

# Properties of the z-Transform

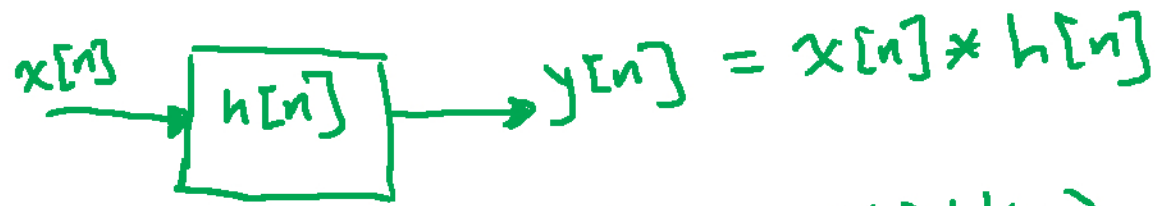
z-transform properties are analogous to DTFT properties

### 1) Linearity

$$ax_1[n] + bx_2[n] \xleftrightarrow{z} aX_1(z) + bX_2(z) \quad R_{x_1} \cap R_{x_2}$$

### 2) Convolution

$$x_1[n] * x_2[n] \xleftrightarrow{z} X_1(z)X_2(z) \quad R_{x_1} \cap R_{x_2}$$



$$X(z) \quad H(z)$$

$$Y(z) = X(z) \underbrace{H(z)}_{\text{system function transfer function}}$$

$$R_x \cap R_h$$

system function  
transfer function

### 3) Differentiation of $X(z)$

$$n x[n] \xleftrightarrow{z} -z \frac{d}{dz} X(z) \quad \text{ROC } R_x$$

$$n a^n u[n] \xleftrightarrow{z} -z \frac{d}{dz} \left( \frac{1}{1-az^{-1}} \right) = \frac{az^{-1}}{(1-az^{-1})^2}, \quad |z| > |a|$$

### 4) Multiplication by exponential sequence

$$z_0^n x[n] \xleftrightarrow{z} X(z/z_0) \quad \text{ROC } |z_0| \cdot R_x$$

if  $X(z)$  has a pole/zero at  $\alpha$  -

$X(z/z_0)$  has a pole/zero at  $z_0 \alpha$

$$z_0 = r e^{j\phi} \Rightarrow z_0 \alpha = r |\alpha| e^{j \arg\{\alpha\} + \phi}$$

Scale radius by  $r$ , shift angle by  $\phi$

## 5) Time-shift property

$$x[n-n_0] \xleftrightarrow{z} z^{-n_0} X(z) \quad \text{ROC } R_x \quad (\text{except possibly } z=0 \text{ or } z=\infty)$$

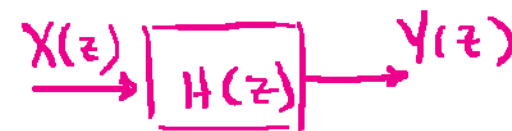
System described by difference equation

$$y[n] + a_1 y[n-1] + \dots + a_N y[n-N] = b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

$$\sum_{k=0}^N a_k y[n-k] = \sum_{k=0}^M b_k x[n-k]$$

$$\sum_{k=0}^N a_k z^{-k} Y(z) = \sum_{k=0}^M b_k z^{-k} X(z)$$

$$Y(z) = \underbrace{\frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}}}_{H(z)} X(z)$$



system/transfer function

Copyright 2012  
Barry Van Veen