Properties of the z-Transform

z-transform properties are analogous to DTFT properties

1) Linearity

$$ax_1[n] + bx_2[n] \overset{z}{\longleftrightarrow} aX_1(z) + bX_2(z) \qquad R_{x_1} \cap R_{x_2}$$

2) Convolution

$$x_1[n] * x_2[n] \overset{z}{\longleftrightarrow} X_1(z)X_2(z) \qquad R_{x_1} \cap R_{x_2}$$

3) Differentiation of $X(z)$

$$n\,x[n] \overset{z}{\longleftrightarrow} -z\frac{d}{dz}X(z) \qquad \text{ROC } R_x$$

$$n a^n u[n] \overset{z}{\longleftrightarrow} -z\frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right) = \frac{az^{-1}}{(1-az^{-1})^2}, \quad |z|>|a|$$

4) Multiplication by exponential sequence

$$z_0^n\,x[n] \overset{z}{\longleftrightarrow} X(z/z_0) \qquad \text{ROC } |z_0|\cdot R_x$$

if $X(z)$ has a pole/zero at α —

$X(z/z_0)$ has a pole/zero at $z_0\alpha$

$z_0 = re^{j\phi} \Rightarrow z_0\alpha = r|\alpha|e^{j\arg\{\alpha\}+\phi}$.

scale radius by r, shift angle by ϕ

5) Time-shift property

$$x[n-n_0] \xleftrightarrow{z} z^{-n_0} X(z) \quad \text{ROC } R_x \text{ (except possibly } z=0 \text{ or } z=\infty)$$

System described by difference equation

$$y[n] + a_1 y[n-1] + \cdots a_N y[n-N] = b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$\sum_{k=0}^{N} a_k z^{-k} Y(z) = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$Y(z) = \underbrace{\frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}}_{H(z)} X(z)$$

$H(z)$ system/transfer function