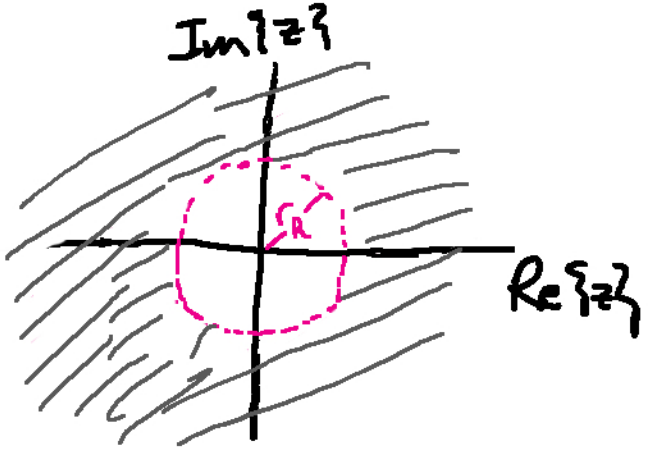
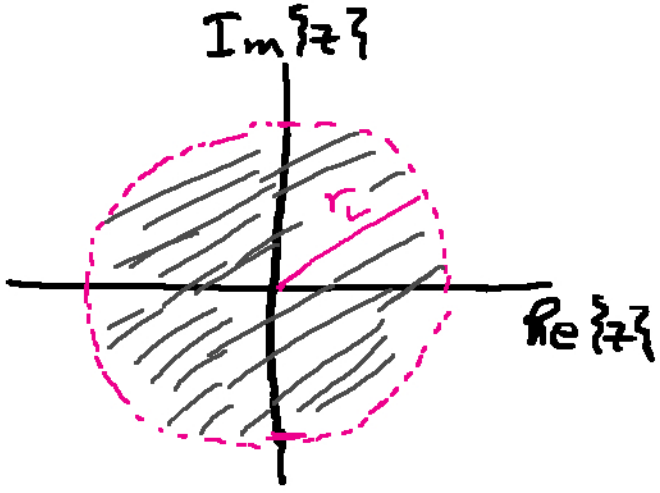
Properties of the ROC

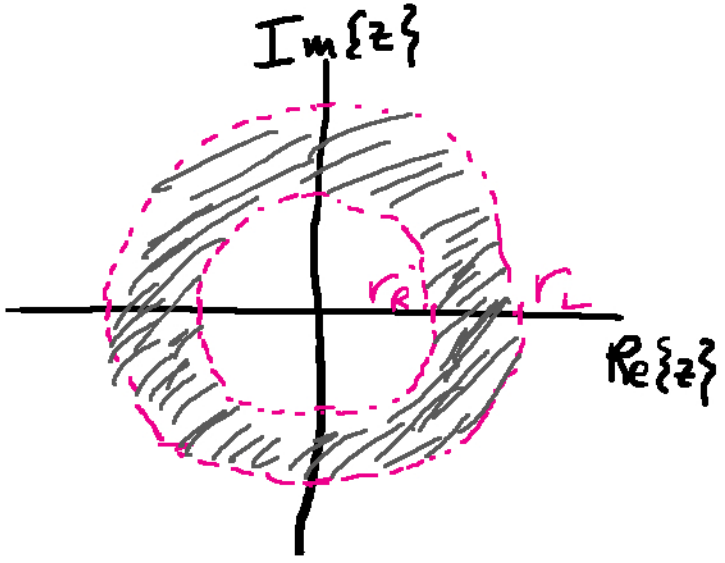
1) Three possibilities



$|z| > r_R$



$|z| < r_L$



$r_R < |z| < r_L$

2) ROC cannot contain any poles.

3) DTFT exists (absolutely summable)

$\Longleftrightarrow$ ROC includes $|z| = 1$ → unit circle

4) If $x[n]$ is finite duration $\Rightarrow$

$$x[n] = 0 \quad , \quad n < N_1 \; , \; n > N_2$$

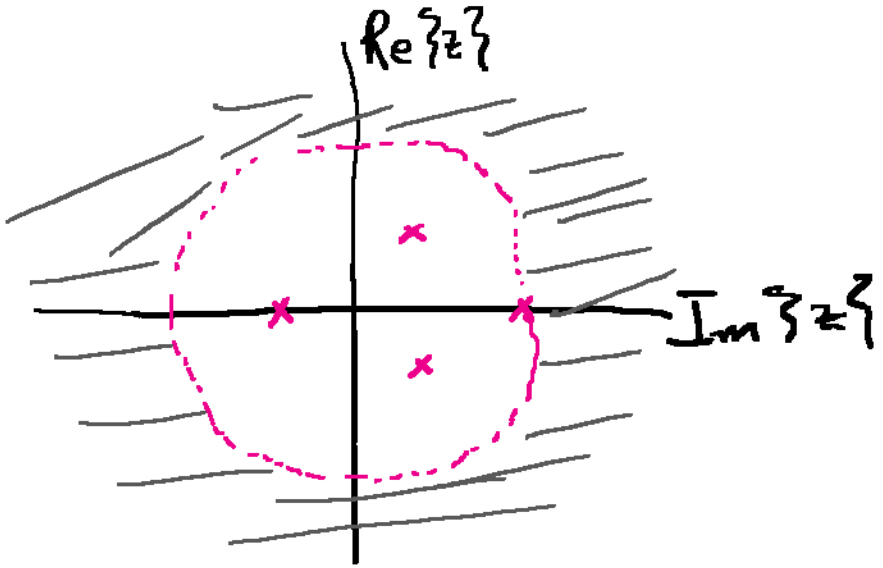
then ROC is all z except possibly $z = 0, \infty$

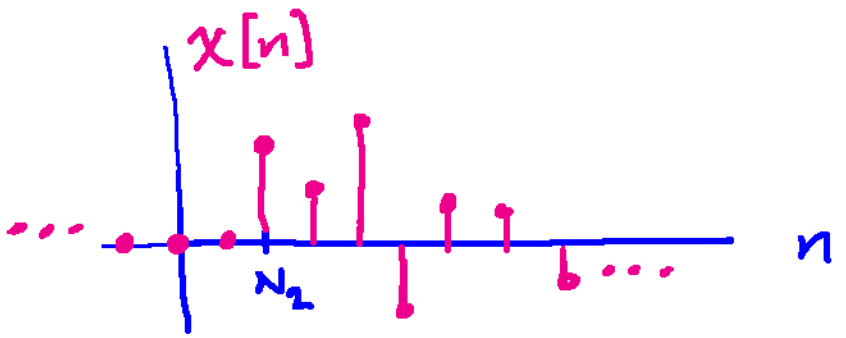
$$\sum_{n=N_1}^{N_2} |x[n] z^{-n}| < \infty$$

$z \neq 0$ if $N_2 > 0$

$z \neq \infty$ if $N_1 < 0$

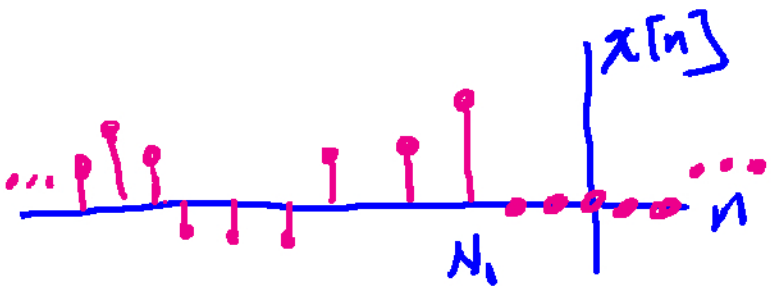
5) Right-sided signal

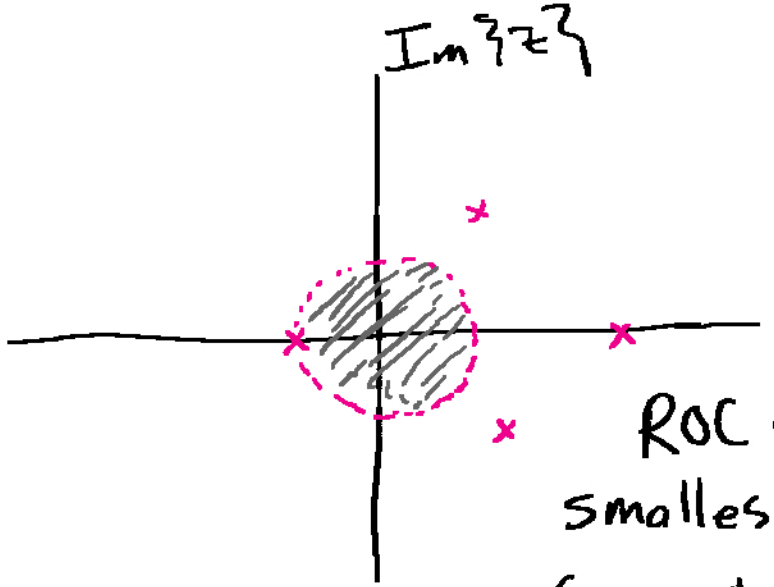




ROC extends out from largest magnitude pole (except possibly $z = \infty$)

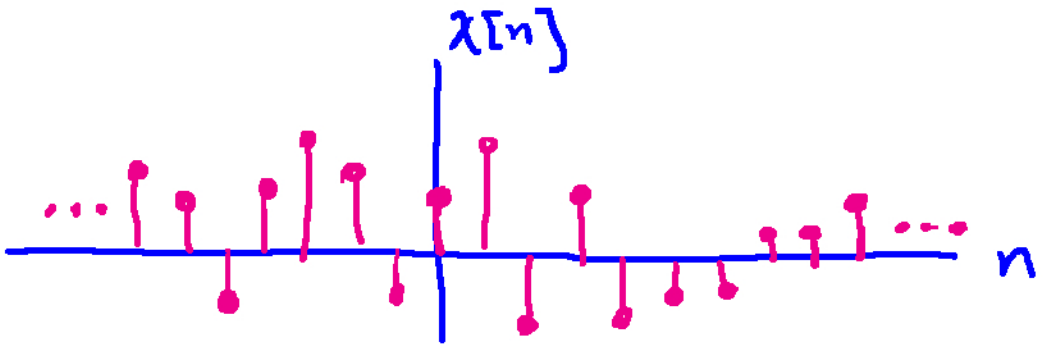
6) Left-sided signal

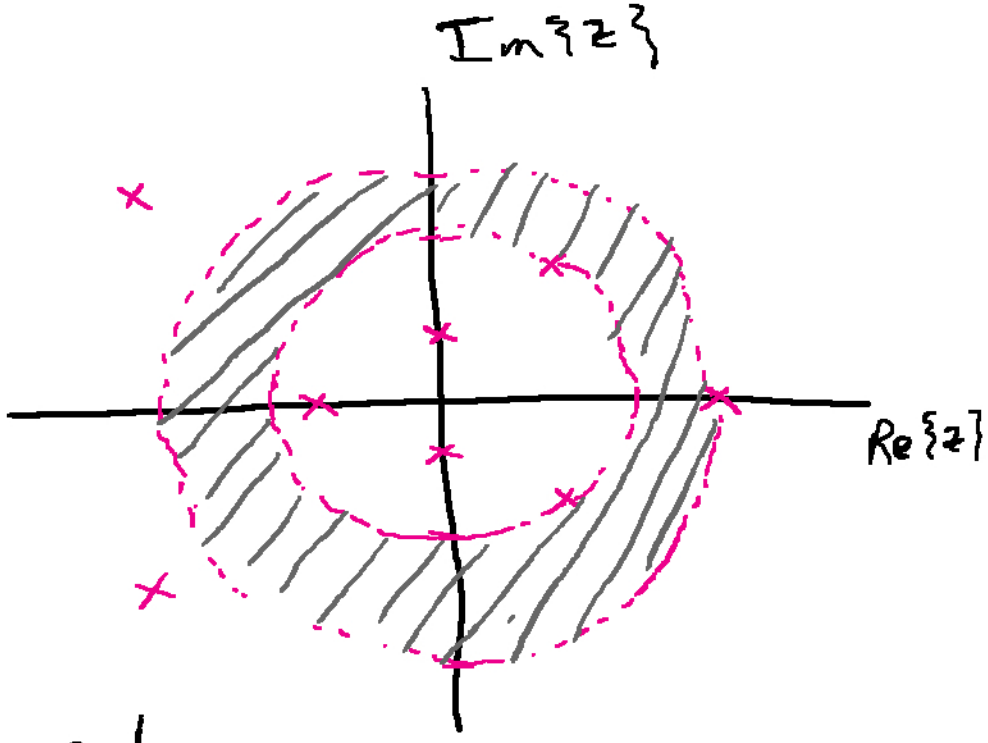




ROC extends in from smallest magnitude pole (except possibly $z = 0$)

7) Two-sided signal





ROC lies in a ring
bounded inside and outside by poles

Not all two-sided signals have z-transforms