Introduction to the z-Transform

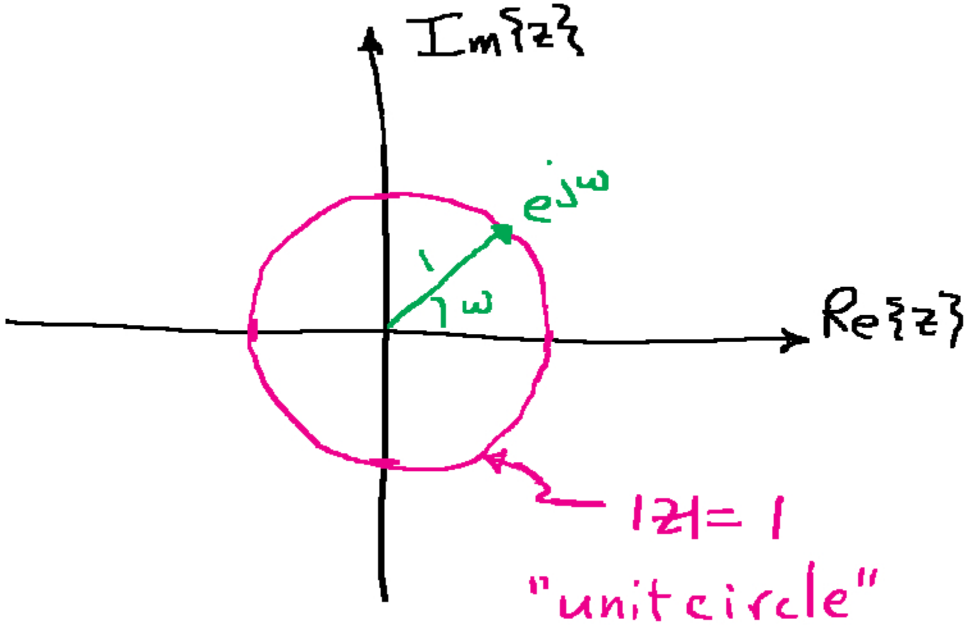
- Generalization of the DTFT
  - signals for which DTFT doesn't exist
  - New ideas: stability causality
- Discrete-time counterpart to the Laplace transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad z \in \mathbb{C} \quad \text{(complex numbers)}$$

DTFT $\quad X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \implies X(e^{j\omega}) = X(z)\big|_{z=e^{j\omega}}$

- Complex valued fcn of real-valued ω
- $X(z)$ is complex-valued fcn of complex-valued z

The Complex Plane (z-plane)



- $X(z)$ is defined on the plane
- $X(e^{j\omega})$ is defined only on the unit circle $|z| = 1$ or $z = e^{j\omega}$

- As $-\pi < \omega < \pi$, $z = e^{j\omega}$ goes once around the unit circle
- Confirms 2π periodicity of the DTFT

Notation: $x[n] \xleftrightarrow{z} X(z)$; $z\{x[n]\} = X(z)$

Examples:

$X(z) = z$

$X(z) = \frac{1}{z - 0.58}$