z-transform Analysis of LTI Systems

LTI systems described by difference equations play an important role in signal processing

1) Filters: infinite impulse response (IIR)
finite impulse response (FIR)

a) Filter design

b) Filter implementation

2) Models:

a) for a system introducing distortion

b) for characteristics of a random process

General form $$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k], \quad a_0 \neq 0$$

$$\sum_{k=0}^{N} a_k z^{-k} Y(z) = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$Y(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} X(z)$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} = \underbrace{\frac{b_0}{a_0} \frac{\prod_{k=1}^{M}(1-c_k z^{-1})}{\prod_{k=1}^{N}(1-d_k z^{-1})}}_{\text{assumes } b_0 \neq 0}$$

transfer or system function

c_k: zeros

d_k: poles

Example: Given $H(z)$, find difference equation

$$H(z) = \frac{(z+1)^2}{(z-\frac{1}{2})(z+\frac{1}{4})}$$

write in terms of z^{-1}

$$= \frac{\cancel{z^2}(1+z^{-1})^2}{\cancel{z^2}(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$$

zeros: $z = -1 \quad (\times 2)$

poles: $z = \frac{1}{2}, -\frac{1}{4}$

multiply terms

$$= \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$b_0 = 1$, $b_1 = 2$, $b_2 = 1$

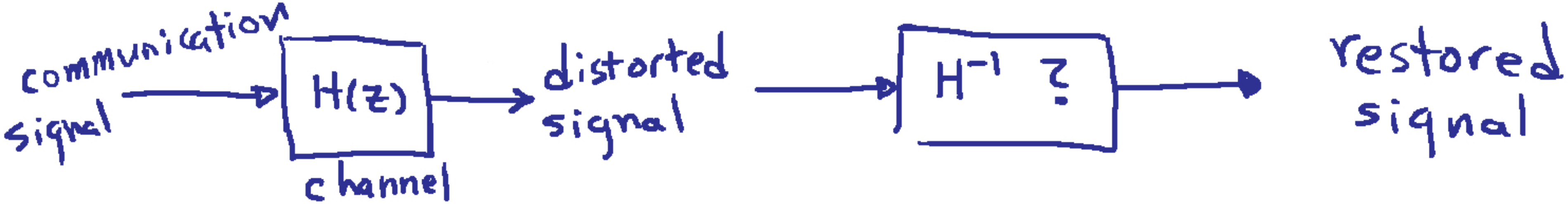
$a_0 = 1$, $a_1 = -\frac{1}{4}$, $a_2 = -\frac{1}{8}$

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + 2x[n-1] + x[n-2]$$

Key questions –

1) What are the conditions for a stable system?
2) What are the conditions for a causal system?

3) If $H(z)$ is a model for a system that introduces distortion, can it be reversed?



4) How does the frequency response relate to the parameters of the difference equation?