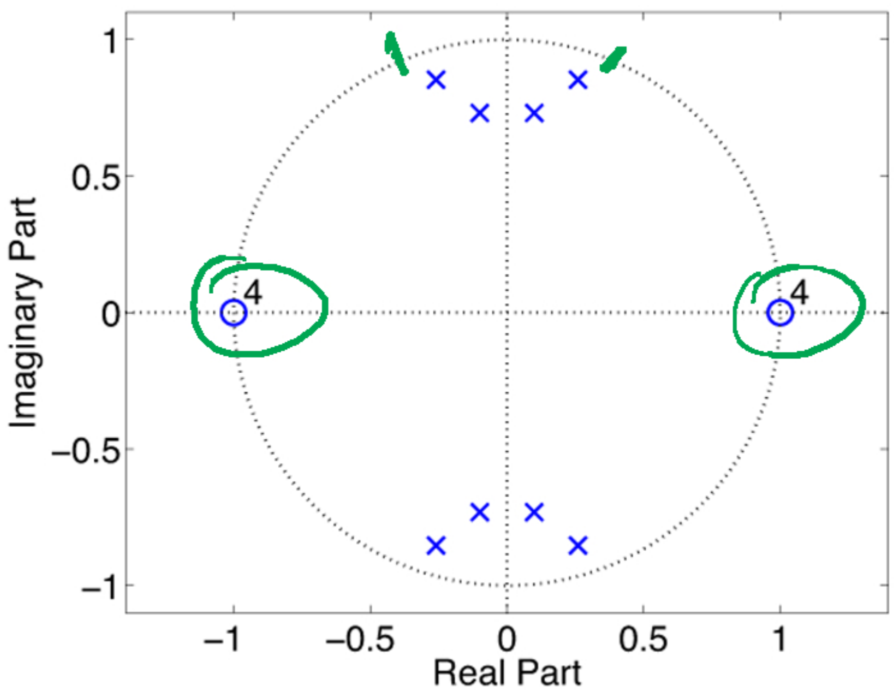


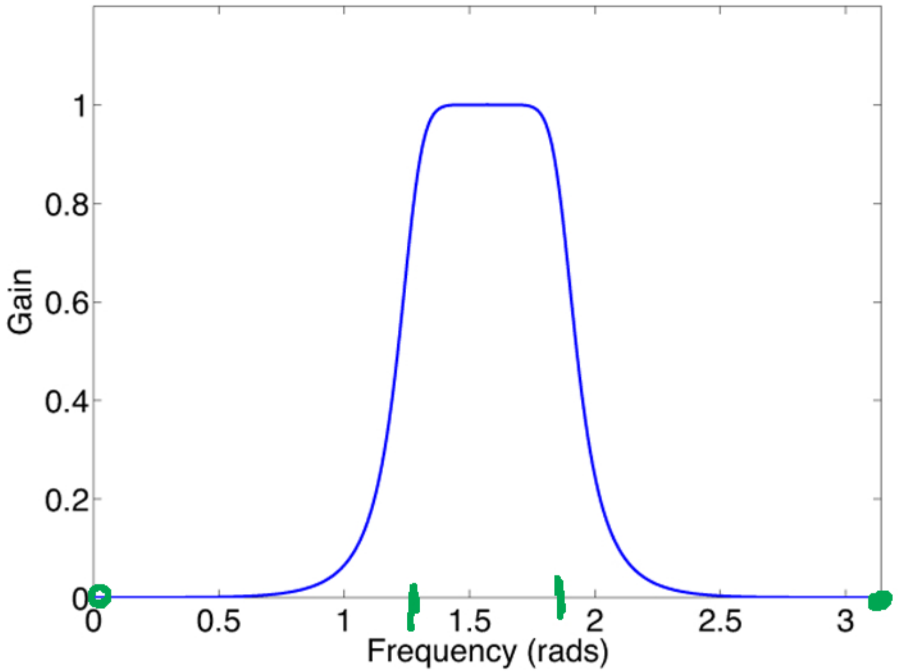
poles push magnitude response up

zeros pull magnitude response down

Can infer filter characteristics from pole/zero plot

Example:





Example:

$$|H(e^{j\omega})| = |b_0| \prod_{i=1}^{20} L_i$$